Int J Adv Manuf Technol (2009) 40:819–836
DOI 10.1007/s00170-008-1389-5

ORIGINAL ARTICLE

A new algorithm for optimum tolerance allocation of complex assemblies with alternative processes selection

M. Siva Kumar · S. M. Kannan · V. Jayabalan

Received: 17 December 2005 / Accepted: 7 January 2008 / Published online: 13 March 2008

Abstract Allocating tolerance to sub-components of a complex assembly with alternative processes selection by using Lagrange's multiplier method is tedious as well as difficult. The present work is aims to solve the problem with simple effort in three stages. In the first stage, the maximum of two processes are selected from the alternative processes of each component and these two processes correspond to the smaller sum of difference in manufacturing cost. A hybrid optimum tolerance allocation method is developed in a second and third stage by combining Tabu search (TS) and heuristic approach. Application of the proposed algorithm is demonstrated on complex tolerancing products like knuckle joint and wheel mounting assembly. For the same manufacturing conditions, compared with tolerance synthesis by Singh method, the proposed method saved nearly $74,880 and $479,520, respectively, per year in manufacturing costs of knuckle joint and wheel mounting assembly.

Keywords Tolerance synthesis · Manufacturing costs · Non-traditional optimization techniques · Tolerance cost curves · Manufacturing processes

M. S. Kumar (✉)
Department of Mechanical Engineering,
National Engineering College,
K.R. Nagar,
Kovilpatti, 628 503 Tamilnadu, India
e-mail: lawan_sisa@yahoo.com

S. M. Kannan
Velammal College of Engineering & Technology,
Madurai, India
e-mail: soma_kannan@yahoo.co.in

V. Jayabalan
Department of Manufacturing Engineering, Anna University,
Chennai, India

Nomenclature

Symbol	Meaning, Unit
a	Subassembly index
t_{alo}	Allocated tolerance, mm
t_{ralo}	Reallocated tolerance, mm
N	Number of components in subassembly
t_{sasm}	Subassembly tolerance, mm
t_{csasm}	Calculated subassembly tolerance, mm
$P1...P4$	Process number from 1 to 4

1 Introduction

A part cannot be manufactured exactly to the nominal dimensions due to inherent variability in the manufacturing process. Tolerance plays a vital role to get the desired fit as well as performance of the product. Proper allocation of tolerance among the components of a mechanical assembly reduces the manufacturing cost in a large extent and critical clearance is also maintained for part interchangeability. Considerable research work has been published on optimal tolerance synthesis for simple and complex assemblies. Ostwald [1] introduced a mathematical formulation of advanced tolerance synthesis problem into 0-1 integer programming problem and considered a discrete cost function to select an optimal tolerance. Lee [4] reported a branch-and-bound algorithm that is more efficient than Bala's algorithm to handle both the linear and non-linear assemblies in selecting optimum tolerance with process limits, and interrelated dimensional chain. Chase [5] obtained optimum tolerances by using four different optimization tools considering both discrete and continuous cost function and reported that an exhaustive search based on Lagrange's multiplier (ESLM) approach is the most reliable technique to obtain the exact global optima. Zhang [6] introduced simulated annealing, a non-traditional optimization tool to obtain global optimum in advanced tolerance synthesis problems for continuous cost function. Wu [8] presented a design method for allocating dimensional tolerances of product with asymmetric quality losses and computed average quality losses of batch products according to distribution of functional characteristics. Chase [9] described several algorithms to perform optimum tolerance allocation automatically based on optimization techniques for both worst-case and root-sum square method. Diplaris [10] formulated a new analytical cost tolerance model closer to industrial practice based on an available industrial knowledge and earlier published data. Monica [11] developed a methodology to allow an automatic tolerance allocation capable of minimizing manufacturing costs based on Monte Carlo simulation. Ye [12] introduced a new concurrent engineering method for tolerance allocation and constructed a nonlinear optimization model to implement the method. The model minimized quality loss and manufacturing cost simultaneously in a single objective function by setting both process tolerances and design tolerances. Singh [13] demonstrated the application of GA on complex tolerancing problems.

2 Problem definition and solving methodology

The market stability of the company is determined by the ability of the company to produce quality product with an attractive price. Distributing tolerance optimally to interrelated dimensional chains product with alternative processes selection is tedious with the LM method. In this present work, this issue is solved in three stages. In the first stage, a maximum of two processes are selected from each component's alternative processes based on sum of difference between minimum and actual manufacturing cost of the processes. In the second stage, the assembly tolerance/constraint is considered as an in equality constraint and component's tolerances are allocated by TS. In the last stage, the allocated tolerances from stage II are adjusted to meet closer to assembly specification with in the process precision limits based on sum of difference between minimum and actual manufacturing cost of the process. The implementation of the present work is explained with the help of two interrelated dimensional chain products.

2.1 Stage - I: selection of the best processes

The list of components, its alternative processes, cost function constants, and precision limits of each process are assumed as available/known data. The tolerance limit of the process is divided into n delta values (small equal parts) by using expression (1) and the delta tolerance is computed from Eq. (2).

$$t_{del,ij} = \frac{t_{\max,ij} - t_{\min,ij}}{n} \tag{1}$$

$$t_{ijk} = t_{\min,ij} + (k-1)^{*} t_{del,ij} \tag{2}$$

where,

$t_{del,ij}$	Delta tolerance of ith component for jth process in mm
$t_{max,ij}$	Maximum process tolerance of ith component for jth process in mm
$t_{min,ij}$	Minimum process tolerance of ith component for jth process in mm
i	Component number index
j	Process number index
k	Discrete point index
n	Number of discrete points
t_{ijk}	Tolerance of ith component for jth process at kth discrete point in mm

The manufacturing cost of the ith component by using jth process for t_{ijk} tolerance is determined by the following expression (3).

$$C_{ijk} = C0_{ij}{}^{*}E^{\left(-C1_{ij}{}^{*}t_{ijk}\right)} + C2_{ij} \tag{3}$$

where,

$C0_{ij}$,$C1_{ij}$ and $C2_{ij}$	Exponential cost function constants of ith component for jth process
C_{ijk}	Manufacturing cost at t_{ijk} in $US

The sum of difference between actual manufacturing cost (C_{ijk}) and the minimum manufacturing cost ($C_{min,ijk}$) is calculated by using Eqs. (4) and (5).

$$C_{dif,ijk} = C_{ijk} - C_{\min,ijk} \tag{4}$$

$$C_{dif,ij} = \sum_{k=1}^{n} C_{dif,ijk} \tag{5}$$

where,

$C_{dif,ijk}$	Manufacturing cost difference at t_{ijk} in $US
$C_{min,ijk} = \min_{j=1}^{np_i} \left[C_{ijk}\right]$	Minimum manufacturing cost difference at t_{ijk} in $US
$C_{dif,ij}$	Sum of difference in manufacturing cost of ith component for jth process in $US
np_i	Number of process for ith component

The alternative processes of each component are arranged in ascending order based on $C_{dif,ij}$ from which the first two process are selected for tolerance allocation. The method proposed in this paper reduced the search space to maximum extent by selecting an optimum process. For example, a product consists of five components and each component has six different alternative processes, then the exhaustive search space contains 7,776 ($\prod_{i=1}^{nc} np_i$) combinations but the present method has only 32 (2^{nc}) combinations.

2.2 Stage - II: initial tolerance allocation by Tabu search

Metaheuristic superimposed on another heuristic (Glover [3]), designed for the solution of hard optimization problems popularly known as Tabu search. The basic principle of Tabu search is to pursue local search whenever it encounters a local optimum by allowing non-improving moves; *cycling* back to previously visited solutions is prevented by the use of *memories*, called *Tabu lists* that record the recent history of the search, a key idea that can be linked to artificial intelligence concepts. A very simple memory mechanism is described in Glover [2] to implement the oscillating assignment heuristic. TS has been applied in different fields like mobile radio networks (Jin-Kao [7]), bicriteria flowshop problem (Vinicius [14]) and job shop scheduling (Eugeniusz [15] and Chao[16]) . In this paper, TS is implemented in obtaining optimum tolerance allocation. The tolerance space between maximum and minimum process tolerance of the components are divided into discrete values based on the expression (6). The scheme of TS is represented in Fig. 25. (Appendix A).

$$t_{dis,ij} = \frac{t_{\max,ij} - t_{\min,ij}}{2^{nb} - 1} \tag{6}$$

where,

nb Bit length

$t_{dis,ij}$ Discrete tolerance of ith component for jth process in mm

The initial tolerance allocation is obtained from Eq. (7) in which *de* was obtained from converting the binary number (considered as allocated tolerance) into a decimal value. The assembly tolerance is estimated by expression (8).

$$t_{IA,ij} = t_{\min,ij} + t_{dis,ij} * de \tag{7}$$

$$t_{asm} \geq t_{asmI} \geq \sum_{i=1}^{nc} t_{IA,ij} \tag{8}$$

where,

$t_{IA,ij}$ Initial allocated tolerance of ith component for jth process in mm

de Decimal equivalent from binary string conversion

t_{asm} Assembly tolerance in mm

t_{asmI} Initial stage assembly tolerance in mm

nc Number of components in an assembly

2.3 Stage - III: final tolerance allocation by heuristic approach

The flow chart of heuristic approach is represented in Fig. 26 (Appendix A). The part dimensions are ranked based on ascending order of sum of cost difference between actual and minimum manufacturing cost. The difference between required and assembly tolerance values are obtained by using expression (9).

$$t_{dif} = t_{asm} - \sum_{i=1}^{nc} t_{IA,ij} \tag{9}$$

where,

t_{dif} Difference in tolerance in mm

The t_{dif} value is added to part dimension's tolerance starting from first rank to last rank part dimension without violating expressions (10) and (11).

$$t_{FA,ij} \leq t_{\max,ij} \tag{10}$$

$$t_{asm} \approx t_{asmF} \approx \sum_{i=1}^{nc} t_{FA,ij} \tag{11}$$

where,

$t_{FA,,ij}$ Final allocated tolerance of ith component for jth process in mm

t_{asmF} Final stage assembly tolerance in mm

The manufacturing cost of the product is calculated with the help of expression (12).

$$C_{asm} = \sum_{i=1}^{nc} C0_{ij} * E^{\left(-C1_{ij} * t_{FA,ij}\right)} + C2_{ij} \tag{12}$$

where,

C_{asm} Cost of the assembly in \$US

3 Case studies

Two case studies are presented in this work by adopting the proposed new algorithm. The tolerance space is divided into ten equal delta values (for demonstration purpose *n* is assumed as 10) and manufacturing cost is calculated. Based on sum of difference in manufacturing cost, maximum of two processes are selected in each component in the first stage. The initial solution is generated randomly for the specified bit length (*nb*). For example the binary number 10101110 *(nb=8)* has bit length of 8. Neighbors are generated with a size of *(nb−1)*. For example, 7 neighbors are generated for the bit length 8. After every *nb* iteration, the Tabu list bit is allowed to produce neighbors. After 1,000 iterations, there is no further improvement in the achieved results. Hence, up to 1,000 iterations are attempted to obtain the best-allocated tolerances for bit length varying from 7 to 19. The best-allocated tolerances from the second stage are redefined by adopting heuristic approach without violating process precision limits and assembly requirements in the third stage. All problems are solved on a Pentium IV PC using C programming.

3.1 Wheel mounting assembly (WMA)

The component and dimension details are shown in Fig. 1. The alternative processes and its exponential cost function constants of the part dimensions are listed in Table 1, which are used by Singh [13].

The dimension of *Y1* and *Y2* are computed from Eqs. (13) and (14). The tolerance on dimension *Y1* and *Y2* are expressed in expressions (15) and (16).

$$Y1 = X2 - X4 \quad (13)$$

$$Y2 = X5 - X1 - X2 - X3 \quad (14)$$

$$t_{Y1} \leq 0.11 \geq t_{X2} + t_{X4} \quad (15)$$

$$t_{Y2} \leq 0.24 \geq t_{X1} + t_{X2} + t_{X3} + t_{X5} \quad (16)$$

where,

X1,X2 ... X5	Dimension of the components of WMA in mm
Y1 and *Y2*	Critical dimensions in mm
t_{X1}, t_{X2} ... t_{X5}	Tolerance on dimensions X1, X2 ... X5 in mm
t_{Y1} and t_{Y2}	Tolerance on dimensions Y1 and Y2 in mm

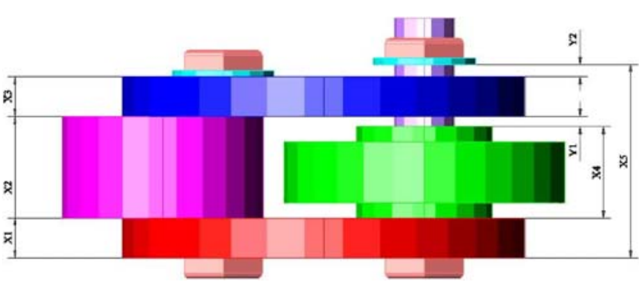


Fig. 1 Wheel mounting assembly

Table 1 Exponential cost function constants of wheel mounting assembly Singh [13]

Part dimension	Process no.	Cost model constants			Precision limits (mm)	
		C0	C1	C2	t_{min}	t_{max}
X1,X2 & X3	1	241.00	55.80	28.20	0.006	0.08
	2	260.00	52.00	29.80	0.006	0.08
	3	286.40	59.50	25.82	0.006	0.08
	4	271.50	57.64	23.00	0.006	0.08
X4	1	312.84	105.66	42.20	0.002	0.06
	2	352.43	92.70	35.00	0.002	0.06
X5	1	208.25	62.45	22.50	0.010	0.10
	2	240.43	66.70	20.20	0.010	0.10
	3	211.42	40.05	25.05	0.010	0.10
	4	214.16	58.82	300.00	0.010	0.10

The steps are given below in detail to demonstrate the proposed algorithm.

Stage I:

Step 1: The precision limits of components presented in Table 1 are divided into ten equal divisions by using expression (1). For example, the precision limits of process 1 of part dimension X1 are substituted in expression (1) and $t_{del,X11}$ is equal to

$$t_{del,X11} = \frac{0.08 - 0.006}{10} = 0.0074$$

Step 2: The delta tolerance is computed based on expression (2). The delta tolerance of first division (k=1) for process 1 of part dimension X1 is

$$t_{X111} = 0.006 + (1 - 1)*0.0074 = 0.006$$

Step 3: The manufacturing cost of the component X1 for t_{X111} tolerance is obtained by substituting C0, C1, and C2 values (read from Table 1) in expression (3).

$$C_{X111} = 214* \exp(-55.8*0.006) + 28.2$$
$$= \$200.63$$

In similar way, the manufacturing cost of X1 is determined for other processes 2, 3, and 4 by substituting corresponding C0, C1, and C2 values.

$$C_{X121} = 260* \exp(-52.0*0.006) + 29.5 = \$220.12$$

$$C_{X131} = 286.4* \exp(-59.5*0.006) + 25.82 = \$226.23$$

$$C_{X141} = 271.5* \exp(-57.64*0.006) + 23.0 = \$215.15$$

Table 2 Manufacturing cost of wheel mounting assembly's dimensions X1,X2, and X3

Division no.	Tolerance (mm)	Manufacturing cost for various processes (in $US)				Difference in cost (in $US)			
		P1	P2	P3	P4	P1-C_{min}	P2-C_{min}	P3-C_{min}	P4-C_{min}
1	0.0060	200.63	220.12	226.23	215.12	0.00	19.48	25.60	14.49
2	0.0134	142.30	159.33	154.86	148.41	0.00	17.03	12.56	6.11
3	0.0208	103.70	117.95	108.90	104.86	0.00	14.25	5.20	1.16
4	0.0282	78.16	89.80	79.31	76.44	1.72	13.36	2.87	0.00
5	0.0356	61.26	70.63	60.26	57.88	3.38	12.75	2.38	0.00
6	0.0430	50.08	57.59	47.99	45.77	4.31	11.82	2.22	0.00
7	0.0504	42.68	48.71	40.10	37.86	4.81	10.85	2.23	0.00
8	0.0578	37.78	42.67	35.01	32.70	5.08	9.97	2.31	0.00
9	0.0652	34.54	38.56	31.74	29.33	5.21	9.23	2.40	0.00
10	0.0726	32.39	35.76	29.63	27.13	5.26	8.63	2.50	0.00
11	0.0800	30.98	33.86	28.27	25.70	5.28	8.16	2.57	0.00
Sum of difference in manufacturing cost						35.04	135.53	62.85	21.76

Table 3 Manufacturing cost of wheel mounting assembly's dimension X4

Tolerance (mm)	Manufacturing cost for various processes (in $US)		Difference in cost (in $US)	
	P1	P2	P1-C_{min}	P2-C_{min}
0.0020	295.45	327.79	0.00	32.34
0.0078	179.41	206.02	0.00	26.61
0.0136	116.54	134.90	0.00	18.35
0.0194	82.48	93.35	0.00	10.87
0.0252	64.03	69.08	0.00	5.06
0.0310	54.03	54.91	0.00	0.88
0.0368	48.61	46.63	1.98	0.00
0.0426	45.67	41.79	3.88	0.00
0.0484	44.08	38.97	5.11	0.00
0.0542	43.22	37.32	5.90	0.00
0.0600	42.75	36.35	6.40	0.00
Sum of difference in manufacturing cost			23.27	94.11

Step 4: The difference in manufacturing cost between minimum and actual is computed based on expression (4).

$$C_{dif,X111} = C_{X111} - \min(C_{X111}, C_{X121}, C_{X131}, C_{X141})$$

$$C_{dif,X111} = 200.63 - \min(200.63, 220.12, 226.23, 215.12) = 0.0$$

In a similar way,

$$C_{dif,X121} = 220.12 - \min(200.63, 220.12, 226.23, 215.12) = 19.48$$

$$C_{dif,X131} = 226.23 - \min(200.63, 220.12, 226.23, 215.12) = 25.6$$

Table 4 Manufacturing cost of wheel mounting assembly's dimension X5

Tolerance (mm)	Manufacturing cost for various processes (in $US)				Difference in cost (in $US)			
	P1	P2	P3	P4	P1-C_{min}	P2-C_{min}	P3-C_{min}	P4-C_{min}
0.010	134.02	143.60	166.70	418.93	0.00	9.58	32.67	284.90
0.019	86.07	87.90	123.83	370.05	0.00	1.83	37.76	283.97
0.028	58.74	57.34	93.94	341.25	1.39	0.00	36.59	282.52
0.037	43.16	40.58	73.09	324.30	2.58	0.00	32.51	283.72
0.046	34.28	31.38	58.55	314.31	2.89	0.00	27.17	282.93
0.055	29.21	26.33	48.41	308.43	2.88	0.00	22.08	282.09
0.064	26.33	23.57	41.34	304.96	2.76	0.00	17.78	281.40
0.073	24.68	22.05	36.41	302.92	2.63	0.00	14.36	280.88
0.082	23.74	21.21	32.97	301.72	2.53	0.00	11.76	280.51
0.091	23.21	20.76	30.58	301.01	2.45	0.00	9.82	280.26
0.100	22.90	20.50	28.90	300.60	2.40	0.00	8.40	280.09
Sum of difference in manufacturing cost					22.52	11.41	250.89	3103.27

Table 5 Selected process for tolerance allocation – wheel mounting assembly

Part dimension	Selected process number for optimum tolerance allocation
X1, X2, and X3	1 and 4
X4	1 and 2
X5	1 and 2

$$C_{dif,X141} = 215.12 - \min(200.63, 220.12, 226.23, 215.12) = 14.49$$

Step 5: The steps starting from 2 to 4 are repeated until *k* reaches 11 and the results are tabulated in Table 2

Step 6: The sum of difference between minimum and actual manufacturing cost of each process are determined by using Eq. (5).

$$C_{dif,X11} = \text{Sum of column } (P1 - C_{\min}) \text{ from Table 2} = 35.04$$

$$C_{dif,X12} = \text{Sum of column } (P3 - C_{\min}) \text{ from Table 2} = 135.53$$

$$C_{dif,X12} = \text{Sum of column } (P3 - C_{\min}) \text{ from Table 2} = 62.85$$

$$C_{dif,X12} = \text{Sum of column } (P4 - C_{\min}) \text{ from Table 2} = 21.76$$

Step 7: The processes 1 and 4 are selected for tolerance allocation of part dimension X1, X2, and X3, shown in Table 2.

Step 8: The steps are repeated from 1 to 7 for other part dimensions X4 and X5. The results are presented in Tables 3 and 4.

Step 9: The outcome of the first stage is presented in Table 5.

Table 6 Manufacturing details – parts of wheel mounting assembly

Part dimension	C0	C1	C2	t_{min}	t_{max}	t_{dis}
X1	271.50	57.64	23.0	0.006	0.08	0.0002902
X2	271.50	57.64	23.0	0.006	0.08	0.0002902
X3	271.50	57.64	23.0	0.006	0.08	0.0002902
X4	352.43	92.70	35.0	0.002	0.06	0.0002275
X5	240.43	66.70	20.2	0.010	0.10	0.0003529

It is observed from the above Table 5 that the wheel mounting assembly can be produced any of the process combinations from 111, 112, 411, 412, 121, 122, 421 and 422.

Stage II:

Step 10: To demonstrate the application of the second stage, it is assumed that the process combination 422 is selected for allocating tolerance to part dimensions of the assembly. The tolerance requirement on *Y1* and *Y2* are assumed to be 0.11 and 0.24 mm, respectively. Table 6 shows the selected process details for the part dimensions to allocate tolerance.

Step 11: Tolerance limits of the process is divided into small discrete points based on Eq. (6) in which number of bit (*nb*) is assumed to be 8 and tolerance limits are read from Table 6. For example, $t_{dis,X14}$ (discrete tolerance of part dimension X1 for process 4) is equal to

$$t_{dis,X14} = \frac{0.08 - 0.006}{2^8 - 1} = 0.0002902$$

Similarly for other part dimensions, the discrete tolerances are calculated and presented in Table 6.

Step 12: An eight-digit binary number is generated randomly using C program for each part dimensions and presented in Table 7.

Step 13: The part dimension tolerances are estimated by using expression (7) and listed in Table 7.

$$de = 1*2^0 + 1*2^1 + 0*2^2 + 1*2^3 + 0*2^4 + 0*2^5 + 0*2^6 + 1*2^7 = 139$$

$$t_{IA,X14} = 0.006 + 0.0002902*139 = 0.046337$$

The manufacturing cost is computed by using given below expression.

$$C_{X14} = C0_{X14}*E^{\left(-C1_{X14}*t_{IA,X14}\right)} + C2_{X14}$$

$$C_{X14} = 271.5*E^{(-57.64*0.046337)} + 23 = 41.79$$

Table 7 Initial solution for Tabu search

Part dimension	Binary no.	De	t_{IA}	Manufacturing cost
X1	10001011	139	0.046337	41.79
X2	10011100	156	0.051271	37.14
X3	11111100	252	0.079129	25.84
X4	10111100	188	0.044761	40.56
X5	10010110	150	0.062941	23.81
t_{y1}=0.096031				
t_{y2}=0.239678				
C_{asm}=169.13				

Table 8 First iteration binary numbers

Iteration	Bit	X1	X2	X3	X4	X5
1	1	00001011	00011100	01111100	00111100	00010110
	2	11001011	11011100	10111100	11111110	11010110
	3	10101011	10111100	11011100	10011100	10110110
	4	10011011	10001100	11101100	10101100	10000110
	5	10000011	10010100	11110100	10110100	10011110
	6	10001011	10011000	11111000	10111000	10010010
	7	10001001	10011110	11111110	10111110	10010100

The other part dimension's tolerances are allocated by following the above step. The t_{y1} and t_{y2} are calculated based on Eqs. (15) and (16) and the results are presented in Table. 7.

$$t_{y1} = 0.051271 + 0.044761 = 0.096031$$

$$t_{y2} = 0.046337 + 0.051271 + 0.079129 + 0.062941$$
$$= 0.239678$$

Step 14: Either converting 0 to 1 or 1 to 0 from the initial solution, seven neighbors ($nb-1$) are generated for each component. For example, part dimension X1, the first bit neighbor is

Initial value: **1**0001011	Neighbor value: **0**0001011
Similarly for other bit's neighbors are,	
Initial value: 1**0**001011	Neighbor value: 1**1**001011
Initial value: 10**0**01011	Neighbor value: 10**1**01011
Initial value: 100**0**1011	Neighbor value: 100**1**1011
Initial value: 1000**1**011	Neighbor value: 1000**0**011
Initial value: 10001**0**11	Neighbor value: 10001**1**11
Initial value: 100010**1**1	Neighbor value: 100010**0**1

In a similar way, the neighbors of part dimensions X2, X3, X4, and X5 are obtained and listed in Table 8.

Similar to step 13, the allocated tolerances are obtained for each neighbor and tabulated in Table 9. It is observed from Table 9 that the minimum manufacturing cost of the assembly that meets assembly specification (t_{y1}=0.11 and t_{y2}=0.24 mm) on dimension Y1 and Y2 is 169.2. The string corresponding to that is considered as initial solution and step 14 is repeated until specific iterations are reached. The result of first iteration is shown in bold values in Table 9. The results up to eight iterations are presented in Table 16 (Appendix B). It is assumed that the initial allocated tolerance for part dimensions X1, X2, X3, X4, and X5 are 0.046337, 0.051271, 0.079129, 0.044761, and 0.062941 mm (from stage II), respectively, for demonstrating purpose.

Stage III:

Step 15: Steps starting from 1 to 6 are repeated for the values given in Table 6. The results are presented in Table 10. Based on the sum of difference in manufacturing cost, the ranking of part dimensions are made for final tolerance allocation.

Step 16: The tolerance difference (t_{dif}) between required and calculated values is determined for t_{y1} and t_{y2}.

$$t_{dif\,y1} = 0.11 - (0.051271 + 0.044761) = 0.013968$$

$$t_{dif\,y2} = 0.24$$
$$- (0.046337 + 0..051271 + 0.079129 + 0.062941)$$
$$= 0.00032$$

Step 17: Since Y1 is related with part dimension X2 and X4, the $t_{dif,y1}$ value is added with $t_{IA,X42}$ (part dimension X4 occupy 2nd rank and X2 occupy 3rd rank).

$$t_{FA,X42} = t_{IA,X42} + t_{dif\,y1}$$

$$t_{FA,X42} = 0.044761 + 0.013968 = 0.058729 \leq 0.06$$

$$t_{y1} = 0.051271 + 0.058729 = 0.11$$

Similarly, $t_{dif,y2}$ value is added with $t_{IA,X52}$.

$$t_{FA,X52} = t_{IA,X52} + t_{dif\,y2}$$

Table 9 Evaluation details for first iteration

Iteration	Bit	tx1	tx2	tx3	tx4	tx5	ty1	ty2	Manufacturing Cost
1	1	0.009192	0.014125	0.041984	0.015647	0.017765	0.029773	0.083067	584.60
Bit 7 in Tabu list and it is not allowed to produce neighbor until 8th iteration	2	0.064910	0.069843	0.060557	0.059773	0.085529	0.129616	0.280839	145.95
	3	0.055624	0.060557	0.069843	0.037482	0.074235	0.098039	0.260259	160.94
	4	0.050980	0.046627	0.074486	0.041122	0.057294	0.087749	0.229388	173.81
	5	0.044016	0.048949	0.076808	0.042941	0.065765	0.091890	0.235537	174.65
	6	0.046337	0.050110	0.077969	0.043851	0.061529	0.093961	0.235945	171.15
Bit 7 Tabu	7	0.045760	0.051850	0.079710	0.045220	0.062240	0.097070	0.239550	169.20

Table 10 Details of sum of difference between actual and minimum manufacturing cost – wheel mounting assembly

Division no.	Tolerance (mm)	C_{X14}, C_{X24}, C_{X34}	C_{X42}	C_{X52}	C_{min}	C_{X14}, C_{X24}, $C_{X34} - C_{min}$	$C_{X42} - C_{min}$	$C_{X52} - C_{min}$
1	0.0060	215.12	237.08	181.33	181.33	33.79	55.75	0.00
2	0.0114	163.73	157.50	132.60	132.60	31.14	24.90	0.00
3	0.0168	126.09	109.25	98.60	98.60	27.49	10.65	0.00
4	0.0222	98.52	80.01	74.89	74.89	23.63	5.12	0.00
5	0.0276	78.32	62.29	58.35	58.35	19.97	3.94	0.00
6	0.0330	63.52	51.54	46.81	46.81	16.71	4.73	0.00
7	0.0384	52.68	45.03	38.76	38.76	13.92	6.26	0.00
8	0.0438	44.74	41.08	33.15	33.15	11.60	7.93	0.00
9	0.0492	38.93	38.68	29.23	29.23	9.70	9.45	0.00
10	0.0546	34.67	37.23	26.50	26.50	8.17	10.73	0.00
11	0.0600	31.55	36.35	24.60	24.60	6.95	11.76	0.00
Sum of difference in manufacturing cost						203.05	151.21	0.00
Ranking of tolerance allocation for part dimension in final allocation						3	2	1

$$t_{FA,X52} = 0.062941 + 0.00032 = 0.063261 \leq 0.1$$

$$t_{y2} = 0.046337 + 0.051271 + 0.079129 + 0.063261 = 0.239998$$

Step 18: The details of initial and final allocated tolerance and its manufacturing cost of part dimensions are listed in Table 11.

Step 19: Steps 11 to 18 are repeated again for various bit length (minimum of 7 to maximum of 19 is assumed based on accuracy requirement). The t_{y1}, t_{y2} and manufacturing cost of the assembly for various bit length are shown graphically in Fig. 2. The minimum manufacturing cost of the product is obtained in bit length 13.

Step 20: Steps 10 to 19 are repeated for other process combinations. The values of allocated tolerance of part dimension and cost of the product with respect ot number of iterations are shown in Figs. 3, 4, 5, 6, 7, and 8 for process combination of 121, 421, and 422. In which the combination of process 422 provided a better result (minimum product cost) than the other process combinations.

Table 11 Details of initial and final allocated tolerance

Part dimension	Initial allocation		Final allocation	
	$t_{IA,ij}$	Manufacturing cost	$t_{FA,ij}$	Manufacturing cost
X5	0.062941	23.81	0.063261	23.74
X4	0.044761	40.56	0.058729	36.52
X1	0.079129	25.84	0.079129	25.84
X2	0.051271	37.14	0.051271	37.14
X3	0.046337	41.79	0.046337	41.79
t_{y1}	0.096032		0.11	
t_{y2}	0.239678		0.239998	
Total manufacturing cost	169.14		165.03	

Figure 9 represents the manufacturing cost of the wheel mounting assembly for various process combinations. The figure shows that the process combination 422 produces better tolerance allocation with minimum manufacturing cost.

The comparison of allocated tolerance and its manufacturing cost for the proposed and Singh [13] method are shown in Figs. 10 and 11.

3.2 Knuckle joint assembly (KJA)

There are five components (rod, fork, pin, collar, and taper pin) and six dimensions involved in knuckle joint

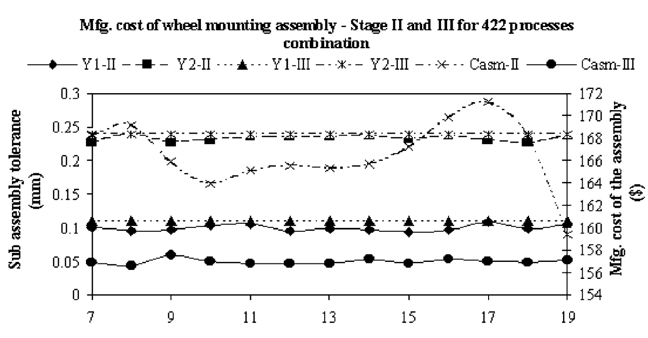


Fig. 2 Manufacturing cost of wheel mounting assembly for various bit length –422 processes combination

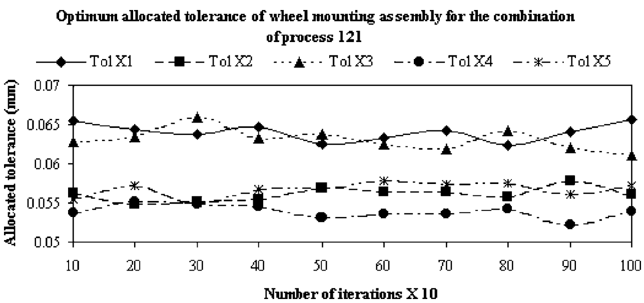


Fig. 3 Tolerance allocations for 121 processes combination of wheel mounting

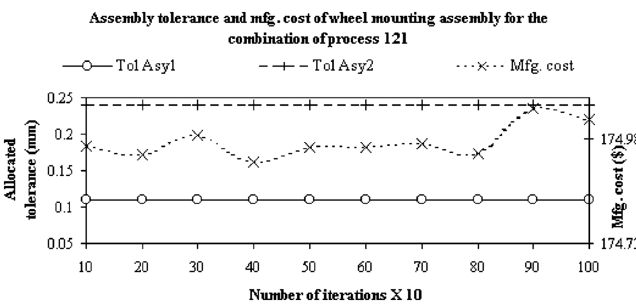


Fig. 4 Assembly Tolerance and manufacturing cost for 121 processes combination

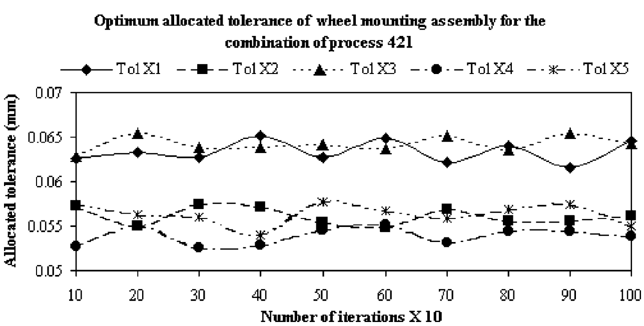


Fig. 5 Tolerance allocations for 421 processes combination of wheel mounting

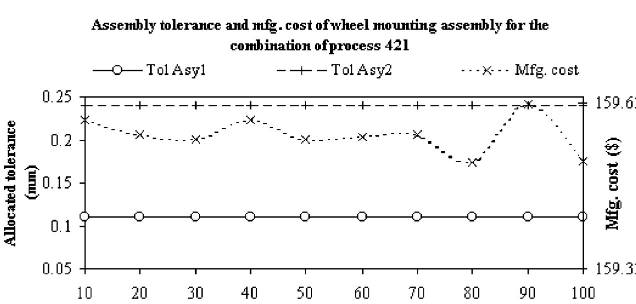


Fig. 6 Assembly Tolerance and manufacturing cost for 421 processes combination

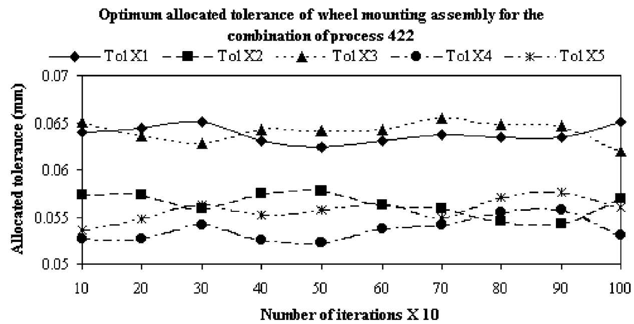


Fig. 7 Tolerance allocation for 422 processes combination of wheel mounting

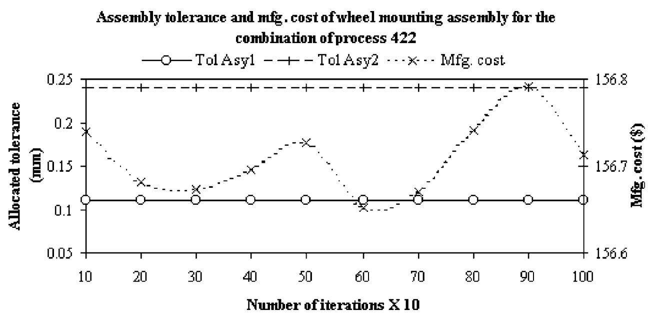


Fig. 8 Assembly tolerance and manufacturing cost for 422 processes combination

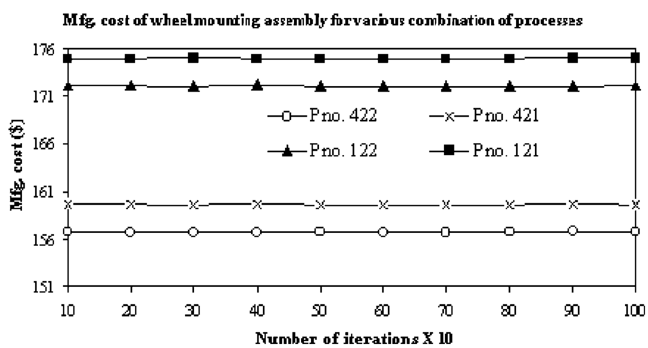


Fig. 9 Manufacturing cost of the wheel mounting assembly for various combinations of processes

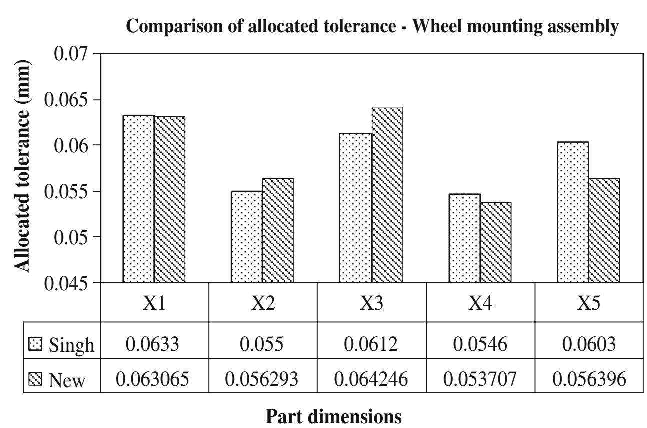


Fig. 10 Wheel mounting assembly t_{alo} comparison

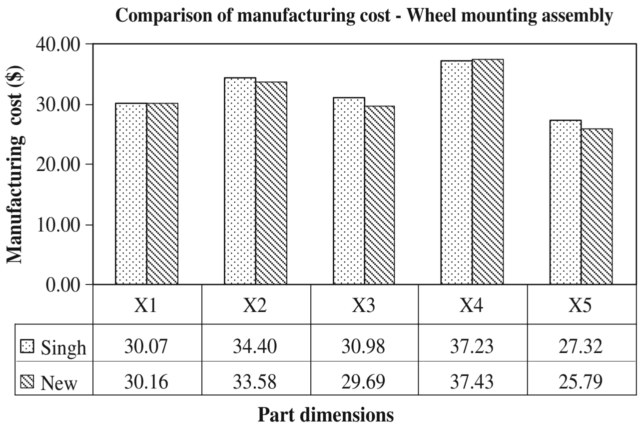


Fig. 11 Wheel mounting assembly manufacturing cost comparison

assembly as shown in Fig. 12. The manufacturing details and the exponential cost function constants are presented in Table 12. Out of six dimensions, the fork has three dimensions, namely, *d1*, *d2*, and *d3*. It is assumed that all these three dimensions have equal tolerances and are produced in a single manufacturing process. The tolerance cost (TC) curves of all the dimensions with alternative processes are represented in Figs. 13, 14, 15, and 16. The dimensions of *Y1* and *Y2* (interrelated dimensional chains) are computed based on Eqs. (17) and (18). The tolerance on dimensions *Y1* and *Y2* are less than or equal to 0.3 mm.

$$Y1 = d5 - d1 - d2 - d3 - d4 \quad (17)$$

$$Y2 = d2 - d6 \quad (18)$$

Similar representations of Figs. 3, 4, 5, 6, 7, and 8 are shown for knuckle joint assembly in Figs. 17, 18, 19, and 20. Two combinations of processes, namely, 1321 and 1322 are obtained from the stage I to produce knuckle joint assembly with the given alternative processes. The tolerances are allocated with the proposed hybrid algorithm in

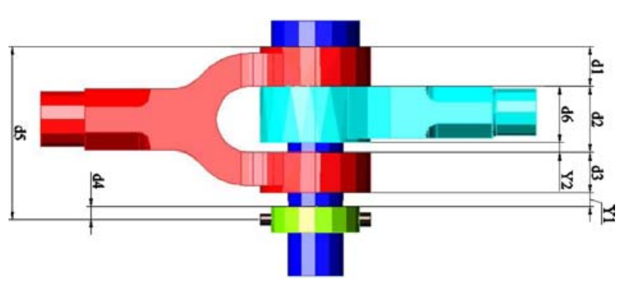


Fig. 12 Knuckle joint assembly

Table 12 Exponential cost function constants for knuckle joint assembly Singh [13]

Part dimension	Process no.	Cost model constants			Precision limits (mm)	
		C0	C1	C2	t_{min}	t_{max}
d6	1	296.40	19.50	23.82	0.01	0.15
	2	331.50	17.64	20.00	0.01	0.15
d1,d2&d3	1	311.50	15.80	24.20	0.01	0.15
	2	280.00	14.00	19.80	0.01	0.15
	3	296.40	19.50	23.82	0.01	0.15
	4	331.50	17.64	20.00	0.01	0.15
d5	1	92.84	13.66	17.20	0.02	0.20
	2	82.43	16.70	21.00	0.02	0.20
d4	1	128.25	82.45	32.50	0.01	0.10
	2	160.43	86.70	29.20	0.01	0.10
	3	231.42	50.05	28.05	0.01	0.10
	4	134.16	78.82	500.00	0.01	0.10

the second and third stages. From Figs. 17, 18, 19, and 20, very little variation is observed in allocated tolerance of part dimensions.

The assembly tolerance and the manufacturing cost of the assembly for various bit length of 1322 processes combination are presented in Fig. 21. It is observed that the minimum manufacturing cost of the assembly is achieved in bit length 14. Figure 22 represents the comparison of manufacturing cost between the combination of processes 1321 and 1322. It is observed that 1322 combination of processes is allocated tolerance with the minimum manufacturing cost. The allocated tolerance and manufacturing cost of components are compared with Singh [13] method and shown in Figs. 23 and 24.

4 Results

The optimum allocated tolerance and the corresponding processes of knuckle joint assembly's components are presented in Table 13. Similar results for wheel mounting assembly are shown in Table 14. A comparison between the present and Singh method are given in Table 15.

The manufacturing cost saving in wheel mounting assembly and knuckle joint assembly for the following manufacturing conditions are

20 products per hour; eight hour per shift;
3 shift per day and 300 days per year

$$MCS_{WMA} = (159.98 - 156.65)*20*8*3*300 = \$479520/\text{year}$$
$$MCS_{KJA} = (407.44 - 406.92)*20*8*3300 = \$74880/\text{year}$$

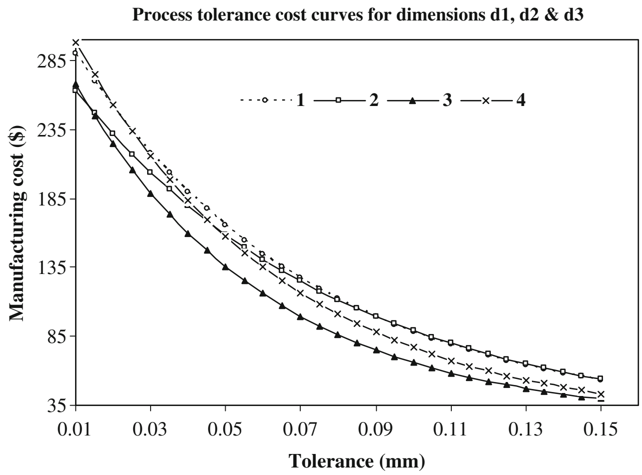


Fig. 13 TC curves for various processes of dimensions d1, d2, and d3

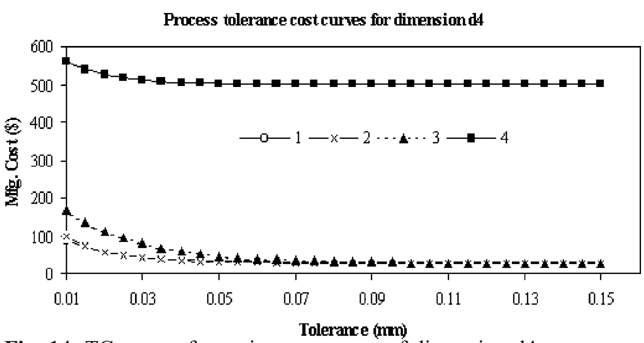


Fig. 14 TC curves for various processes of dimension d4

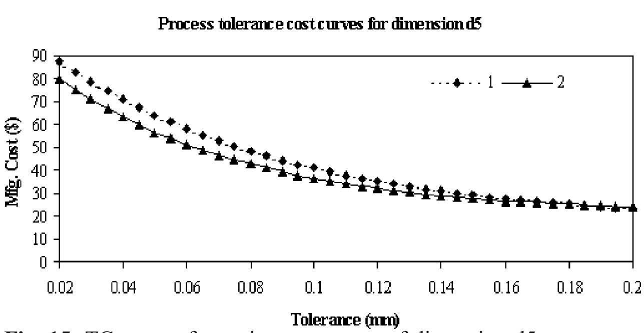


Fig. 15 TC curves for various processes of dimension d5

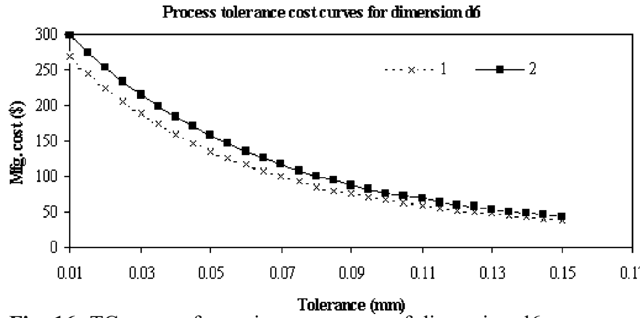


Fig. 16 TC curves for various processes of dimension d6

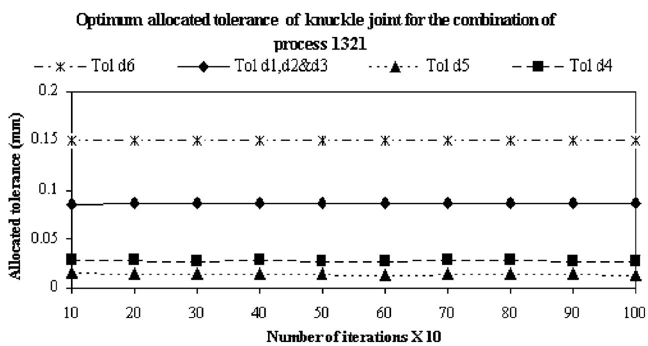


Fig. 17 Tolerance allocation for 1321 processes combination of knuckle joint

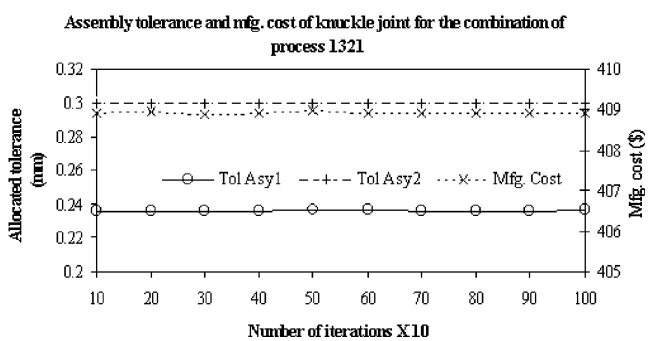


Fig. 18 Assembly tolerance and manufacturing cost for 1321 processes combination

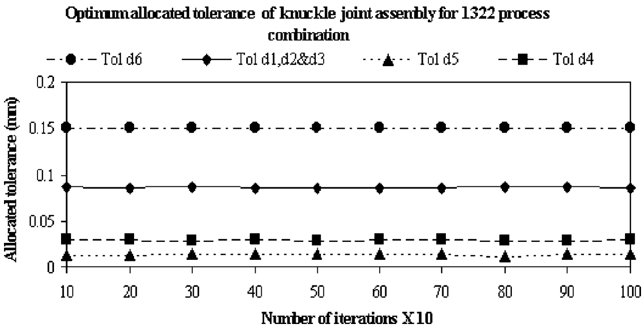


Fig. 19 Tolerance allocation for 1322 processes combination of knuckle joint

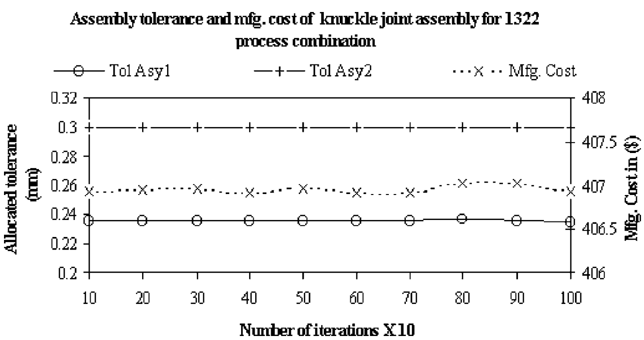


Fig. 20 Assembly tolerance and manufacturing cost for 1322 processes combination

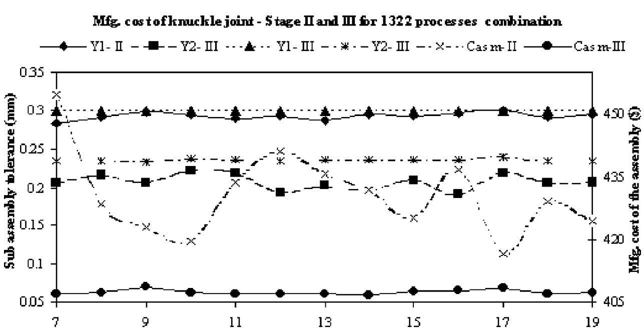


Fig. 21 t_{y1}, t_{y2} and C_{asm} for various bit length of 1322 processes

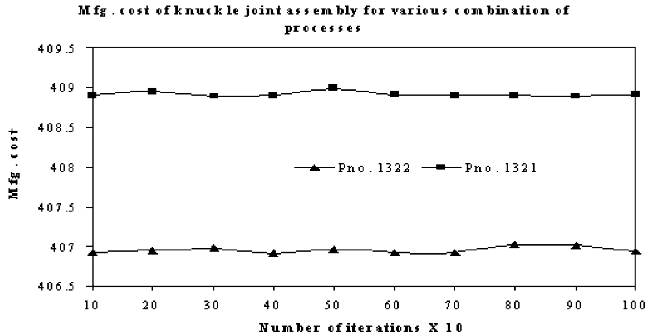
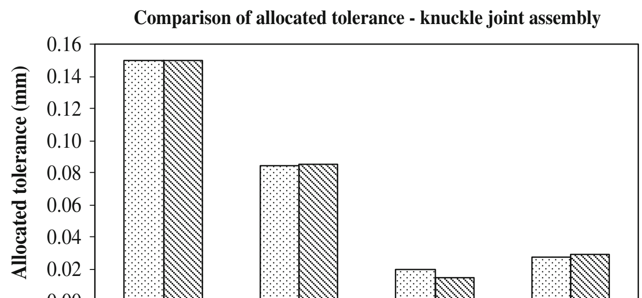


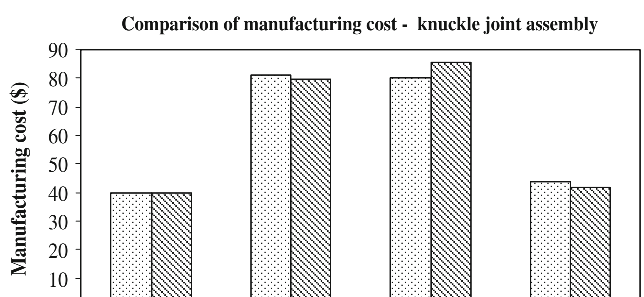
Fig. 22 Manufacturing cost of the knuckle joint for various processes



	d6	d1,d2&d3	d5	d4
Singh	0.15000	0.08410	0.0200	0.02760
New	0.15	0.085437	0.014561	0.029128

Part dimension

Fig. 23 Knuckle joint assembly t_{alo} comparison



	d6	d1,d2&d3	d5	d4
Singh	39.73	81.32	80.02	43.86
New	39.73	79.84	85.64	42.04

Part dimension

Fig. 24 Knuckle joint assembly manufacturing cost comparison

Table 13 Optimum manufacturing process and allocated tolerance for knuckle joint dimensions

Part dimension	Process no.	t_{alo} (mm)	Manufacturing cost (in $US)
d6	1	0.150000	39.73
d1,d2&d3	3	0.085765	79.48
d5	2	0.013529	86.76
d4	2	0.029176	41.99
Y1		0.235765	
Y2		0.300000	
Manufacturing cost			406.92

Table 14 Optimum manufacturing process and allocated tolerance for wheel mounting dimensions

Part dimension	Process no.	T_{alo} (mm)	Manufacturing cost (in $US)
X1	4	0.063065	30.16
X2		0.056293	33.58
X3		0.064246	29.69
X4	2	0.053707	37.43
X5	2	0.056396	25.79
Y1		0.110000	
Y2		0.240000	
Manufacturing cost			156.65

Table 15 Comparison between Singh method and the proposed method

Method/Product	Knuckle joint assembly			Wheel mounting assembly		
	Combination of processes	Manufacturing cost (in $US)	CPU time (s)	Combination of Processes	Manufacturing cost (in $US)	CPU time (s)
Singh	1322	407.44	[b]5.49	421	159.98	5.37
New	1322	406.92	[a][0.6+1.52+3.3]=5.42	421	159.52	[0.54+1.52+3.2]=5.26
			5.42	422	156.65	5.26

[a] CPU time for three stages [I+II+III]
[b] Singh GA parameters are applied

5 Conclusions

The following conclusions can be drawn:

- There is no chance to omit the best process in alternative process for optimum tolerance allocation.
- The search space to obtain processes combination to allocate tolerance optimally for the components are reduced to a large extent.
- The developed method was tested successfully in complex assemblies with alternative processes selection.
- The observation from the tested products showed a saving of $479,520 and $74,880 in manufacturing cost of wheel mounting and knuckle joint assemblies, respectively.
- The present method proved better than GA by obtaining 2.1% of saving in manufacturing cost of wheel mounting assembly.
- Even though, the method seems to take time; the CPU time (1,000 iterations have been tried out for about 20 times) is more or less same as with genetic algorithm. The CPU time for both the Singh method and the proposed method is presented in Table 15.

Appendix A

Fig. 25 Scheme of Tabu search

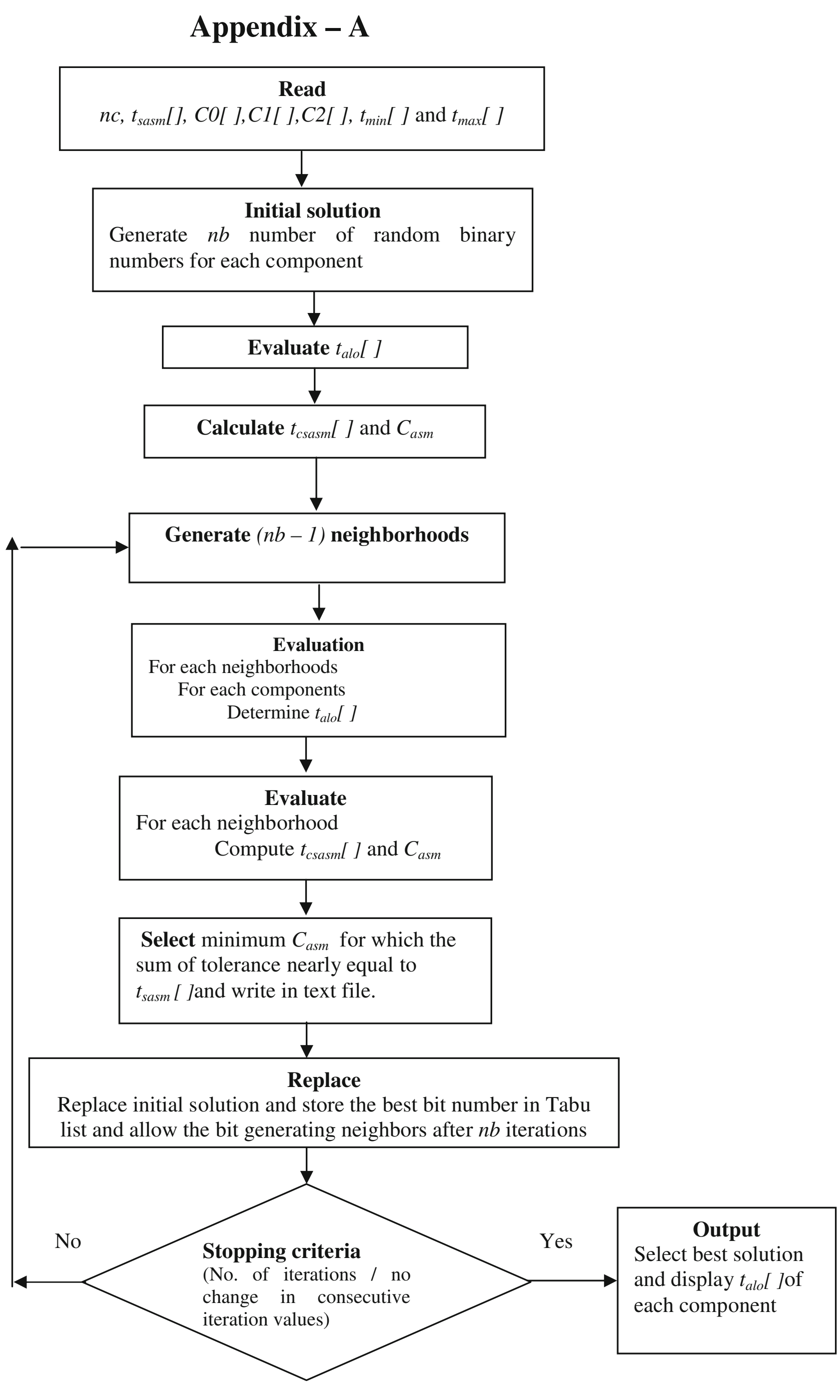
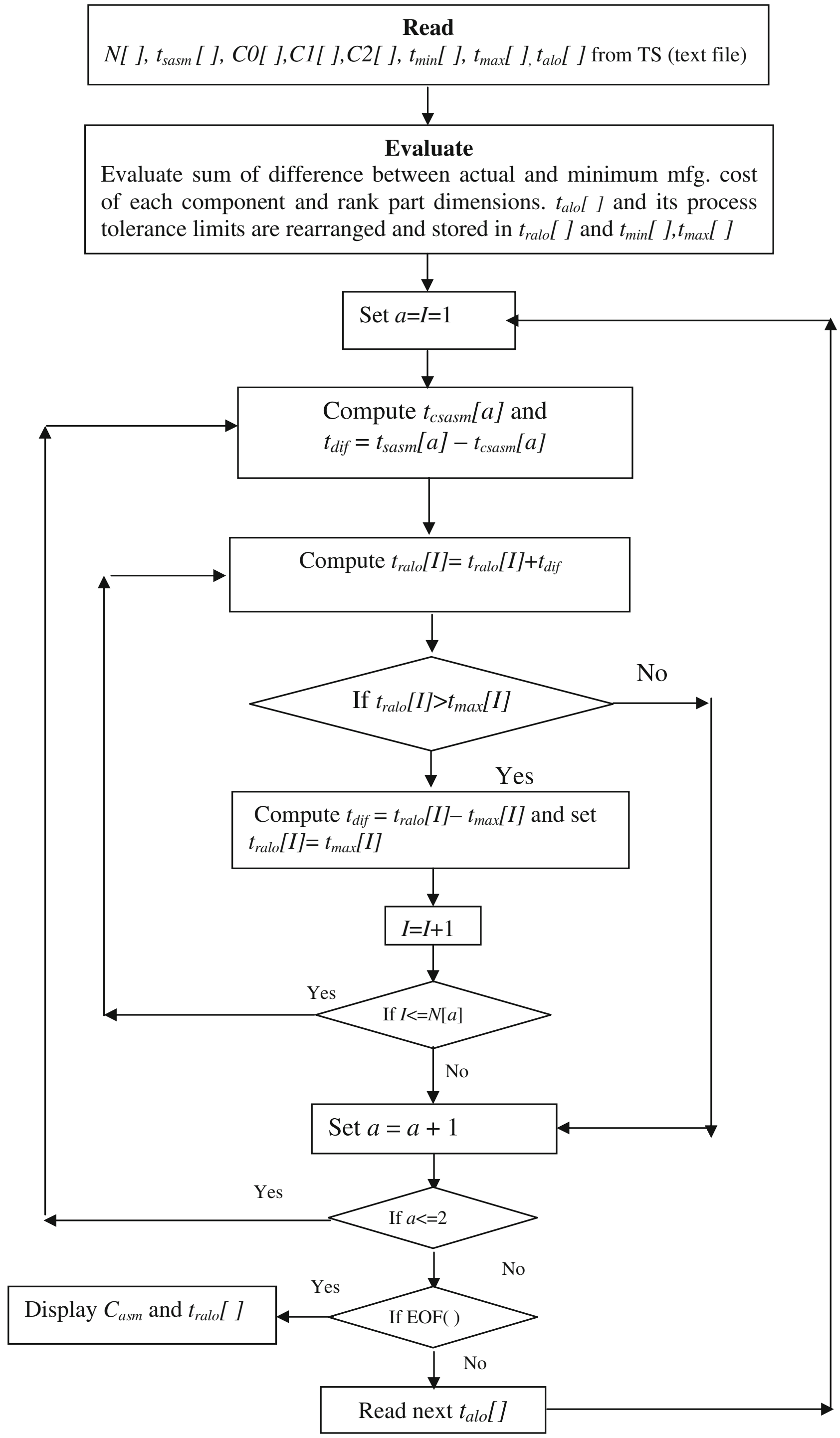


Fig. 26 Scheme of heuristic approach



Appendix B

Table 16 Tabu search example

Particulars	Bit	X1	X2	X3	X4	X5	tx1	tx2	tx3	tx4	tx5	ty1	ty2	Manufacturing cost
Initial Solution		10001011	10011100	11111100	10111100	10010110	0.046337	0.051271	0.079129	0.044761	0.062941	0.096031	0.239678	169.13
1	1	00001011	00011100	01111100	00111100	00010110	0.009192	0.014125	0.041984	0.015647	0.017765	0.029773	0.083067	584.60
Bit 7 in Tabu list and it is not	2	11001011	11011100	10111100	11111110	11010110	0.064910	0.069843	0.060557	0.059773	0.085529	0.129616	0.280839	145.95
allowed to produce neighbor	3	10101011	10111100	11011100	10011100	10110110	0.055624	0.060557	0.069843	0.037482	0.074235	0.098039	0.260259	160.94
until 8th iteration	4	10011011	10001100	11101100	10101100	10000110	0.050980	0.046627	0.074486	0.041122	0.057294	0.087749	0.229388	173.81
	5	10000011	10010100	11110100	10110100	10011110	0.044016	0.048949	0.076808	0.042941	0.065765	0.091890	0.235537	174.65
	6	10001011	10011000	11111000	10111000	10010010	0.046337	0.050110	0.077969	0.043851	0.061529	0.093961	0.235945	171.15
Bit 7 Tabu	7	10001001	10011110	11111110	10111110	10010100	0.045760	0.051850	0.079710	0.045220	0.062240	0.097070	0.239550	169.20
2	8	10001000	10011111	11111111	10111111	10010101	0.045467	0.052141	0.080000	0.045443	0.062588	0.097584	0.240196	169.01
Bit 6 in Tabu list and it is not	1	00001001	00011110	01111110	00111110	00010100	0.008612	0.014706	0.042565	0.016102	0.017059	0.030808	0.082941	585.41
allowed to produce neighbor	2	11001001	11011110	10111110	11111110	11010100	0.064329	0.070424	0.061137	0.059773	0.084824	0.130196	0.280714	145.77
until 9th iteration	3	10101001	10111110	11011110	10011110	10110100	0.055043	0.061137	0.070424	0.037937	0.073529	0.099075	0.260133	160.51
	4	10011001	10001110	11101110	10101110	10000100	0.050400	0.047208	0.075067	0.041576	0.056588	0.088784	0.229263	173.50
	5	10000001	10010110	11110110	10110110	10011100	0.043435	0.049529	0.077388	0.043396	0.065059	0.092925	0.235412	174.62
Bit 6 Tabu	6	10001101	10011010	11111010	10111010	10010000	0.046920	0.050690	0.078550	0.044310	0.060820	0.095000	0.236980	169.90
3	(Tabu) 7													
Bit 8 in Tabu list and it is not	8	10001100	10011011	11111011	10111011	10010001	0.046630	0.050980	0.078840	0.044530	0.061180	0.095510	0.237620	169.70
allowed to produce neighbor	1	00001101	00011010	01111010	00111010	00010000	0.009773	0.013545	0.041404	0.015192	0.015647	0.028737	0.080369	598.96
until 10th iteration	2	11001101	11011010	10111010	11111010	11010000	0.065490	0.069263	0.059976	0.058863	0.083412	0.128125	0.278141	146.42
	3	10101101	10111010	11011010	10011010	10110000	0.056204	0.059976	0.069263	0.037027	0.072118	0.097004	0.257561	161.75
	4	10011101	10001010	11101010	10101010	10000000	0.051561	0.046047	0.073906	0.040667	0.055176	0.086714	0.226690	175.23
Bit 8 Tabu	5	10000101	10010010	11110010	10110010	10011000	0.044596	0.048369	0.076227	0.042486	0.063647	0.090855	0.232839	175.34
4	(Tabu) 6													
Bit 4 in Tabu list and it is not	(Tabu) 7													
allowed to produce neighbor	(Tabu) 8													
until 11th iteration	1	00001100	00011011	01111011	00111011	00010001	0.009482	0.013835	0.041694	0.015420	0.016000	0.029255	0.081012	595.32
	2	11001100	11011011	10111011	11111011	11010001	0.065200	0.069553	0.060267	0.059090	0.083765	0.128643	0.278784	146.25
	3	10101100	10111011	11011011	10011011	10110001	0.055914	0.060267	0.069553	0.037255	0.072471	0.097522	0.258204	161.42
	4	10011100	10001011	11101011	10101011	10000001	0.05127	0.04634	0.07420	0.040890	0.05553	0.08723	0.22733	174.80
Bit 4 Tabu														
5	5	10010100	10000011	11100011	10100011	10001001	0.04895	0.04402	0.07187	0.039070	0.05835	0.08309	0.22319	180.50

Bit 5 in Tabu list and it is not	(Tabu) 6													
allowed to produce neighbor	(Tabu) 7													
until 12th iteration	(Tabu) 8													
	1	00011100	00001011	01101011	00101011	00000001	0.014125	0.009192	0.037051	0.011780	0.010353	0.020973	0.070722	675.17
	2	11011100	11001011	10101011	11101011	11000001	0.069843	0.064910	0.055624	0.055451	0.078118	0.120361	0.268494	149.86
Bit 5 Tabu	3	10111100	10101011	11001011	10001011	10100001	0.060557	0.055624	0.064910	0.033616	0.066824	0.089239	0.247914	168.33
6	(Tabu) 4													
Bit 1 in Tabu list and it is not	(Tabu) 5													
allowed to produce neighbor	(Tabu) 6													
until 13th iteration	(Tabu) 7													
	(Tabu) 8													
	1	00010100	0000011	01100011	00100011	00001001	0.011800	0.006870	0.034730	0.009960	0.013180	0.016830	0.066580	720.90
Bit 1 Tabu	2	11010100	11000011	10100011	11100011	11001001	0.067522	0.062588	0.053302	0.053631	0.080941	0.116220	0.264353	153.21
7														
Bit 3 in Tabu list and it is not	3	00110100	00100011	01000011	00000011	00011001	0.021090	0.016160	0.025440	0.002680	0.018820	0.018840	0.081510	717.70
allowed to produce neighbor	(Tabu) 4													
until 14th iteration	(Tabu) 5													
	(Tabu) 6													
	(Tabu) 7													
	(Tabu) 8													
Bit 3 Tabu	(Tabu) 1													
8	2	01110100	01100011	00000011	01000011	01011001	0.039660	0.034730	0.006870	0.017240	0.041410	0.051970	0.122670	457.70
Bit 2 in Tabu list and it is not	(Tabu) 3													
allowed to produce neighbor	(Tabu) 4													
until 15th iteration	(Tabu) 5													
	(Tabu) 6													
	7	00110110	00100001	01000001	00000001	000011011	0.021671	0.015576	0.024863	0.002227	0.019529	0.017804	0.081639	729.49
Bit 2 Tabu	(Tabu) 8													

References

1. Ostwald PF, Huang J (1977) A method for optimal tolerance selection. T ASME J Eng Ind 99:558–565
2. Glover F (1977) Heuristics for integer programming using surrogate constraints. Decis Schemes 8(1):156–166
3. Glover F (1986) Future paths for integer programming and links to artificial intelligence. Comput Oper Res 13(5):533–549
4. Lee WJ, Woo TC (1989) Optimum selection of discrete tolerances. T ASME J Mech Transmissions Autom Des 111:243–251
5. Chase KW, Greenwood WH, Loosli BG, Hauglund LF (1990) Least cost tolerance allocation for mechanical assemblies with automated process selection. Manuf Rev 3(1):49–59
6. Zhang C, Wang HP (1993) The discrete tolerance optimization problem. Manuf Rev 6(1):60–71
7. Hao J-K, Dorne R, Galinier P (1998) Tabu search for frequency assignment in mobile radio networks. J Heuristics 4(1):47–62
8. Wu CC, Tang GK (1998) Tolerance designs for products with asymmetric quality losses. Int J Prod Res 36(9):2529–2541
9. Chase KW (1999) Minimum cost tolerance allocation. ADCATS Report No. 99-5, Brigham Young University, Provo, Utah
10. Diplaris SC, Sfantsikopoulos MM (2001) Cost – tolerance function: a new approach for cost optimum machining accuracy. Int J Adv Manuf Technol 16(1):32–38
11. Carfagni M, Governi L, Fhiesi F (2001) Development of a method for automatic tolerance allocation. Proc. of the XII ADM International Conference, Italy, pp. D1-20–D1-27
12. Ye B, Salustri FA (2003) Simultaneous tolerance synthesis for manufacturing and quality. Research in Engineering Design, University of Windor
13. Singh PK, Jain SC, Jain PK (2004) A GA-based solution to optimum tolerance synthesis of mechanical assemblies with alternate manufacturing processes: focus on complex tolerancing problems. Int J Prod Res 42(24):5185–5215
14. Armentano VA, Claudio JE (2004) An application of a multi-objective Tabu search algorithm to a bicriteria flowshop problem. J Heuristics 10(5):463–481
15. Nowicki E, Smuntnicki C (2005) An advanced Tabu search algorithm for the job shop problem. J Sched 8(2):145–159
16. Zhang CY, Li PG, Guan ZL, Rao YQ (2007) A Tabu search algorithm with a new neighborhood structure for the job shop scheduling problem. Comput Operat Res 34(11):3229–3242