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ORIGINAL ARTICLE

Comparison of fuzzy AHP and fuzzy TOPSIS methods for facility location selection

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Abstract Facility location selection is a multi-criteria decision problem and has a strategic importance for many companies. The conventional methods for facility location selection are inadequate for dealing with the imprecise or vague nature of linguistic assessment. To overcome this difficulty, fuzzy multi-criteria decision-making methods are proposed. The aim of this study is to use fuzzy analytic hierarchy process (AHP) and the fuzzy technique for order preference by similarity to ideal solution (TOPSIS) methods for the selection of facility location. The proposed methods have been applied to a facility location selection problem of a textile company in Turkey. After determining the criteria that affect the facility location decisions, fuzzy AHP and fuzzy TOPSIS methods are applied to the problem and results are presented. The similarities and differences of two methods are also discussed.

Keywords Facility location selection · Fuzzy logic · Multi-criteria decision-making · Fuzzy AHP · Fuzzy TOPSIS

1 Introduction

Facility location selection is the determination of a geographic site for a firm's operations. The facility location decision involves organizations seeking to locate, relocate or expand their operations. The facility location decision process encompasses the identification, analysis, evaluation and selection among alternatives [1]. Selecting a plant location is a very important decision for firms because they are costly and difficult to reverse, and they entail a long-term commitment. And also location decisions have an impact on operating costs and revenues. For instance, a poor choice of location might result in excessive transportation costs, a shortage of qualified labor, lost of competitive advantage, inadequate supplies of raw materials, or some similar condition that would be detrimental to operations [2].

The general procedure for making location decisions usually consists of the following steps:

1. Decide on the criteria that will be used to evaluate location alternatives
2. Identify criteria that are important
3. Develop location alternatives
4. Evaluate the alternatives and make a selection [2]

There are many criteria that influence the location decisions of firms. However some criteria are so important that they tend to dominate the decision. In our study, we take five criteria into consideration. These are favorable labor climate, proximity to markets, community considerations, quality of life, proximity to suppliers and resources [3].

Favorable labor climate Labor climate is an important criterion for location decisions. Labor climate is a function of wage rates, training requirements, attitudes toward work, worker productivity, and union strength. Many executives believe that weak unions or a low probability of a union organizing is a major advantage. Especially labor-intensive firms give strong consideration to labor climate.

Proximity to markets After determining where the demand for goods and services is greatest, management must select a location for the facility that will supply that demand. Locating

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near markets is particularly important when goods are bulky or heavy and outbound transportation rates are high.

Community considerations Many communities actively try to attract new businesses because they are viewed as potential sources of future tax revenues, and as sources of new job opportunities. Community related criteria involve the cost and availability of utilities, environmental regulations, taxes, and existence of development support [2].

Quality of life Quality of life is an important criterion to attract and keep a qualified staff. Quality of schools, recreational facilities, and an attractive life style can make the difference in their locational decisions.

Proximity to suppliers and resources It is an important criterion for the industries dependent on bulky and heavy raw materials. In such cases inbound transportation costs become a dominant criterion, forcing these firms to locate facilities near suppliers [3].

The conventional approaches for facility location problems like locational cost volume analysis, factor rating, and center of gravity method [2] tend to be less effective in dealing with the imprecise or vague nature of the linguistic assessment [4]. In real life, the evaluation data of plant location suitability for various subjective criteria and the weights of the criteria are usually expressed in linguistic terms. And also, to efficiently resolve the ambiguity frequently arising in available information and do more justice to the essential fuzziness in human judgment and preference, the fuzzy set theory has been used to establish an ill-defined multiple criteria decision-making problems [5]. Thus in this paper, fuzzy AHP and fuzzy TOPSIS methods are proposed for facility location selection, where the ratings of various alternative locations under various subjective criteria and the weights of all criteria are represented by fuzzy numbers.

There are studies in the literature that use the fuzzy TOPSIS method [6, 7] and other fuzzy multi-criteria decision-making methods [8–11] for facility location selection. But differently from other studies, fuzzy AHP and fuzzy TOPSIS methods are proposed for facility location selection and the results are compared in this study.

The remainder of this paper is organized as follows. Fuzzy sets, linguistic variables and fuzzy numbers are briefly explained in Sect. 2. Then in Sect. 3, fuzzy AHP method is introduced. In Sect. 4, fuzzy TOPSIS method is explained and the steps of proposed method are summarized. In Sect. 5, a numerical example is given to illustrate the proposed methods and the results that are gained with these methods are compared. And finally Sect. 6 concludes the paper.

2 Fuzzy sets

In order to deal with vagueness of human thought, Zadeh [12] first introduced the fuzzy set theory. A fuzzy set is a class of objects with a continuum of grades of membership. Such a set is characterized by a membership function which assigns to each object a grade of membership ranging between zero and one [12]. A fuzzy set is an extension of a crisp set. Crisp sets only allow full membership or non-membership at all, whereas fuzzy sets allow partial membership. In other words, an element may partially belong to a fuzzy set [13]. Fuzzy sets and fuzzy logic are powerful mathematical tools for modeling: uncertain systems in industry, nature and humanity; and facilitators for common-sense reasoning in decision-making in the absence of complete and precise information. Their role is significant when applied to complex phenomena not easily described by traditional mathematical methods, especially when the goal is to find a good approximate solution [14]. Fuzzy sets theory providing a more widely frame than classic sets theory, has been contributing to capability of reflecting real world [15]. Modeling using fuzzy sets has proven to be an effective way for formulating decision problems where the information available is subjective and imprecise [16].

2.1 Linguistic variable

A linguistic variable is a variable whose values are words or sentences in a natural or artificial language [17]. As an illustration, *age* is a linguistic variable if its values are assumed to be the fuzzy variables labeled *young, not young, very young, not very young,* etc. rather than the numbers 0, 1, 2, 3.. [18].

The concept of a linguistic variable provides a means of approximate characterization of phenomena which are too complex or too ill-defined to be amenable to description in conventional quantitative terms. The main applications of the linguistic approach lie in the realm of humanistic systems-especially in the fields of artificial intelligence, linguistics, human decision processes, pattern recognition, psychology, law, medical diagnosis, information retrieval, economics and related areas [17].

2.2 Fuzzy numbers

A fuzzy number $\widetilde{M}$ is a convex normalized fuzzy set $\widetilde{M}$ of the real line R such that [16]:

- It exists such that one $x_0 \in R$ with $\mu_{\widetilde{M}}(x_0) = 1$ (x_0 is called mean value of $\widetilde{M}$)
- $\mu_{\widetilde{M}}(x)$ is piecewise continuous.

It is possible to use different fuzzy numbers according to the situation. In applications it is often convenient to work with triangular fuzzy numbers (TFNs) because of their computational simplicity, and they are useful in promoting representation and information processing in a fuzzy environment. In this study TFNs are adopted in the fuzzy AHP and fuzzy TOPSIS methods.

Triangular fuzzy numbers can be defined as a triplet (l, m, u). The parameters l, m, and u. respectively, indicate the smallest possible value, the most promising value, and the largest possible value that describe a fuzzy event. A triangular fuzzy number $\widetilde{M}$ is shown in Fig. 1 [19].

There are various operations on triangular fuzzy numbers. But here, only important operations used in this study are illustrated. If we define, two positive triangular fuzzy numbers (l_1, m_1, u_1) and (l_2, m_2, u_2) then:

$$(l_1,\ m_1,\ u_1) + (l_2,\ m_2,\ u_2) = (l_1 + l_2,\ m_1 + m_2,\ u_1 + u_2) \tag{1}$$

$$(l_1,\ m_1,\ u_1).(l_2,\ m_2,\ u_2) = (l_1.l_2,\ m_1.m_2,\ u_1.u_2) \tag{2}$$

$$(l_1,\ m_1,\ u_1)^{-1} \approx (1/u_1, 1/m_1, 1/l_1) \tag{3}$$

$$(l_1, m_1, u_1).k = (l_1k,\ m_1k,\ u_1k) \tag{4}$$

(k is a positive real number)

The distance between two triangular fuzzy numbers can be calculated by vertex method [20]:

$$d_v(\widetilde{m}, \widetilde{n}) = \sqrt{\frac{1}{3}\left[(l_1 - l_2)^2 + (m_1 - m_2)^2 + (u_1 - u_2)^2\right]} \tag{5}$$

3 Fuzzy analytic hierarchy process

First proposed by Thomas L. Saaty [21], the analytic hierarchy process (AHP) is a widely used multiple criteria decision-making tool. The analytic hierarchy process, since its invention, has been a tool at the hands of decision-makers and researchers, becoming one of the most widely used multiple criteria decision-making tools [22]. Although the purpose of AHP is to capture the expert's knowledge, the traditional AHP still cannot really reflect the human thinking style [4]. The traditional AHP method is problematic in that it uses an exact value to express the decision-maker's opinion in a comparison of alternatives [23]. And AHP method is often criticized, due to its use of unbalanced scale of judgments and its inability to adequately handle the inherent uncertainty and imprecision in the pair-wise comparison process [19]. To overcome all these shortcomings, fuzzy analytical hierarchy process was developed for solving the hierarchical problems. Decision-makers usually find that it is more accurate to give interval judgments than fixed value judgments. This is because usually he/she is unable to make his/her preference explicitly about the fuzzy nature of the comparison process [4].

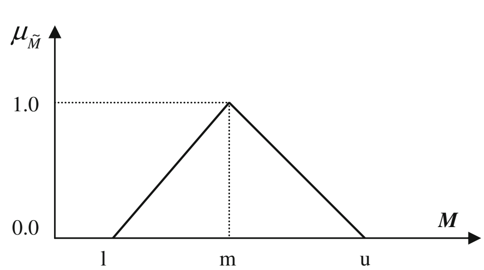

Fig. 1 Triangular fuzzy number,

The first study of fuzzy AHP is proposed by Van Laarhoven and Pedrycz [24], which compared fuzzy ratios described by triangular fuzzy numbers. Buckley [25] initiated trapezoidal fuzzy numbers to express the decision-maker's evaluation on alternatives with respect to each criterion Chang [26] introduced a new approach for handling fuzzy AHP, with the use of triangular fuzzy numbers for pair-wise comparison scale of fuzzy AHP, and the use of the extent analysis method for the synthetic extent values of the pair-wise comparisons. Fuzzy AHP method is a popular approach for multiple criteria decision-making and has been widely used in the literature [19, 27–44].

In this study the extent fuzzy AHP is utilized, which was originally introduced by Chang [26]. Let $X = \{x_1,\ x_2,\ x_3, \ldots\ldots, x_n\}$ an object set, and $G = \{g_1, g_2, g_3, \ldots\ldots, g_n\}$ be a goal set. Then, each object is taken and extent analysis for each goal is performed, respectively. Therefore, m extent analysis values for each object can be obtained, with the following signs:

$$M^1_{gi}, M^2_{gi}, \ldots\ldots M^m_{gi}, \qquad i = 1, 2, \ldots., n,$$

where $M^j_{gi}(j = 1,\ 2,\ \ldots, m)$ all are TFNs. The steps of Chang's [26] extent analysis can be given as in the following:

Step 1: The value of fuzzy synthetic extent with respect to the ith object is defined as

$$S_i = \sum_{j=1}^{m} M^j_{gi} \otimes \left[\sum_{i=1}^{n}\sum_{j=1}^{m} M^j_{gi}\right]^{-1} \tag{6}$$

To obtain $\sum_{j=1}^{m} M^j_{gi}$, the fuzzy addition operation of m extent analysis values for a particular matrix is performed such as:

$$\sum_{j=1}^{m} M^j_{gi} = \left(\sum_{j=1}^{m} l_j, \sum_{j=1}^{m} m_j, \sum_{j=1}^{m} u_j\right) \tag{7}$$

and to obtain $\left[\sum_{j=1}^{n}\sum_{j=1}^{m} M_{gi}^{j}\right]^{-1}$, the fuzzy addition operation of M_{gi}^{j} $(j = 1, 2, ..., m)$ values is performed such as:

$$\sum_{i=1}^{n}\sum_{j=1}^{m} M_{gi}^{j} = \left(\sum_{i=1}^{n} l_i, \sum_{i=1}^{n} m_i, \sum_{i=1}^{n} u_i\right) \tag{8}$$

and then the inverse of the vector above is computed, such as:

$$\left[\sum_{i=1}^{n}\sum_{j=1}^{m} M_{gi}^{j}\right]^{-1} = \left(\frac{1}{\sum_{i=1}^{n} u_i}, \frac{1}{\sum_{i=1}^{n} m_i}, \frac{1}{\sum_{i=1}^{n} l_i}\right). \tag{9}$$

Step 2: As $M_1 = (l_1, m_1, u_1)$ and $M_2 = (l_2, m_2, u_2)$ are two triangular fuzzy numbers, the degree of possibility of $M_2 = (l_2, m_2, u_2) \geq M_1 = (l_1, m_1, u_1)$ is defined as:

$$V(M_2 \geq M_1) = \sup_{y \geq x}\left\lfloor \min\left(\mu_{M_1}(x), \mu_{M_2}(y)\right)\right\rfloor \tag{10}$$

and can be expressed as follows:

$$V(M_2 \geq M_1) = hgt(M_1 \cap M_2) = \mu_{M_2}(d) \tag{11}$$

$$= \begin{cases} 1, & if \quad m_2 \geq m_1 \\ 0, & if \quad l_1 \geq u_2 \\ \frac{l_1 - u_2}{(m_2 - u_2) - (m_1 - l_1)}, & otherwise \end{cases} \tag{12}$$

Figure 2 illustrates Eq. (11) where d is the ordinate of the highest intersection point D between μ_{M_1} and μ_{M_2}. To compare M_1 and M_2, we need both the values of $V(M_1 \geq M_2)$ and $V(M_2 \geq M_1)$

Step 3: The degree possibility for a convex fuzzy number to be greater than k convex fuzzy M_i (i=1, 2, k) numbers can be defined by

$$V(M \geq M_1, M_2, ...M_k) \tag{13}$$

$$= V[(M \geq M_1)\ and\ (M \geq M_2)\ and....and\ (M \geq M_k)]$$

$$= \min \quad V(M \geq M_i), \quad i = 1, 2, 3, ..., k$$

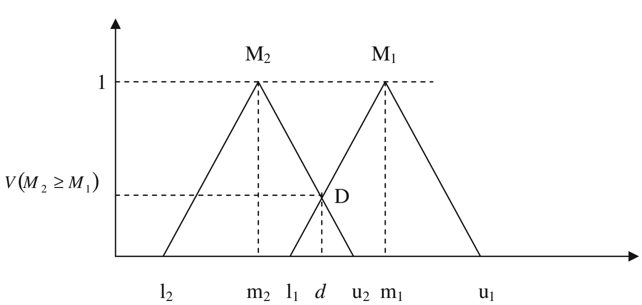

Fig. 2 The intersection between M_1 and M_2 [26]

Assume that $d(A_i) = \min V(S_i \geq S_k)$ for k=1, 2,...., n; $k \neq i$. Then the weight vector is given by

$$W' = (d'(A_1), d'(A_2), ..., d'(A_n))^T \tag{14}$$

where A_i= (i=1, 2,... n) are n elements.

Step 4: Via normalization, the normalized weight vectors are

$$W = (d(A_1), d(A_2), ..., d(A_n))^T \tag{15}$$

where W is a non-fuzzy number.

4 Fuzzy TOPSIS method

The TOPSIS method was firstly proposed by Hwang and Yoon [45]. The basic concept of this method is that the chosen alternative should have the shortest distance from the positive ideal solution and the farthest distance from negative ideal solution. Positive ideal solution is a solution that maximizes the benefit criteria and minimizes cost criteria, whereas the negative ideal solution maximizes the cost criteria and minimizes the benefit criteria [46]. In the classical TOPSIS method, the weights of the criteria and the ratings of alternatives are known precisely and crisp values are used in the evaluation process. However, under many conditions crisp data are inadequate to model real-life decision problems. Therefore, the fuzzy TOPSIS method is proposed where the weights of criteria and ratings of alternatives are evaluated by linguistic variables represented by fuzzy numbers to deal with the deficiency in the traditional TOPSIS.

There are many applications of fuzzy TOPSIS in the literature. For instance, Triantaphyllou and Lin [47] developed a fuzzy version of the TOPSIS method based on fuzzy arithmetic operations, which leads to a fuzzy relative closeness for each alternative. Chen [20] extended the TOPSIS to the fuzzy environment and gave a numerical example of system analysis engineer selection for a software company. Tsaur et al. [48] applied fuzzy set theory to evaluate the service quality of airline. Chu [6] presented a fuzzy TOPSIS model under group decisions for solving the facility location selection problem. Chu and Lin [49] proposed the fuzzy TOPSIS method for robot selection. Abo-Sinna and Amer [50] extended the TOPSIS approach to solve the multi-objective large-scale nonlinear programming problems with block angular structure. Saghafian and Hejazi [51] proposed a modified fuzzy TOPSIS method for the multi-criteria selection problem when there is a group of decision-makers. And they proposed a new distance measure for fuzzy TOPSIS. Wang and Elhag [46] proposed a fuzzy TOPSIS method based on alpha level sets and presented a nonlinear programming solution procedure for bridge risk assessment. Jahanshahloo et al. [52] extended the TOPSIS method to decision-making problems with fuzzy data and they used the concept of α-cuts to normalize fuzzy numbers. Chen et al. [53] presented a fuzzy TOPSIS approach to deal with the supplier selection problem in supply chain system. Bottani and Rizzi [54] proposed a mult-attribute approach based on TOPSIS technique and fuzzy set theory for the selection and ranking of the most suitable service provider. Wang and Chang [55] developed an evaluation approach based on the fuzzy TOPSIS to help the Air Force Academy in Taiwan to choose initial training aircraft. Li [56] gave a comparative analysis of compromise ratio method and the extended fuzzy TOPSIS method and illustrated a numerical example by showing their similarity and differences. Benitez et al. [57] presented a fuzzy TOPSIS method for measuring quality of service in the hotel industry. Yang and Hung [58] proposed to use TOPSIS and fuzzy TOPSIS methods for plant layout design problem. Wang and Lee [59] generalized TOPSIS to fuzzy multiple-criteria group decision-making in a fuzzy environment. They proposed two operators Up and Lo that are employed to find ideal and negative ideal solutions.

In this paper, the extension of TOPSIS method is considered which was proposed by Chen [20] and Chen et al. [53] The algorithm of this method can be described as follows:

Step 1: First of all a committee of decision-makers is formed. In a decision committee that has K decision-makers; fuzzy rating of each decision-maker $D_k = (k=1, 2,...K)$ can be represented as triangular fuzzy number $\widetilde{R}_k = (k = 1, 2, \ldots K)$ with membership function $\mu_{\widetilde{R}_k}(x)$.

Step 2: Then evaluation criteria are determined.

Step 3: After that, appropriate linguistic variables are chosen for evaluating criteria and alternatives.

Step 4: Then the weight of criteria are aggregated [53].

If the fuzzy ratings of all decision-makers are described as triangular fuzzy numbers $\widetilde{R}_k = (a_k, b_k, c_k)$, k=1,2,..., K., then the aggregated fuzzy rating can be determined as $\widetilde{R} = (a, b, c)$, k=1,2,..., K. Here;

$$a = \min_k\{a_k\}, \quad b = \frac{1}{K}\sum_{k=1}^{K} b_k, \quad c = \max_k\{c_k\} \tag{16}$$

If the fuzzy rating and importance weight of the kth decision-maker are $\widetilde{x}_{ijk} = (a_{ijk}, b_{ijk}, c_{ijk})$ and $\widetilde{w}_{ijk} = (w_{jk1}, w_{jk2}, w_{jk3})$, i=1,2,...m, j=1,2,...n respectively, then the aggregated fuzzy ratings $(\widetilde{x}_{ij})$ of alternatives with respect to each criterion can be found as $(\widetilde{x}_{ij}) = (a_{ij}, b_{ij}, c_{ij})$

Here,

$$a_{ij} = \min_k\{a_{ijk}\}, \quad b_{ij} = \frac{1}{K}\sum_{k=1}^{K} b_{ijk}, \quad c_{ij} = \max_k\{c_{ijk}\} \tag{17}$$

Then the aggregated fuzzy weights $(\widetilde{w}_{ij})$ of each criterion are calculated as:

$$(\widetilde{w}_j) = (w_{j1}, w_{j2}, w_{j3}) \tag{18}$$

Here,

$$w_{j1} = \min_k\{w_{jk1}\}, \quad w_{j2} = \frac{1}{K}\sum_{k=1}^{K} w_{jk2}, \quad w_{j3} = \max_k\{w_{jk3}\}$$

Step 5: Then the fuzzy decision matrix is constructed as:

$$\widetilde{D} = \begin{bmatrix} \widetilde{x}_{11} & \widetilde{x}_{12} & \cdots & \widetilde{x}_{1n} \\ \widetilde{x}_{21} & \widetilde{x}_{22} & \cdots & \widetilde{x}_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ \widetilde{x}_{m1} & \widetilde{x}_{m2} & \cdots & \widetilde{x}_{mn} \end{bmatrix},$$

$$\widetilde{W} = [\widetilde{w}_1, \ \widetilde{w}_2, \cdots \ \widetilde{w}_n]$$

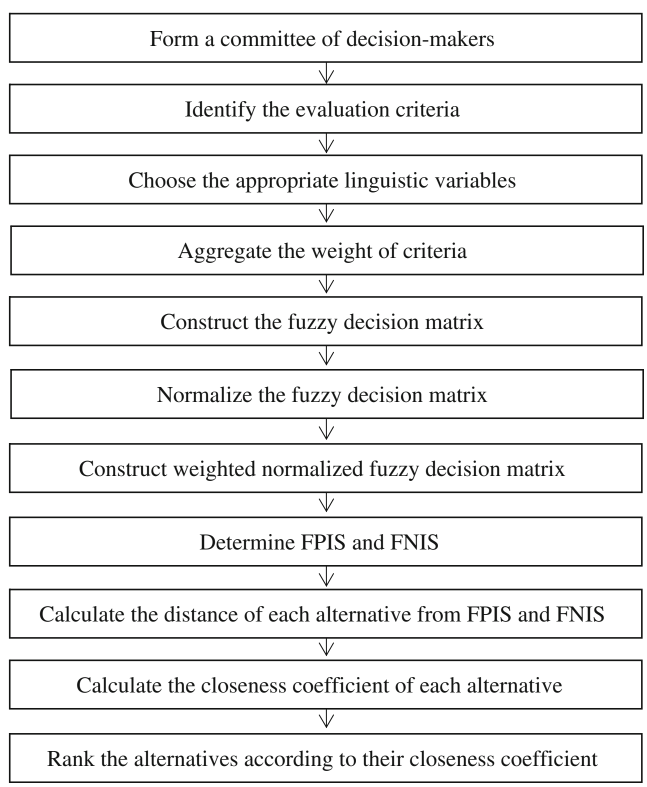

Fig. 3 The steps of the proposed method

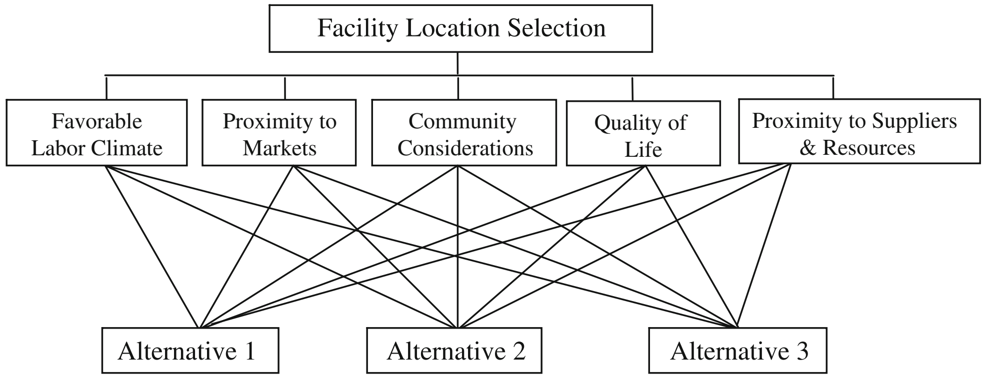

Fig. 4 Hierarchical structure of facility location selection process

Here $\widetilde{x}_{ij} = (a_{ji}, b_{ij}, c_{ij})$ and $\widetilde{w}_j = (w_{j1}, w_{j2}, w_{j3})$; i=1, 2,... m, j=1, 2,...n can be approximated by positive triangular fuzzy numbers.

Step 6: After constructing the fuzzy decision matrix, it is normalized. Instead of using complicated normalization formula of classical TOPSIS, the linear scale transformation can be used to transform the various criteria scales into a comparable scale. Therefore, we can obtain the normalized fuzzy decision matrix $\widetilde{R}$ [20].

$$\widetilde{R} = [\widetilde{r}_{ij}]_{mxn} \quad i = 1, 2, \ldots, m; \; j = 1, 2, \ldots n \tag{19}$$

where:

$$\widetilde{r}_{ij} = \left(\frac{a_{ij}}{c_j^*}, \frac{b_{ij}}{c_j^*}, \frac{c_{ij}}{c_j^*} \right),$$

$$c_j^* = \max_i c_{ij}$$

Step 7: Considering the different weight of each criterion, the weighted normalized decision matrix is computed by multiplying the importance weights of evaluation criteria and the values in the normalized fuzzy decision matrix. The weighted normalized decision matrix $\widetilde{V}$ is defined as:

$$\widetilde{V} = [\widetilde{v}_{ij}]_{mxn} \quad i = 1, 2, \ldots, m \quad j = 1, 2, \ldots, n \tag{20}$$

$$\widetilde{v}_{ij} = \widetilde{r}_{ij}(.)\widetilde{w}_j$$

here $\widetilde{w}_j$ represents the importance weight of criterion C_j.

According to the weighted normalized fuzzy decision matrix, normalized positive triangular fuzzy numbers can also approximate the elements $\widetilde{v}_{ij}$, $\forall i, j$.

Step 8: Then, the fuzzy positive ideal solution (FPIS, A^*) and fuzzy negative ideal solution (FNIS, A^-) are determined as [53]:

$$A^* = (\widetilde{v}_1^*, \widetilde{v}_2^*, \ldots \widetilde{v}_n^*), \tag{21}$$

$$A^- = (\widetilde{v}_1^-, \widetilde{v}_2^-, \ldots \widetilde{v}_n^-), \tag{22}$$

where

$$\widetilde{v}_j^* = \max_i \{v_{ij3}\} \quad \text{and} \quad \widetilde{v}_j^- = \min_i \{v_{ij1}\}$$

$$i = 1, 2, \ldots m, \; j = 1, 2, \ldots n.$$

Table 1 Linguistic variables for importance weight of each criterion

Linguistic variables	Triangular fuzzy numbers
Very low (VL)	(0, 0, 0.2)
Low (L)	(0.1, 0.2, 0.3)
Medium low (ML)	(0.2, 0.35, 0.5)
Medium (M)	(0.4, 0.5 ,0.6)
Medium high (MH)	(0.5, 0.65, 0.8)
High (H)	(0.7, 0.8, 0.9)
Very high (VH)	(0.8, 1,1)

Table 2 Importance weight of criteria from three decision-makers

Criteria	Decision-makers		
	D_1	D_2	D_3
C_1	VH	VH	VH
C_2	H	VH	VH
C_3	VH	H	H
C_4	MH	H	MH
C_5	H	H	H

Table 3 Linguistic variables for ratings

Linguistic variables	Triangular fuzzy numbers
Very poor (VP)	(0, 0, 2)
Poor (P)	(1, 2, 3)
Medium poor (MP)	(2, 3.5, 5)
Fair (F)	(4, 5, 6)
Medium good (MG)	(5, 6.5, 8)
Good (G)	(7, 8, 9)
Very good (VG)	(8, 10, 10)

Step 9: Then the distance of each alternative from FPIS and FNIS are calculated as:

$$d_i^* = \sum_{j=1}^{n} d_v\left(\widetilde{v}_{ij}, \widetilde{v}_j^*\right), \quad i = 1, 2, \ldots m \tag{23}$$

$$d_i^- = \sum_{j=1}^{n} d_v\left(\widetilde{v}_{ij}, \widetilde{v}_j^*\right), \quad i = 1, 2, \ldots m \tag{24}$$

where $d_v(.,.)$ is the distance measurement between two fuzzy numbers.

Step 10: A closeness coefficient (CC_i) is defined to rank all possible alternatives. The closeness coefficient represents the distances to the fuzzy positive ideal solution (A^*) and fuzzy negative ideal solution (A^-) simultaneously. The closeness coefficient of each alternative is calculated as [20]:

$$CC_i = \frac{d_i^-}{d_i^* + d_i^-}, \quad i = 1, 2, \ldots m \tag{25}$$

Table 4 Ratings of the three alternatives by decision-makers under five criteria

Criteria	Alternatives	Decision-makers		
		D_1	D_2	D_3
C_1	A_1	VG	G	VG
	A_2	G	VG	G
	A_3	MG	MG	G
C_2	A_1	G	VG	VG
	A_2	MG	F	F
	A_3	MG	F	MG
C_3	A_1	F	MG	MG
	A_2	G	VG	VG
	A_3	VG	G	VG
C_4	A_1	G	VG	G
	A_2	MG	MG	MG
	A_3	F	F	MG
C_5	A_1	G	G	VG
	A_2	G	G	MG
	A_3	MG	MG	G

Table 5 Fuzzy decision matrix and fuzzy weights of three alternatives

	A_1	A_2	A_3	Weight
C_1	(7, 9.33, 10)	(7, 8.67, 10)	(5, 7, 9)	(0.8, 1, 1)
C_2	(7, 9.33, 10)	(4, 5.5, 8)	(4, 6, 8)	(0.7, 0.93, 1)
C_3	(4, 6, 8)	(7, 9.33, 10)	(7, 9.33, 10)	(0.7, 0.87, 1)
C_4	(7, 9.33,10)	(5, 6.5, 8)	(4, 5.5, 8)	(0.5, 0.7, 0.9)
C_5	(7, 8.67,10)	(5, 7.5, 9)	(5, 7, 9)	(0.7, 0.8, 0.9)

Step 11: According to the closeness coefficient, the ranking of the alternatives can be determined. Obviously, according to Eq. (25) an alternative A_i would be closer to FPIS and farther from FNIS as CC_i approaches to 1.

The general steps of fuzzy TOPSIS method [20] can be summarized as in the Fig. 3.

5 Application in a textile company

Our application is related with the facility location problem of an integrated Turkish Textile Company which is specialized in home-textile. This company experienced a growth in the demand for its products and has also unsatisfied from the expansion of existing location. Company desires to find a new location and it has three alternatives (A_1, A_2, A_3). First of all, a committee of decision-makers is formed. There are three decision-makers (D_1, D_2, D_3) in the committee. Then evaluation criteria are determined as favorable labor climate (C_1), proximity to markets (C_2), community considerations (C_3), quality of life (C_4), proximity to suppliers and resources (C_5). The hierarchical structure for the selection of the best facility location is seen as in Fig. 4.

5.1 Application with TOPSIS method

In this section fuzzy TOPSIS method is proposed for the facility location selection problem of the textile company. Firstly, three decision-makers evaluated the importance of criteria by using the linguistic variables in Table 1. The importance weights of the criteria determined by these three decision-makers are shown in Table 2.

Table 6 Normalized fuzzy decision matrix

	A_1	A_2	A_3
C_1	(0.7, 0.93, 1)	(0.7, 0.87, 1)	(0.5, 0.7, 0.9)
C_2	(0.7, 0.93, 1)	(0.4, 0.55, 0.8)	(0.4, 0.6, 0.8)
C_3	(0.4, 0.6, 0.8)	(0.7, 0.93, 1)	(0.7, 0.93, 1)
C_4	(0.7, 0.93,1)	(0.5, 0.65, 0.8)	(0.4, 0.55, 0.8)
C_5	(0.7, 0.87,1)	(0.5, 0.75, 0.9)	(0.5, 0.7, 0.9)

Table 7 Weighted normalized fuzzy decision matrix

	A_1	A_2	A_3
C_1	(0.56, 0.93, 1)	(0.56, 0.87, 1)	(0.4, 0.7, 0.9)
C_2	(0.49, 0.87, 1)	(0.28, 0.51, 0.8)	(0.28, 0.56, 0.8)
C_3	(0.28, 0.52, 0.8)	(0.49, 0.81, 1)	(0.49, 0.81, 1)
C_4	(0.35, 0.65, 0.9)	(0.25, 0.46, 0.72)	(0.2, 0.39, 0.72)
C_5	(0.49, 0.69, 0.9)	(0.35, 0.6, 0.81)	(0.35, 0.56, 0.81)

Three decision-makers use the linguistic variables shown in Table 3 to evaluate the ratings of alternatives with respect to each criterion. The ratings of three alternatives under five criteria are shown in Table 4.

Then linguistic variables shown in Tables 2 and 4 are converted into triangular fuzzy numbers to form fuzzy decision matrix as shown in Table 5.

The normalized fuzzy decision matrix is formed as in Table 6. Then weighted normalized fuzzy decision matrix is formed as in Table 7.

After a weighted normalized fuzzy decision matrix is formed, fuzzy positive ideal solution (FPIS) and fuzzy negative ideal solution (FNIS) are determined as in the following:

$$A^* = [(1,\ 1,\ 1,),\ (1,\ 1,\ 1),\ (1,\ 1,\ 1),\ (0.9,\ 0.9,\ 0.9),\ (0.9,\ 0.9,\ 0.9)]$$

$$A^- = [(0.4, 0.4, 0.4), (0.28, 0.28, 0.28), (0.28, 0.28, 0.28), (0.2, 0.2, 0.2)., (0.35, 0.35, 0.35)]$$

Then the distance of each alternative from FPIS and FNIS with respect to each criterion are calculated by using vertex method as:

$$d(A_1, A^*) = \sqrt{\frac{1}{3}\left[(1-0.56)^2 + (1-0.93)^2 + (1-1)^2\right]} = 0.26$$

$$d(A_1, A^-) = \sqrt{\frac{1}{3}\left[(0.4-0.56)^2 + (0.4-0.93)^2 + (0.4-1)^2\right]} = 0.47$$

Here only the calculation of the distance of the first alternative to FPIS and FNIS for the first criterion is shown, as the calculations are similar in all steps. The results of all alternatives' distances from FPIS and FNIS are shown in Tables 8 and 9.

Table 8 Distances between $A_i (i=1, 2, 3)$ and A^* with respect to each criterion

	C_1	C_2	C_3	C_4	C_5
$d(A_1, A^*)$	0.26	0.30	0.51	0.35	0.27
$d(A_2, A^*)$	0.27	0.52	0.31	0.47	0.37
$d(A_3, A^*)$	0.39	0.50	0.31	0.51	0.38

Table 9 Distances between $A_i (i=1 ,2, 3)$ and A^- with respect to each criterion

	C_1	C_2	C_3	C_4	C_5
$d(A_1, A^-)$	0.47	0.55	0.33	0.49	0.38
$d(A_2, A^-)$	0.45	0.33	0.53	0.34	0.30
$d(A_3, A^-)$	0.34	0.34	0.53	0.32	0.29

d_i^* and d_i^- of three alternatives are shown in Table 10. Then closeness coefficients of three alternatives are calculated by this formula:

$$CC_i = \frac{d_i^-}{d_i^* + d_i^-}, \quad i = 1, 2, \ldots\ldots\ldots m$$

Calculation of closeness coefficient for the alternatives is as follows:

$$CC_1 = \frac{2.23}{1.69 + 2.23} = 0.57 \quad CC_2 = \frac{1.95}{1.95 + 1.93} = 0.50$$

$$CC_3 = \frac{1.82}{2.10 + 1.82} = 0.46$$

According to the closeness coefficient of three alternatives, the ranking order of three alternatives is determined as $A_1 > A_2 > A_3$. The first alternative is determined as the most appropriate facility location for the textile company. In other words, the first alternative is closer to the FPIS and farther from the FNIS.

5.2 Application with fuzzy AHP method

In this section, fuzzy AHP method is proposed for the same problem of the textile company. We proposed a group decision based on fuzzy AHP. Firstly each decision-maker (D_p), individually carry out pair-wise comparison by using Saaty's 1–9 scale as in Eq. (26). [60]:

$$D_p = \begin{bmatrix} b_{11p} & b_{12p} & \cdots & b_{1mp} \\ b_{21p} & b_{22p} & \cdots & b_{2mp} \\ \vdots & \vdots & \ddots & \vdots \\ b_{m1p} & b_{m2p} & \cdots & b_{mmp} \end{bmatrix} \quad p = 1, 2, \ldots.t \qquad (26)$$

Table 10 Computations of d_i^*, d_i^- and CC_i

	A_1	A_2	A_3	Ranking order
d_i^*	1.69	1.93	2.10	
d_i^-	2.23	1.95	1.82	
CC_i	0.57	0.50	0.46	$A_1 > A_2 > A_3$

Table 11 The fuzzy evaluation matrix with respect to goal

	C_1	C_2	C_3	C_4	C_5
C_1	(1,1,1)	(1, 1.67, 3)	(1, 2.33, 3)	(3, 4.33, 5)	(3, 3, 3)
C_2	(0.33, 0.77, 1)	(1,1,1)	(0.33, 2.11, 3)	(3, 3.67, 5)	(1, 2.33, 3)
C_3	(0.33, 0.56, 1)	(0.33, 1.22, 3)	(1,1,1)	(1,3,5)	(1, 1.67, 3)
C_4	(0.20, 0.24, 0.33)	(0.20,0.28, 0.33)	(0.20, 0.51, 1)	(1,1,1)	(0.33, 0.55, 1)
C_5	(0.33, 0.33, 0.33)	(0.33, 0.55, 1)	(0.33, 0.77, 1)	(1, 2.33, 3)	(1,1,1)

Three decision-makers' pair-wise comparisons of for the five criteria are as follows:

$$D_1 = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \end{array} \begin{array}{c} \begin{array}{ccccc} C_1 & C_2 & C_3 & C_4 & C_5 \end{array} \\ \begin{bmatrix} 1 & 3 & 1 & 5 & 3 \\ 1/3 & 1 & 1/3 & 3 & 1 \\ 1 & 3 & 1 & 5 & 3 \\ 1/5 & 1/3 & 1/5 & 1 & 1/3 \\ 1/3 & 1 & 1/3 & 3 & 1 \end{bmatrix} \end{array}$$

$$D_2 = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \end{array} \begin{array}{c} \begin{array}{ccccc} C_1 & C_2 & C_3 & C_4 & C_5 \end{array} \\ \begin{bmatrix} 1 & 1 & 3 & 3 & 3 \\ 1 & 1 & 3 & 3 & 3 \\ 1/3 & 1/3 & 1 & 1 & 1 \\ 1/3 & 1/3 & 1 & 1 & 1 \\ 1/3 & 1/3 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

$$D_3 = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \end{array} \begin{array}{c} \begin{array}{ccccc} C_1 & C_2 & C_3 & C_4 & C_5 \end{array} \\ \begin{bmatrix} 1 & 1 & 3 & 5 & 3 \\ 1 & 1 & 3 & 5 & 3 \\ 1/3 & 1/3 & 1 & 3 & 1 \\ 1/5 & 1/5 & 1/3 & 1 & 1/3 \\ 1/3 & 1/3 & 1 & 3 & 1 \end{bmatrix} \end{array}$$

Then, a comprehensive pair-wise comparison matrix is built as in Table 11 by integrating three decision-makers' grades through Eq. (27) [60]. By this way, decision-makers' pair-wise comparison values are transformed into triangular fuzzy numbers.

$$l_{je} = \min\left(b_{jep}\right), \quad m_{je} = \frac{\sum_{p=1}^{t} b_{jep}}{p}, \tag{27}$$

$$u_{je} = \max\left(b_{jep}\right), \quad p = 1, 2, \ldots.t \quad j = 1, 2, \ldots.m$$

$$e = 1, 2, \ldots m$$

$$\widetilde{b}_{je} = \left(l_{je}, m_{je}, u_{je}\right), \quad j = 1, 2, \ldots..m \quad e = 1, 2, \ldots.m$$

From Table 11, according to extent analysis synthesis values respect to main goal are calculated like in Eq. (6):

$$S_{C_1} = (9,\ 12.33,\ 15) \otimes (1/50.99, 1/37.22, 1/23.24)$$
$$= (0.177,\ 0.331,\ 0.645)$$

$$S_{C_2} = (5.66,\ 9.88,\ 13) \otimes (1/50.99, 1/37.22, 1/23.24)$$
$$= (0.111,\ 0.265,\ 0.559)$$

$$S_{C_3} = (3.66,\ 7.45,\ 13) \otimes (1/50.99, 1/37.22, 1/23.24)$$
$$= (0.072,\ 0.200,\ 0.559)$$

$$S_{C_4} = (1.93,\ 2.58,\ 3.66) \otimes (1/50.99, 1/37.22, 1/23.24)$$
$$= (0.038,\ 0.069,\ 0.157)$$

$$S_{C_5} = (2.99,\ 4.98,\ 6.33) \otimes (1/50.99, 1/37.22, 1/23.24)$$
$$= (0.059,\ 0.134,\ 0.272)$$

These fuzzy values are compared by using Eq. (12), and these values are obtained:

$$V(S_{C_1} \geq S_{C_2}) = 1, \quad V(S_{C_1} \geq S_{C_3}) = 1$$

$$V(S_{C_1} \geq S_{C_4}) = 1 \quad V(S_{C_1} \geq S_{C_5}) = 1$$

$$V(S_{C_2} \geq S_{C_1}) = 0.853, \quad V(S_{C_2} \geq S_{C_3}) = 1$$

$$V(S_{C_2} \geq S_{C_4}) = 1 \quad V(S_{C_2} \geq S_{C_5}) = 1$$

Table 12 The fuzzy evaluation matrix with respect to C_1

C_1	A_1	A_2	A_3
A_1	(1,1,1)	(0.33, 2.11, 3)	(3, 3.67, 5)
A_2	(0.33, 1.22, 3)	(1,1,1)	(1, 3, 5)
A_3	(0.2, 0.29, 0.33)	(0.33, 0.51, 1)	(1,1,1)

Table 13 The fuzzy evaluation matrix with respect to C_2

C_2	A_1	A_2	A_3
A_1	(1,1,1)	(3, 5.67, 7)	(3, 5, 7)
A_2	(0.14, 0.21, 0.33)	(1,1,1)	(0.33, 0.77, 1)
A_3	(0.14, 0.23, 0.33)	(1, 1.67, 3)	(1,1,1)

Table 14 The fuzzy evaluation matrix with respect to C_3

C_3	A_1	A_2	A_3
A_1	(1,1,1)	(0.2, 0.2, 0.2)	(0.14, 0.23, 0.33)
A_2	(5, 5, 5)	(1,1,1)	(0.33, 1.44, 3)
A_3	(3, 5, 7)	(0.33, 1.44, 3)	(1,1,1)

Table 16 The fuzzy evaluation matrix with respect to C_5

C_5	A_1	A_2	A_3
A_1	(1,1,1)	(1, 2.33, 5)	(3, 3, 3)
A_2	(0.2, 0.73, 1)	(1,1,1)	(0.33, 2.11, 3)
A_3	(0.33, 0.33, 0.33)	(0.33, 1.22, 3)	(1,1,1)

$$V(S_{C_3} \geq S_{C_1}) = 0.741, \quad V(S_{C_3} \geq S_{C_2}) = 0.873$$

$$V(S_{C_3} \geq S_{C_4}) = 1 \quad V(S_{C_3} \geq S_{C_5}) = 1$$

$$V(S_{C_4} \geq S_{C_1}) = 0 \quad V(S_{C_4} \geq S_{C_2}) = 0.190$$

$$V(S_{C_4} \geq S_{C_3}) = 0.393 \quad V(S_{C_4} \geq S_{C_5}) = 0.532$$

$$V(S_{C_5} \geq S_{C_1}) = 0.286, \quad V(S_{C_5} \geq S_{C_2}) = 0.551$$

$$V(S_{C_5} \geq S_{C_3}) = 0.752 \quad V(S_{C_5} \geq S_{C_4}) = 1$$

Then priority weights are calculated by using Eq. (13):

$$d'(C_1) = \min(1,\ 1,\ 1,\ 1) = 1$$

$$d'(C_2) = \min(0.853,\ 1,\ 1,\ 1) = 0.853$$

$$d'(C_3) = \min(0.741,\ 0.873,\ 1,\ 1) = 0.741$$

$$d'(C_4) = \min(0,\ 0.190,\ 0.393,\ 0.592) = 0$$

$$d'(C_5) = \min(0.286,\ 0.551,\ 0.752,\ 1) = 0.286$$

Priority weights form $W' = (1,\ 0.853,\ 0.741,\ 0,\ 0.286)$ vector. After the normalization of these values priority weight respect to main goal is calculated as (0.347, 0.269, 0.257, 0, 0.099).

After the priority weights of the criteria are determined, the priority of the alternatives will be determined for each criterion. From the pair-wise comparisons of the decision-makers for three alternatives, evaluation matrixes are formed as in Tables 12, 13, 14, 15 and 16. Then, priority weights of alternatives for each criterion are determined by making the same calculation like in Table 17.

Table 15 The fuzzy evaluation matrix with respect to C_4

C_4	A_1	A_2	A_3
A_1	(1,1,1)	(3, 3, 3)	(0.33, 2.78, 5)
A_2	(0.33, 0.33, 0.33)	(1,1,1)	(0.2, 1.4, 3)
A_3	(0.2, 1.17, 3)	(0.33, 2.11, 5)	(1,1,1)

The weight vector from Table 12 is calculated as (0.488, 0.432, 0.080).

The weight vector from Table 13 is calculated as (0.879, 0, 0.121).

The weight vector from Table 14 is calculated as (0.110, 0.445, 0.445).

The weight vector from Table 15 is calculated as (0.452, 0.165, 0.383).

The weight vector from Table 16 is calculated as (0.447, 0.302, 0.221).

Alternative A_1 which has the highest priority weight is selected as a best facility location for the textile company. The ranking order of the alternatives with fuzzy AHP method is $A_1 > A_2 > A_3$. We have reached the same result with fuzzy TOPSIS.

The company management found the application results satisfactory and decided to select the first alternative for facility location. Fuzzy AHP and fuzzy TOPSIS methods are both appropriate for the selection of a facility location or other multi-criteria decision-making problems of the company. But these two methods have some limitations and advantages. According to the problem the most appropriate method should be chosen.

We can summarize the differences and similarities between fuzzy AHP and fuzzy TOPSIS methods as follows:

- When these two methods are compared with respect to the amount of computations, fuzzy AHP requires more complex computations than fuzzy TOPSIS.
- Pair-wise comparisons for criteria and alternatives are made in fuzzy AHP, while there is no pair-wise comparison in fuzzy TOPSIS [61].
- TOPSIS has been proved to be one of the best methods addressing rank reversal issue that is the change in the ranking of the alternatives when a non-optimal alternative is introduced.
- Fuzzy TOPSIS works well for one-tier decision tree, while fuzzy AHP is preferable for widely spread

Table 17 Summary of priority weights of the main-attributes of the goal

	C_1	C_2	C_3	C_4	C_5	Alternative priority weight
Weight	0.347	0.296	0.257	0.000	0.099	
Alternative						
A_1	0.488	0.879	0.110	0.452	0.477	0.505
A_2	0.432	0.000	0.445	0.165	0.302	0.295
A_3	0.080	0.121	0.445	0.383	0.221	0.200

hierarchies, where few importance/rating pair-wise comparisons are required at lower level trees [54]. But, Kahraman et al. [61] proposed hierarchical fuzzy TOPSIS method for the problems that have complex structure.

- Through fuzzy AHP, the decision-maker is only asked to give judgments about either the relative importance of one criterion against another or its preference of one alterative on one criterion against another. However, when the number of alternatives and criteria grows, the pair-wise comparison process becomes cumbersome, and the risk of inconsistencies grows [54].
- In the extent analysis of fuzzy AHP [26] the priority weights of criterion or alternative can be equal to zero. In this situation, we do not take this criterion or alternative into consideration. This is the one of the disadvantages of this method.
- Fuzzy TOPSIS ranks alternatives measuring their relative distances to positive ideal solution and negative ideal solutions, providing then a meaningful performance measurement for each alternative. In fuzzy AHP, decision-makers make pair-wise comparisons and priority weights of alternatives are determined by the extent analysis method for the synthetic extent values of these values.
- Both in fuzzy AHP and fuzzy TOPSIS we can adopt linguistic variables.
- The ranking results of the fuzzy AHP and fuzzy TOPSIS are the same. This shows that when the decision-makers are consistent with himself/herself in determining the data, two method independently, the ranking results will be same.

6 Conclusions

Decision-makers face up to the uncertainty and vagueness from subjective perceptions and experiences in the decision-making process [62]. By using fuzzy AHP and fuzzy TOPSIS, uncertainty and vagueness from subjective perception and the experiences of decision-maker can be effectively represented and reached to a more effective decision.

In this study facility location selection with fuzzy AHP and fuzzy TOPSIS method has been proposed. The decision criteria were favorable labor climate, proximity to markets, community considerations, quality of life, proximity to suppliers and resources. These criteria were evaluated to determine the order of location alternatives for selecting the most appropriate one. Although two methods have the same objective of selecting the best facility location for the company, they have differences. In fuzzy TOPSIS decision-makers used the linguistic variables to asses the importance of the criteria and to evaluate the each alternative with respect to each criterion. These linguistic variables converted into triangular fuzzy numbers and fuzzy decision matrix was formed. Then normalized fuzzy decision matrix and weighted normalized fuzzy decision matrix were formed. After FPIS and FNIS were defined, distance of each alternative to FPIS and FNIS were calculated. And then the closeness coefficient of each alternative was calculated separately. According to the closeness coefficient of three alternatives, the ranking order of three alternatives has been determined as $A_1 > A_2 > A_3$. In fuzzy AHP, decision-makers made pair-wise comparisons for the criteria and alternatives under each criterion. Then these comparisons integrated and decision-makers' pair-wise comparison values are transformed into triangular fuzzy numbers. The priority weights of criteria and alternatives are determined by Chang's [26] extent analysis. According to the combination of the priority weights of criteria and alternatives, the best alternative is determined. According to the fuzzy AHP, the best alternative is A_1 and the ranking order of the alternatives is $A_1 > A_2 > A_3$ the same as fuzzy TOPSIS. Companies should choose the appropriate method for their problem according to the situation and the structure of the problem they have.

In future studies, other multi-criteria methods like fuzzy PROMETHEE and ELECTRE can be used to handle facility location selection problems. And also the proposed methods can be applied to other multi-criteria decision problems like supplier selection, personnel selection, software selection, project selection and machine selection of companies.

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