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ORIGINAL ARTICLE

Confidence interval for optimal preventive maintenance interval and its applications in maintenance planning

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Abstract Replacement problems of deteriorating systems have been extensively studied. Typically, the time between failures is characterized by lifetime distribution in which the parameters are estimated from historical data. On the other hand, in most cases, the work focuses on determining the optimal replacement schedule by assuming that model parameters are constant. Here, the issues arising from the use of estimated parameters are studied and the results are applied to opportunistic replacement. Also, a graphical approach is proposed to obtain the confidence limits for the optimal replacement time, considering the key parameters of the two popular replacement models, namely, the age replacement model and the block replacement model. The applications of the proposed confidence interval are presented, namely, determination of the window of opportunity for minimum cost, maintenance scheduling given erratic customer demand, and opportunistic maintenance for multi-component systems.

Keywords Preventive maintenance planning · Age replacement policy · Block replacement policy · Optimal replacement interval · Confidence limits · Opportunistic replacement

Notations

$R(t)$	Reliability of the system over time t
$F(t)$	Unreliability of the system over time t, $F(t) = 1 - R(t)$
$f(t)$	Failure density of the system, $f(t) = - R'(t)$
$h(t)$	Hazard rate of the system, $h(t) = f(t) / R(t)$
$H(T)$	Cumulative hazard rate of the system, $H(T) = \int_0^T h(t)dt$
n	Number of failure in one preventive replacement cycle
$F^{(n)}(T)$	n-fold Stieltjes convolution of $F(T)$ with finite mean
$M(T)$	Renewal function of block replacement policy: expected number of failures on [0, T], $M(T) = \sum_{n=1}^{\infty} F^{(n)}(T) \approx F(T)$
$m(T)$	Renewal density of block replacement policy, $m(T) \equiv dM(T) / dT \approx f(T)$
μ	MTTF of the system, $\mu = \int_0^{\infty} R(t)dt$
C_p	Cost of system preventive replacement
C_f	Cost of system failure
C_r	Cost ratio of system preventive replacement to cost of system failure
$C(T)$	Total cost per unit time when T is the replacement time
η	Weibull distribution scale parameter
β	Weibull distribution shape parameter
$\beta*$	Shape parameter that corresponds to minimum $T*$ for a given C_r
T_η^*	Cost-optimal time between preventive replacements
$T*$	Normalized cost-optimal time between preventive replacements, $T* = T_\eta^* \big/ \eta$

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1 Introduction

In today's manufacturing companies, there is a growing dependence upon smoothing operations of systems consist-

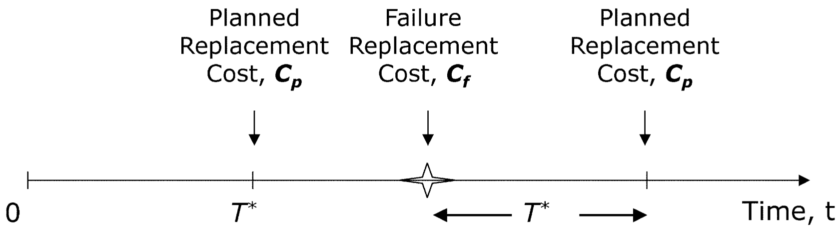

Fig. 1 Age replacement schematic

ing of software, people, and equipment. Failure of some of these operations often leads to grave economic losses. Recognizing the importance of ensuring failure-free and economically viable operations, comprehensive maintenance programs have been put in place for many systems in which preventive maintenance (PM) is a key component. However, maintenance is often a business process that has not been optimized, and thus considered a liability of business operations (Jardine and Tsang [1]). As highlighted by Tu et al. [2], effective control of maintenance cost can be used as a competitive edge over other competitors.

Over the years, many PM models have been developed, based on economics, reliability and operational considerations (Tam et al. [3]; Lai et al. [4]). Optimal maintenance schedules have been derived using techniques ranging from fundamental renewal process (Barlow and Proschan [5]; Jardine [6]) to the more recent genetic algorithms (Shum and Gong [7], Ilgin and Tunali [8]; Robert and Shahabudeen [9]; Marsequerra and Podofillini [10]). In these models, one of the key decision variables is the time interval after which PM should be performed. For example, the recent survey by Wang [11] reveals that the central theme in most research publications is to determine the optimal PM interval under various maintenance policies.

Common parameters in these PM models are the cost of preventive replacement, C_p, and the cost of failure, C_f. C_p is usually easy to quantify as it typically consists of the cost of new replacement component and installation cost. But, C_f cannot be easily determined as it is related to the cost due to loss in production, downstream delay, etc. As a result, it is often an estimated value from repair cost data. Another source of inaccuracy comes from the inter-failure time model and its parameters. In most existing literature, it is assumed that these parameters are known; however, in practice, they must be estimated from failure data. As a result, the optimal PM interval computed is exposed to sampling risk as the repair cost and failure data used for estimation are only incidents experienced thus far. In many practical situations, these data are highly censored due to issues related to data collection and unobserved failures. Fortunately, this sampling risk can suitably be addressed through the use of statistical confidence interval in which the uncertainty of the cost and parameter estimates are conveyed through ranges of possible values, also known as interval estimates, for the unknown cost and parameters. In essence, sampling variations in cost and parameter estimates are transmitted to that of the optimal PM interval and the latter can be computed from those of the former. In this paper, we present a graphical approach to obtaining the confidence interval for the optimal PM interval.

Publications dealing with the effect of sampling variability on optimal replacement interval are scarce. Leger and Cleroux [12] used the bootstrap methodology of Efron [13] to construct a confidence interval for the actual cost of using a given nonparametric estimate of the optimal age replacement strategy. Gaver et al. [14] addressed the issue of Weibull parameters variability by conducting sensitivity studies on the long-run average costs of the system. So far, there *appears to be* no work that derives the confidence interval for the optimal PM interval, considering the variability involved in estimating the requisite input parameters.

In this paper, we derive the confidence interval of the optimal PM interval under two common replacement

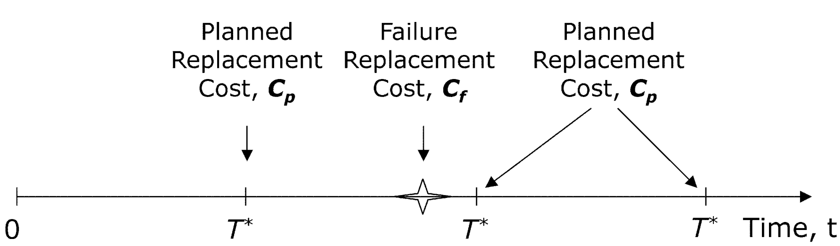

Fig. 2 Block replacement schematic

Table 1 Replacement model and key expressions

	Age replacement	Block replacement
Replacement cost per unit time, $C(T)$ where $C_r = C_f / C_p$	$\frac{[1+(C_r-1)(F(T))]}{\int_0^T [1-F(t)]dt}$	$\frac{1+C_r M(T)}{T}$
Functional optimal replacement interval, T^*; $\frac{d(C(T))}{dT}=0$	$h(T^*)\int_0^{T^*}[1-F(t)]dt - F(T^*) = \frac{1}{C_r-1}$	$T^* f(T^*) - F(T^*) = \frac{1}{C_r}$
Functional T^* for the case of a two-parameter Weibull function with η=1	$\beta(T^*)^{(\beta-1)}\int_0^{T^*}\exp\left\{-(t)^\beta\right\}dt + \exp\left\{-(T^*)^\beta\right\} = \frac{C_r}{C_r-1}$	$\left[1+\beta(T^*)^\beta\right]\exp\left\{-(T^*)^\beta\right\} = 1+\frac{1}{C_r}$

policies, namely age replacement policy and block replacement policy (Wang [11]; Yoo et al. [15]). Based on the concept of equivalence set (Lehmann [16]), the confidence interval for the optimal PM interval is constructed from those of the parameters involved in determining the point estimate of the PM interval. As confidence interval is dependent on the sample size and the sampling distribution of the statistics used in estimating the parameters, it only captures variability due to estimation, and does not address possible error in model selection. For illustration purposes, it is assumed that the inter-failure time follows a two-parameter Weibull distribution in which the parameters are estimated.

There are many applications related to the use of confidence interval for the optimal PM interval. For example, from the confidence intervals, a simple opportunistic PM strategy for multiple components systems can easily be determined.

This paper is organized as follows. In the following section, popular replacement models whose failure density function follows that of a Weibull distribution with η=1, are reviewed. For each type of model, the relationship between C_r, T^* and β is derived mathematically and then depicted graphically. Exploiting the unique graphical property, the confidence limits for T^* and T^*_η are derived. Finally, two practical scenarios are presented in which the results are useful.

2 Preliminary results

In this section, the common replacement models—age replacement and block replacement—are reviewed. The models are optimized based on minimization of cost per unit time that is determined using the renewal-reward theorem (Tijms [17]). This is followed by illustrating the case of Weibull failure distribution, and reviewing the relationship between its parameters and replacement model parameters.

In age replacement policy, as proposed by Barlow and Proschan [5], a system is replaced upon failure or after T^* units of operation, whichever comes first. This is illustrated in Fig. 1, which shows the cost that will be incurred during failure and replacement.

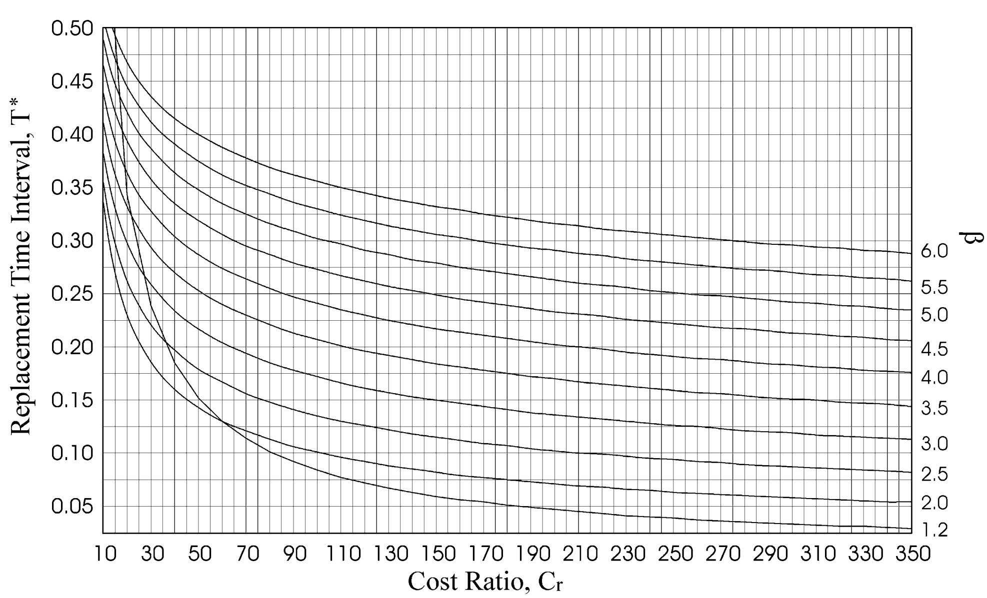

Fig. 3 Age replacement: Behaviour of C_r, T^* and β using C_r as x-axis

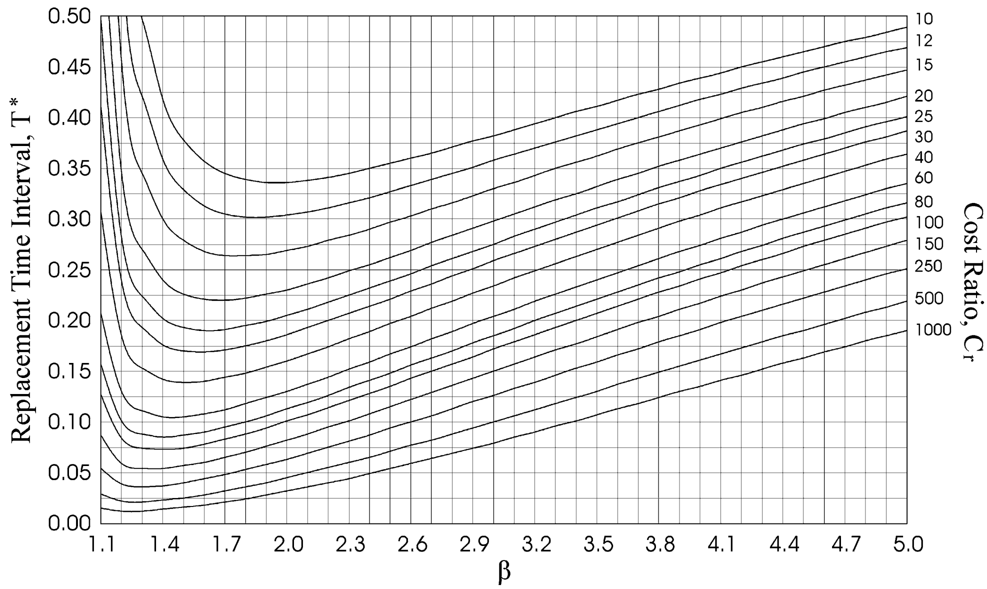

Fig. 4 Age replacement: Behaviour of C_r, T^* and β using β as x-axis

Figure 2 illustrates the cost concept of block replacement. For every T^* units of operation, the system is replaced by a new one from the same population. At each failure, a new one also replaces the system. However, the count down to the next preventive replacement is not reset at each unscheduled failure replacement. It is assumed that the probability of having more than one failure in one replacement cycle is negligible, thus the renewal function, $M(T)$, can be approximated by $F(T)$ (Gertsbakh [18]).

Table 1 summarizes the cost expression for each model and readers can refer to Gertsbakh [18] for a detailed derivation. For illustration purpose, the inter-failure times are assumed to follow a two-parameter Weibull distribution with increasing failure rate. Without loss of generality and simplicity, Weibull scale parameter is set to $\eta=1$, so that the resulting optimal replacement interval is now normalized by the actual scale parameter, η. The relationship between cost of failure replacement, C_r, the optimal replacement interval T^* and the distribution parameter β is shown in Table 1.

3 Methodology

Optimal replacement time is a function of C_r and β as shown in Table 1. Theoretically, by the principle of equivariant confidence sets (Lehmann [16]), the confidence

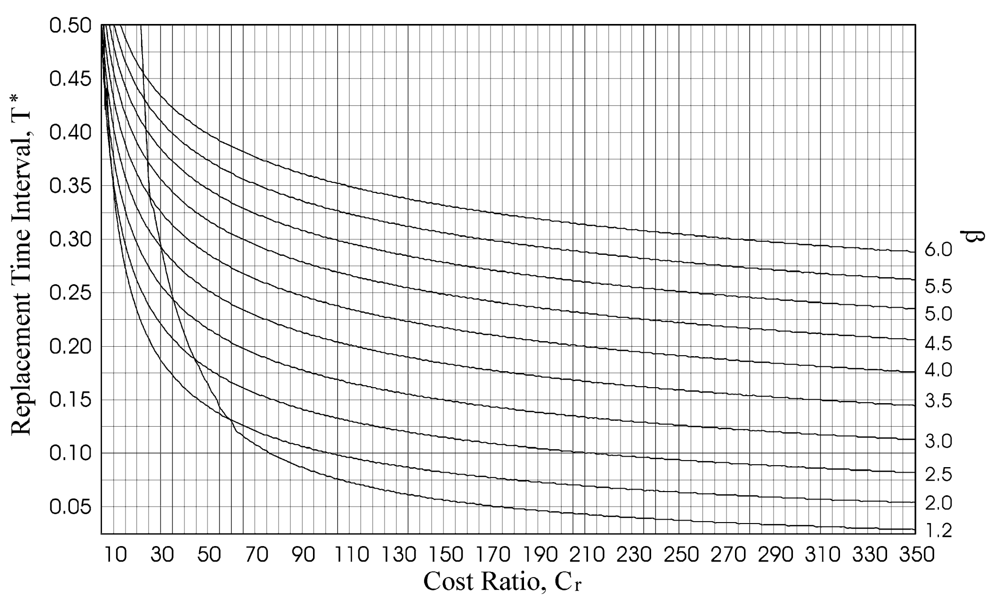

Fig. 5 Block replacement: Behaviour of C_r, T^* and β using C_r as x-axis

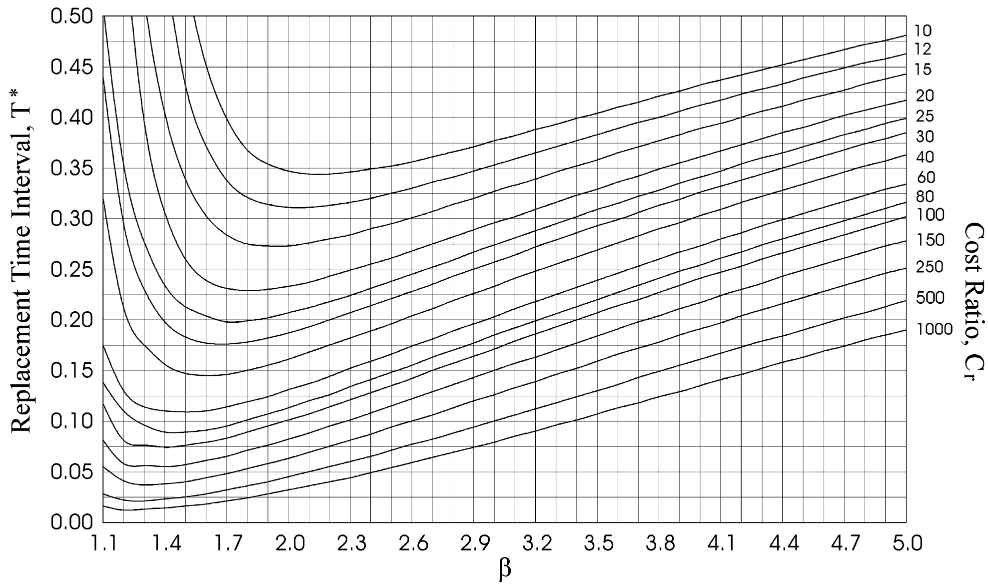

Fig. 6 Block replacement: Behaviour of C_r, T^* and β using β as x-axis

limits for T^* can be constructed from those of C_r and β. Closed form solution for the confidence limits can be expressed easily if T^* can be explicitly evaluated in closed form. However, this is not the case for both the replacement models. Therefore, the alternative approach is to numerically integrate and graph the functional T^* of the replacement models. The confidence limits for T^* are then obtained by projecting the confidence bounds for C_r and β on the graph. In this section, their graphical relationship will be shown and later exploited to derive the confidence limits for T^*.

3.1 Graphing C_r, T^* and β

The functional T^* contour plots of the age replacement model on two different axes (Figs. 3 and 4) reveal some interesting and useful properties. It can be seen that for each (β, C_r) there exists a unique value of T^*. Similarly, (β, T^*) uniquely determines C_r. Figure 3 reveals that for each constant β, T^* is monotonic decreasing with respect to C_r. This unique relationship between C_r, T^* and β facilitates the derivation of the confidence limits for T^* from the confidence limits for C_r. Figure 3 shows non-monotonic

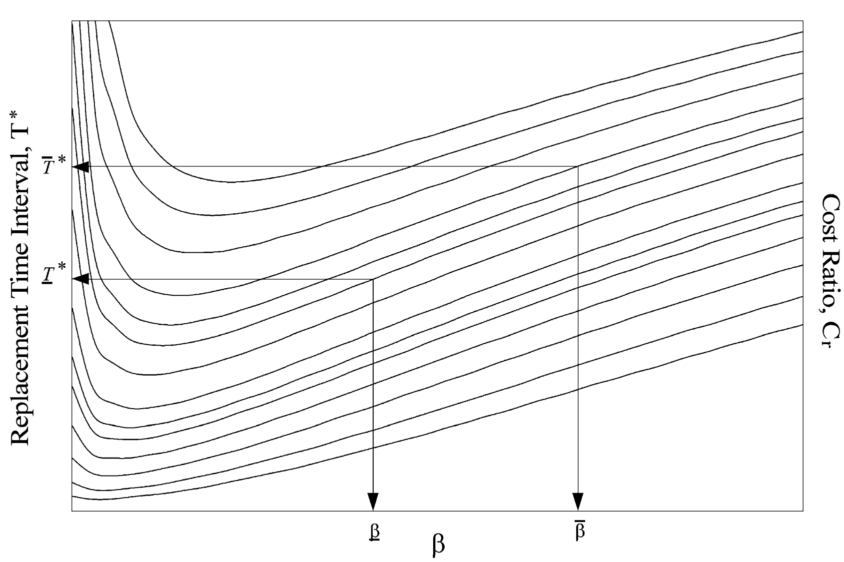

Fig. 7 Graphical representation of Eqs. (8) and (9)

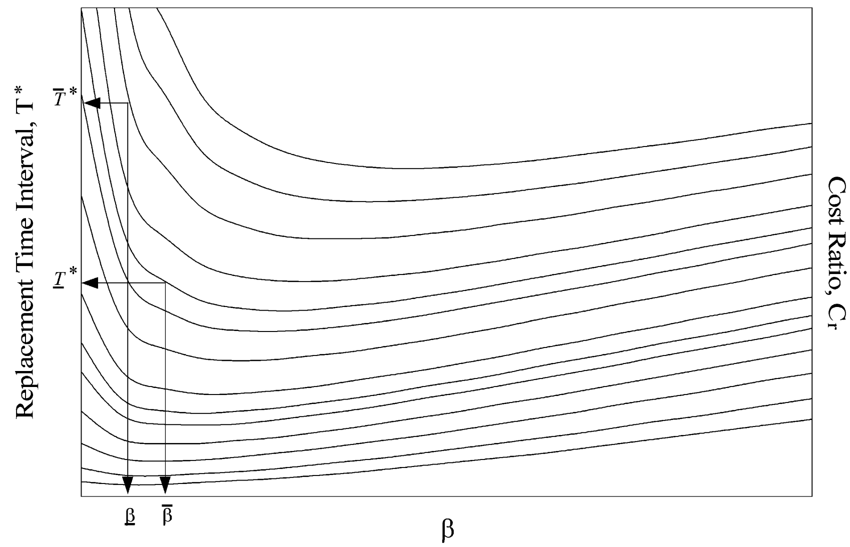

Fig. 8 Graphical representation of Eqs. (10) and (11)

relationship between T^* and β. Also, C_r is not monotonic in β. This is illustrated by the curve belonging to β=1.2, where it intersects through other family of curves.

Figure 4 confirms that for each constant C_r, T^* is not monotonic in β. Similarly, for each constant T^*, C_r is not monotonic in β. Nevertheless, for any interval of β, the minimum T^* always occurs at either of the extremes.

Similarly, the functional T^* contour plots of the block replacement model are shown in Figs. 5 and 6. It is observed that the relationship between C_r, T^* and β exhibits the same graphical characteristic for both the replacement models. This unique characteristic will be utilized in determining the confidence limits of T^*.

Table 2 Failure time data (in ascending order)

Relay failure times (in thousand switch cycle)	
3647.9	13692.9
4708.4	14453.1
5123.1	14462.4
6556.3	15472.1
7206.7	16906.7
7790.5	18594.4
8550.4	18661.8
8804.5	19068.5
8879.9	19764.7
10466.4	19777.3
10479.4	20007.9
10611.5	22817.2
11627	23174.2
12046.3	23274.8
12418.7	24651.9

3.2 Confidence limits for C_r, β and η

In practice, it is difficult to estimate the value of C_f. However, when historical data of C_f for a sufficiently large sample is available and C_p is a constant, one can approximate the 100(1-α) % confidence interval for C_r thus:

$$\left[\underline{C}_r, \overline{C}_r\right] \equiv \left[\widehat{C}_r - Z_{\alpha/2}\frac{s}{\sqrt{n}}, \widehat{C}_r + Z_{\alpha/2}\frac{s}{\sqrt{n}}\right] \tag{1}$$

where, $Z_{\alpha/2}$ is the (1-α/2) fractile of the standard normal variate, s the sample standard deviation and $\widehat{C}_r$ the average cost ratio.

For large, uncensored data, the upper and lower confidence limits ($\overline{\beta}$ and $\underline{\beta}$) of β can be obtained as follows, using the following equation of Abernethy et al. [19] for the approximate calculation of 100(1-α) % confidence interval:

$$\left[\underline{\beta}, \overline{\beta}\right] \equiv \left[\widehat{\beta}\exp\left(-\frac{0.78Z_{\alpha/2}}{\sqrt{n}}\right), \widehat{\beta}\exp\left(\frac{0.78Z_{\alpha/2}}{\sqrt{n}}\right)\right] \tag{2}$$

where, $Z_{\alpha/2}$ is the (1-α/2) fractile of the standard normal variate and $\widehat{\beta}$ the estimated Weibull shape parameter, which can be derived from probability plotting.

Table 3 Key parameters of example 1 and their confidence limits

β			η (Thousand of switch cycle)		
Estimate	Lower C.L	Upper C.L	Estimate	Lower C.L	Upper C.L
2.5	1.98	3.16	15585.5	13738.63	17680.64

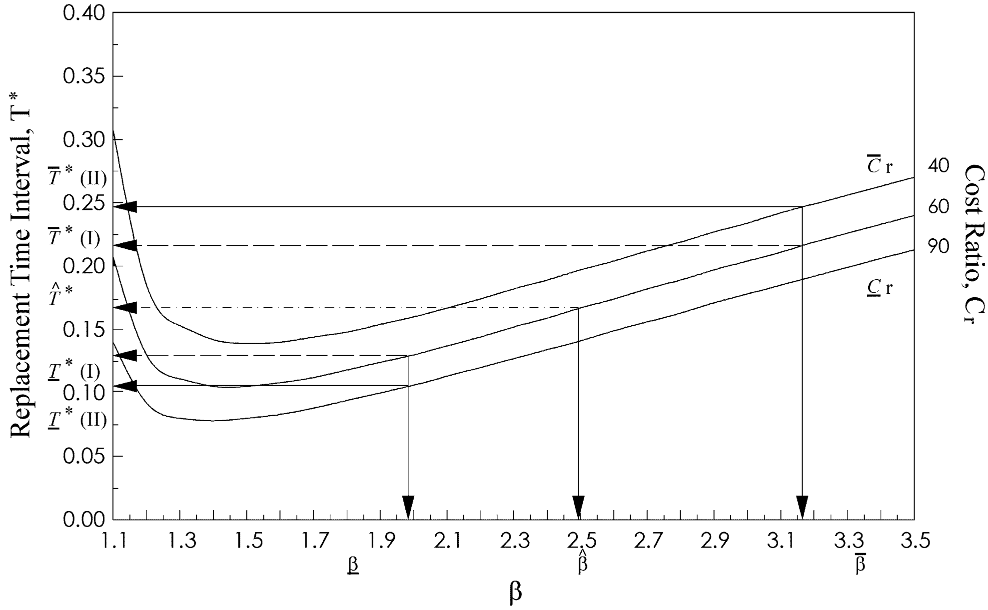

Fig. 9 Graphical solution to example 1

Applicable, as is Eq. (2), to large uncensored data, Abernethy et al. [19] also gives the 100(1-α) % confidence interval for the characteristic life η as shown in Eq. (3).

$$\left[\underline{\eta}, \overline{\eta}\right] \equiv \left[\widehat{\eta} \exp\left(-\frac{1.05 Z_{\alpha/2}}{\widehat{\beta}\sqrt{n}}\right), \widehat{\eta} \exp\left(\frac{1.05 Z_{\alpha/2}}{\widehat{\beta}\sqrt{n}}\right)\right] \quad (3)$$

where, $Z_{\alpha/2}$ is the (1-α/2) fractile of the standard normal variate and the estimated Weibull scale parameter, which can be derived from probability plotting.

3.3 Confidence limits for T* and T^*_η

Treating T^* as the statistic of interest, one can write thus:

$$L_1(T^* : \beta, C_r) = \beta (T^*)^{(\beta-1)} \int_0^{T^*} \exp\left\{-(t)^\beta\right\} dt + \exp\left\{-(T^*)^\beta\right\} - \frac{C_r}{C_r - 1} \quad (4)$$

$$L_2(T^* : \beta, C_r) = \left[1 + \beta (T^*)^\beta\right] \exp\left\{-(T^*)^\beta\right\} - 1 - \frac{1}{C_r} \quad (5)$$

where, Eqs. (4) and (5) apply to age and block replacement model, respectively.

Previously, it has been shown that all the three replacement models share the same relational traits between C_r, T^* *and* β. Hence, it is now possible to generalize the approach for determining the confidence limits for T^*.

By utilizing the unique and monotonic relationship between T^* and C_r for each β, one can determine the upper and lower confidence limits for T^*; $\left(\overline{T}^*, \underline{T}^*\right)$ can be obtained using $\underline{C}_r$ and $\overline{C}_r$, respectively. However, the relationship between T^* and β, given C_r, follows a concave function. Therefore, combining the relationship between C_r, T^* and β, and by principle of equivariant, the 100(1-α) % confidence limits of T^* can be expressed mathematically thus:

$$\underline{T}^* = \arg\min\left\{T^* : L_i\left(T^* : \beta, \overline{C}_r\right) = 0\right\}, \ \forall i = 1 \ \& \ 2 \quad (6)$$

$$\overline{T}^* = \arg\max\left\{T^* : L_i(T^* : \beta, \underline{C}_r) = 0\right\}, \ \forall i = 1 \ \& \ 2 \quad (7)$$

For each curve, in Figs. 4 and 6, that corresponds to a constant C_r, there exists a β^* that gives the minimum T^*. In particular, when $\underline{\beta} \geq \beta^*$, the unique solutions of Eqs. (6)

Table 4 90% Confidence limits for T^* under different scenarios of Example 1

Case	η=1			$\eta \equiv \widehat{\eta}$			$\eta \equiv [\underline{\eta}, \overline{\eta}]$		
	$\widehat{T}^*$	$\underline{T}^*$	$\overline{T}^*$	$\widehat{T}^*_\eta$	$\underline{T}^*_\eta$	$\overline{T}^*_\eta$	$\widehat{T}^*_\eta$	$\underline{T}^*_\eta$	$\overline{T}^*_\eta$
(I) Bounded β with constant C_r	0.167	0.129	0.216	2595.517	2008.883	3361.98	2595.517	1770.832	3813.928
(II) Bounded β and C_r	0.167	0.105	0.246	2595.517	1631.075	3833.176	2595.517	1349.345	4348.465

Table 5 Components' failure time data (in ascending order)

Component A failure times (days)		Component B failure times (days)		Component C failure times (days)	
35.576	127.413	29.401	180.389	30.078	93.104
42.817	132.012	68.014	182.389	40.752	94.273
48.201	132.889	79.946	205.1	44.355	94.364
55.373	133.512	85.768	227.249	46.395	95.238
69.158	136.137	106.538	234.491	48.003	98.025
75.925	158.988	116.571	236.361	48.806	98.617
80.837	160.469	117.631	239.619	61.883	102.971
82.286	170.664	120.451	251.017	62.312	103.369
83.119	177.785	127.281	270.106	63.827	104.15
90.297	178.758	143.668	277.514	64.002	107.215
94.177	182.181	147.417	303.833	69.294	108.782
95.377	186.489	149.77	314.077	73.579	117.42
105.673	191.408	154.627	345.372	73.898	118.536
107.7	199.818	156.196	377.557	73.974	123.454
114.738	214.809			77.358	124.801
121.307	218.889			83.14	134.148
122.081	253.689			87.545	135.996
122.41	259.501			88.08	155.166
				89.715	158.21
				90.257	175.463

and (7) are shown graphically in Fig. 7, and mathematically thus:

$$\underline{T}^* = \left\{T^* : L_i\left(T^* : \underline{\beta}, \overline{C}_r\right) = 0\right\},\ \forall i = 1\ \&\ 2 \tag{8}$$

$$\overline{T}^* = \left\{T^* : L_i\left(T^* : \overline{\beta}, \underline{C}_r\right) = 0\right\},\ \forall i = 1\ \&\ 2 \tag{9}$$

Similarly, when $\overline{\beta} \leq \beta^*$, the unique solutions of Eqs. (6) and (7) are shown graphically in Fig. 8 and mathematically thus:

$$\underline{T}^* = \left\{T^* : L_i\left(T^* : \overline{\beta}, \overline{C}_r\right) = 0\right\},\ \forall i = 1\ \&\ 2 \tag{10}$$

$$\overline{T}^* = \left\{T^* : L_i\left(T^* : \underline{\beta}, \underline{C}_r\right) = 0\right\},\ \forall i = 1\ \&\ 2 \tag{11}$$

It is assumed that the characteristic life η=1. As η has a linear scaling effect on T^*, the confidence limits for T^*_η are as shown in Eq. (12), when variability of η is negligible.

$$\left[\underline{T}^*_\eta, \overline{T}^*_\eta\right] \equiv \left[\hat{\eta}\underline{T}^*, \hat{\eta}\overline{T}^*\right] \tag{12}$$

Equation (13) gives the confidence limits for T^*_η, taking the confidence interval of η into consideration.

$$\left[\underline{T}^*_\eta, \overline{T}^*_\eta\right] \equiv \left[\underline{\eta}\underline{T}^*, \overline{\eta}\overline{T}^*\right] \tag{13}$$

4 Illustrative examples

The following examples serve to illustrate the usage of confidence limits for T^* through two common applications.

Example 1 Relays are used in many data acquisition equipments. As relays are electromechanical devices, they

Table 6 Key parameters of example 2 and their confidence limits

Component	β			η (days)		
	Estimate	Lower C.L	Upper C.L	Estimate	Lower C.L	Upper C.L
A	2.50	2.02	3.10	149.40	133.15	167.63
B	2.30	1.80	2.93	211.80	183.78	244.09
C	3.00	2.45	3.67	102.50	93.58	112.27

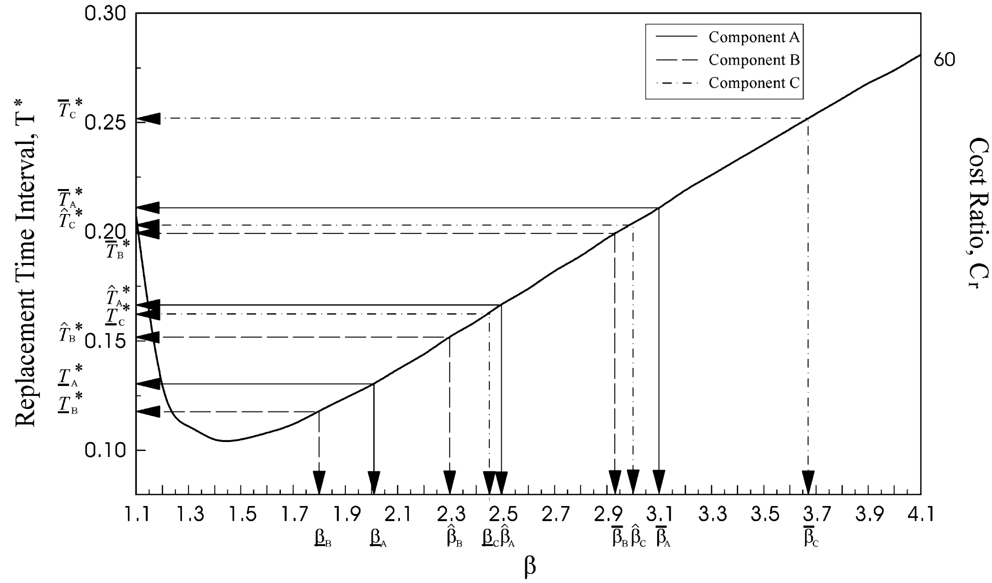

Fig. 10 Deriving confidence limits for T^* of each component

eventually wear out. A relay operates on a continuous basis (7 days a week, 24 hours a day). The switching frequency is a better measure of wear and tear than calendar time. The excellent equipment reliability information system makes available historical data for the past 3 years. The sorted failure data are shown in Table 2. From the probability plot of historical failure data, it is observed that the two-parameter Weibull life distribution best describes it. The estimated value and 90% confidence interval of the Weibull parameters are given in Table 3.

Assuming that the age replacement model is used as the preventive replacement policy and the estimated value of C_r is 60, the normalized optimal replacement interval, T^*, is 0.1665. Including the effect of $\widehat{\eta} = 15585.5k$ switch cycle, the optimal replacement interval, T^*_η, is 2595.5k switch cycle. However, this may not be the optimal replacement interval, because the estimated value of C_r and β may not reflect the true value of the component.

The relationship between C_r, T^* and β is shown in Fig. 9. From Fig. 9, using the proposed graphical approach, one can derive the confidence limit for T^*. Table 4 gives the possible 90% confidence limits of T^* and T^*_η for the case when C_r is constant, and when it is bounded within a range of likely values, like [40, 90] as in this case.

Through comparing the initial estimate of $T^*_\eta = 2595.5$k switch cycle, which was obtained without considering the varying effect of C_r and β , with that of Table 4, it can be seen that a large disparity in replacement interval is caused by ignoring the confidence limits for C_r and β.

As the confidence interval for T^*_η provides the PM interval range, executing the PM is more flexible. This is useful especially in the manufacturing scenario when an urgent unplanned customer order arrives at a time that might overlap with the scheduled PM. If the job can be completed within the time frame $\left[\underline{T}^*_\eta, \overline{T}^*_\eta\right]$, PM can be delayed up to $\overline{T}^*_\eta$, thus allowing the job to be completed without interruption.

Example 2 Based on real-life lift failure data, the three most common types of component replacement failure are car light, LED display and indicator/button light. Table 5 shows the sorted historical data of failure time for each component over the past 5 years. It has been shown that the components' inter-failure time adheres to the two-parameter Weibull distribution. For the sake of simplicity, the car light, LED display and indicator/button light are designated as component A, B and C, respectively. The estimated value and 90% confidence interval of the Weibull param-

Table 7 90% Confidence limits for T^* under different scenarios of example 2

Component	$\eta=1$			$\eta \equiv \widehat{\eta}$			$\eta \equiv [\underline{\eta}, \overline{\eta}]$		
	$\widehat{T}^*$	$\underline{T}^*$	$\overline{T}^*$	$\widehat{T}^*_\eta$	$\underline{T}^*_\eta$	$\overline{T}^*_\eta$	$\widehat{T}^*_\eta$	$\underline{T}^*_\eta$	$\overline{T}^*_\eta$
A	0.167	0.132	0.211	24.880	19.667	31.529	24.880	17.528	35.376
B	0.152	0.118	0.199	32.123	24.997	42.113	32.123	21.690	48.535
C	0.204	0.163	0.252	20.905	16.680	25.86	20.905	15.229	28.324

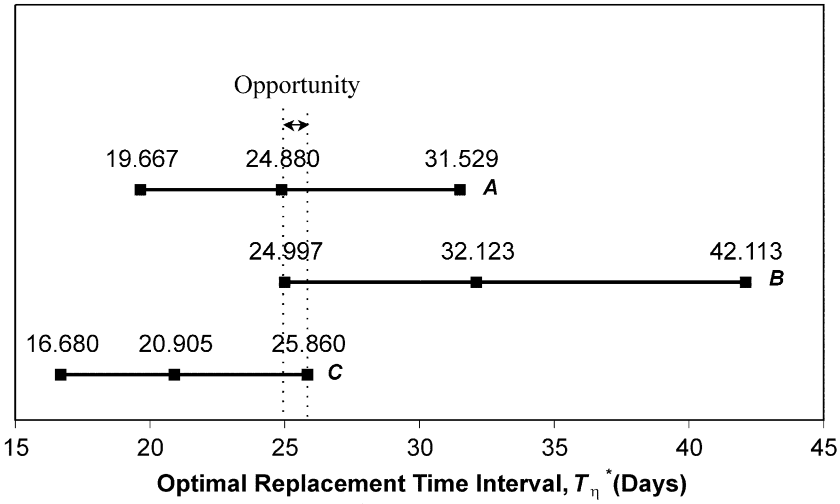

Fig. 11 Confidence limits for T^*_η of each component when $\eta \equiv \widehat{\eta}$

eters for each component are given in Table 6. The estimated value of C_r is 60 and equal.

Assuming that the age replacement model is used, the normalized optimal replacement time interval, T^*, is 0.167, 0.152 and 0.204 for components A, B and C, respectively. Including the effect of their individual $\widehat{\eta}$, the optimal replacement interval (in days) is 24.88, 32.123 and 20.905 for components A, B and C, respectively.

Using the confidence limits for β, one can derive the confidence limits for T^* of each component as illustrated in Fig. 10. Confidence limits for T^* and T^*_η of each component, including the effect of their individual $\widehat{\eta}$, are given in Table 7.

The spread of confidence bounds for T^*_η of each component is shown in Figs. 11 and 12. The confidence bounds for T^*_η in Fig. 11 assumes $\eta \equiv \widehat{\eta}$ while that in Fig. 12 assumes $\eta \equiv \left[\underline{\eta}, \overline{\eta}\right]$.

There are regions where each component's confidence bounds for T^*_η overlap each other. This enables the PM to be carried out concurrently for all the three components, thus resulting in lesser scheduled down time. In this example, the opportunistic replacement time interval is [24.997, 25.860] days when one assumes that $\eta \equiv \overline{\eta}$ or [21.690, 28.324] days when one assumes that $\eta \equiv \left[\underline{\eta}, \overline{\eta}\right]$.

5 Conclusions

A graphical method is proposed for determining the confidence interval for the optimal replacement interval, T^*, under age replacement and block replacement models in which the inter-failure times follow a two-parameter Weibull distribution. If one is armed with the confidence

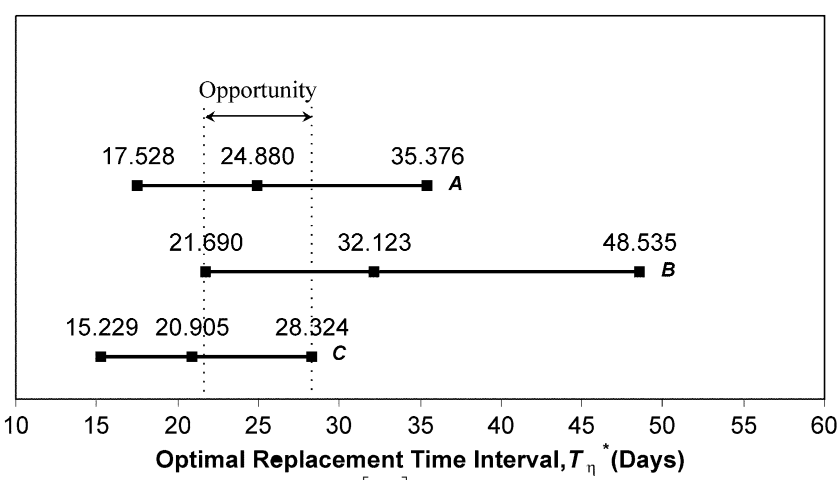

Fig. 12 Confidence limits for T^*_η of each component when $\eta \equiv \left[\underline{\eta}, \overline{\eta}\right]$

limits for T^*, scheduling PM can be more flexible. When faced with multiple components replacement situation, an opportunistic replacement strategy can easily be formulated based on the confidence intervals for T^* of these components. Although the current replacement models seek to optimize the cost, the method proposed here can be easily extended to those aimed at maximizing availability.

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