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ORIGINAL ARTICLE

A new mathematical modeling and a genetic algorithm search for milk run problem (an auto industry supply chain case study)

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Abstract The idea of Milk Run has been used in the context of logistic and supply chain problems in order to manage the transportation of materials. In this paper, we propose a new Milk Run method, as a mixed integer approach, to manage supply chain problems. Since the resulted problem formulation is NP-Hard, we use some meta-heuristic and compare the results with the optimal solutions of the proposed Milk Run method. The mathematical modeling of this paper is purposely customized for a special case of an auto industry. We implement the mathematical formulation and the meta-heuristic using some actual data and compare the results with the current strategy. The preliminary results indicate that the proposed method could provide a practical tool to significantly reduce the cost of logistic.

Keywords Logistic · Milk run system · Vehicle Routing Problem (VRP)

1 Introduction

During the past decade, many have focused on marginal issues of production planning problem such as supply chain and logistics [1, 2]. The idea of lean production has increasingly become popular, and there is a high degree of correlation between a lean production plan and a good logistic strategy. The implementation of a good logistic strategy on production planning plays an important role on the success of the whole management.

The first step towards bringing the logistics and pure objectives in one line is to handle the supply network control from the suppliers and make it observable. Without having a good observation and control, a pure network cannot be achieved. Implementing the pure logistic almost requires changes in the routes and transportation methods as well as amending the transportation schedule. These changes often are difficult to perform in an organization.

In a pure logistic network, time intervals for loading parts into the transporting vehicle, the period the transporting vehicle arrives at the destination factory, and even the unloading of the transporting vehicle in the destination factory are difficult to do in real world specially when we intend to have a precise plan similar to what we have in flight schedule of the airlines. Vehicle Routing Problem (VRP) is one of the most interesting areas of research in many industries. There are normally a few trucks and suppliers in a classical VRP. The trucks are responsible to ship goods from suppliers to a demand center. The primary goal of VRP is to minimize the transportation expenditures. However, there are other objectives involved in VRP such as due date for deliveries, etc. These objectives could also be treated as multi-objective VRP (for a good survey, see [2]). There is also a more complex form of VRP where a supplier must be visited within a specified time interval called VRP with time window (VRPTW). There are two different VRPTW: soft time window and hard time window [3]. In the case of soft time windows, a violation of the time window constraints is accepted with a penalty but in the case of hard one, customer cannot get a service from supplier later than the window time and when a customer reaches sooner, the customer must wait. The case of time window leads us to have a more complicated mathematical formulation. Dantzig

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and Ramser [4] are believed to be the first people who introduced the idea of VRP. They propose a method for routing of a fleet of gasoline delivery trucks between a bulk terminal and a number of service stations supplied by the terminal. The VRP is normally formulated as Mixed Integer Problem (MIP), and there have been many branch and bound method to solve such problem [5]. However, many VRP which are formulated as MIP are classified as NP-Hard problem. There are other mathematical methods which are implemented such as set covering, shortest path, etc. [6]. One of the challenging aspects of VRP is the desirability to find an optimal in short amount of time. This is in contrast to the nature of the problem which is NP-Hard. Therefore, in many cases, we may be interested in looking for a near optimal solution which leads us for heuristic or meta-heuristic approaches such as Genetic Algorithm (GA), Ant colony, Simulated annealing, etc. Alexandre and Teodor [7] propose a cooperative parallel meta-heuristic for the vehicle routing problem with time windows. The method is based on the solution warehouse strategy, in which several search threads cooperate by asynchronously exchanging information on the best solutions identified and uses tabu search procedure. Reimann et al. [8] propose an Ant colony to find a near optimal solution for vehicle routing problem. Jeon et al. [9] develop a vehicle-routing problem and use a hybrid GA to find a near optimal solution and compare them with the actual optimal. There are also tremendous works on using GA for vehicle routing [10–12]. There is a special case of VRP problem called Milk Run which would be the main focus of this paper [13]. We present different forms of this problem and propose a new mathematical model which is more suitable for an automaker called SAIPA in Iran. The primary objective of the proposed method of this paper is to consider pickup time as part of the model. In fact, the modeling determines the exact time that a particular supplier should be ready to ship the order. The other important factor in our model is the ability to diversify orders among suppliers. The proposed model is also capable of assigning different routes for a truck. We present a practical MIP to find the optimal transportation of different parts for an auto industry based on the idea of Milk Run concept. The resulted problem formulation is run with an actual data using a real world case study. The results are discussed and compared with the actual data. Since the proposed method of the paper is MIP and is hard to solve for real world problem, we also use a meta-heuristic approach to solve the resulted problem. The proposed method of this paper is purposely designed for one of the biggest auto maker in the world called SAIPA. According to the company's financial statement for the fiscal year ending in March 2008, the company assembles about half a million cars, which is almost half of Iran's market share. The proposed method of this paper is more customized to take into account the management strategy which did not explicitly exist in previous research. We hope the proposed method of this paper could also be used for many other industries around the world. This paper is organized as follows. We first study the Milk Run problem in Section 2. Section 3 presents a mathematical model, and a meta-heuristic approach is presented in Section 4. The results are compared in Section 5, and finally, conclusion remarks are given at the end in order to summarize the contribution of the paper.

2 Milk Run systems

One of the most important and well-known pure logistic research tools is Milk Run System. The name of this system comes from the traditional system for selling milk in the West, in which the milkman used to walk to the doors of the customers' houses with his dray in a specified route and deliver the milk containing bottles to his customers and finally take back the empty bottles. This system has been performed in miscellaneous industries and the auto manufacturing companies of the world have been (and are) the most important clients of this system (Fig. 1) [13].

Milk Run System determines the route, the time schedule, the type, and the number of parts that different trucks must choose in order to receive the orders from various suppliers with the primary assumption that all trucks must return the empty palates to the demand center (e.g. auto maker). There are different objectives involved in this kind of modeling which need to be minimized such as inventory and transportation costs.

The cost advantages of this system for short distances and consignments with high delivery frequency and value is extremely remarkable, and we will show this with some

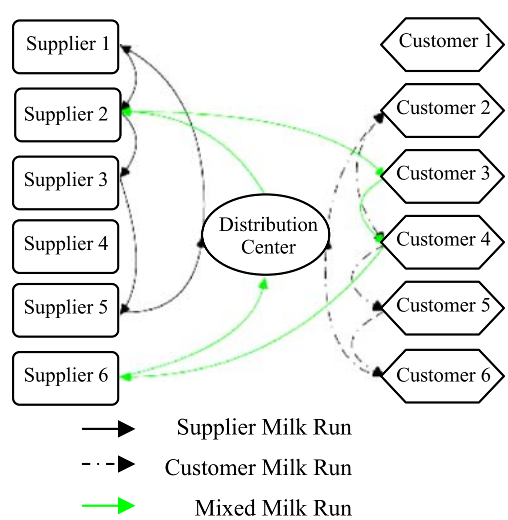

Fig. 1 Supplier, customer, and mixed milk runs transportation network

actual data. As the distance between the suppliers and the manufacturers increases, the advantages of performing this system increase at a declining speed. Also, through increasing such parameters like suppliers' geographical density, value of parts, expenses for keeping the inventory, and production circulation, the advantages of running this system increase. In this system, the frequency of the consignments delivery depends on the price and the size of the parts. Large size and valuable parts have to be transported with more frequency to prevent increasing of the inventory expenses. In this system, the company, based on material requirements planning system output, determines the weekly requirements of every supplier and the time schedule. We also determine the coherence and route of collecting the parts.

Running pure logistic network requires three principal changes in the system:

1. Reducing the number of orders or stockpiles
2. Increasing the number and the frequency of delivery to the factory
3. Making a smooth course of materials inflow to the factory

The most important advantages of running Milk Run system include:

- Materials and parts inflow to the production line is becoming easier.
- The performance of the supply chain and the logistic is improved due to effectively using of the transporting vehicles' spaces, controlling the transport charges, as well as reducing the level of parts inventory and their maintenance costs.
- The space of the valuable warehouse in a factory is reduced.
- The total number of required palates in the supply chain is reduced and the capital expenditures, maintenance, and repairs costs and operating expenses of the palates are less than the other methods.
- It reduces the lack of confidence at the time of delivery of parts and consequently reduces the parts stock level.
- It Increases the capital turnover.
- It Increases the flexibility in supplying the parts.
- It smoothes and establishes a good discipline in logistic operations (e.g., loading and unloading)

3 Other logistic systems

A logistic pattern means the form of the parts process and storage during the supply chain. The process form means the relocation of the production supplies (transport routes, the cargo size or frequency of shipment, form or method of transportation, cargo combination, and...) and storage form the means the process of storing them (locating the middle storehouses, determination of the minimum and maximum inventory, determination of the quantity of the cycle inventory and gaining assurance, determination of inventory review policy, and...) during the chain.

Different types of logistic patterns are divided into four main categories [14, 15]:

1. Direct shipment from supplier to client
2. Indirect shipment using a third party
3. Indirect shipment by middle storehouses
4. Collecting from suppliers and direct shipment to client

3.1 Direct shipment

In this method, the consignments are directly delivered to the manufacturers from the suppliers, and the company is responsible for storing the general inventory. This logistic pattern is traditionally used for supplying all parts. In the future, all parts of Categories A and C and some parts of Category B of an auto company shall be also supplied with this method (Fig. 2).

3.2 Indirect shipment

In this method, the suppliers deliver their consignments to the storehouses of a third company, and the said company shall be responsible for feeding the storehouses of the manufacturing company. In this method, under the Vendor Managed Inventory system, the inventory of the third company shall be under the control of the suppliers, and the total inventory is divided between the third company and the manufacturing company (Fig. 3).

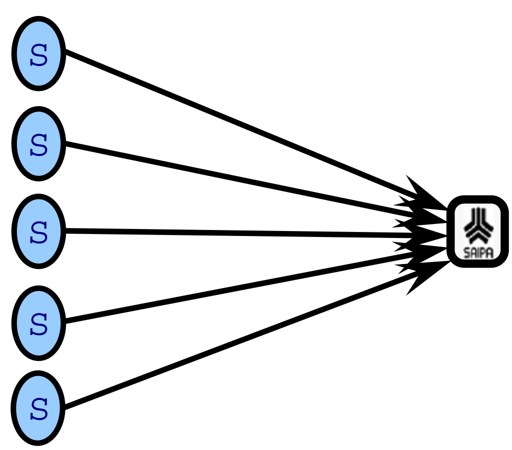

Fig. 2 All shipments are carried out directly

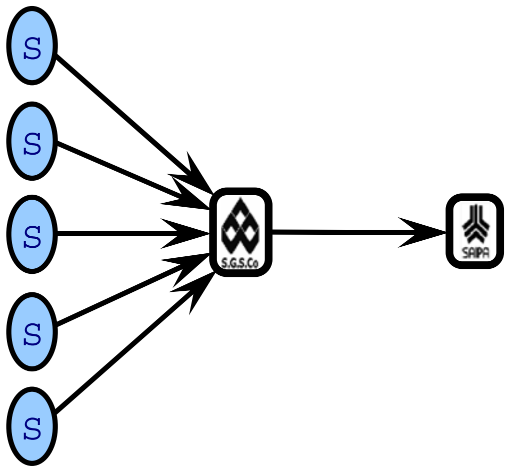

Fig. 3 All shipments are carried out indirectly

3.3 Indirect shipment through storerooms

In this method, the suppliers deliver their consignments to the storerooms constructed by the third company and located adjacent to them. Then, the third company delivers the large consignments consisting of various parts to the manufacturing company from these storerooms, according to a specified schedule (Fig. 4).

3.4 Collection from the suppliers and direct shipment

In this method, the third company divides the different suppliers into bunches through a dynamic planning and allocates one transporting vehicle to each bunch for collecting the suppliers' consignments. In the next step, each transporting vehicle starts loading the suppliers' consignments according to a specified time schedule and under a predefined order (Fig. 5).

The possible question here arisen is that considering the current conditions, which one of the aforementioned patterns is better to be implemented? To answer this question, we should say that every one of the said patterns

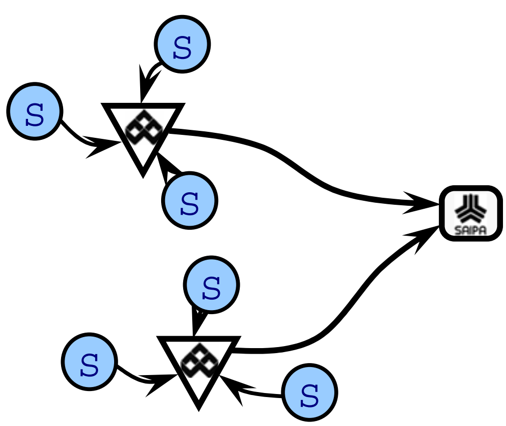

Fig. 4 All shipments are carried out through storerooms

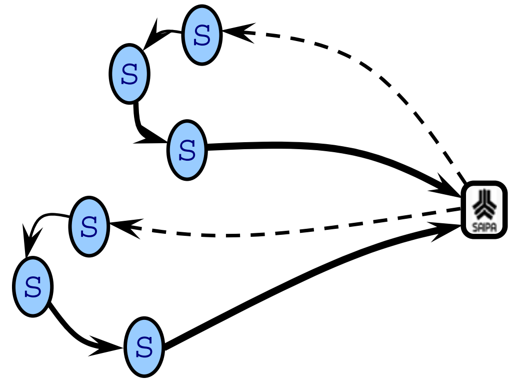

Fig. 5 Collection from suppliers and shipping them to client

has its own specifications (features); the volume of the required investment, operating expenses, level of service rendered to customers, and the complications of the execution of every pattern is different from others. Therefore, the logistic patterns must be designed with due consideration of these features. These features can be divided into three categories [5]:

- Economic features such as transportation costs, inventories, storekeeping, loading and unloading, man force planning and investment costs, and required equipment
- Technical features such as complications of execution, required coordination, required technology, the road conditions, and cultural acceptability
- Performance features like level of service rendered to customer, line stoppage risk, and volume of flexibility

There are numerous factors which may influence the features of the logistic patterns and make a relative superiority of some of these patterns over other ones. For example, the lack of readiness of information infrastructures may lead to priority of direct shipment pattern over other ones because the reliance of this pattern on the information infrastructures is not so much vital. Also, the low level of the production or when the supplied parts have a small size may be strong for those patterns such as shipment through storerooms or collecting from suppliers and shipment to the manufacturing company because, in this situation, other patterns impose high level of inventory on the system. These factors and their influences on different logistic patterns can be illustrated in the following conceptual model framework.

4 The proposed Milk Run model

In this section, we deal with the milk system modeling. As we already explained, the proposed method of this paper is a special case of Milk Run which is purposely designed and customized for SAIPA auto maker. In fact, there are three main issues which need to be handled in the proposed method. The first requirement is to use different suppliers to provide some particular parts. In fact, the company monitors the service level for some selected suppliers, and based on the quality of their services, management makes some necessary decisions to adjust the quantity of each vendor. The other problem is that the model must determine the exact time that a supplier would ship the parts. In other words, every supplier needs to know the exact time to ship a particular part, and this could help the system to preschedule the production plan. Finally, the proposed model of this paper is capable of changing a truck to work in different routes in various time schedules. In the following section, we first introduce the decision variables and the parameters used in our problem formulation.

4.1 Decision variables and parameters

x_{tkpj}	: The number of shipping palates of part p transported with vehicle k from supplier j with the due date of time t
V_k	: Truck capacity k
$y_{tkij}: \begin{cases} 1 \; if \; x_{tkpj} > 0 \\ 0 \; if \; x_{tkpj} = 0 \end{cases}$	: The minimum inventory level of part $pc_p^{\min}$
V_P^{PL}	: The maximum capacity of palate for part p
U_{tp}	: The average consumption of part p in time t
x_{tp}^M	: The remaining part p at the end of time t
H_p	: The inventory cost per hour of each palate containing part p in warehouse
C_{kij}	: The cost of the truck k moving from supplier i to supplier j
γ_{pj}	: The percentage of part p allocated to supplier j
$\widehat{C}$	: The fixed cost of waiting a truck at each supplier

4.2 The mathematical model

Since the main goal of implementation of the Milk Run System is to decrease the transportation costs and reduce the level of inventory of parts at the warehouse, we consider the following objective function,

$$\min \sum_t \sum_k \sum_i \sum_j \left[C_{kij} + \widehat{C}\right] y_{tkij} + \sum_t \sum_p H_p \times x_{tp}^M$$

The first part of the objective function explains the total costs of transportation. This function states that if a transporting vehicle moves from one supplier to another one, the transportation cost with the vehicle and the fixed cost of the loading must be considered.

The second part is used for inventory costs. In summary, the basis of the objective function is to simultaneously reduce the transportation costs as well as the inventory expenditures.

The first constraint is associated with truck capacity.

$$\sum_p \sum_j x_{tkpj} \times V_p^{PL} \leq V_k \qquad \forall(t,k) \qquad (1)$$

This constraint investigates that the number of palates collected from the suppliers for transportation to warehouses from a proper volume of parts for collecting and transporting to warehouses would not exceed the number of transporting vehicles.

$$\sum_t \sum_k x_{tkpj} = \gamma_{pj} \qquad \forall(p,j) \qquad (2)$$

This constraint states that every supplier is only allowed to transfer the volume of parts committed in the contract. This constraint is stated here because the companies who are relying on the suppliers usually supply their similar type parts from two or several suppliers.

$$x_{tp}^M \geq c_p^{\min} \qquad \forall(t,p) \qquad (3)$$

Each supplier may set a minimum inventory level for its own parts which is defined according to various factors. This amount of the inventory is usually determined for some working days (2 or 3 days)

$$x_{tp}^M = \sum_k \sum_j x_{tkpj} + x_{(t-1)p}^M - U_{tp} \qquad \forall(t,p) \qquad (4)$$

The total number of each parts is equal to the total number of parts arrive at the end of the time t, plus the inventory of the parts at the end of time $t-1$ minus the number of parts used in time t. This constraint studies that in each moment what quantity of the parts are available to avoid shortage of the parts in the warehouse.

$$\sum_k \sum_j y_{tkij} \leq 1 \qquad \forall i \geq 2, \forall t \qquad (5)$$

$$\sum_k \sum_i y_{tkij} \leq 1 \qquad \forall j \geq 2, \forall t \qquad (6)$$

These constraints use one and only one transporting vehicle for each time schedule t. In this model, it is presumed that two or several transporting vehicles may not be used simultaneously. In this model, the warehouse of the manufacturing company has been marked with $i=1$ number. Note that since the company's warehouse is also considered as a supplier, we must separate these two constraints. Therefore, this issue is explained in the next two constraints. The first constraint states that in one moment, at most, one transporting vehicle can enter to supplier j from the supplier i and in the next constraint, only one transporting vehicle can exit the supplier i.

$$\sum_j y_{tk1j} \leq 1 \qquad \forall(t,k) \tag{7}$$

$$\sum_i y_{tki1} \leq 1 \qquad \forall(t,k) \tag{8}$$

Equations 7 and 8 are similar to Eqs. 5 and 6, but the difference is that these constraints are exclusively related to the warehouse of the manufacturing company.

$$\sum_i y_{tkiq} = \sum_j y_{tkqj} \qquad \forall(t,k), \forall q > 1 \tag{9}$$

This constraint, which is designed for sequencing the routes, states that if a vehicle gets to a knot, it must exit from it. The purpose of this constraint is to avoid the stoppage of the vehicle in the place of one supplier and each transporting vehicle, entering any place, must exit from that place. Note that the transporting vehicle only stops when it arrives at the supplier's warehouse, and this condition will be stated in the following constraint,

$$\sum_{i \in S} \sum_{j \in S} y_{tkij} \leq |S| - 1 \; S \subseteq \{2, 3, \ldots, KT\} \qquad \forall(t,k) \tag{10}$$

(KT is the total number of the suppliers)

Equation 10 states that each transporting vehicle starts from the supplier's warehouse and its destination is the same warehouse. It is stated to avoid creation of laps in the transportation route. This constraint states that the transporting vehicle never returns to the place it has referred to before in this route.

$$x_{tkpj} \leq M \times \sum_i y_{tkij} \qquad \forall(t,k,p,j) \tag{11}$$

(M is a big number)

The Eq. 11 states that we can bring parts from the supplier j if the transporting vehicle is able to enter from one supplier to this supplier. This constraint is important because it states the relationship between two alternatives of x and y.

Table 1 The computational results of MIP and GA

The number of		Time horizon	The Proposed MIP method		The GA implementation	
Supplier	Parts		CPU	Cost	CPU	Cost
2	5	5	0.3	3.6E+4	0.0	3.6E+4
2	5	10	0.37	7.2E+4	0.0	7.2E+4
3	10	15	2.37	32E+4	0.0	32E+4
3	10	20	5.47	42E+4	0.0	41.8E+4
4	15	30	10.58	13E+5	0.07	13E+5
4	15	40	17.85	17E+5	0.14	17E+5
5	20	60	37.5	38E+5	0.39	38E+5
5	20	80	68.9	49E+5	1.23	50E+5
6	25	150	158	106E+5	5.68	116E+5
6	25	200	454	135E+5	12.69	150E+5
7	30	280	1026	253E+5	29.67	275E+5
7	30	350	2845	316E+5	46.59	348E+5
8	40	450	–	–	83.54	510E+5
8	40	600	–	–	98.29	656E+5
10	50	720	–	–	138.5	922E+5

Table 2 The relative gap between MIP versus GA and GA versus actual production plan

The number of		Time horizon	The Percentage gap between	
Supplier	Parts		MIP vs. GA	GA vs. Actual
2	5	5	0	22
2	5	10	0	26
3	10	15	0	24
3	10	20	0	25
4	15	30	0	24
4	15	40	0	24
5	20	60	0	24
5	20	80	1	25
6	25	150	9.4	15
6	25	200	11.1	18
7	30	280	8.6	18
7	30	350	10.1	17
8	40	450	–	16
8	40	600	–	19
10	50	720	–	15

5 Computational results

In order to analyze the performance of the proposed method, we have solved the resulted model using MIP software package using some real data from SAIPA Complex for thee groups of variables: the number of suppliers, the number of parts, and the time horizon for the planning horizon. As we can observe, when the number of suppliers and parts are increased from two and five to seven and 30, respectively, the CPU time is increased almost exponentially. This comes from the fact that there are many binary variables in our proposed model that we may not be able to run the resulted model for large-scale problems. Therefore, we use a meta-heuristic approach to solve the problem. We have used the GA approach in order to compare the optimal solutions in relatively small-scale problems. The details of the computations are explained in [16] and are not repeated here. Table 1 shows the details of the implementation for both methods. The execution CPU times are given in minutes while the objective function which estimates the cost given on columns 5 and 7 are in local Iranian currencies, Rial. Table 2 compares the relative gap between the MIP versus GA and GA versus the actual plan. As we can see, there is no difference between the optimal solutions of the proposed method for some small-scale problems. Also, the gap between the GA and the actual plan changes from 15% to 25%, which indicates that even a near-optimal solution could reduce the cost of supply chain significantly. As we could observe from the CPU time on Table 1, the CPU time needed to solve the proposed method is relatively acceptable for some small-scale problems but for real-world case problem, we may not be able to find the optimal solution very easily. In summary, we can conclude that the proposed MIP method could help us find a master plan, and for more detailed planning, one may use GA method.

We have compared the results from the implementation of GA with the actual supply chain plan summarized in Table 2.

6 Conclusions

We have proposed a new mathematical model for Milk Run problem. The proposed method of this paper is different from the other existing methods. The new proposed method is formulated as a Mixed Integer problem, and the resulted problem was solved using an optimization tools using some actual information. Since there are many binary variables involved with this problem and the optimal solution cannot be found for real world application when the number of suppliers and parts are more than ten, we have used a GA method to find a near-optimal solution. We have implemented the problem formulation using the real data gathered from SAIPA Group in the City of Tehran. The GA implementation indicates that we could use this method practically, and the near optimal solution could significantly lower supply chain expenditures.

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