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A note on single-machine makespan problem with general deteriorating function

Wen-Chiung Lee · Chin-Chia Wu · Han-Chu Liu

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Abstract Recently, deteriorating job scheduling problems have received increasing attention. However, the majority of the research assumes that the actual job processing time is a function of its starting time. In this note, we develop a new deterioration model where the actual job processing time is a function of jobs already processed. We show that the single-machine makespan problem remains polynomially solvable under the proposed model.

Keywords Scheduling · Single-machine · Deteriorating jobs · Makespan

1 Introduction

For many years, job processing times are assumed to be known and fixed from the first job to be processed until the last job to be completed. However, there are many situations in which a job that is processed later consumes more time than the same job when processed earlier. For example, such a problem arises in steel production, where ingot batches must be preheated by gas to the required temperature in soaking pits before they can be hot-rolled by a blooming mill (critical machine) [1, 2]. Janiak [3] also pointed out that problems of sequencing jobs on machines with simultaneous allocation of constrained resources exist in steel and copper plants. The time and effort required to control the fire increase if there is a delay in the start of the fire fighting effort [4]. Other examples also appear in maintenance scheduling and cleaning assignments. Scheduling in this setting is known as scheduling deteriorating jobs.

Gupta and Gupta [2] and Browne and Yechiali [5] were the pioneers in studying deteriorating job scheduling problems. Gupta and Gupta [2] introduced the problem with polynomial processing time functions and proposed the branch-and-bound and heuristic algorithms to search for the optimal and near-optimal solutions for the makespan problem. Browne and Yechiali [5] introduced the makespan problems with exponential job processing times and provided insight into problem solutions. Mosheiov [6] studied a linear deterioration model where jobs have only job-dependent growth rates. He showed that the problems of minimizing the makespan, the total flow time, the sum of weighted sum of completion times, the total lateness, the maximum lateness, the maximum tardiness, and the number of tardy jobs remain polynomially solvable. Sundararaghavan and Kunnathur [7] proposed optimal and heuristic algorithms to minimize the makespan and the total weighted completion time, respectively. Chen [8] considered a single-processor scheduling model where the execution time of a task is a decreasing linear function of its starting time, and presented an $O(n^2)$-time dynamic programming algorithm to minimize the number of late tasks. Cheng and Ding [9] considered a family of scheduling problems for a set of start-time-dependent tasks with release times and linearly increasing/decreasing processing rates on a single machine to minimize the makespan. Bachman et al. [10] proved that the total weighted completion time problem is NP-hard when the processing time is a linear function of its starting time. Ng et al. [11] investigated three scheduling problems with deteriorating jobs to minimize the total completion time on a single machine. Cheng and Ding [12] considered a piecewise-linear model where each task has a normal processing time that deteriorates as a step function if its

W.-C. Lee · C.-C. Wu (✉) · H.-C. Liu
Department of Statistics, Feng Chia University,
Taichung, Taiwan, Republic of China
e-mail: cchwu@fcu.edu.tw

starting time is beyond a given deterioration date. Cheng *et al.* [13] studied scheduling problems for a set of non-preemptive jobs on single- or multi-machines without idle times where the processing time of a job is a piecewise non-increasing function of its starting time. Cheng and Ding [14] studied the feasibility problem of scheduling a set of starting time dependent tasks on a single machine with known deadlines and processing rates and identical initial processing times. Wu et al. [15] investigated a single-machine problem in which processing times of jobs are starting time dependent and the aim is to minimize the total weighted completion time.

In addition, Shiau et al. [16] considered a simple linear deterioration model in a two-machine flowshop where the objective is to minimize the mean flow time. Guo and Wang [17] investigated a single machine problem where the actual processing time is given by $p_{ij}(t) = p_{ij}(a + bt)$. They showed that the makespan problem is polynomially solvable under the group technology assumption. Under the same model, Xu et al. [18] proved that the total weighted completion time problem remains polynomially solvable. Moreover, Wang et al. [19] showed that single-machine group scheduling problems are polynomially solvable where the objectives are to minimize the makespan and the total completion time under the model $p_{ij}(t) = a_{ij} - b_{ij}t$. An extensive survey of different models and criteria can be found in Alidaee and Womer [20] and Cheng et al. [21]. Recently, Toksari and Güner [22] introduced a mixed nonlinear integer programming formulation for the parallel machine earliness/tardiness (ET) scheduling problem with simultaneous effects of learning and linear deterioration, sequence dependent setups, and a common due-date for all jobs. Wang et al. [23] considered a case that both the group setup times and job processing times are increasing functions of their starting times. They showed that the makespan and the total weighted completion time problems remain solvable in polynomial times.

Although the deteriorating job scheduling problems have been extensively studied in various machine settings and performance measures, most of the researchers assume that the actual job processing time is a function of its starting time. In this note, we propose a new model where the deterioration phenomenon is expressed as a function of the processing times of jobs already processed. This model is motivated by the idea of Koulamas and Kyparisis [24], who consider a sum-of-processing-time-based learning effect model. It is seen from this model that the more jobs already processed, the steeper deteriorating effect for all subsequence jobs not processed yet. Specifically, we consider a new deterioration model where the actual job processing time depends on the processing times of the jobs already processed. In this short note, we study the single-machine makespan problem. The rest of this paper is organized in three sections. The problem formulation is given in the next section. The problem under the proposed model is discussed in Section 3. The conclusion is given in the last section.

2 Problem formulation

There are n jobs ready to be processed on a single machine. All jobs are available at time zero. Let P_j denote the normal processing time of job j for j= 1, 2, …, n. In addition, let $P_{[r]}$ denote the processing time of the job scheduled in the rth position in a job sequence. The normal processing time of a job is incurred if the job is scheduled first in a sequence. The processing time of a later job becomes longer than its normal processing time due to the deterioration effect. We define a new sum-of-processing-time-based deterioration model as follows. If job j is scheduled in the rth position in a sequence, then its actual processing time is

$$p_{jr} = p_j \left(1 + \frac{\sum_{l=1}^{r-1} p_{[l]}}{\sum_{l=1}^{n} p_l} \right)^a \qquad (1)$$

where the deterioration rate a lies between 0 and 1. Under this deterioration model, the actual processing time of job j is affected by the previous (r-1) jobs.

3 Makespan problem

In this section, we will show that the optimal solution for the proposed problem can be obtained by the largest processing time (LPT) first rule. Before developing the results, we first derive the following two lemmas that are essential in the proof of the theorem.

Lemma 1 *Let* $f(x) = (1 + x)^a - 1 - ax(1 + x)^{a-1}$. *Then* $f(x) \geq 0$ *for* $x \geq 0$ *and* $0 \leq a \leq 1$.

Proof To show $f(x) \geq 0$, we take the first derivative of $f(x)$ and yield

$$f'(x) = a(1 + x)^{a-1} - a(1 + x)^{a-1} - ax(a - 1)(1 + x)^{a-2}$$
$$= -a(a - 1)x(1 + x)^{a-2}.$$

Since $0 \leq a \leq 1$, it implies that $f'(x) \geq 0$ for $x \geq 0$ and $0 \leq a \leq 1$. Thus, $f(x)$ is an increasing function on $x \geq 0$. Since $f(0) = 0$, we have $f(x) \geq 0$ for $x \geq 0$ and $0 \leq a \leq 1$. This completes the proof. □

Lemma 2 *$[1-(1+\lambda x)^a]-\lambda[1-(1+x)^a]\geq 0$ for $x\geq 0$, $\lambda\geq 1$, and $0\leq a\leq 1$.*

Proof In order to show the above inequality is non-negative, we consider the following equation

$$g(\lambda)=[1-(1+\lambda x)^a]-\lambda[1-(1+x)^a].$$

Taking the first and the second derivatives of $g(\lambda)$, we obtain

$$g'(\lambda)=-ax(1+\lambda x)^{a-1}-[1-(1+x)^a]$$

and

$$g''(\lambda)=-a(a-1)x^2(1+\lambda x)^{a-2}.$$

Since $x\geq 0$, $\lambda\geq 1$, and $0\leq a\leq 1$, we have $g''(\lambda)\geq 0$ for $\lambda\geq 1$. It implies that $g'(\lambda)$ is an increasing function on $\lambda\geq 1$. From Lemma 1, we have $g'(\lambda)\geq g'(1)\geq 0$.

Therefore, $g(\lambda)$ is also an increasing function for $\lambda\geq 1$. Since $g(1)=0$, it implies that

$$g(\lambda)=[1-(1+\lambda x)^a]-\lambda[1-(1+x)^a]\geq 0$$

for $x\geq 0$, $\lambda\geq 1$, and $0\leq a\leq 1$. This completes the proof. □

Theorem 1 *Under the sum-of-processing-time-based deterioration model, the optimal schedule for the makespan problem is obtained if jobs are ordered according to the largest processing time (LPT) first rule.*

Proof We will prove the theorem by contradiction. Suppose that there is an optimal solution that does not follow LPT rule. In this schedule, there is at least two adjacent jobs, say job i followed by job j such that $p_i < p_j$. Furthermore, we assume that job i is scheduled in the rth position and job j is scheduled in the $(r+1)$th position in S. In addition, let B be the starting time for job i in S. We now perform an adjacent pairwise interchange of jobs i and j, leaving the remaining jobs in their original positions, to derive a new sequence S'. We will show the pairwise interchange of jobs i and j does not increase the makespan of sequence S, and it leads to a contradiction of the optimality of S.

It is derived that the completion times of jobs i and j in S are

$$C_i(S)=B+p_i\left(1+\frac{\sum_{l=1}^{r-1}p_{[l]}}{\sum_{l=1}^{n}p_l}\right)^a$$

and

$$C_j(S)=B+p_i\left(1+\frac{\sum_{l=1}^{r-1}p_{[l]}}{\sum_{l=1}^{n}p_l}\right)^a+p_j\left(1+\frac{\sum_{l=1}^{r-1}p_{[l]}+p_i}{\sum_{l=1}^{n}p_l}\right)^a.$$

Similarly, the completion times of jobs j and i in S' are

$$C_j(S')=B+p_j\left(1+\frac{\sum_{l=1}^{r-1}p_{[l]}}{\sum_{l=1}^{n}p_l}\right)^a$$

and

$$C_i(S')=B+p_j\left(1+\frac{\sum_{l=1}^{r-1}p_{[l]}}{\sum_{l=1}^{n}p_l}\right)^a+p_i\left(1+\frac{\sum_{l=1}^{r-1}p_{[l]}+p_j}{\sum_{l=1}^{n}p_l}\right)^a.$$

Taking the difference between $C_j(S)$ and $C_i(S')$, one derives that

$$\begin{aligned}C_j(S)-C_i(S')=&\left(p_i-p_j\right)\left(1+\frac{\sum_{l=1}^{r-1}p_{[l]}}{\sum_{l=1}^{n}p_l}\right)^a\\&-p_i\left(1+\frac{\sum_{l=1}^{r-1}p_{[l]}+p_j}{\sum_{l=1}^{n}p_l}\right)^a\\&+p_j\left(1+\frac{\sum_{l=1}^{r-1}p_{[l]}+p_i}{\sum_{l=1}^{n}p_l}\right)^a.\end{aligned}\quad(2)$$

Substituting $P=\sum_{l=1}^{n}p_l$, $P_r=\sum_{l=1}^{r-1}p_{[l]}$, $t=1+\frac{P_r}{P}$, $\lambda=\frac{p_j}{p_i}$ and $x=\frac{p_i}{P+P_r}$ into Eq. (2), we have from Lemma 2

$$C_j(S)-C_i(S')=p_it^a\{[1-(1+\lambda x)^a]-\lambda[1-(1+x)^a]\}\geq 0.$$

Thus, the makespan of S is greater than or equal to that of S'. This contradicts the optimality of S and proves that jobs are ordered according to LPT rule. □

4 Conclusion

The contribution of this note is to propose a new sum-of-processing-time-based deterioration model. We showed that the largest processing times first rule provides the optimal solution for the single-machine makespan problem. The consideration of other criteria or the extension to flowshop problems might be interesting issues for future research.

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