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ORIGINAL ARTICLE

Variable sampling inspection plans with screening for assuring average outgoing surplus quality loss limit indexed by Taguchi's loss

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Abstract Taguchi has proposed a variable-quality evaluation called "Taguchi's quality loss" instead of the attribute quality evaluation such as the proportion of nonconforming items. Arizono et al. have proposed a single acceptance sampling plan based on operating characteristics from the viewpoint of assuring Taguchi's quality loss. This sampling plan is designed to guarantee the constraints of the prescribed acceptance probabilities for respective lots with the allowable quality loss limit and the unallowable quality loss limit. However, in the acceptance sampling plan based on operating characteristics, the corrective action for rejected lot is not prescribed. On the other hand, the sampling inspection plan with screening is well known as the sampling scheme with the corrective action for rejected lots. Then, there is the attribute sampling inspection scheme with screening in order to guarantee the expectation of the proportion of nonconforming items in the shipping lot. However, the variable sampling inspection scheme with screening has not yet been prescribed. Then, in this article, we propose a variable sampling scheme with screening procedure for the purpose of assuring the upper limit of the maximum expected surplus loss indexed by Taguchi's loss.

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1 Introduction

Taguchi [9, 10] has presented a new approach to quality improvement where the reduction of deviation from a specified target value is the guiding principle. In this approach, Taguchi has proposed the variable quality evaluation, which is called "Taguchi's quality loss" instead of the attribute quality evaluation such as the proportion of nonconforming items. According to the theory of the Taguchi method, the loss caused by the deviation from its target value is expressed as a quadratic form with respect to the difference between the actual value x and the target value μ_0. Then, this quality evaluation as the quality loss has been applied in many fields of decision-making about the quality improvement [2, 3, 5, 6, 11]. However, the concept of "quality loss" has hardly been applied to the design of acceptance sampling plans for assuring the lot quality.

When the mean and variance of the lot quality are given as μ and σ^2, the expected loss in the Taguchi method can be evaluated as $k\left\{\sigma^2+(\mu-\mu_0)^2\right\}$, where k denotes the proportional coefficient based on the functional limit of items and the monetary loss brought by the product which cannot fulfill its fundamental function. In the viewpoint of the Taguchi's expected quality loss, every lot is not necessarily uniform in the loss even though the respective lots have the same proportion of nonconforming items. Arizono et al. [1] proposed a single acceptance sampling plan based on operating characteristics in the viewpoint of assuring Taguchi's quality loss. This sampling plan is designed for guaranteeing that the acceptance probabilities against the respective lots with the allowable quality loss limit and the unallowable quality loss limit are less than or equal to the prescribed producer's risk and consumer's risk.

By the way, the corrective action for the rejected lot isn't prescribed in the sampling inspection scheme based on operating characteristics. On the other hand, the corrective action for the rejected lot is prescribed in the sampling inspection scheme with screening. However, although the existent sampling inspection scheme with screening has been designed for the attribute quality evaluation such as the proportion of nonconforming items, there is no sampling inspection plan with screening for the variable quality evaluation.

In this article, we adopt the quality evaluation based on Taguchi's quality loss instead of the traditional quality evaluation such as the proportion of nonconforming items. Accordingly, we consider the variable acceptance sampling inspection plan with screening. In this scheme, the sampling plan is decided so that the upper limit of the maximum expected surplus loss (the average outgoing surplus quality loss limit; AOSQLL) indexed by Taguchi's loss after the corrective action is less than or equal to the permissible average outgoing surplus quality loss; PAOSQL. Then, we develop the decision algorithm for determining the economical sample size and the criterion of acceptance rule. The effectiveness of the variable sampling inspection plan with screening proposed in this article is explained and is verified through some numerical examples.

2 Notations

In this section, we define and explain some symbols and abbreviations in this article.

μ and σ^2	Mean and variance
$N(\mu, \sigma^2)$	Normal distribution with mean μ and variance σ^2
k	Proportional coefficient for Taguchi's quality loss
τ^2	Taguchi's quality loss per item
μ_0 and σ_0^2	Mean and variance in ideal quality characteristic distribution
τ_0^2	Unavoidable quality loss per item
n and D	Sample size and acceptance criterion of variable sampling plan
$Q(\tau^2)$	Probability of acceptance in the variable sampling plan (n, D)
T^2	Expected quality loss per item for the lot after screening and replacing
L	Expected quality loss per item of the outgoing lot
L_{upper}	Expected quality loss limit
S	Average outgoing surplus quality loss
S_{limit}	Specified permissible average outgoing surplus quality loss
χ_ϕ^2	Central Chi-square distribution with ϕ degrees of freedom
$\chi_\phi^2(\alpha)$	Upper 100α percentile of χ_ϕ^2 distribution
u_γ	Upper 100γ percentile of standard normal distribution

Abbreviations

AOSQL	Average outgoing surplus quality loss
AOSQLL	Average outgoing surplus quality loss limit
PAOSQL	Specified permissible average outgoing surplus quality loss

3 Concept of assurance for the average outgoing surplus quality loss

Suppose that the quality characteristics obey a normal distribution $N(\mu, \sigma^2)$. According to the concept of Taguchi's quality loss, the loss is expressed as the quadratic form with respect to the difference between the actual value x and target value μ_0. Then, the expected loss per item is obtained as follows:

$$E\left[k(x-\mu_0)^2\right] = k\left\{(\mu-\mu_0)^2+\sigma^2\right\}. \tag{1}$$

Simply, we denote k=1 without the loss of generality because k is constant. Then let $(\mu,\sigma^2) = (\mu_0,\sigma_0^2)$ be the mean and variance in the ideal quality characteristic distribution. In this case, from Eq. (1), we have

$$E\left[(x-\mu_0)^2\middle|(\mu,\sigma^2) = (\mu_0,\sigma_0^2)\right] = \sigma_0^2 \equiv \tau_0^2.$$

Then, τ_0^2 can be interpreted as the unavoidable quality loss in the ideal quality characteristic distribution.

Because we define σ_0^2 as the variance in the ideal quality characteristic distribution, we assume the variance of quality characteristics to be as follows:

$$\sigma^2 \geq \sigma_0^2. \tag{2}$$

Then, the following relation for τ^2 has to be satisfied:

$$\tau^2 = (\mu-\mu_0)^2+\sigma^2 \geq \tau_0^2(=\sigma_0^2). \tag{3}$$

Equation (3) denotes that the expected loss per item is always more than or equal to that in the ideal situation.

Since Q (τ^2) means the acceptance probability of lots with $\tau_0^2, \left\{1-\mathrm{Q}\left(\tau_0^2\right)\right\}$ means the rejection probability of lots with τ_0^2 and is less than or equal to the specified producer's risk α. Then we inspect the items in the rejected lots and all the discovered nonconforming items are replaced by the conforming items. When we let T^2 and L be the expected loss per item on the screened lot and the expected loss per item on all lots after screening and replacing, the following relation is derived:

$$L = \tau^2 Q(\tau^2) + T^2\{1 - Q(\tau^2)\}. \quad (4)$$

Then, the difference $L - \tau_0^2$ means the avoidable quality loss, and is defined as the average outgoing surplus quality loss (AOSQL) in this article. However, it is not easy to obtain the value L, because it is difficult to evaluate the value of T^2 exactly.

We should discuss the screening rule. The quality loss τ_0^2 is the expected loss on the ideal quality characteristics, therefore, we decide the screening rule so that the expected loss after screening and replacing falls below τ_0^2. In the case that the quality of items is evaluated by whether it conforms or not, the lot after screening and replacing do not include the nonconforming item. Then it is obvious that the proportion of nonconforming items is 0, and the lot quality is simply evaluated. On one hand, the expected loss after screening and replacing depends on the quality loss of the exchanged items in the case that the quality of items is evaluated by the Taguchi's quality loss.

Accordingly, in order to ensure that the expected loss after screening and replacing is less than or equal to τ_0^2, we adopt the following screening procedure as the simplest and easiest procedure, that is, we employ the screening rule that the items having a quality loss more than τ_0^2 detected in the total inspection are replaced with the items with the quality loss less than τ_0^2. Therefore, the item is not replaced when the following relation for x is satisfied

$$(x - \mu_0)^2 \le \tau_0^2 (= \sigma_0^2),$$

where this expression is rewritten as:

$$\mu_0 - \sigma_0 \le x \le \mu_0 + \sigma_0. \quad (5)$$

Then the screening rule for the actual value x can be presented as:

$$\begin{cases} \text{if } x \text{ satisfy the relation of Eq.(5), then ship it,} \\ \qquad \text{otherwise, replace it by the item} \\ \qquad\qquad \text{satisfying the relation of Eq.(5).} \end{cases}$$

From the screening rule mentioned above, it is clear that the value T^2 is less than or equal to τ_0^2 from Eq. (5). If T^2 is replaced by τ_0^2 on Eq. (4), L is modified to the upper limit of the expected quality loss on the lot after screening and replacing, i.e., L_{upper}. Then, we have

$$L_{\text{upper}} = \tau^2 Q(\tau^2) + \tau_0^2\{1 - Q(\tau^2)\}. \quad (6)$$

Further, we can obtain the following relation:

$$L_{\text{upper}} - \tau_0^2 = (\tau^2 - \tau_0^2) Q(\tau^2) \equiv S. \quad (7)$$

Then, the value S in Eq. (7) is interpreted to be the maximum value of the surplus expected quality loss (in what follows, we call "average outgoing surplus quality loss limit (AOSQLL)"). We consider the acceptance sampling plan assuring that the value S is less than or equal to a specified PAOSQL S_{limit}. Consequently, let α be the specified producer's risk on the expected loss τ_0^2, in this article. We propose the variable acceptance sampling plan that S is less than or equal to S_{limit}, that is, the algorithm for deciding the sampling plan satisfying the following relation:

$$(\tau^2 - \tau_0^2) Q(\tau^2) \le S_{\text{limit}}, \quad (8)$$

is developed.

4 Proposal of design procedure for variable sampling plans with screening inspection

Let x_j, j=1,2,...,n, be random samples from a normal distribution $N(\mu, \sigma^2)$, but μ and σ^2 is unknown. Taguchi has proposed the estimator $\widehat{\tau}^2$ of the expected loss defined by

$$\widehat{\tau}^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \mu_0)^2 = (\overline{x} - \mu_0)^2 + s^2, \quad (9)$$

where $\overline{x}$ and s^2 denote the maximum likelihood estimates of μ and σ^2 calculated as follows:

$$\overline{x} = \frac{1}{n}\sum_{i=1}^{n} x_i,$$

$$s^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \overline{x})^2.$$

It is known that the statistic $n\widehat{\tau}^2/\sigma^2$ obeys a noncentral Chi-square distribution with n degrees of freedom and noncentrality parameter $n\xi^2$ [8], where ξ^2 is defined as

$$\xi^2 = \frac{(\mu - \mu_0)^2}{\sigma^2}. \quad (10)$$

Then, we discuss a statistic ρ, where ρ is defined as

$$\rho = \frac{1 + \xi^2}{1 + 2\xi^2}\frac{n\widehat{\tau}^2}{\sigma^2}. \quad (11)$$

Based on the noncentral Chi-square distribution with n degrees of freedom and noncentrality parameter $n\xi^2$, the mean and variance of the statistic ρ are respectively given by

$$E[\rho] = \frac{1+\xi^2}{1+2\xi^2} E\left[\frac{n\widehat{\tau}^2}{\sigma^2}\right] = \frac{n(1+\xi^2)^2}{1+2\xi^2},$$

$$V[\rho] = \left(\frac{1+\xi^2}{1+2\xi^2}\right)^2 V\left[\frac{n\widehat{\tau}^2}{\sigma^2}\right] = \frac{2n(1+\xi^2)^2}{1+2\xi^2},$$

and coincide with those of the central Chi-square distribution with ϕ degrees of freedom, where

$$\phi = \frac{n\left(1+\xi^2\right)^2}{1+2\xi^2}. \tag{12}$$

Accordingly, the central Chi-square distribution with ϕ degrees of freedom can be employed as the approximate distribution of ρ.

From Eq. (3) and Eqs. (10)–(12), we have the following relation

$$\rho = \phi\frac{\hat{\tau}^2}{\tau^2}. \tag{13}$$

Then, by using the Patnaik's approximation [7], we obtain the approximation model:

$$\hat{\tau}^2_\phi \sim \frac{\tau^2\chi^2_\phi}{\phi}, \tag{14}$$

where χ^2_ϕ means the central Chi-square distribution with ϕ degrees of freedom. By making use of this approximation model, we formulate the inspection plan. Let D be the acceptance value, the acceptance rule is constructed as

$$\begin{cases} \text{if } \hat{\tau}^2 < \text{D, then accept the lot,} \\ \quad \text{otherwise, reject the lot.} \end{cases}$$

And the rejected lot is inspected totally. When the expected loss on the lot in the ideal quality characteristic distribution is τ_0^2, we design the acceptance sampling plan in order that the producer's risk is less than or equal to the specified value α. Then, let us consider two cases, $\tau^2 = \tau_0^2$ and $\tau^2 = \tau_1^2 > \tau_0^2$. In the case of $\tau^2 = \tau_0^2$, since the distribution of the quality characteristics is defined as $N\left(\mu_0, \sigma_0^2\right)$, the approximation model is given as

$$\hat{\tau}^2 \sim \frac{\tau_0^2\chi_n^2}{n}. \tag{15}$$

Then, ϕ_0, which is the degrees of freedom on the approximation model, satisfies the relation of $\phi_0 = n$, because of the condition of $\xi^2 = 0$ from Eq. (10). Therefore, the acceptance value $D \equiv \tau_0^2\chi_n^2(\alpha)/n$ is derived as the percentile for the producer's risk α by the approximation model on Eq. (15), where $\chi_n^2(\alpha)$ denotes the upper 100α% percentile of the central Chi-square distribution with n degrees of freedom. Accordingly, the acceptance rule is determined as

$$\text{if } \hat{\tau}^2 \le \frac{\chi_n^2(\alpha)}{n}\tau_0^2, \text{ then accept the lot.} \tag{16}$$

In the case of $\tau^2 = \tau_1^2$, the probability that the lot on τ_1^2 is accepted, $\mathrm{Q}\left(\tau_1^2\right)$, is derived from the condition that AOSQLL is less than or equal to PAOSQL S_{limit} as follows:

$$\mathrm{Q}\left(\tau_1^2\right) \le \frac{S_{\text{limit}}}{\tau_1^2 - \tau_0^2}. \tag{17}$$

There are innumerable pairs of $\left(\mu_1, \sigma_1^2\right)$ with the same τ_1^2 due to the relation of $\tau_1^2 = \sigma_1^2 + (\mu_1 - \mu_0)^2$. Naturally, $\mathrm{Q}\left(\tau_1^2\right)$ is a function for $\left(\mu_1, \sigma_1^2\right)$ because τ_1^2 is a function for $\left(\mu_1, \sigma_1^2\right)$. Since the relation of Eq. (17) should be satisfied for all pairs of $\left(\mu_1, \sigma_1^2\right)$ with same τ_1^2, Eq. (17) needs to be rewritten as follows:

$$\max_{\left(\mu_1, \sigma_1^2\right)\in\tau_1^2} \mathrm{Q}\left(\tau_1^2\right) \le \frac{S_{\text{limit}}}{\tau_1^2 - \tau_0^2}. \tag{18}$$

When ϕ_1 is the degree of freedom in the distribution of $\hat{\tau}^2$ for given τ_1^2, we obtain the approximation model:

$$\hat{\tau}^2 \sim \frac{\tau_1^2\chi^2_{\phi_1}}{\phi_1}, \tag{19}$$

where

$$\phi_1 = \frac{n\left(1+\xi_1^2\right)^2}{1+2\xi_1^2}, \xi_1^2 = \frac{(\mu_1 - \mu_0)^2}{\sigma_1^2}.$$

The probability that the lot with the quality loss τ_1^2 is rejected by the acceptance rule of Eq. (16) has to satisfy the following relation:

$$\min_{\left(\mu_1, \sigma_1^2\right)\in\tau_1^2} \Pr\left(\hat{\tau}^2 \ge \frac{\chi_n^2(\alpha)}{n}\tau_0^2\right) \ge 1 - \frac{S_{\text{limit}}}{\tau_1^2 - \tau_0^2}.$$

Therefore, we can obtain the relation as

$$\frac{\chi_n^2(\alpha)}{n}\tau_0^2 \le \min_{\left(\mu_1, \sigma_1^2\right)\in\tau_1^2} \frac{\chi^2_{\phi_1}(\varepsilon)}{\phi_1}\tau_1^2, \tag{20}$$

where

$$\varepsilon \equiv 1 - \frac{S_{\text{limit}}}{\tau_1^2 - \tau_0^2}.$$

In this case, by applying the Wilson-Hilferty approximation [4], we consider the behavior of $\chi^2_{\phi_1}(\varepsilon)/\phi_1$ in ϕ_1, that is, a pair of $\left(\mu_1, \sigma_1^2\right)$. Next, in order to obtain the pair of $\left(\mu_1, \sigma_1^2\right)$ minimizing the right side of Eq. (20), we have further three sub-cases as follows:

a) $\tau_1^2 \le \tau_0^2 + S_{\text{limit}}$

It means $\varepsilon \le 0$. Then, because the expected loss is always less than or equal to S_{limit}, it isn't necessary to consider this sub-case on designing the sampling plan.

b) $\tau_0^2 + S_{\text{limit}} < \tau_1^2 \le \tau_0^2 + 2S_{\text{limit}}$

Then, $0 < \varepsilon \le 0.5$. Based on the Wilson-Hilferty approximation, this case can correspond to $0 \le u_\varepsilon < \infty$, where $u_{1-\theta}$ represents the upper 100(1−θ) percentile of the standard normal distribution. Then, in the range:

$$0 \le u_\varepsilon < \sqrt{\frac{8}{9n}},$$

we know that the function $\chi^2_{\phi_1}(\varepsilon)/\phi_1$ is concave. While, the function $\chi^2_{\phi_1}(\varepsilon)/\phi_1$ is monotonous decreasing function for ϕ_1 in the range:

$$\sqrt{\frac{8}{9n}} \le u_\varepsilon < \infty.$$

See Appendix A. Therefore, we must have further two conditions as follows, where γ is defined by the following relation:

$$u_\gamma = \sqrt{\frac{8}{9n}}.$$

b–i) $\tau_0^2 + S_{\text{limit}} < \tau_1^2 \le \tau_0^2 + \frac{S_{\text{limit}}}{\gamma}$

Then, $0 < \varepsilon \le 1 - \gamma$. Under this condition, from Appendix A, $\chi^2_{\phi_1}(\varepsilon)/\phi_1$ is monotonous decreasing function in ϕ_1. Therefore, $\chi^2_{\phi_1}(\varepsilon)/\phi_1$ is the minimum value when ϕ_1 has the maximum value in the range $\phi_1 \ge n$. In this case, ϕ_1 is monotonous increasing function in ξ_1 and ξ_1 is expressed by Eq. (12) as follows:

$$\xi_1^2 = \frac{\tau_1^2}{\sigma_1^2} - 1.$$

For fixed τ_1^2, ξ_1 has the maximum value in the condition of $\sigma_1^2 = \sigma_0^2$. Therefore, when the parameter ξ_1^2 is defined by the following pair of (μ_1, σ_1^2) for fixed τ_1^2:

$$(\mu_1, \sigma_1^2) = \left(\mu_0 \pm \sqrt{\tau_1^2 - \sigma_0^2}, \sigma_0^2\right).$$

When the maximum value of ϕ_1 for given condition is denoted as $\phi_{1_{\max}}$, $\chi^2_{\phi_{1\max}}(\varepsilon)\big/\phi_{1_{\max}}$ reaches the minimum value. Then the following inequality regarding sample size n is derived from Eq. (20):

$$\frac{\tau_0^2}{\tau_1^2} \le \frac{n}{\chi_n^2(\alpha)} \frac{\chi^2_{\phi_{1_{\max}}}(\varepsilon)}{\phi_{1_{\max}}}. \tag{21}$$

b–ii) $\tau_0^2 + \frac{S_{\text{limit}}}{\gamma} < \tau_1^2 \le \tau_0^2 + 2S_{\text{limit}}$

Then, $1 - \gamma < \varepsilon \le 0.5$. Under this condition, from Appendix A, $\chi^2_{\phi_1}(\varepsilon)\big/\phi_1$ is concave in ϕ_1. Then, let $\phi_{1_{\min}}$ and $\phi_{1_{\max}}$ be the minimum value and maximum value of ϕ_1, respectively. Then, $\chi^2_{\phi_1}(\varepsilon)\big/\phi_1$ takes the minimum value when ϕ_1 is either $\phi_{1_{\min}}$ or $\phi_{1_{\max}}$. Since ϕ_1 is the monotonous increasing function in $\xi_1^2(\ge 0)$, ϕ_1 takes the minimum value $\phi_{1_{\min}} = n$ in the case of $\xi_1^2 = 0$, that is, $(\mu_1, \sigma_1^2) = (\mu_0, \tau_1^2)$. While, $\phi_{1_{\max}}$ is obtained in the case of $\sigma_1^2 = \sigma_0^2$:

$$(\mu_1, \sigma_1^2) = \left(\mu_0 \pm \sqrt{\tau_1^2 - \sigma_0^2}, \sigma_0^2\right).$$

Consequently, the following inequality regarding sample size n is derived

$$\frac{\tau_0^2}{\tau_1^2} \le \frac{n}{\chi_n^2(\alpha)} \min\left(\frac{\chi^2_{\phi_{1_{\max}}}(\varepsilon)}{\phi_{1_{\max}}}, \frac{\chi_n^2(\varepsilon)}{n}\right). \tag{22}$$

c) $\tau_0^2 + 2S_{\text{limit}} < \tau_1^2$

Then, it is obvious that $0.5 < \varepsilon$. Under this condition, $\chi^2_{\phi_1}(\varepsilon)\big/\phi_1$ is the monotonous increasing function in ϕ_1 from Appendix B. Therefore, $\chi^2_{\phi_1}(\varepsilon)\big/\phi_1$ is minimized in the range $\phi_1 \ge n$ under the following condition:

$$(\mu_1, \sigma_1^2) = (\mu_0, \tau_1^2).$$

Then we have $\phi_1 = \phi_{1_{\min}} = n$, and the following inequality regarding sample size n is derived

$$\frac{\tau_0^2}{\tau_1^2} \le \frac{n}{\chi_n^2(\alpha)} \frac{\chi_n^2(\varepsilon)}{n} = \frac{\chi_n^2(\varepsilon)}{\chi_n^2(\alpha)}. \tag{23}$$

As mentioned above, the purpose of this article is to provide the design procedure for the sampling inspection plan that S is less than or equal to S_{limit} for any $\tau_1^2 (> \tau_0^2)$. Therefore, we should employ the maximum value among sample sizes obtained by using Eqs. (21)–(23). In this case, we denote the adopted sample size as $n_{\max}$. we can obtain the following acceptance rule:

$$\text{if } \hat{\tau}^2 \le \frac{\chi^2_{n_{\max}}(\alpha)}{n_{\max}} \tau_0^2 \equiv D, \text{ the lot is accepted.} \tag{24}$$

By the above, we can decide the acceptance sampling plan for assuring that AOSQLL is less than or equal to PAOSQL under the given producer's risk α on τ_0^2.

5 Numerical examples

In order to illustrate the validity of the provided procedure, we show some numerical examples. Let $N(\mu_0, \sigma_0^2) = N(0.0, 1.0)$, and then the ideal expected loss is given as $\tau_0^2 = 1.0$. Then, let α=0.05 and S_{limit}=0.25. Figure 1 shows the required sample size for each of the expected loss τ_1^2. In this case, we can obtain sample size $n_{\max}$=30 and the acceptance value D=1.459 for satisfying that AOSQLL is less than or equal to a given value PAOSQL. Similarly, Fig. 2 shows the required sample size for each of the expected loss τ_1^2 in the case of α=0.05 and S_{limit}=0.35. Furthermore, the required sample size for each of the expected loss τ_1^2 in the case of α=0.01 and S_{limit}=0.25 is illustrated by Fig. 3. In these cases, we can obtain sample size $n_{\max}$=17 and $n_{\max}$=63, respectively. We can confirm that $n_{\max}$ in Fig. 2 is smaller than that in Fig. 1 because the expected loss in Fig. 2 is larger than that in Fig. 1. On the

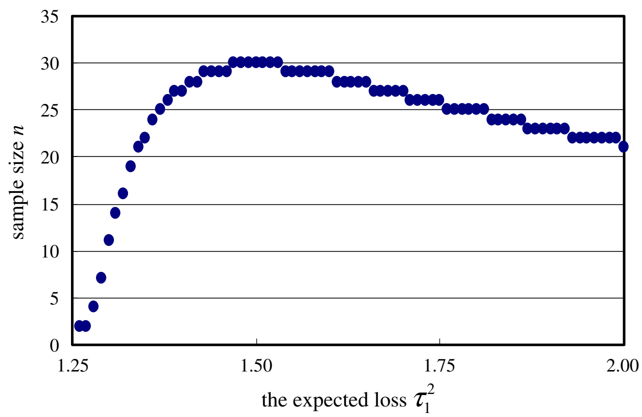

Fig. 1 The required sample size for each of the expected loss $\tau_1^2(S_{\text{limit}} = 0.25, \alpha = 0.05)$

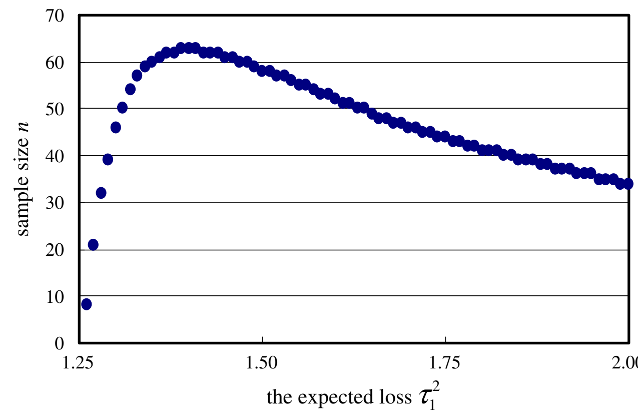

Fig. 3 The required sample size for each of the expected loss $\tau_1^2(S_{\text{limit}} = 0.25, \alpha = 0.01)$

other hand, we can confirm that $n_{\max}$ in Fig. 3 is larger than that in Fig. 1 because a producer's risk in Fig. 3 is smaller than that in Fig. 1.

Next we verify that the AOSQLL for each τ_1^2 is satisfied by applying the proposed sampling plan ($n_{\max}$, D). The approximation distribution $\widehat{\tau}^2$ for τ_1^2 is given in Eq. (19). Let S* be AOSQLL for each τ_1^2. Then S* should satisfy the following condition in Eq. (20):

$$\frac{\chi^2_{\phi_0}(\alpha)}{n_{\max}}\tau_0^2 = \frac{\chi^2_{\phi_1}(\varepsilon *)}{\phi_1}\tau_1^2,$$

where

$$\varepsilon * = 1 - \frac{S*}{\tau_1^2 - \tau_0^2}.$$

Therefore, we obtain the following relation:

$$\chi^2_{\phi_1}(\varepsilon *) = \frac{\phi_1 \tau_0^2}{n_{\max}\tau_1^2}\chi^2_{\phi_0}(\alpha).$$

Furthermore, we can obtain $u_{\varepsilon *}$ by using the Wilson-Hilferty approximation [4] as follows:

$$u_{\varepsilon *} = \sqrt{\frac{9\phi_1}{2}}\left(\sqrt[3]{\frac{\chi^2_{\phi_1}(\varepsilon *)}{\phi_1}} + \frac{2}{9\phi_1} - 1\right).$$

In this case, we can evaluate the AOSQLL S* for each τ_1^2 as

$$S* = \left(\tau_1^2 - \tau_0^2\right)\left(1 - \Phi(u_{\varepsilon *})\right), \tag{25}$$

where $\Phi(u)$ denotes the distribution function of the standard normal distribution.

Then, in Figs. 4, 5, and 6, the AOSQLL S* for each τ_1^2 realized by the proposed inspection plan derived from Eq. (25) is illustrated. The setting parameters in Figs. 4, 5, and 6 correspond to those of Figs. 1, 2, and 3, respectively. In Figs. 4, 5, and 6, we can find that the AOSQLL S* is less than or equal to the PAOSQL S_{limit} on the sampling plan $n_{\max}$.

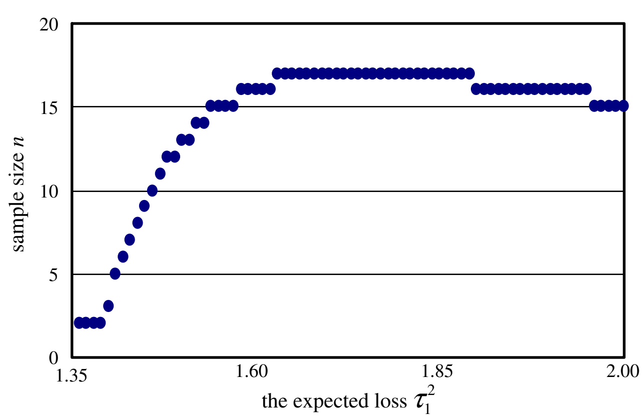

Fig. 2 The required sample size for each of the expected loss $\tau_1^2(S_{\text{limit}} = 0.35, \alpha = 0.05)$

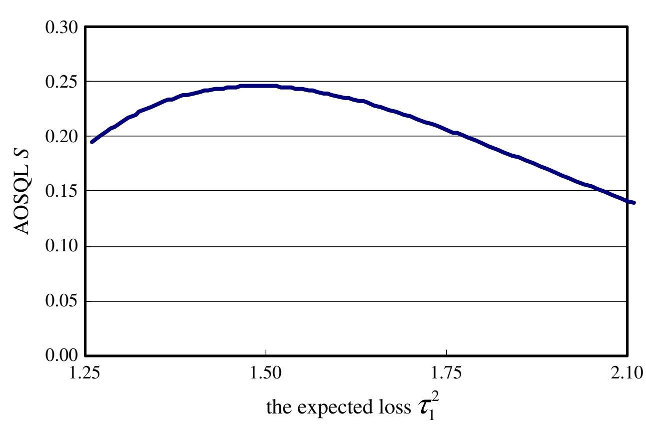

Fig. 4 AOSQL for τ_1^2 under $n_{\max}$ (S_{limit}=0.25, α=0.05)

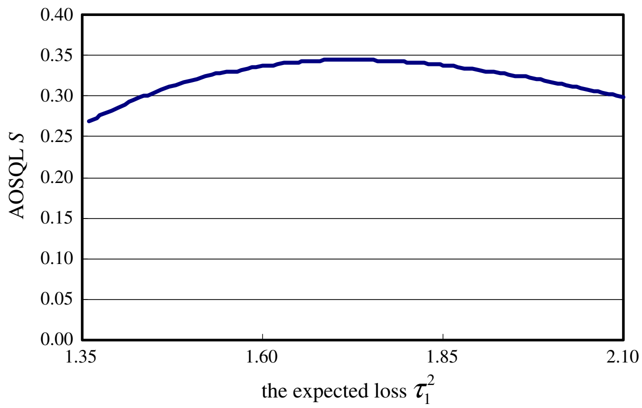

Fig. 5 AOSQL for τ_1^2 under $n_{\max}$ (S_{limit}=0.35, α=0.05)

6 Concluding remarks

In this article, we have propose the acceptance sampling scheme with screening to assure that the AOSQLL based on Taguchi's quality loss is always less than or equal to the specified PAOSQL, and provided the design procedure for this sampling plan. In the Taguchi's quality loss, there are innumerable pairs of the mean and variance in the qulaity characteristic distribution for the given expected quality loss. Then, we have defined the AOSQL, and further the AOSQLL has been derived as the upper limit of AOSQL. Finally, we have verified that the AOSQLL is less than or equal to the specified permissible average outgoing surplus quality loss PAOSQL through some numerical examples.

In the Taguchi's quality loss based on the deviation from the target value of the quality characteristic, the loss is evaluated even if the quality characteristics is in the range of the standard specification limits. In this point, the quality control technique based on the concept of the Taguchi's quality loss is different from the quality control technique based on the concept of the traditional loss by the nonconforming item. The quality control technique based on the Taguchi's quality loss is understood as the quality control technique to aim at higher quality. Therefore, we are convinced that the proposed inspection scheme will very much contribute as a technique for quality control in a real industrial environment. In addition, through the case study in the real industrial environment, the authors would like to present the benefit of the proposed inspection scheme. Unfortunately, we don't yet have an opportunity to apply the proposed inspection plan in the real industrial environment. Therefore, we would like to make this problem a subject future study.

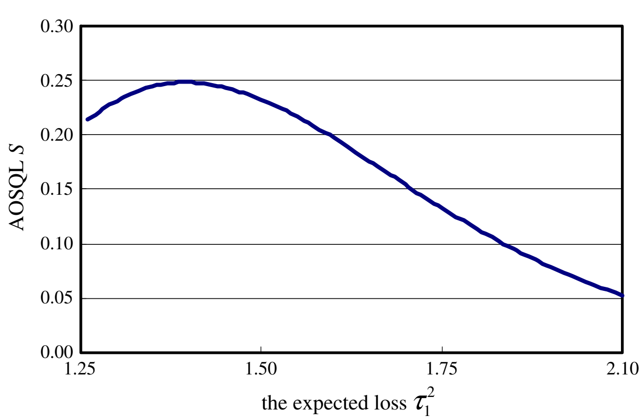

Fig. 6 AOSQL for τ_1^2 under $n_{\max}$ (S_{limit}=0.25, α=0.01)

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Appendix A: Behavior of $\chi^2_{\phi_1}(\varepsilon)\big/\phi_1$ for ϕ_1 in $\tau_0^2 + S_{\text{limit}} \le \tau_1^2 < \tau_0^2 + 2S_{\text{limit}}$

We employ the Wilson-Hilferty approximation for the upper percentile of central Chi-square distribution:

$$\chi^2_{\phi_1}(1-\theta) = \phi_1\left\{1 - \frac{2}{9\phi_1} + u_{1-\theta}\sqrt{\frac{2}{9\phi_1}}\right\}^3,$$

where $u_{1-\theta}$ for θ ($\geq$ 0.5) denotes the upper 100(1−θ) percentile of the standard normal distribution. We consider the function:

$$\frac{\chi^2_{\phi_1}(1-\theta)}{\phi_1} = \left\{1 - \frac{2}{9\phi_1} + u_{1-\theta}\sqrt{\frac{2}{9\phi_1}}\right\}^3.$$

Then we have

$$\frac{d}{d\phi_1}\frac{\chi^2_{\phi_1}(1-\theta)}{\phi_1} = \frac{\sqrt{2}}{6\phi_1^{\frac{3}{2}}}\left\{\sqrt{\frac{8}{9\phi_1}} - u_{1-\theta}\right\} \cdot\left\{1 - \frac{2}{9\phi_1} + u_{1-\theta}\sqrt{\frac{2}{9\alpha_1}}\right\}^2.$$

Further, since $\phi_1 \geq n$, in the case that $u_{1-\theta} > \sqrt{8/(9n)}$, it is obvious that

$$\sqrt{\frac{8}{9\phi_1}} - u_{1-\theta} < 0.$$

Then, we know that the function of $\chi^2_{\phi_1}(1-\theta)\big/\phi_1$ is the monotonous decreasing function in ϕ_1. On the other hand, in the case that $u_{1-\theta} \leq \sqrt{8/(9n)}$, we know that the function $\chi^2_{\phi_1}(1-\theta)\big/\phi_1$ is the concave in ϕ_1.

Appendix B: Behavior of $\chi^2_{\phi_1}(\varepsilon)\Big/\phi_1$ for ϕ_1 in $\tau_0^2 + 2S_{\text{limit}} \leq \tau_1^2$

Based on the approximation of the upper percentile of central Chi-square distribution for θ<0.5, we have also

$$\frac{\chi^2_{\phi_1}(1-\theta)}{\phi_1} = \left\{1 - \frac{2}{9\phi_1} - u_{1-\theta}\sqrt{\frac{2}{9\phi_1}}\right\}^3.$$

Then the differential coefficient for ϕ_1 is derived as

$$\frac{d}{d\phi_1}\frac{\chi^2_{\phi_1}(1-\theta)}{\phi_1} = \frac{\sqrt{2}}{6\phi_1^{\frac{3}{2}}}\left\{\sqrt{\frac{8}{9\phi_1}} + u_{1-\theta}\right\} \cdot\left\{1 - \frac{2}{9\phi_1} - u_{1-\theta}\sqrt{\frac{2}{9\phi_1}}\right\}^2.$$

Since ϕ_1, $u_\theta > 0$, it is obvious that

$$\frac{d}{d\phi_1}\frac{\chi^2_{\phi_1}(1-\theta)}{\phi_1} > 0.$$

Therefore, we know that the function $\chi^2_{\phi_1}(1-\theta)\Big/\phi_1$ is the monotonous increasing function in ϕ_1.

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