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ORIGINAL ARTICLE

Single-machine group scheduling problems with deteriorating jobs

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Abstract This paper investigates single-machine scheduling problems with deteriorating jobs and the group technology (GT) assumption. By deteriorating jobs and the group technology assumption, we mean that the group setup times and job processing times are both increasing functions of their starting times, i.e., the group setup times and job processing times are both described by a function which is proportional to a linear function of time. The two objectives of scheduling problems are to minimize the makespan and the total weighted completion time, respectively. We show that these problems remain solvable in polynomial time when deterioration and group technology are considered simultaneously.

Keywords Scheduling · Single-machine · Deteriorating jobs · Group technology

1 Introduction

Traditional scheduling problems usually involve jobs with constant, independent processing times. In practice, however, we often encounter settings in which the job processing times vary with time. Hence, there is a growing interest in the literature to study scheduling problems involving deteriorating jobs, i.e., jobs whose processing times are increasing functions of their starting times. Job deterioration appears, e.g., in scheduling maintenance jobs, steel production, national defense, emergency medicine, or cleaning assignments, where any delay in processing a job is penalized by incurring additional time for accomplishing the job. Extensive surveys of different scheduling models and problems involving jobs with start-time-dependent processing times can be found by Alidaee and Womer [1] and Cheng et al. [2]. More recent papers that have considered scheduling jobs with deterioration effects include Guo and Wang [3], Janiak and Kovalyov [4], Wang et al. [5], Wang and Xia [6–8], Xu et al. [9], Gawiejnowicz et al. [10], Gawiejnowicz [11], Wang [12], Wang et al. [13], Wu et al. [14, 15], Shiau et al. [16], and Wang et al. [17].

Recently, an important class of scheduling problem has been characterized by the group technology (GT) assumption, i.e., the jobs are classified into groups by their similar production requirements. No machine setups are needed between two consecutively scheduled jobs from the same group, although an independent setup is required between jobs of different groups. In group technology, it is conventional to schedule continuously all jobs from the same group. Group technology that groups similar products into families helps increase the efficiency of operations and decrease the requirement of facilities. Hence, the scheduling in a group technology environment results in a new stream of research (Potts and Van Wassenhove [18]). To the best of our knowledge, only a few results concerning scheduling problems with deteriorating jobs and group technology simultaneously are known. Guo and Wang [3], who considered the makespan minimization problem under group technology, assumed that the setup times are constant and that the actual processing time of a job is a proportional linear function of its starting time. Under the same model, Xu et al. [9] proved that the total weighted completion time minimization problem can be solved in polynomial time. Wang et al. [13], who considered the makespan minimization problem and the total completion time minimization

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problem under group technology, assumed that the setup times are constant and that the actual processing time of a job is a general linear decreasing function of its starting time. They showed that these problems can be solved in polynomial time. Since longer setup or preparation might be necessary as food quality deteriorates or a patient's condition worsens, Wu et al. [14] considered a situation where the setup time grows and jobs deteriorate as they wait for processing, i.e., group setup times and job processing times are both described by a simple linear deterioration function. For single-machine group scheduling, they proved that the makespan minimization problem and the total completion time minimization problem can be solved in polynomial time.

In this paper, we continue the work of Wu et al. [14], focusing instead on a proportional linear deterioration under the group technology assumption and starting time-dependent setup times. The remaining part of the paper is organized as follows. In the next section, a precise formulation of the problem is given. The problems of minimizing the makespan and the total weighted completion time are given in Sects. 3 and 4, respectively. The last section outlines our conclusions.

2 Problem formulation

There are n jobs grouped into f groups, and these n jobs are to be processed on a single machine. All jobs are available at time t_0, where $t_0 \geq 0$. Jobs are processed one by one in groups on the machine and a setup time is required if the machine switches from one group to another. We assume that the setup times are time-dependent and that the processing of a job may not be interrupted. Let n_i be the number of jobs belonging to group G_i, thus, $n_1 + n_2 + \ldots + n_f = n$; J_{ij} denotes the jth job in group G_i, i=1, 2,.., f; j=1, 2,.., n_i. Let p_{ij} be the actual processing time of job J_{ij}. The general model is:

$$p_{ij} = \alpha_{ij} + \beta_{ij} t$$

where α_{ij} is the basic processing time of job J_{ij}, β_{ij} is its deterioration rate, and t is its start time. In this paper, we consider a new model where $\beta_{ij}=b\alpha_{ij}$ and $p_{ij}=\alpha_{ij}(1+bt)$. In fact, we consider the following general proportional model:

$$p_{ij} = \alpha_{ij}(a + bt)$$

As in the above proportional model, we also assume that the setup time of group G_i is a proportional model, that is:

$$s_i = \delta_i(a + bt)$$

The objectives are to minimize the makespan and the total weighted completion time, respectively.

For a given schedule π, $C_{ij}(\pi)$ represents the completion time of job J_{ij} in group G_i under schedule π. $C_{\max} = \max\left\{C_{ij} \middle| i = 1, 2, \ldots, f; j = 1, 2, \ldots, n_i\right\}$ and $\sum w_{ij} C_{ij}$ represent the makespan and total weighted completion time of a given schedule, respectively. In the remaining part of the paper, all of the problems considered will be denoted using the three-field notation schema $\alpha|\beta|\gamma$ introduced by Graham et al. [19].

3 Makespan minimization problem

In this section, we consider a single-machine group scheduling problem with deteriorating jobs. The objective function is to minimize the makespan of all jobs.

Theorem 1 *For the problem* $1|p_{ij} = \alpha_{ij}(a + bt),\ s_i = \delta_i(a + bt),\ GT|C_{\max}$, *the optimal schedule can be obtained if the group sequence and the job sequence in each group are arranged in any order.*

Proof For a given schedule π, let $C_{[i][j]}(\pi)$ represent the completion time of the job scheduled in the j position and in the ith group under schedule π. Then, the completion times for jobs in the first group $G_{[1]}$ are:

$$\begin{aligned}
C_{[1][1]} &= t_0 + \delta_{[1]}(a + bt_0) + \alpha_{[1][1]}\left(a + b\left(t_0 + \delta_{[1]}(a + bt_0)\right)\right) \\
&= \left(t_0 + \tfrac{a}{b}\right)\left(1 + b\delta_{[1]}\right)\left(1 + b\alpha_{[1][1]}\right) - \tfrac{a}{b} \\
C_{[1][2]} &= C_{[1][1]} + \alpha_{[1][2]}\left(a + bC_{[1][1]}\right) \\
&= \left(t_0 + \tfrac{a}{b}\right)\left(1 + b\delta_{[1]}\right)\left(1 + b\alpha_{[1][1]}\right)\left(1 + b\alpha_{[1][2]}\right) - \tfrac{a}{b} \\
&\vdots \\
C_{[1][n_1]} &= \left(t_0 + \tfrac{a}{b}\right)\left(1 + b\delta_{[1]}\right)\prod_{k=1}^{n_{[1]}}\left(1 + b\alpha_{[1]k}\right) - \tfrac{a}{b}
\end{aligned}$$

The completion times for jobs in the second group $G_{[2]}$ are:

$$\begin{aligned}
C_{[2][1]} &= C_{[1][n_1]} + \delta_{[1]}\left(a + bC_{[1][n_1]}\right) \\
&\quad + \alpha_{[1][1]}\left(a + b\left(C_{[1][n_1]} + \delta_{[1]}\left(a + bC_{[1][n_1]}\right)\right)\right) \\
&= \left(t_0 + \tfrac{a}{b}\right)\left(1 + b\delta_{[1]}\right)\left(1 + b\delta_{[2]}\right) \\
&\quad \prod_{k=1}^{n_{[1]}}\left(1 + b\alpha_{[1]k}\right)\left(1 + b\alpha_{[2][1]}\right) - \tfrac{a}{b} \\
C_{[2][2]} &= \left(t_0 + \tfrac{a}{b}\right)\left(1 + b\delta_{[1]}\right)\left(1 + b\delta_{[2]}\right) \\
&\quad \prod_{k=1}^{n_{[1]}}\left(1 + b\alpha_{[1]k}\right)\left(1 + b\alpha_{[2][1]}\right)\left(1 + b\alpha_{[2][2]}\right) - \tfrac{a}{b} \\
&\vdots \\
C_{[2][n_2]} &= \left(t_0 + \tfrac{a}{b}\right)\left(1 + b\delta_{[1]}\right)\left(1 + b\alpha_{[2]}\right) \\
&\quad \prod_{k=1}^{n_{[1]}}\left(1 + b\alpha_{[1]k}\right)\prod_{k=1}^{n_{[2]}}\left(1 + b\alpha_{[2]k}\right) - \tfrac{a}{b}
\end{aligned}$$

The completion times for jobs in the last group $G_{[f]}$ are:

$$\begin{aligned} C_{[f][1]} &= C_{[f-1][n_{f-1}]} + \delta_{[f]}\left(a + bC_{[f-1][n_{f-1}]}\right) \\ &\quad + \alpha_{[f][1]}\left(a + b\left(C_{[f-1][n_{f-1}]} + \delta_{[f]}\left(a + bC_{[f-1][n_{f-1}]}\right)\right)\right) \\ &= \left(t_0 + \tfrac{a}{b}\right)\prod_{i=1}^{f}\left(1 + b\delta_{[i]}\right) \\ &\quad \prod_{i=1}^{f-1}\prod_{k=1}^{n_{[i]}}\left(1 + b\alpha_{[i]k}\right)\left(1 + b\alpha_{[f][1]}\right) - \tfrac{a}{b} \\ C_{[f][2]} &= \left(t_0 + \tfrac{a}{b}\right)\prod_{i=1}^{f}\left(1 + b\delta_{[i]}\right)\prod_{i=1}^{f-1}\prod_{k=1}^{n_{[i]}}\left(1 + b\alpha_{[i]k}\right) \\ &\quad \left(1 + b\alpha_{[f][1]}\right)\left(1 + b\alpha_{[f][2]}\right) - \tfrac{a}{b} \\ &\vdots \\ C_{[f][n_f]} &= \left(t_0 + \tfrac{a}{b}\right)\prod_{i=1}^{f}\left(1 + b\delta_{[i]}\right)\prod_{i=1}^{f-1}\prod_{k=1}^{n_{[i]}}\left(1 + b\alpha_{[i]k}\right) \\ &\quad \left(1 + b\alpha_{[f][1]}\right)\left(1 + b\alpha_{[f][2]}\right)\dots\left(1 + b\alpha_{[f][n_f]}\right) - \tfrac{a}{b} \\ &= \left(t_0 + \tfrac{a}{b}\right)\prod_{i=1}^{f}\left(1 + b\delta_{[i]}\right)\prod_{i=1}^{f}\prod_{k=1}^{n_{[i]}}\left(1 + b\alpha_{[i]k}\right) - \tfrac{a}{b} \end{aligned}$$

Hence, the makespan of all jobs is:

$$\begin{aligned} C_{[f][n_f]} &= \left(t_0 + \tfrac{a}{b}\right)\prod_{i=1}^{f}\left(1 + b\delta_{[i]}\right)\prod_{i=1}^{f}\prod_{k=1}^{n_{[i]}}\left(1 + b\alpha_{[i]k}\right) - \tfrac{a}{b} \\ &= \left(t_0 + \tfrac{a}{b}\right)\prod_{i=1}^{f}\left(1 + b\delta_{i}\right)\prod_{i=1}^{f}\prod_{k=1}^{n_i}\left(1 + b\alpha_{ik}\right) - \tfrac{a}{b} \end{aligned} \tag{1}$$

Since the term $\prod_{i=1}^{f}(1 + b\delta_i)$ is independent of the permutation of group sequence, then the term $\prod_{i=1}^{f}\prod_{k=1}^{n_i}(1 + b\alpha_{ik})$ is independent of the permutation of the job sequence in each group. Thus, the makespan is independent of the permutation of the group sequence and the job sequence in each group. □

4 Total weighted completion time minimization problem

Theorem 2 *For the problem* $1|p_{ij} = \alpha_{ij}(a + bt),\ s_i = \delta_i(a + bt),\ GT|\sum w_{ij}C_{ij}$, *the optimal schedule satisfies the following:*

1. The job sequence in each group is in nondecreasing order of $\frac{\alpha_{ij}}{w_{ij}(1+b\alpha_{ij})}$, i.e.:

$$\begin{aligned} &\frac{\alpha_{i(1)}}{w_{i(1)}\left(1 + b\alpha_{i(1)}\right)} \leq \frac{\alpha_{i(2)}}{w_{i(2)}\left(1 + b\alpha_{i(2)}\right)} \leq \dots \\ &\leq \frac{\alpha_{i(n_i)}}{w_{i(n_i)}\left(1 + b\alpha_{i(n_i)}\right)} \\ &i = 1,\ 2, \dots,\ f \end{aligned}$$

2. The groups are arranged in nondecreasing order of:

$$\frac{(1 + b\delta_i)\prod_{j=1}^{n_i}\left(1 + b\alpha_{ij}\right) - 1}{(1 + b\delta_i)\sum_{k=1}^{n_i} w_{i(k)}\prod_{j=1}^{k}\left(1 + b\alpha_{i(j)}\right)}$$

Proof In the same group, the result of item 1 above can be easily obtained by using simple interchanging technology.

Next, we consider the case in item 2. Let π and π' be two job schedules where the difference between π and π' is a pairwise interchange of two adjacent groups G_i and G_j, that is, $\pi = [S_1,\ G_i,\ G_j,\ S_2]$, $\pi' = [S_1,\ G_j,\ G_i,\ S_2]$, where S_1 and S_2 are partial sequences. Furthermore, we assume that t denotes the completion time of the last job in S_1. To show that π dominates π', it suffices to show that $C_{jn_j}(\pi) \leq C_{in_i}(\pi')$ and $\sum w_{ij}C_{ij}(\pi) \leq \sum w_{ij}C_{ij}(\pi')$. Under π, the completion time for the kth job in group G_i is:

$$C_{i[k]}(\pi) = \left(t + \frac{a}{b}\right)(1 + b\delta_i)\prod_{l=1}^{k}\left(1 + b\alpha_{i(l)}\right) - \frac{a}{b}$$

and the completion time for the kth job in group G_j is:

$$\begin{aligned} C_{j[k]}(\pi) &= \left(t + \frac{a}{b}\right)(1 + b\delta_i)\left(1 + b\delta_j\right) \\ &\quad \prod_{l=1}^{n_i}\left(1 + b\alpha_{i(l)}\right)\prod_{l=1}^{k}\left(1 + b\alpha_{j(l)}\right) - \frac{a}{b} \end{aligned} \tag{2}$$

Under π', the completion times of the kth job in groups G_j and G_i are:

$$C_{j[k]}(\pi') = \left(t + \frac{a}{b}\right)\left(1 + b\delta_j\right)\prod_{l=1}^{k}\left(1 + b\alpha_{j(l)}\right) - \frac{a}{b}$$

and:

$$\begin{aligned} C_{i[k]}(\pi') &= \left(t + \frac{a}{b}\right)\left(1 + b\delta_j\right)(1 + b\delta_i) \\ &\quad \times \prod_{l=1}^{n_j}\left(1 + b\alpha_{j(l)}\right)\prod_{l=1}^{k}\left(1 + b\alpha_{i(l)}\right) - \frac{a}{b} \end{aligned} \tag{3}$$

respectively. From Eqs. (2) and (3), we have:

$$C_{jn_j}(\pi) = C_{in_i}(\pi')$$

and:

$$\sum w_{ij}C_{ij}(\pi) - \sum w_{ij}C_{ij}(\pi')$$

$$= \left(t + \frac{a}{b}\right)\left(1 + b\delta_j\right)\sum_{k=1}^{n_j} w_j(k)$$

$$\prod_{l=1}^{k}\left(1 + b\alpha_{j(l)}\right)\left(\left(1 + b\delta_i\right)\prod_{l=1}^{n_i}\left(1 + b\alpha_{il}\right) - 1\right)$$

$$- \left(t + \frac{a}{b}\right)\left(1 + b\delta_i\right)\sum_{k=1}^{n_i} w_i(k)$$

$$\prod_{l=1}^{k}\left(1 + b\alpha_{i(l)}\right)\left(\left(1 + b\delta_j\right)\prod_{l=1}^{n_j}\left(1 + b\alpha_{jl}\right) - 1\right)$$

If:

$$\frac{\left(1 + b\delta_i\right)\prod_{l=1}^{n_i}\left(1 + b\alpha_{il}\right) - 1}{\left(1 + b\delta_i\right)\sum_{k=1}^{n_i} w_{i(k)}\prod_{l=1}^{k}\left(1 + b\alpha_{i(l)}\right)}$$

$$\leq \frac{\left(1 + b\delta_j\right)\prod_{l=1}^{n_j}\left(1 + b\alpha_{jl}\right) - 1}{\left(1 + b\delta_j\right)\sum_{k=1}^{n_j} w_{j(k)}\prod_{l=1}^{k}\left(1 + b\alpha_{j(l)}\right)}$$

then:

$$\sum w_{ij}C_{ij}(\pi) - \sum w_{ij}C_{ij}(\pi') \leq 0$$

This completes the proof. □

From Theorem 2, the problem $1|p_{ij} = \alpha_{ij}(a + bt),\ s_i = \delta_i(a + bt),\ \mathrm{GT}|\sum w_{ij}C_{ij}$ can be solved by the algorithm presented in the following

Algorithm 1 Step 1 Jobs in each group are scheduled in nondecreasing order of $\frac{\alpha_{ij}}{w_{ij}\left(1+b\alpha_{ij}\right)}$, i.e.:

$$\frac{\alpha_{i(1)}}{w_{i(1)}\left(1 + b\alpha_{i(1)}\right)} \leq \frac{\alpha_{i(2)}}{w_{i(2)}\left(1 + b\alpha_{i(2)}\right)}$$

$$\leq \ldots \leq \frac{\alpha_{i(n_i)}}{w_{i(n_i)}\left(1 + b\alpha_{i(n_i)}\right)}$$

$$i = 1,\ 2, \ldots,\ f$$

Step 2 Calculate:

$$\rho(G_i) = \frac{\left(1 + b\delta_i\right)\prod_{j=1}^{n_i}\left(1 + b\alpha_{ij}\right) - 1}{\left(1 + b\delta_i\right)\sum_{k=1}^{n_i} w_{i(k)}\prod_{j=1}^{k}\left(1 + b\alpha_{i(j)}\right)}$$

$$i = 1,\ 2, \ldots,\ f.$$

Step 3 Groups are scheduled in nondecreasing order of $\rho(G_i)$, i.e.:

$$\rho(G_1) \leq \rho(G_2) \leq \ldots \leq \rho\left(G_f\right)$$

Clearly, the total time for Algorithm 1 is O($n$log$n$). In addition, we demonstrate the algorithm in the following example.

Example 1 Let n=1, f=2, a=b=1, and t_0=0. Also, G_1: {J_{11}, J_{12}}, δ_1=1, α_{11}=3, α_{12}=5, w_{11}=2, w_{12}=5, G_2: {J_{21}, J_{22}}, δ_2=2, α_{21}=3, α_{22}=2, w_{21}=6, and w_{22}=2.

Solution According to Algorithm 1, we solve Example 1 as follows:

Step 1 In group G_1, the optimal job sequence is [J_{12}, J_{11}]. In group G_2, the optimal job sequence is [J_{21}, J_{22}].

Steps 2 and 3 $\rho(G_1) = \frac{47}{156} > \rho(G_2) = \frac{37}{144}$. Hence, the optimal group sequence is [G_2, G_1]. Therefore, the optimal schedule is [J_{21}, J_{22}, J_{12}, J_{11}]. Consequently, the optimal value of the total weighted completion time is 5,745.

5 Conclusions

In this paper, we have considered single-machine scheduling problems with deterioration jobs and the group technology assumption. By deteriorating jobs and the group technology assumption, we mean that the group setup times and job processing times are both described by a function which is proportional to a linear function of time. We showed that the makespan minimization problem and the total weighted completion time minimization problem remain polynomially solvable. In addition, we proposed algorithms to solve these problems.

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