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ORIGINAL ARTICLE

Double sampling $\overline{x}$ control chart for a first-order autoregressive moving average process model

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Abstract In this paper, we consider the double sampling (DS) $\overline{X}$control chart for monitoring processes in which the observations can be represented as a first-order autoregressive moving average (ARMA(1, 1)) model. The properties of the DS $\overline{X}$ control chart with the sampling intervals driven by the rational subgroup concept are studied and compared with the Shewhart chart and the variable sample size (VSS) chart, both properly modified to account for the serial correlation. Numerical results show that the correlation within subgroups has a significant impact on the properties of the charts. For processes with low to moderate correlation levels, the DS $\overline{X}$chart is substantially more efficient in detecting process mean shifts.

Keywords Autocorrelation · Average run length · Control chart · Double sampling · First-order autoregressive moving average process · Statistical process control

1 Introduction

A Shewhart control chart is a statistical decision rule applied to data from a process to determine if a quality characteristic has shifted from its target setting [1]. This form of process monitoring is widely used in industry to distinguish common causes from special causes of variation (often responsible for process shifts) and also to identify the time of a process change in order to facilitate understanding its root cause and improving the process by preventing its future occurrence.

Despite being a powerful problem-solving and process-improvement tool of inherently simple application, one of the major drawbacks of the Shewhart $\overline{X}$ chart is its slowness to detect small shifts in the process mean (large out-of-control average run length [ARL]). For this reason, the utilization of adaptive or dynamic control charts has progressively grown in the last two decades, contributing to a remarkable improvement in the effectiveness of process monitoring.

A control chart is considered adaptive if at least one of its parameters—sampling interval, sample size, or control limits width—is allowed to change in real time based on the actual values of the sample statistics which provide current information about the status of the process [2]. Considerable efforts have already been undertaken on the research of the statistical properties of the adaptive Shewhart charts with variable sampling interval [3–9], with variable sample size [10–14], and with both the sampling interval and the sample size being variable [15–24].

The double sampling (DS) procedure is another alternative that has been used to improve the performance of the traditional Shewhart charts [25–27]. He and Grigoryan [28, 29] considered the joint double sampling $\overline{X}$and s charts and extended the double sampling procedure to study multivariate multiple sampling charts.

The usual way to evaluate the statistical performance of control charts with fixed sampling interval is to measure their average run length (ARL). The DS $\overline{X}$ chart offers faster process shift detection than the standard Shewhart chart, but at the expense of a larger administration burden. The DS $\overline{X}$ chart also outperforms the variable sampling

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Table 1 Autocorrelation coefficient at lag 1 for the selected combinations of autoregressive and moving average coefficients

ϕ	θ	ρ_1
0.950	0.900	0.073
	0.450	0.824
	0.000	0.950
	−0.450	0.971
	−0.900	0.975
0.475	0.450	0.025
	0.000	0.475
	−0.450	0.689
	−0.900	0.737
0.000	0.000	0.000
−0.475	−0.450	0.374
	−0.900	0.497
	−0.900	0.255

interval $\overline{X}$ chart and performs better than the static exponentially weighted moving average (EWMA) and cumulative sum (CUSUM) charts for large shifts, although it is not as effective in detecting small shifts.

Control charts are usually designed and evaluated assuming that the process observations are independent and identically distributed (i.i.d.). However, this assumption is often violated in practice, as, in reality, most process data are serial correlated. The design of tolerance limits also needs to be properly modified when the quality characteristic to be monitored follows an autocorrelated process [30].

If the autocorrelation found in process monitoring is a special cause, then attempts should be made to eliminate it. On the other hand, if it is an inherent part of the common cause variability and cannot be disregarded, then it must be accounted for in the design of the control chart to avoid biased estimates of process parameters that greatly increase the rate of false alarms and the time, or the number of samples, required to detect shifts in the process mean [31–34]. To deal with autocorrelated data, various charting techniques have been proposed. In many applications, the dynamics of the process induces correlation in observations closely spaced in time. If the interval between observations is short enough to produce correlation, then one simple approach is to skip some of them, with the consequent disadvantage of disregarding the concept of rational subgroups.

A second alternative would be to map the problem back to uncorrelated data and apply traditional control charts [35–37]. The option of widening the control limits has also been suggested as a remedial method to deal with the data autocorrelation [38]. Vasilopoulos and Stamboulis [39] formulated control charts with the control limits width tailored to processes described by a first- or second-order autoregressive model.

A more typical approach is to fit an appropriate time-series model to the observations and charting the i.i.d. forecast errors or residuals [40–44]. In addition to the fact that forecasts "recover" from abrupt changes and, thus, leave only a limited "window of opportunity" for detection [31], this approach is not well-suited for detecting small shifts in processes with mild positive autocorrelation [32, 45, 46], users may have difficulty interpreting residual control charts [47], and, besides this, fitting and maintaining an appropriate time-series model for each required process variable can be cumbersome. Control charts based on residuals seem to work better when the level of correlation is high. Thus, there will be

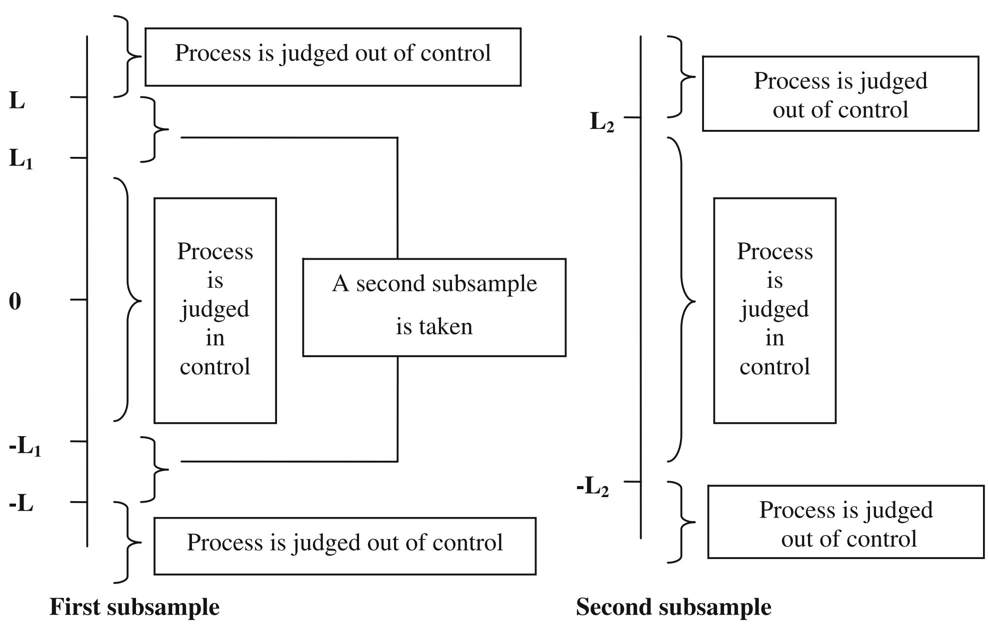

Fig. 1 Graphical view of the double sampling (DS) chart

Table 2 Control limits coefficients for the DS chart (n_1=1)

		$\overline{n}=3$			$\overline{n}=4$			$\overline{n}=5$		
		n_2=4	n_2=6	n_2=8	n_2=6	n_2=8	n_2=12	n_2=9	n_2=12	n_2=18
L_1		0.6745	0.9674	1.1503	0.6745	0.8871	1.1503	0.7647	0.9674	1.2206
ϕ	θ	L_2			L_2			L_2		
0.950	0.900	2.9497	2.8697	2.7900	2.9240	2.8612	2.7409	2.8790	2.8019	2.6663
	0.450	2.9861	2.9398	2.8772	2.9685	2.9240	2.8173	2.9307	2.8637	2.7280
	0.000	2.9919	2.9520	2.8917	2.9759	2.9342	2.8286	2.9387	2.9170	2.7362
	−0.450	2.9937	2.9565	2.8974	2.9785	2.9381	2.8330	2.9419	2.8763	2.7393
	−0.900	2.9947	2.9591	2.9003	2.9800	2.9401	2.8353	2.9436	2.8781	2.7410
0.475	0.450	2.9380	2.8456	2.7542	2.9085	2.8350	2.7039	2.8567	2.7722	2.6281
	0.000	2.9616	2.8733	2.7797	2.9263	2.8537	2.7172	2.8700	2.7827	2.6362
	−0.450	2.9688	2.8831	2.7882	2.9327	2.8600	2.7218	2.8744	2.7864	2.6390
	−0.900	2.9724	2.8874	2.7922	2.9355	2.8629	2.7243	2.8763	2.7885	2.6399
0.000	0.000	2.9360	2.8420	2.7521	2.9059	2.8338	2.7011	2.8554	2.7696	2.6274
	−0.450	2.9457	2.8527	2.7610	2.9130	2.8403	2.7061	2.8602	2.7736	2.6308
	−0.900	2.9496	2.8569	2.7642	2.9157	2.8424	2.7089	2.8622	2.7760	2.6313
−0.475	−0.900	2.9384	2.8465	2.7556	2.9091	2.8361	2.7034	2.8569	2.7715	2.6284

Table 3 Control limits coefficients for the DS chart (n_1=2)

		$\overline{n}=3$			$\overline{n}=4$			$\overline{n}=5$		
		n_2=4	n_2=6	n_2=8	n_2=6	n_2=8	n_2=12	n_2=9	n_2=12	n_2=18
L_1		1.1503	1.3830	1.5341	0.9674	1.1503	1.3830	0.9674	1.1503	1.3880
ϕ	θ	L_2			L_2			L_2		
0.950	0.900	2.9539	2.8740	2.7877	2.9438	2.8885	2.7701	2.9093	2.8391	2.7018
	0.450	2.9947	2.9611	2.9040	2.9886	2.9595	2.8692	2.9667	2.9153	2.7895
	0.000	2.9960	2.9704	2.9181	2.9908	2.9658	2.8794	2.9708	2.9220	2.7972
	−0.450	2.9984	2.9758	2.9256	2.9938	2.9707	2.8858	2.9749	2.9274	2.8028
	−0.900	2.9997	2.9787	2.9296	2.9954	2.9733	2.8893	2.9771	2.9302	2.8059
0.475	0.450	2.9366	2.8354	2.7303	2.9213	2.8505	2.7113	2.8770	2.7927	2.6419
	0.000	2.9660	2.8741	2.7676	2.9435	2.8747	2.7323	2.8947	2.8091	2.6537
	−0.450	2.9738	2.8860	2.7794	2.9501	2.8826	2.7387	2.8998	2.8141	2.6576
	−0.900	2.9801	2.8943	2.7872	2.9558	2.8886	2.7442	2.9051	2.8191	2.9576
0.000	0.000	2.9327	2.8297	2.7266	2.9173	2.8477	2.7080	2.8741	2.7899	2.6398
	−0.450	2.9465	2.8462	2.7411	2.9279	2.8579	2.7173	2.8820	2.7976	2.6453
	−0.900	2.9491	2.8492	2.7438	2.9283	2.8583	2.7176	2.8819	2.7970	2.6443
−0.475	−0.900	2.9352	2.8351	2.7300	2.9201	2.8489	2.7102	2.8757	2.7912	2.6402

Table 4 Probability of taking the second subsample (DS chart, n_1=1)

δ	$\overline{n}=3$			$\overline{n}=4$			$\overline{n}=5$		
	n_2=4	n_2=6	n_2=8	n_2=6	n_2=8	n_2=12	n_2=9	n_2=12	n_2=18
0.00	0.50	0.33	0.25	0.50	0.38	0.25	0.44	0.33	0.22
0.25	0.51	0.35	0.26	0.51	0.39	0.26	0.46	0.35	0.24
0.50	0.55	0.39	0.31	0.55	0.43	0.31	0.50	0.39	0.28
0.75	0.61	0.46	0.37	0.61	0.50	0.37	0.56	0.46	0.34
1.00	0.67	0.54	0.46	0.67	0.57	0.46	0.63	0.54	0.43
1.25	0.74	0.62	0.55	0.74	0.66	0.55	0.71	0.62	0.52
1.50	0.81	0.71	0.64	0.81	0.74	0.64	0.78	0.71	0.61
1.75	0.87	0.78	0.73	0.87	0.81	0.73	0.84	0.79	0.70
2.00	0.91	0.85	0.80	0.91	0.87	0.80	0.89	0.85	0.78

Table 5 Average run length (ARL) for the DS chart ($\overline{n} = 3,\ n_1 = 1$)

n_2	ϕ	θ	δ								
			0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
4	0.950	0.900	370.4	153.9	43.4	14.8	6.3	3.3	2.0	1.5	1.2
		0.450	370.4	267.9	137.3	68.0	35.3	19.5	11.5	7.2	4.8
		0.000	370.4	316.6	216.3	135.8	84.5	53.5	34.8	23.3	16.1
		−0.450	370.4	338.4	266.8	193.6	136.1	95.4	67.3	48.2	35.0
		−0.900	370.4	349.5	298.0	236.9	181.5	136.8	102.9	77.7	59.1
	0.475	0.450	370.4	142.0	37.6	12.5	5.3	2.8	1.8	1.4	1.2
		0.000	370.4	224.9	91.5	38.6	18.0	9.3	5.4	3.4	2.4
		−0.450	370.4	275.6	147.5	75.5	40.3	22.7	13.6	8.6	5.7
		−0.900	370.4	305.3	194.6	114.9	68.2	41.7	26.5	17.4	11.8
	0.000	0.000	370.4	136.5	35.1	11.5	4.9	2.6	1.7	1.3	1.2
		−0.450	370.4	196.3	68.9	26.5	11.7	6.0	3.5	2.3	1.7
		−0.900	370.4	240.4	106.2	47.3	22.9	12.1	7.0	4.4	3.0
	−0.475	−0.900	370.4	179.5	57.8	21.1	9.2	4.7	2.8	1.9	1.4
6	0.950	0.900	370.4	132.8	33.7	11.1	4.8	2.6	1.8	1.4	1.2
		0.450	370.4	270.7	141.1	70.8	37.2	20.8	12.3	7.8	5.2
		0.000	370.4	320.6	224.9	145.4	93.0	60.5	40.3	27.6	19.4
		−0.450	370.4	341.2	274.6	204.8	148.4	107.1	77.9	57.3	42.7
		−0.900	370.4	351.6	304.4	247.6	194.7	151.1	116.9	90.8	71.0
	0.475	0.450	370.4	116.0	27.0	8.7	3.8	2.2	1.6	1.3	1.2
		0.000	370.4	202.7	73.6	28.9	13.0	6.6	3.8	2.5	1.8
		−0.450	370.4	256.8	124.0	58.9	29.8	16.2	9.5	6.0	4.0
		−0.900	370.4	289.1	167.6	91.6	51.4	30.2	18.7	12.1	8.2
	0.000	0.000	370.4	110.4	24.9	8.0	3.6	2.1	1.5	1.3	1.2
		−0.450	370.4	168.4	51.3	18.3	7.9	4.1	2.5	1.7	1.4
		−0.900	370.4	213.2	81.7	33.2	15.1	7.8	4.5	2.9	2.0
	−0.475	−0.900	370.4	151.0	42.1	14.4	6.2	3.2	2.1	1.5	1.3
8	0.950	0.900	370.4	120.2	28.8	9.5	4.2	2.4	1.7	1.4	1.3
		0.450	370.4	271.9	142.8	72.2	38.2	21.4	12.8	8.1	5.4
		0.000	370.4	320.6	225.4	146.5	94.5	62.2	42	29.1	20.7
		−0.450	370.4	340.2	272.3	202.7	147.4	107.3	79.1	59.1	44.8
		−0.900	370.4	350.0	300.2	242.3	190.2	148.3	116.0	91.4	72.6
	0.475	0.450	370.4	99.5	21.6	7.0	3.3	2.1	1.6	1.4	1.2
		0.000	370.4	185.5	61.7	23.1	10.1	5.2	3.1	2.1	1.6
		−0.450	370.4	240.6	106.5	47.5	23.0	12.2	7.1	4.4	3.0
		−0.900	370.4	274.3	146.1	74.7	39.9	22.6	13.6	8.7	5.8
	0.000	0.000	370.4	94.1	19.8	6.5	3.1	2	1.6	1.4	1.2
		−0.450	370.4	149.1	41.3	14.2	6.1	3.3	2.1	1.6	1.3
		−0.900	370.4	193.3	66.9	25.6	11.3	5.8	3.4	2.3	1.7
	−0.475	−0.900	370.4	131.9	33.5	11.2	4.9	2.7	1.9	1.5	1.3

Table 6 ARL for the DS chart ($\overline{n} = 4,\ n_1 = 1$)

n_2	ϕ	θ	δ								
			0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
6	0.950	0.900	370.4	130.0	32.3	10.5	4.5	2.4	1.6	1.3	1.1
		0.450	370.4	272.5	143.4	72.5	38.2	21.4	12.7	8.0	5.4
		0.000	370.4	322.9	229.9	150.4	97.0	63.3	42.3	28.9	20.3
		−0.450	370.4	343.3	280.1	211.7	154.8	112.4	81.9	60.3	44.9
		−0.900	370.4	353.2	309.4	254.9	202.5	158.2	122.9	95.6	74.8
	0.475	0.450	370.4	112.7	25.5	8.0	3.5	2.0	1.4	1.2	1.1
		0.000	370.4	202.8	73.6	28.9	12.9	6.6	3.8	2.5	1.8
		−0.450	370.4	259.2	126.8	60.7	30.9	16.9	9.9	6.2	4.2

Table 6 (continued)

n_2	ϕ	θ	δ								
			0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
		−0.900	370.4	292.6	173.0	96.0	54.4	32.2	19.9	12.9	8.7
	0.000	0.000	370.4	106.9	23.4	7.3	3.2	1.9	1.4	1.2	1.1
		−0.450	370.4	167.0	50.4	17.8	7.6	3.9	2.4	1.7	1.3
		−0.900	370.4	213.9	82.2	33.4	15.3	7.9	4.5	2.9	2.0
	−0.475	−0.900	370.4	148.9	40.8	13.8	5.9	3.1	1.9	1.4	1.2
8	0.950	0.900	370.4	116.4	27.1	8.7	3.8	2.2	1.6	1.3	1.2
		0.450	370.4	274.3	146.0	74.5	39.7	22.4	13.4	8.5	5.7
		0.000	370.4	324.1	232.9	154.2	100.9	66.9	45.4	31.5	22.4
		−0.450	370.4	343.5	281.1	214.0	158.3	116.6	86.4	64.8	49.1
		−0.900	370.4	353.0	309.1	255.3	204.3	161.4	127.3	100.7	80.1
	0.475	0.450	370.4	95.2	19.8	6.3	2.9	1.8	1.4	1.2	1.2
		0.000	370.4	184.5	61.0	22.7	9.9	5.0	3.0	2.0	1.5
		−0.450	370.4	242.6	108.4	48.7	23.7	12.6	7.3	4.6	3.1
		−0.900	370.4	277.9	151.0	78.4	42.3	24.2	14.6	9.3	6.3
	0.000	0.000	370.4	89.6	18.1	5.7	2.7	1.8	1.4	1.2	1.2
		−0.450	370.4	146.4	39.8	13.4	5.7	3.0	1.9	1.5	1.2
		−0.900	370.4	192.8	66.4	25.3	11.2	5.7	3.3	2.2	1.6
	−0.475	−0.900	370.4	128.5	31.8	10.4	4.5	2.5	1.7	1.3	1.2
12	0.950	0.900	370.4	101.0	22.0	7.2	3.4	2.1	1.6	1.4	1.2
		0.450	370.4	275.0	147.1	75.4	40.4	22.9	13.8	8.8	5.9
		0.000	370.4	322.1	228.9	150.7	98.6	65.9	45.2	31.9	23.0
		−0.450	370.4	340.0	272.2	203.3	148.9	109.7	82.0	62.3	48.1
		−0.900	370.4	348.9	297.3	238.8	187.5	147.0	116.2	92.9	75.1
	0.475	0.450	370.4	74.3	14.3	4.9	2.6	1.9	1.6	1.4	1.2
		0.000	370.4	158.4	45.9	16.1	6.9	3.6	2.3	1.7	1.3
		−0.450	370.4	217.0	84.7	34.7	16.0	8.2	4.7	3.0	2.1
		−0.900	370.4	254.4	121.1	56.8	28.5	15.4	9.0	5.7	3.8
	0.000	0.000	370.4	69.3	13.1	4.5	2.5	1.9	1.6	1.4	1.2
		−0.450	370.4	119.7	28.6	9.4	4.2	2.4	1.7	1.4	1.3
		−0.900	370.4	163.6	48.7	17.2	7.4	3.9	2.4	1.7	1.4
	−0.475	−0.900	370.4	102.9	22.7	7.4	3.4	2.1	1.6	1.4	1.2

Table 7 ARL for the DS chart ($\overline{n} = 5,\ n_1 = 1$)

n_2	ϕ	θ	δ								
			0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
9	0.950	0.900	370.4	108.9	24.2	7.6	3.4	2.0	1.4	1.2	1.1
		0.450	370.4	276.8	149.3	77.0	41.3	23.4	14.1	8.9	6.0
		0.000	370.4	326.5	238.2	160.2	106.1	71.1	48.6	34.0	24.3
		−0.450	370.4	345.2	286.0	220.8	165.4	123.1	92.1	69.5	53.1
		−0.900	370.4	354.3	313.0	261.6	211.8	169.0	134.4	107.2	85.8
	0.475	0.450	370.4	85.8	16.8	5.3	2.5	1.6	1.3	1.2	1.1
		0.000	370.4	176.3	55.8	20.2	8.7	4.5	2.7	1.8	1.4
		−0.450	370.4	236.8	102.6	45.1	21.7	11.4	6.6	4.1	2.8
		−0.900	370.4	274.1	145.6	74.2	39.5	22.3	13.4	8.5	5.7
	0.000	0.000	370.4	80.3	15.3	4.8	2.3	1.6	1.3	1.2	1.1
		−0.450	370.4	136.5	35.1	11.6	4.9	2.7	1.7	1.3	1.2
		−0.900	370.4	183.8	60.5	25.6	9.8	5.0	2.9	2.0	1.5
	−0.475	−0.900	370.4	118.3	27.7	8.8	3.8	2.2	1.5	1.3	1.1
12	0.950	0.900	370.4	97.9	20.7	6.6	3.1	1.9	1.5	1.3	1.2
		0.450	370.4	277.4	150.2	77.8	41.9	23.9	14.4	9.2	6.2
		0.000	370.4	326.1	237.5	160.1	106.8	72.3	50.1	35.6	25.8

Table 7 (continued)

n_2	ϕ	θ	δ								
			0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
		−0.450	370.4	343.1	280.5	214.0	159.6	119.0	89.6	68.4	52.9
		−0.900	370.4	351.8	305.7	251.2	201.1	159.9	127.6	102.6	83.2
	0.475	0.450	370.4	70.9	13.1	4.3	2.3	1.7	1.4	1.3	1.2
		0.000	370.4	156.8	45.0	15.6	6.7	3.5	2.2	1.6	1.3
		−0.450	370.4	217.7	85.2	35.0	16.1	8.3	4.8	3.1	2.1
		−0.900	370.4	256.6	123.7	58.5	29.5	16.1	9.4	5.9	4.0
	0.000	0.000	370.4	65.9	11.9	4.0	2.2	1.6	1.4	1.3	1.2
		−0.450	370.4	117.0	27.4	8.8	3.9	2.2	1.6	1.3	1.2
		−0.900	370.4	162.2	47.8	16.8	7.2	3.7	2.3	1.6	1.3
	−0.475	−0.900	370.4	99.9	21.4	6.8	3.1	1.9	1.5	1.3	1.2
18	0.950	0.900	370.4	85.8	17.6	5.9	3.0	2.0	1.6	1.4	1.3
		0.450	370.4	276.0	148.3	76.4	41.0	23.4	14.1	9.0	6.1
		0.000	370.4	320.6	226.0	147.9	96.6	64.6	44.6	31.7	23.1
		−0.450	370.4	337.1	265.0	194.5	140.9	103.3	77.3	59.0	46.0
		−0.900	370.4	345.3	287.2	224.9	173.2	134.3	105.8	84.7	69.0
	0.475	0.450	370.4	54.1	9.9	3.8	2.4	1.9	1.6	1.4	1.3
		0.000	370.4	131.0	33.2	11.1	4.9	2.7	1.9	1.5	1.3
		−0.450	370.4	190.6	64.9	24.5	10.8	5.5	3.3	2.2	1.6
		−0.900	370.4	231.1	96.8	41.4	19.5	10.2	5.8	3.7	2.5
	0.000	0.000	370.4	49.9	9.0	3.6	2.4	1.9	1.6	1.4	1.3
		−0.450	370.4	93.2	19.7	6.5	3.2	2.1	1.7	1.4	1.3
		−0.900	370.4	134.2	34.5	11.6	5.1	2.8	1.9	1.5	1.3
	−0.475	−0.900	370.4	78.0	15.4	5.3	2.8	2.0	1.6	1.4	1.3

Table 8 ARL and probability of taking the second subsample for the DS chart ($\overline{n} = 3,\ n_1 = 2,\ n_2 = 4$)

Chart property	ϕ	θ	δ								
			0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
ARL	0.950	0.900	370.4	139.3	36.3	11.9	5.0	2.6	1.7	1.3	1.1
		0.450	370.4	271.8	142.3	71.5	37.6	20.9	12.4	7.8	5.2
		0.000	370.4	321.7	226.9	146.8	93.5	78.3	39.8	27.0	18.7
		−0.450	370.4	342.5	277.7	207.9	150.3	107.7	77.5	56.4	41.5
		−0.900	370.4	352.4	307.1	251.0	197.5	152.6	117.3	90.2	69.7
	0.475	0.450	370.4	124.7	30.0	9.6	4.1	2.2	1.5	1.2	1.1
		0.000	370.4	214.3	82.5	33.6	15.4	7.9	4.5	2.9	2.0
		−0.450	370.4	270.1	140.2	70.0	36.6	20.0	12.0	7.6	5.0
		−0.900	370.4	303.1	190.3	110.6	64.7	39.0	24.4	15.9	10.7
	0.000	0.000	370.4	118.9	27.7	8.7	3.7	2.1	1.4	1.2	1.1
		−0.450	370.4	181.8	59.2	21.8	9.5	4.8	2.9	1.9	1.5
		−0.900	370.4	229.4	95.7	41.1	19.4	10.1	5.8	3.7	2.5
	−0.475	−0.900	370.4	163.7	48.7	17.1	7.4	3.8	2.3	1.6	1.3
Probability of taking the second subsample	0.950	0.900	0.25	0.28	0.36	0.47	0.60	0.72	0.82	0.90	0.95
		0.450	0.25	0.27	0.32	0.40	0.50	0.60	0.70	0.79	0.86
		0.000	0.25	0.26	0.30	0.35	0.43	0.51	0.59	0.68	0.75
		−0.450	0.25	0.26	0.28	0.33	0.38	0.44	0.51	0.58	0.65
		−0.900	0.25	0.26	0.28	0.31	0.35	0.40	0.45	0.51	0.58
	0.475	0.450	0.25	0.28	0.36	0.47	0.60	0.73	0.83	0.90	0.95
		0.000	0.25	0.27	0.32	0.40	0.50	0.61	0.71	0.79	0.86
		−0.450	0.25	0.26	0.30	0.36	0.43	0.51	0.60	0.68	0.76
		−0.900	0.25	0.26	0.28	0.33	0.38	0.45	0.52	0.59	0.66
	0.000	0.000	0.25	0.28	0.36	0.48	0.61	0.73	0.83	0.91	0.95

Table 8 (continued)

Chart property	ϕ	θ	δ								
			0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
		−0.450	0.25	0.27	0.32	0.41	0.51	0.61	0.71	0.80	0.87
		−0.900	0.25	0.26	0.30	0.36	0.43	0.52	0.60	0.69	0.76
	−0.475	−0.900	0.25	0.27	0.32	0.41	0.51	0.62	0.72	0.81	0.87

many applications for which it will be desirable, from both practical and statistical considerations, to apply a standard control chart to the original correlated observations; in this case, being necessary to determine the effect of the correlation and how to adjust the chart parameters to account for the correlation [33].

It has already been shown that, when the correlation ranges from low to moderate, the $\overline{X}$ chart with variable sampling interval detects process shifts faster than the $\overline{X}$ chart with fixed sampling interval [48]. However, to our knowledge, the application of the DS scheme for the monitoring of autocorrelated processes has not been studied yet.

2 The process model

The primary purpose of this paper is to address the monitoring of autocorrelated processes with the observations collected in rational subgroups, as is often the case in the application of statistical process control, particularly in process industries. The objective in forming rational subgroups is not only to capture the inherent short-term variability within the subgroups, but also to allow all potential sources of variation to act between subgroups [1].

As the process autocorrelation attenuates to zero when the time between samples increases, only the within-

Table 9 ARL and probability of taking the second subsample for the DS chart ($\overline{n}=3,\ n_1=2,\ n_2=6$)

Chart property	ϕ	θ	δ								
			0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
ARL	0.950	0.900	370.4	122.6	29.3	9.4	4.1	2.2	1.5	1.2	1.1
		0.450	370.4	274.4	146.0	74.4	39.6	21.6	13.3	8.4	5.6
		0.000	370.4	325.6	235.8	156.8	102.5	67.7	45.6	31.5	22.2
		−0.450	370.4	345.5	286.2	220.2	163.6	120.3	88.7	65.9	49.5
		−0.900	370.4	354.8	314.3	262.8	212.0	167.9	132.1	103.9	82.0
	0.475	0.450	370.4	104.2	22.6	7.1	3.2	1.9	1.4	1.2	1.1
		0.000	370.4	194.7	67.8	26.0	11.6	5.9	3.4	2.3	1.7
		−0.450	370.4	254.3	121.1	56.8	28.5	15.4	9.0	5.6	3.8
		−0.900	370.4	290.9	170.0	93.0	52.0	30.3	18.5	11.8	7.9
	0.000	0.000	370.4	98.5	20.8	6.5	2.9	1.8	1.4	1.2	1.1
		−0.450	370.4	159.3	46.4	16.3	7.0	3.7	2.3	1.6	1.3
		−0.900	370.4	208.5	78.2	31.5	14.4	7.4	4.3	2.8	2.0
	−0.475	−0.900	370.4	141.5	37.8	12.8	5.5	3.0	2.0	1.5	1.3
Probability of taking the second subsample	0.950	0.900	0.17	0.19	0.26	0.37	0.50	0.63	0.75	0.85	0.92
		0.450	0.17	0.18	0.23	0.30	0.40	0.50	0.61	0.71	0.80
		0.000	0.17	0.18	0.21	0.26	0.33	0.41	0.50	0.59	0.67
		−0.450	0.17	0.17	0.20	0.24	0.29	0.35	0.41	0.49	0.56
		−0.900	0.17	0.17	0.19	0.22	0.26	0.30	0.36	0.42	0.48
	0.475	0.450	0.17	0.19	0.27	0.38	0.51	0.64	0.76	0.86	0.92
		0.000	0.17	0.18	0.23	0.31	0.40	0.51	0.62	0.72	0.81
		−0.450	0.17	0.18	0.21	0.26	0.33	0.41	0.50	0.59	0.68
		−0.900	0.17	0.17	0.20	0.24	0.29	0.35	0.42	0.49	0.57
	0.000	0.000	0.17	0.19	0.27	0.38	0.52	0.65	0.77	0.86	0.93
		−0.450	0.17	0.18	0.23	0.31	0.41	0.52	0.63	0.73	0.81
		−0.900	0.17	0.18	0.21	0.27	0.34	0.42	0.51	0.60	0.68
	−0.475	−0.900	0.17	0.18	0.23	0.31	0.41	0.52	0.63	0.74	0.82

Table 10 ARL and probability of taking the second subsample for the DS chart ($\overline{n}=3,\ n_1=2,\ n_2=8$)

Chart property	ϕ	θ	δ								
			0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
ARL	0.950	0.900	370.4	112.5	25.7	8.3	3.7	2.1	1.5	1.3	1.1
		0.450	370.4	274.6	146.5	75.0	40.0	22.7	13.6	8.6	5.8
		0.000	370.4	325.8	236.4	158.0	104.1	69.4	47.3	32.9	23.5
		−0.450	370.4	345.4	286.2	220.6	164.6	120.8	90.7	68.1	47.5
		−0.900	370.4	354.6	313.8	262.5	212.2	168.8	133.7	106.0	84.5
	0.475	0.450	370.4	91.2	18.8	6.1	2.9	1.8	1.4	1.2	1.1
		0.000	370.4	179.7	58.1	21.5	9.4	4.8	2.9	2.0	1.5
		−0.450	370.4	241.0	106.9	47.8	23.2	12.3	7.1	4.5	3.1
		−0.900	370.4	280.0	153.6	80.0	43.1	24.5	14.7	9.3	6.2
	0.000	0.000	370.4	85.9	17.3	5.6	2.7	1.8	1.4	1.2	1.1
		−0.450	370.4	144.8	39.0	13.4	5.9	3.2	2.1	1.6	1.3
		−0.900	370.4	193.1	67.3	26.1	11.8	6.2	3.7	2.5	1.9
	−0.475	−0.900	370.4	126.7	31.7	10.7	4.8	2.7	1.9	1.5	1.3
Probability of taking the second subsample	0.950	0.900	0.13	0.15	0.21	0.31	0.44	0.58	0.7	0.81	0.89
		0.450	0.13	0.14	0.18	0.25	0.34	0.44	0.55	0.66	0.75
		0.000	0.13	0.13	0.16	0.21	0.27	0.35	0.44	0.53	0.61
		−0.450	0.13	0.13	0.15	0.19	0.23	0.29	0.36	0.43	0.5
		−0.900	0.13	0.13	0.15	0.17	0.21	0.25	0.3	0.36	0.42
	0.475	0.450	0.13	0.15	0.21	0.32	0.45	0.58	0.71	0.82	0.9
		0.000	0.13	0.14	0.18	0.25	0.34	0.45	0.56	0.67	0.76
		−0.450	0.13	0.13	0.16	0.21	0.28	0.35	0.44	0.53	0.62
		−0.900	0.13	0.13	0.15	0.19	0.23	0.29	0.36	0.43	0.51
	0.000	0.000	0.13	0.15	0.22	0.32	0.45	0.59	0.72	0.83	0.9
		−0.450	0.13	0.14	0.18	0.26	0.35	0.46	0.57	0.67	0.77
		−0.900	0.13	0.14	0.17	0.21	0.28	0.36	0.45	0.54	0.63
	−0.475	−0.900	0.13	0.14	0.19	0.26	0.35	0.46	0.58	0.68	0.78

subgroup autocorrelation needs to be modeled (the correlation between subgroups is considered to be negligible).

Throughout this paper, we assume that the observations of the quality characteristic to be monitored can be represented by a first-order autoregressive moving average (ARMA(1, 1) model that has frequently been used in applications [45, 49–52] and is equivalent to the AR(1) process with an additional random error [53], which is considered to be the natural model for representing the process observations in most of the applications [32–34, 48]. The observations X_t for an ARMA(1, 1) process are given by:

$$X_t=(1-\phi)\mu+\phi X_{t-1}-\theta\varepsilon_{t-1}+\varepsilon_t,\quad t=1,\ 2,\ldots,\ n \tag{1}$$

where ϕ is the autoregressive parameter of the AR(1) process, μ is the process mean, θ is the moving average parameter, and the ε_ts are the random shock components of the ARMA(1, 1) process, which are i.i.d. normal random variables with mean zero and variance one. Additionally, we consider that the time-series model is accurate. The numerical results presented in this paper are for the case of positive autocorrelation, which are the most likely to be found in actual manufacturing environments. The selected combinations of ϕ and θ, borrowed from a previous work by Wardell et al. [49], are shown in Table 1 and the autocorrelation coefficient at lag 1 was obtained through the expression $\rho_1=\frac{(1-\phi_1\theta_1)(\phi_1-\theta_1)}{1+\theta_1^2-2\phi_1\theta_1}$, given by Box et al. [53].

3 The double sampling $\overline{X}$ chart for autocorrelated processes

The DS procedure assumes that one master sample, comprising two successive subsamples, can be taken at each sampling without any intervening time and, therefore, come from the same probability distribution. The DS procedure is carried out as follows.

First stage Take a subsample of size n_1 and compute the mean $\overline{X}_1$. Let $Z_1=\left(\overline{X}_1-\mu_0\right)/\sigma_{\overline{X}_1}$ where $\overline{X}_1\sim N\left(\mu_0,\sigma_{\overline{X}_1}\right)$, with μ_0 being the in-control process mean and $\sigma_{\overline{X}_1}$ the standard deviation of the first subsample (see Appendix). When the process is out of control, μ=μ_1=μ_0±$\delta\sigma$, σ being the standard deviation of observations spaced enough to eliminate the

Table 11 ARL and probability of taking the second subsample for the DS chart ($\overline{n}=4,\ n_1=2,\ n_2=6$)

Chart property	ϕ	θ	δ								
			0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
ARL	0.950	0.900	370.4	119.4	27.9	8.8	3.7	2.1	1.4	1.2	1.1
		0.450	370.4	276.8	149.2	76.7	41.0	23.1	13.8	8.7	5.8
		0.000	370.4	327.5	240.0	161.3	106.2	70.4	47.5	32.8	23.1
		−0.450	370.4	346.9	290.0	222.9	168.6	176.2	92.2	68.6	51.5
		−0.900	370.4	355.6	317.2	267.3	217.3	173.1	136.7	107.8	85.2
	0.475	0.450	370.4	100.1	21.0	6.4	2.8	1.7	1.3	1.1	1.0
		0.000	370.4	180.7	67.0	25.6	11.3	5.7	3.3	2.2	1.6
		−0.450	370.4	255.0	121.8	57.3	28.7	15.5	9.0	5.7	3.8
		−0.900	370.4	292.2	172.0	94.7	53.2	30.1	19.0	12.2	8.2
	0.000	0.000	370.4	94.2	19.1	5.8	2.6	1.6	1.2	1.1	1.0
		−0.450	370.4	161.6	44.4	15.3	6.5	3.4	2.1	1.5	1.2
		−0.900	370.4	205.8	76.0	30.2	13.7	7.0	4.1	2.6	1.9
	−0.475	−0.900	370.4	137.0	35.4	11.7	5.0	2.7	1.7	1.3	1.2
Probability of taking the second subsample	0.950	0.900	0.33	0.36	0.44	0.55	0.67	0.78	0.87	0.93	0.96
		0.450	0.33	0.35	0.40	0.48	0.58	0.67	0.76	0.84	0.90
		0.000	0.33	0.35	0.38	0.44	0.51	0.59	0.67	0.74	0.81
		−0.450	0.33	0.34	0.37	0.41	0.46	0.52	0.59	0.66	0.72
		−0.900	0.33	0.34	0.36	0.39	0.43	0.48	0.54	0.59	0.65
	0.475	0.450	0.33	0.36	0.44	0.55	0.68	0.79	0.87	0.93	0.97
		0.000	0.33	0.35	0.41	0.49	0.58	0.68	0.77	0.84	0.90
		−0.450	0.33	0.35	0.38	0.44	0.51	0.59	0.67	0.75	0.81
		−0.900	0.33	0.34	0.37	0.41	0.46	0.53	0.59	0.66	0.73
	0.000	0.000	0.33	0.36	0.44	0.56	0.68	0.79	0.88	0.93	0.97
		−0.450	0.33	0.35	0.41	0.49	0.58	0.68	0.77	0.85	0.90
		−0.900	0.33	0.35	0.38	0.44	0.51	0.59	0.68	0.75	0.82
	−0.475	−0.900	0.33	0.35	0.41	0.49	0.59	0.69	0.78	0.85	0.91

autocorrelation. Without loss of generality, we considered σ=1.0.

At this stage, there are three possibilities:

- If $|Z_1| \leq L_1$, conclude that the process mean is in control and the DS procedure finishes at this stage.
- If $|Z_1| > L$, with L>L_1>0, conclude that the process mean is off-target and the DS procedure finishes at this stage.
- If $L > |Z_1| > L_1$, the DS procedure goes to the second stage.

Second stage Take a subsample of size n_2 and compute the mean $\overline{X}_2$. Let $Z_2 = \left(\overline{Y}-\mu_0\right)/\sigma_{\overline{Y}}$, where $\overline{Y}$ is the combined master sample mean given by $\overline{Y} = \left(n_1\overline{X}_1 + n_2\overline{X}_2\right)/(n_1+n_2)$ and $\sigma_{\overline{Y}}$ is its standard deviation.

At this stage, there are only two possibilities:

- If $|Z_2| \leq L_2$, conclude that the process mean is in control.
- If $|Z_2| > L_2$, conclude that the process is out of control.

Figure 1 shows the graphical representation of this procedure.

4 Properties of the double sampling $\overline{X}$ control chart

The following intervals and unions of intervals can be defined:

$$I_1 = \left[\mu_0 - L_1\sigma_{\overline{X}_1};\ \mu_0 + L_1\sigma_{\overline{X}_1}\right]$$
$$I_2 = \left[\left(\mu_0 - L\sigma_{\overline{X}_1};\ \mu_0 - L_1\sigma_{\overline{X}_1}\right) \cup \left(\mu_0 + L_1\sigma_{\overline{X}_1};\ \mu_0 + L\sigma_{\overline{X}_1}\right)\right]$$
$$I_3 = \left[\left(-\infty;\ \mu_0 - L\sigma_{\overline{X}_1}\right) \cup \left(\mu_0 + L\sigma_{\overline{X}_1};\ +\infty\right)\right]$$
$$I_4 = \left[\mu_0 - L_2\sigma_{\overline{Y}};\ \mu_0 + L_2\sigma_{\overline{Y}}\right]$$

According to Fig. 1, the probability of taking the second subsample is:

$$\Pr\left[\overline{X}_1 \in I_2\right] = \Phi\left(L - \frac{\delta}{\sigma_{\overline{X}_1}}\right) - \Phi\left(L_1 - \frac{\delta}{\sigma_{\overline{X}_1}}\right) + \Phi\left(-L_1 - \frac{\delta}{\sigma_{\overline{X}_1}}\right) - \Phi\left(-L - \frac{\delta}{\sigma_{\overline{X}_1}}\right) \quad (2)$$

The average sample size per sampling is:

$$\overline{n} = n_1 + n_2 \Pr\left[\overline{X}_1 \in I_2\right] \quad (3)$$

Table 12 ARL and probability of taking the second subsample for the DS chart ($\overline{n}=4,\ n_1=2,\ n_2=8$)

Chart property	ϕ	θ	δ								
			0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
ARL	0.950	0.900	370.4	107.8	23.7	7.4	3.2	1.9	1.4	1.1	1.1
		0.450	370.4	278.5	151.8	78.8	42.5	24.2	14.6	9.2	6.2
		0.000	370.4	329.4	244.7	167.1	111.8	105.4	51.6	36.0	25.7
		−0.450	370.4	348.2	294.0	231.4	175.9	132.0	99.1	74.8	57.0
		−0.900	370.4	356.5	320.3	272.8	224.5	181.3	145.1	116.0	92.9
	0.475	0.450	370.4	85.4	16.6	5.1	2.4	1.5	1.2	1.1	1.1
		0.000	370.4	176.9	56.2	20.5	8.9	4.5	2.7	1.8	1.4
		−0.450	370.4	240.5	106.4	47.5	23.0	12.0	7.0	4.4	3.0
		−0.900	370.4	280.7	154.6	80.7	43.6	24.8	14.9	9.4	6.3
	0.000	0.000	370.4	79.9	15.1	4.7	2.2	1.5	1.2	1.1	1.0
		−0.450	370.4	138.4	36.1	12.0	5.1	2.8	1.8	1.4	1.2
		−0.900	370.4	188.3	63.7	24.1	10.7	5.5	3.3	2.2	1.6
	−0.475	−0.900	370.4	120.1	28.5	9.2	4.0	2.3	1.6	1.3	1.2
Probability of taking the second subsample	0.950	0.900	0.25	0.28	0.36	0.47	0.60	0.72	0.82	0.90	0.95
		0.450	0.25	0.27	0.32	0.40	0.50	0.60	0.70	0.79	0.86
		0.000	0.25	0.26	0.30	0.35	0.43	0.51	0.59	0.68	0.75
		−0.450	0.25	0.26	0.28	0.33	0.38	0.44	0.51	0.58	0.65
		−0.900	0.25	0.26	0.28	0.31	0.35	0.40	0.45	0.51	0.58
	0.475	0.450	0.25	0.28	0.36	0.47	0.60	0.73	0.83	0.90	0.95
		0.000	0.25	0.27	0.32	0.40	0.50	0.61	0.71	0.79	0.86
		−0.450	0.25	0.26	0.30	0.36	0.43	0.51	0.60	0.68	0.76
		−0.900	0.25	0.26	0.28	0.33	0.38	0.45	0.52	0.59	0.66
	0.000	0.000	0.25	0.28	0.36	0.48	0.61	0.73	0.83	0.91	0.95
		−0.450	0.25	0.27	0.32	0.41	0.51	0.61	0.71	0.80	0.87
		−0.900	0.25	0.26	0.30	0.36	0.43	0.52	0.60	0.69	0.76
	−0.475	−0.900	0.25	0.27	0.32	0.41	0.51	0.62	0.72	0.81	0.87

The efficiency of the control chart is generally measured by the ARL, which is the average number of samplings before an out-of-control signal. The ARL is given by:

$$\mathrm{ARL} = \frac{1}{1-P} \quad (4)$$

where P=P_1+P_2, with P_i (i=1, 2) being the probability of deciding that the process is in control at stage i of the sampling procedure. The expressions used to calculate P_1 and P_2 are given in the Appendix. When the process is in control, the ARL is the ARL_0, in this case:

$$P_2 = 1 - P_1 - \frac{1}{\mathrm{ARL}_0} \quad (5)$$

The properties of a DS chart depend on the parameters n_1, n_2, L, L_1, and L_2. Preliminary investigation has shown that the best chart performance is reached with large values of L. Following suggestions given by Daudin [26], we set L=5.00. In other words, the false alarm risk during the first-stage sampling is practically zero. The coefficient L_1 is calculated using Eq. 3 and the coefficient L_2 is derived by considering the process to be in control (δ=0) and Eq. 5. Equation 20 in the Appendix gives the value of P_1 as a function of L_1. According to Eq. 21 in the Appendix, P_2 is an increasing function of L_2; therefore, by grid search, it is always possible to find the value of L_2 that matches both sides of the expression in Eq. 5 for any given ARL_0. The values of the control limits coefficients L_1 and L_2 for the DS $\overline{X}$ chart considering the previously mentioned combinations of ϕ and θ values are shown in Tables 2 and 3, respectively, for n_1=1 and n_1=2. The probabilities of taking the second subsample for the DS scheme with n_1=1 are shown in Table 4, the ARLs for the combination $\overline{n}=3$,n_1=1, n_2=4, 6, and 8 are shown in Table 5, for the combination $\overline{n}=4$, n_1=1, n_2=6, 8, and 12 are shown in Table 6, and for the combination $\overline{n}=5$, n_1=1, n_2=9, 12, and 18 are shown in Table 7.

The ARLs and the probabilities of taking the second subsample for the DS $\overline{X}$ chart with n_1=2, the combinations of ϕ and θ values shown in Table 1, and with $\overline{n}=3$ and n_2=4, 6, and 8 are shown in Tables 8, 9, and 10, respectively, in Tables 11, 12, and 13 for $\overline{n}=4$ and n_2=6, 8, and 12, respectively, and in Tables 14, 15, and 16 for $\overline{n}=5$ and n_2=9, 12, and 18, respectively.

Based on the results presented in Tables 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, and 16, it can be confirmed that the ARL deteriorates when the autocorrelation increases. For example, assuming that n_1=2, n_2=4, $\overline{n}=3$, and δ=0.25, the ARL increases from 139.3 for ρ_1=0.073 (ϕ=0.950 and θ=0.900)

Table 13 ARL and probability of taking the second subsample for the DS chart ($\overline{n}=4,\ n_1=2,\ n_2=12$)

Chart property	ϕ	θ	δ								
			0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
ARL	0.950	0.900	370.4	94.1	19.5	6.2	2.8	1.8	1.4	1.2	1.1
		0.450	370.4	277.6	150.8	78.4	42.5	23.5	14.7	9.4	6.4
		0.000	370.4	328.2	242.2	164.9	110.6	75.0	51.9	36.8	26.6
		−0.450	370.4	346.9	290.5	227.1	172.0	129.4	97.7	74.5	57.5
		−0.900	370.4	355.5	316.9	267.6	218.9	176.2	141.3	113.5	91.7
	0.475	0.450	370.4	67.9	12.4	4.1	2.2	1.6	1.3	1.2	1.1
		0.000	370.4	153.0	43.3	15.0	6.5	3.4	2.2	1.6	1.3
		−0.450	370.4	217.9	85.7	35.5	16.5	8.5	5.0	3.2	2.2
		−0.900	370.4	261.6	129.6	62.6	32.0	17.5	10.3	6.5	4.3
	0.000	0.000	370.4	63.0	11.3	3.8	2.1	1.6	1.3	1.2	1.1
		−0.450	370.4	115.4	27.1	8.9	4.0	2.4	1.7	1.4	1.2
		−0.900	370.4	163.7	49.3	17.8	7.9	4.2	2.7	1.9	1.5
	−0.475	−0.900	370.4	98.7	21.4	7.0	3.4	2.1	1.6	1.4	1.2
Probability of taking the second subsample	0.950	0.900	0.17	0.19	0.26	0.37	0.50	0.63	0.75	0.85	0.92
		0.450	0.17	0.18	0.23	0.30	0.40	0.50	0.61	0.71	0.80
		0.000	0.17	0.18	0.21	0.26	0.33	0.41	0.50	0.59	0.67
		−0.450	0.17	0.17	0.20	0.24	0.29	0.35	0.41	0.49	0.56
		−0.900	0.17	0.17	0.19	0.22	0.26	0.30	0.36	0.42	0.48
	0.475	0.450	0.17	0.19	0.27	0.38	0.51	0.64	0.76	0.86	0.92
		0.000	0.17	0.18	0.23	0.31	0.40	0.51	0.62	0.72	0.81
		−0.450	0.17	0.18	0.21	0.26	0.33	0.41	0.50	0.59	0.68
		−0.900	0.17	0.17	0.20	0.24	0.29	0.35	0.42	0.49	0.57
	0.000	0.000	0.17	0.19	0.27	0.38	0.52	0.65	0.77	0.86	0.93
		−0.450	0.17	0.18	0.23	0.31	0.41	0.52	0.63	0.73	0.81
		−0.900	0.17	0.18	0.21	0.27	0.34	0.42	0.51	0.60	0.68
	−0.475	−0.900	0.17	0.18	0.23	0.31	0.41	0.52	0.63	0.74	0.82

to 303.1 for ρ_1=0.737 (ϕ=0.475 and θ=−0.900). The improvement of the ARL by taking larger samples in the second stage (n_2) stands out only for moderate levels of autocorrelation. For example, considering n_1=2, $\overline{n}=4$, δ=0.50, and ρ_1=0.374 (ϕ=0 and θ=−0.450), the ARL varies from 44.4 when n_2=6 to 27.1 when n_2=12. The ARL is only slightly reduced by larger samples in the first stage (n_1). For example, taking n_2=12, $\overline{n}=5$, δ=1.00, and ρ_1=0.475 (ϕ=0.475 and θ=0), the ARL decreases from 6.7 for n_1=1 to 6.1 for n_1=2. When the measurement of the first subsample is feasible with a go–no-go gage (attribute data), the process monitoring with the two-stage procedure, where n_1=1, can outweigh this minor efficiency loss [14].

In order to compare the DS procedure with the standard Shewhart chart and with the adaptive variable sample size (VSS) scheme, we introduce both the $\overline{X}$ chart and the VSS $\overline{X}$ chart for the process model under study.

5 The VSS $\overline{X}$ chart for autocorrelated processes

When the $\overline{X}$ chart with variable sample size is in use, random samples are taken from the process at a fixed unit of time. The process starts in a state of statistical control with mean μ=μ_0 and standard deviation σ, and, at some random time in the future, the mean shifts from μ_0 to $\mu_0\pm\delta\sigma$, where δ>0. The sample means are plotted on the $\overline{X}$ control chart with warning limits $\mu_0 \pm k_1\sigma_{\overline{X}}$ and action limits $\mu_0 \pm k\sigma_{\overline{X}}$, where $0<k_1<k$ and $\sigma_{\overline{X}}$ is the standard deviation of the sample means. The size of each sample depends on what is observed in the preceding sample. If the standardized sample mean, that is, $Z=\frac{\overline{X}-\mu_0}{\sigma_{\overline{X}}}$, falls within the interval ($-k_1$, k_1), then the next sample size, denoted by n_1, should be small. Alternatively, if the sample mean falls within the intervals ($-k$, $-k_1$] or [k_1, k), then the next sample size, denoted by n_2, should be large.

The properties of the VSS $\overline{X}$ control chart for the model considered in this study were determined using the expressions given by Costa [11], except for the sample standard deviation $\sigma_{\overline{X}}$, which was calculated considering the autocorrelation (see Appendix).

The warning limits (k_1) and the probabilities of the sample point falling within the action region for the VSS $\overline{X}$ control chart with $\overline{n}=3$, 4, and 5 are given in Table 17 and the ARLs with the combinations of ϕ and θ shown in Table 1, n_1=2, and $\overline{n}=3$, 4, and 5 are shown in Tables 18, 19, and 20, respectively.

Again, the increase in the autocorrelation leads to a deterioration of the ARL for all values of δ considered. For

Table 14 ARL and probability of taking the second subsample for the DS chart ($\overline{n}=5,\ n_1=2,\ n_2=9$)

Chart property	ϕ	θ	δ								
			0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
ARL	0.950	0.900	370.4	101.2	21.3	6.5	2.9	1.7	1.3	1.1	1.0
		0.450	370.4	281.0	155.2	81.5	44.3	25.3	15.3	9.7	6.5
		0.000	370.4	331.5	249.6	172.8	116.9	79.5	54.8	38.5	27.6
		−0.450	370.4	349.5	298.1	234.8	182.5	204.8	104.7	79.6	61.0
		−0.900	370.4	357.5	323.4	278.1	231.2	188.4	152.1	122.5	98.7
	0.475	0.450	370.4	77.1	14.2	4.4	2.1	1.4	1.2	1.1	1.0
		0.000	370.4	158.8	51.4	18.3	7.8	4.0	2.4	1.7	1.3
		−0.450	370.4	234.2	100.1	43.7	20.8	10.9	6.3	4.0	2.7
		−0.900	370.4	276.0	148.0	75.7	40.3	22.1	13.5	8.5	5.7
	0.000	0.000	370.4	71.7	12.8	4.0	2.0	1.4	1.2	1.1	1.0
		−0.450	370.4	132.5	31.8	10.3	4.4	2.4	1.6	1.3	1.1
		−0.900	370.4	178.7	57.5	21.2	9.2	4.8	2.8	1.9	1.5
	−0.475	−0.900	370.4	110.3	24.7	7.8	3.4	2.0	1.4	1.2	1.1
Probability of taking the second subsample	0.950	0.900	0.33	0.36	0.44	0.55	0.67	0.78	0.87	0.93	0.96
		0.450	0.33	0.35	0.40	0.48	0.58	0.67	0.76	0.84	0.90
		0.000	0.33	0.35	0.38	0.44	0.51	0.59	0.67	0.74	0.81
		−0.450	0.33	0.34	0.37	0.41	0.46	0.52	0.59	0.66	0.72
		−0.900	0.33	0.34	0.36	0.39	0.43	0.48	0.54	0.59	0.65
	0.475	0.450	0.33	0.36	0.44	0.55	0.68	0.79	0.87	0.93	0.97
		0.000	0.33	0.35	0.41	0.49	0.58	0.68	0.77	0.84	0.90
		−0.450	0.33	0.35	0.38	0.44	0.51	0.59	0.67	0.75	0.81
		−0.900	0.33	0.34	0.37	0.41	0.46	0.53	0.59	0.66	0.73
	0.000	0.000	0.33	0.36	0.44	0.56	0.68	0.79	0.88	0.93	0.97
		−0.450	0.33	0.35	0.41	0.49	0.58	0.68	0.77	0.85	0.90
		−0.900	0.33	0.35	0.38	0.44	0.51	0.59	0.68	0.75	0.82
	−0.475	−0.900	0.33	0.35	0.41	0.49	0.59	0.69	0.78	0.85	0.91

example, taking n_1=2, n_2=8, $\overline{n}=4$, and δ=1.5, the ARL increases from 2.3 for ρ_1=0.255 (ϕ=−0.475 and θ=−0.900) to 12.9 for ρ_1=0.824 (ϕ=0.950 and θ=0.450). When the data autocorrelation is not very high, the ARL is slightly reduced by taking larger samples in the second stage (n_2). For instance, taking n_1=2, $\overline{n}=3$, δ=0.50, and ρ_1=0.255 (ϕ=−0.475 and θ=−0.900), the ARL varies from 79.0 when n_2=4 to 67.6 when n_2=8, and some improvement is obtained by larger values of $\overline{n}$. For example, taking n_1=2, n_2=12, δ=0.75, ρ_1=0.374 (ϕ=0 and θ=−0.450), the ARL changes from 13.5 for $\overline{n}=4$ to 8.2 for $\overline{n}=5$.

6 The $\overline{X}$ chart for autocorrelated processes

The $\overline{X}$ chart for autocorrelated processes is designed following the concepts of the classical methodology applied to independent data, with centerline at μ_0 and control limits (CL) set at $\mu_0 \pm k\sigma_{\overline{X}}$, but with the sample standard deviation $\sigma_{\overline{X}}$ determined taking the autocorrelation into account (see Appendix). The ARLs of the $\overline{X}$ control chart with the combinations of ϕ and θ shown in Table 1 and n=3, 4, and 5 are shown in Table 21.

The effect of the data autocorrelation is again to increase the ARL for all of the process mean shifts under consideration. For example, taking n=5 and δ=1.5, the ARL increases from 1.9 for ρ_1=0.073 (ϕ=0.950 and θ=0.900) to 27.9 when ρ_1=0.737 (ϕ=0.475 and θ=−0.900). The effect of increasing the sample size is to improve the mean shift detection ability, for example, taking δ=0.5 and ρ_1=0.497 (ϕ=0 and θ=−0.900), the ARL is reduced from 146.2 for n=3 to 108.9 for n=5.

The effect of the autocorrelation on the ARL of the Shewhart, VSS, and DS control charts at selected sample sizes and mean shift levels is graphically shown in Fig. 2.

The major findings from the comparison among the three process monitoring strategies are as follows:

- In all cases, when the data autocorrelation increases, the control chart's ability to detect process mean shifts is reduced
- For processes with low or mild autocorrelation levels, the ARL of the DS chart is substantially better than the ARL of both the Shewhart and the VSS control charts, regardless of the magnitude of the process shift and the ability of all the control schemes to detect low to moderate mean shifts is improved with larger $\overline{n}$.

Table 15 ARL and probability of taking the second subsample for the DS chart ($\overline{n}=5,\ n_1=2,\ n_2=12$)

Chart property	ϕ	θ	δ								
			0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
ARL	0.950	0.900	370.4	91.1	18.3	5.7	2.6	1.6	1.3	1.1	1.1
		0.450	370.4	280.8	155.2	81.7	44.6	25.7	15.6	10.0	6.7
		0.000	370.4	331.1	249.0	172.6	117.3	115.5	56.0	39.8	28.8
		−0.450	370.4	349.1	296.9	236.2	181.7	138.3	105.4	80.8	62.6
		−0.900	370.4	357.0	322.1	276.3	229.4	187.2	151.7	122.8	99.7
	0.475	0.450	370.4	64.2	11.2	3.6	1.9	1.4	1.2	1.1	1.1
		0.000	370.4	150.6	41.8	14.3	6.1	3.2	2.0	1.5	1.2
		−0.450	370.4	216.9	84.8	34.9	16.1	8.3	4.8	3.1	2.2
		−0.900	370.4	261.5	129.4	62.4	31.8	17.4	10.2	6.4	4.3
	0.000	0.000	370.4	59.4	10.1	3.3	1.8	1.4	1.2	1.1	1.0
		−0.450	370.4	111.5	25.3	8.1	3.6	2.1	1.5	1.3	1.2
		−0.900	370.4	160.0	47.0	16.6	7.3	3.8	2.4	1.7	1.4
	−0.475	−0.900	370.4	94.4	19.7	6.3	2.9	1.9	1.4	1.2	1.1
Probability of taking the second subsample	0.950	0.900	0.25	0.28	0.36	0.47	0.6	0.72	0.82	0.9	0.95
		0.450	0.25	0.27	0.32	0.4	0.5	0.6	0.7	0.79	0.86
		0.000	0.25	0.26	0.3	0.35	0.43	0.51	0.59	0.68	0.75
		−0.450	0.25	0.26	0.28	0.33	0.38	0.44	0.51	0.58	0.65
		−0.900	0.25	0.26	0.28	0.31	0.35	0.4	0.45	0.51	0.58
	0.475	0.450	0.25	0.28	0.36	0.47	0.6	0.73	0.83	0.9	0.95
		0.000	0.25	0.27	0.32	0.4	0.5	0.61	0.71	0.79	0.86
		−0.450	0.25	0.26	0.3	0.36	0.43	0.51	0.6	0.68	0.76
		−0.900	0.25	0.26	0.28	0.33	0.38	0.45	0.52	0.59	0.66
	0.000	0.000	0.25	0.28	0.36	0.48	0.61	0.73	0.83	0.91	0.95
		−0.450	0.25	0.27	0.32	0.41	0.51	0.61	0.71	0.8	0.87
		−0.900	0.25	0.26	0.3	0.36	0.43	0.52	0.6	0.69	0.76
	−0.475	−0.900	0.25	0.27	0.32	0.41	0.51	0.62	0.72	0.81	0.87

7 Example

The purpose of the following example is to show the effect of the serial correlation on the ability of the $\overline{X}$ Shewhart and DS control charts in detecting process mean shifts. To this end, observations from an ARMA(1, 1) process with the residuals ε_t given by an N(0, 1) distribution, ϕ=−0.475, and θ=0.900 (ρ_1=0.225) were simulated and grouped based on the rational subgroup concept. Assuming initially an in-control process with μ_0=0, samples of size n=4 were plotted on an $\overline{X}$ chart, with the process standard deviation estimated by $\frac{\overline{R}}{d_2}$, with $\overline{R}$ being the sample range mean of 50 samples. The control limits of CL=±1.4376 were determined by $\mathrm{CL}=\overline{\overline{X}}\pm\frac{3}{d_2\sqrt{n}}\overline{R}$ [1], with $\overline{R}=1.972$ and d_2=2.059, ignoring the serial correlation (see Fig. 3). The occurrence of sample points in the action region (samples 2, 32, and 50) indicates that, when the correlation is not recognized in the design of the control limits, they become very tight and the probability of false alarms is high.

Afterwards, using the same data set, new control limits calculated by $\mathrm{CL}=\pm 3\sigma_{\overline{X}}=\pm 1.8774$, with $\sigma^2_{\overline{X}_1}=\left(A_1^2+A_2^2+\ldots+A_{n_1}^2\right)$, where $A_i=\frac{\phi^{n_1-1}+\sum_{i=1}^{n_1-i}(1-\theta)\phi^{i-1}}{n_1}$ (see Eq. 8 in the Appendix), were set to account for the within-subgroup correlation. Additionally, the last 20 samples were simulated again with μ_1=1; that is, we considered the case where the assignable cause shifted the process mean by one standard deviation. This disturbance was signalled at sample 42 (run length=12), see Fig. 4.

Finally, the process monitoring strategy using the DS $\overline{X}$ chart was investigated. A sample size of n_1=2 and n_2=6 was adopted, and $\overline{n}=4$, aiming at a fair comparison between the DS and the Shewhart control charts. With coefficients L_1=0.9674 and L_2=2.9201, the control limits were set at ±0.8420 and ±1.310 for the first and second stages, respectively. The action limits of the first stage with factor L=5.00 are not shown on the chart. The values of $\overline{X}_1$ and $\overline{Y}$ are plotted on the left-hand vertical scale and on the right-hand vertical scale of the chart, respectively. As before, the last 20 master samples were simulated with μ_1=1. The disturbance is now signaled at sample 35 (run length=5), see Fig. 5, substantially faster than in the previous simulation with the $\overline{X}$ chart.

8 Conclusion

Most standard control charting schemes for statistical process control are based on the assumption that measurements of

Table 16 ARL and probability of taking the second subsample for the DS chart ($\overline{n} = 5,\ n_1 = 2,\ n_2 = 18$)

Chart property	ϕ	θ	δ								
			0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
ARL	0.950	0.900	370.4	79.8	15.4	5.0	2.5	1.7	1.3	1.2	1.1
		0.450	370.4	276.8	149.8	77.8	42.2	23.5	14.8	9.5	6.5
		0.000	370.4	326.9	239.5	162.2	108.6	73.8	51.3	36.6	26.7
		−0.450	370.4	345.6	287.0	222.3	167.4	125.5	94.8	72.5	56.2
		−0.900	370.4	354.4	313.4	262.2	212.6	170.1	136.0	109.1	88.3
	0.475	0.450	370.4	49.8	8.7	3.3	2.0	1.6	1.3	1.2	1.1
		0.000	370.4	126.5	31.4	10.4	4.6	2.6	1.8	1.4	1.3
		−0.450	370.4	192.0	66.3	25.5	11.4	5.9	3.5	2.4	1.8
		−0.900	370.4	290.8	159.8	76.4	36.5	18.5	10.1	6.0	3.9
	0.000	0.000	370.4	45.8	7.9	3.1	2.0	1.5	1.3	1.2	1.1
		−0.450	370.4	90.3	18.9	6.3	3.1	2.1	1.6	1.4	1.2
		−0.900	370.4	135.3	35.6	12.3	5.6	3.2	2.2	1.7	1.5
	−0.475	−0.900	370.4	75.4	14.8	5.1	2.8	2.0	1.6	1.4	1.2
Probability of taking the second subsample	0.950	0.900	0.17	0.19	0.26	0.37	0.5	0.63	0.75	0.85	0.92
		0.450	0.17	0.18	0.23	0.3	0.4	0.5	0.61	0.71	0.8
		0.000	0.17	0.18	0.21	0.26	0.33	0.41	0.5	0.59	0.67
		−0.450	0.17	0.17	0.2	0.24	0.29	0.35	0.41	0.49	0.56
		−0.900	0.17	0.17	0.19	0.22	0.26	0.3	0.36	0.42	0.48
	0.475	0.450	0.17	0.19	0.27	0.38	0.51	0.64	0.76	0.86	0.92
		0.000	0.17	0.18	0.23	0.31	0.4	0.51	0.62	0.72	0.81
		−0.450	0.17	0.18	0.21	0.26	0.33	0.41	0.5	0.59	0.68
		−0.900	0.17	0.17	0.2	0.24	0.29	0.35	0.42	0.49	0.57
	0.000	0.000	0.17	0.19	0.27	0.38	0.52	0.65	0.77	0.86	0.93
		−0.450	0.17	0.18	0.23	0.31	0.41	0.52	0.63	0.73	0.81
		−0.900	0.17	0.18	0.21	0.27	0.34	0.42	0.51	0.6	0.68
	−0.475	−0.900	0.17	0.18	0.23	0.31	0.41	0.52	0.63	0.74	0.82

the product quality variable are independent within and between subgroups. However, positive autocorrelation at low lags is commonplace in processes of discrete parts manufacturing where, given the advances in sensor technologies, observations can be taken closer together in time. In the present paper, it is assumed that successive observations collected from the process are fit to a first-order autoregressive moving average (ARMA(1, 1)) model, and, furthermore, that such observations are grouped following the rational subgroup concept leading to within-subgroup autocorrelation (each sample consists of units that were consecutively produced and subgroups are far spaced in time to maximize the chance of detecting differences between samples if an assignable cause is present). It is known that even moderate levels of autocorrelation can have a significant effect on the performance of control charts and when the serial correlation is observed, the traditional control chart methodology should not be applied directly.

Aiming to partially recover the chart efficiency lost due to the autocorrelation, we extend the methodology introduced by Daudin [26] by proposing a double sampling (DS) $\overline{X}$ control chart for a process where the successive observations of the quality characteristic to be monitored are fit to an ARMA(1, 1) model. We have derived suitable control limits and measured the process monitoring efficiency in fair comparison with competing schemes. Having

Table 17 Warning limits (k_1) and probabilities of the sample point falling within the action region for the variable sample size (VSS) $\overline{X}$ control chart

$\overline{n} = 3$			$\overline{n} = 4$			$\overline{n} = 5$		
n_2	k_1	$k_1 < \Pr\lvert Z\rvert < k$	n_2	k_1	$k_1 < \Pr\lvert Z\rvert < k$	n_2	k_1	$k_1 < \Pr\lvert Z\rvert < k$
4	1.1454	0.252	6	0.9638	0.335	9	0.9638	0.335
6	1.3757	0.169	8	1.1454	0.252	12	1.1454	0.252
8	1.5245	0.127	12	1.3757	0.169	18	1.3757	0.169

Table 18 ARL for the VSS control chart ($\overline{n} = 3,\ n_1 = 2$)

n_2	ϕ	θ	δ								
			0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
4	0.950	0.900	370.4	192.0	56.7	17.0	6.5	3.4	2.3	1.8	1.5
		0.450	370.4	267.5	137.2	68.2	35.7	20.0	11.9	7.5	5.0
		0.000	370.4	300.3	186.5	108.8	64.9	40.4	26.3	17.8	12.5
		−0.450	370.4	319.7	223.5	144.5	93.2	61.5	41.9	29.4	21.3
		−0.900	370.4	332.2	251.7	176.0	120.8	83.6	59.1	42.7	31.6
	0.475	0.450	370.4	181.1	48.2	13.5	5.2	2.9	2.1	1.7	1.5
		0.000	370.4	248.5	110.4	46.4	20.5	10.0	5.6	3.6	2.5
		−0.450	370.4	286.2	161.4	84.4	44.7	24.6	14.3	8.8	5.8
		−0.900	370.4	309.5	201.8	120.7	71.7	43.5	27.2	17.6	11.8
	0.000	0.000	370.4	175.8	44.7	12.2	4.8	2.8	2.0	1.7	1.5
		−0.450	370.4	233.0	90.8	32.9	13.1	6.2	3.7	2.6	2.0
		−0.900	370.4	271.3	136.3	61.9	28.3	13.9	7.6	4.7	3.3
	−0.475	−0.900	370.4	222.4	79.0	26.2	10.0	4.9	3.0	2.2	1.8
6	0.950	0.900	370.4	192.1	53.6	14.7	5.6	3.1	2.2	1.8	1.6
		0.450	370.4	268.0	138.0	69.1	36.5	20.6	12.4	7.9	5.3
		0.000	370.4	298.9	184.3	106.9	63.5	39.5	25.7	17.4	12.3
		−0.450	370.4	318.0	219.9	140.9	90.1	59.1	40.1	28.1	20.2
		−0.900	370.4	330.6	247.8	171.4	116.5	80.1	56.2	40.4	29.7
	0.475	0.450	370.4	177.6	42.1	10.6	4.3	2.6	2.0	1.7	1.5
		0.000	370.4	247.4	107.1	42.7	17.7	8.3	4.7	3.1	2.3
		−0.450	370.4	285.0	158.6	81.0	41.5	22.0	12.4	7.5	4.9
		−0.900	370.4	308.3	198.9	117.2	68.3	40.5	24.6	15.5	10.2
	0.000	0.000	370.4	171.9	38.4	9.5	4.0	2.5	2.0	1.7	1.5
		−0.450	370.4	231.2	85.3	28.1	10.4	5.0	3.1	2.3	1.9
		−0.900	370.4	270.2	132.1	56.6	24.0	11.2	6.1	3.9	2.8
	−0.475	−0.900	370.4	220.3	72.8	21.5	7.8	4.0	2.7	2.1	1.8
8	0.950	0.900	370.4	192.5	51.7	13.4	5.1	3.0	2.2	1.8	1.6
		0.450	370.4	268.0	138.2	69.4	36.8	20.9	12.7	8.1	5.5
		0.000	370.4	298.0	182.7	105.5	62.4	38.7	25.1	17.0	12.0
		−0.450	370.4	316.9	217.7	138.5	88.1	57.5	38.8	27.1	19.5
		−0.900	370.4	329.6	245.5	168.7	114.0	77.9	54.4	38.9	28.6
	0.475	0.450	370.4	174.6	37.4	8.9	3.9	2.6	2.0	1.8	1.6
		0.000	370.4	246.5	104.4	39.6	15.6	7.2	4.1	2.8	2.2
		−0.450	370.4	284.2	156.6	78.3	38.9	19.9	10.9	6.5	4.3
		−0.900	370.4	32.6	188.3	107.2	60.7	35.0	20.9	12.9	8.4
	0.000	0.000	370.4	168.4	33.7	8.0	3.6	2.5	2.0	1.7	1.6
		−0.450	370.4	229.7	80.7	24.5	8.8	4.4	2.9	2.3	1.9
		−0.900	370.4	269.4	128.7	52.3	21.0	9.5	5.2	3.5	2.6
	−0.475	−0.900	370.4	218.5	67.6	18.4	6.7	3.7	2.6	2.1	1.9

Table 19 ARL for the VSS control chart ($\overline{n} = 4,\ n_1 = 2$)

n_2	ϕ	θ	δ								
			0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
6	0.950	0.900	370.4	166.9	40.9	11.3	4.6	2.6	1.9	1.6	1.4
		0.450	370.4	270.0	140.7	71.0	37.8	21.4	12.9	8.2	5.5
		0.000	370.4	304.5	194.1	115.9	70.4	44.5	29.4	20.2	14.4
		−0.450	370.4	323.5	231.6	153.3	100.7	67.4	46.5	33.0	24.1
		−0.900	370.4	335.4	259.5	185.5	129.6	91.0	65.0	47.4	35.3
	0.475	0.450	370.4	148.7	30.7	8.0	3.5	2.2	1.8	1.5	1.4

Table 19 (continued)

n_2	ϕ	θ	δ								
			0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
		0.000	370.4	233.8	93.3	35.5	14.7	7.1	4.1	2.8	2.1
		−0.450	370.4	278.7	149.1	73.8	37.0	19.5	11.0	6.8	4.5
		−0.900	370.4	305.2	193.1	111.7	64.1	37.6	22.8	14.4	9.5
	0.000	0.000	370.4	142.1	27.7	7.2	3.2	2.1	1.7	1.5	1.4
		−0.450	370.4	208.2	67.3	21.2	8.2	4.2	2.7	2.0	1.7
		−0.900	370.4	253.2	111.6	44.6	18.7	8.9	5.0	3.3	2.5
	−0.475	−0.900	370.4	192.7	54.6	15.8	6.1	3.3	2.3	1.8	1.6
8	0.950	0.900	370.4	168.1	38.7	10.1	4.1	2.5	1.9	1.6	1.5
		0.450	370.4	270.1	141.1	71.6	38.4	21.9	13.3	8.6	5.8
		0.000	370.4	302.6	190.8	113.0	68.3	43.0	28.4	19.5	13.9
		−0.450	370.4	321.3	227.0	148.5	96.6	64.3	44.1	31.2	22.6
		−0.900	370.4	333.4	254.7	179.7	124.2	86.6	61.5	44.6	33.1
	0.475	0.450	370.4	144.2	26.1	6.6	3.1	2.2	1.8	1.6	1.5
		0.000	370.4	232.2	89.3	31.8	12.5	6.0	3.6	2.5	2.0
		−0.450	370.4	277.2	145.7	69.8	33.6	17.0	9.4	5.7	3.9
		−0.900	370.4	303.8	189.8	107.6	60.2	34.3	20.2	12.4	8.1
	0.000	0.000	370.4	137.0	23.2	5.9	2.9	2.1	1.8	1.6	1.4
		−0.450	370.4	205.6	61.7	17.8	6.7	3.6	2.5	2.0	1.7
		−0.900	370.4	251.7	106.7	39.7	15.6	7.3	4.2	2.9	2.3
	−0.475	−0.900	370.4	189.7	48.9	12.9	5.1	3.0	2.2	1.9	1.7
12	0.950	0.900	370.4	169.4	35.9	8.7	3.8	2.5	2.0	1.7	1.5
		0.450	370.4	269.8	140.8	71.6	38.6	22.2	13.6	8.8	6.0
		0.000	370.4	300.2	186.7	109.3	65.5	41.0	26.9	18.4	13.0
		−0.450	370.4	318.8	221.8	142.9	91.9	60.6	41.2	28.9	20.9
		−0.900	370.4	331.2	249.3	173.3	118.3	81.6	57.4	41.4	30.5
	0.475	0.450	370.4	136.3	20.1	5.2	2.9	2.2	1.9	1.7	1.5
		0.000	370.4	229.6	82.5	26.3	9.6	4.7	3.0	2.3	1.9
		−0.450	370.4	275.2	140.5	63.5	28.4	13.5	7.2	4.5	3.2
		−0.900	370.4	302.0	185.3	101.7	54.3	29.2	16.3	9.7	6.2
	0.000	0.000	370.4	128.4	17.7	4.8	2.8	2.2	1.9	1.7	1.5
		−0.450	370.4	201.3	53.1	13.5	5.2	3.1	2.4	2.0	1.8
		−0.900	370.4	249.2	98.6	32.6	11.8	5.7	3.5	2.7	2.2
	−0.475	−0.900	370.4	184.2	40.5	9.7	4.2	2.8	2.2	2.0	1.8

Table 20 ARL for the VSS control chart ($\overline{n} = 5,\ n_1 = 2$)

n_2	ϕ	θ	δ								
			0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
9	0.950	0.900	370.4	150.4	31.3	8.2	3.6	2.3	1.8	1.6	1.4
		0.450	370.4	272.1	143.8	73.7	39.8	22.9	14.0	9.0	6.0
		0.000	370.4	306.2	197.2	118.9	72.9	46.4	30.8	21.3	15.2
		−0.450	370.4	324.6	234.2	156.1	103.1	69.4	48.0	34.1	24.9
		−0.900	370.4	336.2	261.6	188.0	132.0	93.1	66.7	48.7	36.3
	0.475	0.450	370.4	121.5	19.5	5.2	2.7	1.9	1.7	1.5	1.4
		0.000	370.4	218.8	77.1	26.0	10.2	5.0	3.1	2.2	1.8
		−0.450	370.4	269.6	134.6	61.5	28.6	14.3	7.9	4.9	3.4
		−0.900	370.4	299.4	181.6	99.9	54.4	30.3	17.6	10.8	7.0
	0.000	0.000	370.4	114.0	17.3	4.7	2.5	1.9	1.6	1.5	1.4
		−0.450	370.4	185.3	49.0	13.6	5.4	3.0	2.2	1.8	1.6
		−0.900	370.4	235.6	89.9	31.3	12.2	5.9	3.6	2.6	2.0
	−0.475	−0.900	370.4	166.6	37.4	9.8	4.1	2.6	2.0	1.7	1.6
12	0.950	0.900	370.4	151.3	29.2	7.3	3.3	2.2	1.8	1.6	1.5
		0.450	370.4	271.7	143.4	73.6	40.0	23.1	14.2	9.2	6.3

Table 20 (continued)

n_2	ϕ	θ	δ 0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
		0.000	370.4	303.7	192.8	114.9	69.8	44.2	29.3	20.1	14.4
		−0.450	370.4	322.0	228.6	150.2	98.1	65.4	45.0	31.8	23.1
		−0.900	370.4	333.9	255.9	181.1	125.6	87.7	62.4	45.3	33.6
	0.475	0.450	370.4	114.9	15.8	4.4	2.5	2.0	1.7	1.6	1.4
		0.000	370.4	216.2	71.5	22.1	8.3	4.2	2.7	2.1	1.8
		−0.450	370.4	267.8	130.3	56.7	24.9	11.9	6.5	4.1	2.9
		−0.900	370.4	297.7	177.3	94.6	49.4	26.3	14.7	8.8	5.7
	0.000	0.000	370.4	107.0	13.8	4.0	2.4	1.9	1.7	1.6	1.4
		−0.450	370.4	181.2	42.8	10.9	4.5	2.8	2.1	1.8	1.7
		−0.900	370.4	233.2	83.3	26.4	9.8	4.9	3.1	2.4	2.0
	−0.475	−0.900	370.4	161.6	31.7	7.9	3.6	2.5	2.0	1.8	1.6
18	0.950	0.900	370.4	153.4	27.0	6.6	3.2	2.3	1.9	1.7	1.5
		0.450	370.4	270.6	142.1	72.7	39.5	22.9	14.2	9.3	6.3
		0.000	370.4	300.7	187.7	110.2	66.2	41.6	27.3	18.7	13.3
		−0.450	370.4	319.2	222.5	143.7	92.6	61.1	41.6	29.2	21.1
		−0.900	370.4	331.4	249.9	173.9	118.9	82.1	57.8	41.7	30.7
	0.475	0.450	370.4	103.9	11.7	3.9	2.6	2.2	1.9	1.7	1.5
		0.000	370.4	212.0	62.7	16.9	6.3	3.5	2.5	2.1	1.8
		−0.450	370.4	265.2	122.9	48.8	19.5	8.9	5.0	3.3	2.5
		−0.900	370.4	295.6	171.4	86.7	42.1	20.8	11.1	6.6	4.4
	0.000	0.000	370.4	95.4	10.3	3.7	2.6	2.1	1.9	1.7	1.5
		−0.450	370.4	174.1	34.1	8.2	3.9	2.7	2.2	2.0	1.8
		−0.900	370.4	229.1	73.0	20.1	7.4	4.1	2.9	2.4	2.1
	−0.475	−0.900	370.4	152.9	24.3	6.2	3.4	2.6	2.2	1.9	1.8

Table 21 ARL for the Shewhart control chart (k=3.000)

n	ϕ	θ	δ 0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
3	0.950	0.900	370.4	194.0	67.2	25.6	11.3	5.7	3.3	2.2	1.6
		0.450	370.4	264.1	132.5	64.5	33.1	18.2	10.6	6.7	4.4
		0.000	370.4	303.9	191.8	112.0	65.7	39.7	24.9	16.2	10.9
		−0.450	370.4	326.1	236.6	157.3	102.4	67.2	45.0	30.7	21.5
		−0.900	370.4	339.1	268.7	195.9	138.3	97.0	68.6	49.1	35.7
	0.475	0.450	370.4	188.3	63.4	23.8	10.4	5.3	3.1	2.0	1.5
		0.000	370.4	250.6	116.9	54.0	26.7	14.3	8.3	5.2	3.5
		−0.450	370.4	290.2	168.8	92.0	51.3	29.8	18.1	11.6	7.7
		−0.900	370.4	314.3	211.5	130.8	80.3	50.3	32.4	21.5	14.7
	0.000	0.000	370.4	184.2	60.7	22.5	9.8	4.9	2.9	1.9	1.5
		−0.450	370.4	237.3	103.0	45.3	21.7	11.4	6.6	4.1	2.8
		−0.900	370.4	274.7	146.2	74.3	39.4	22.1	13.1	8.2	5.5
	−0.475	−0.900	370.4	224.5	91.1	38.4	17.9	9.3	5.3	3.4	2.3
4	0.950	0.900	370.4	169.6	51.7	18.4	7.8	4.0	2.4	1.7	1.3
		0.450	370.4	266.0	134.9	66.2	34.2	18.8	11.1	6.9	4.6
		0.000	370.4	311.8	206.6	125.9	76.4	47.4	30.3	20.0	13.6
		−0.450	370.4	333.9	255.3	179.1	122.1	83.2	57.4	40.3	28.8
		−0.900	370.4	345.8	287.1	221.0	164.0	120.1	88.0	65.0	48.6
	0.475	0.450	370.4	160.4	46.6	16.1	6.8	3.5	2.1	1.5	1.2
		0.000	370.4	238.1	103.9	45.8	22.0	11.6	6.7	4.2	2.8
		−0.450	370.4	285.0	160.8	85.6	46.8	26.9	16.2	10.3	6.9
		−0.900	370.4	312.3	207.5	126.8	77.1	47.9	30.7	20.3	13.8
	0.000	0.000	370.4	155.2	43.9	15.0	6.3	3.2	2.0	1.4	1.2

Table 21 (continued)

n	ϕ	θ	δ								
			0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
		−0.450	370.4	215.3	83.3	34.0	15.6	8.0	4.6	2.9	2.1
		−0.900	370.4	258.3	125.6	59.8	30.2	16.4	9.6	6.0	4.0
	−0.475	−0.900	370.4	200.3	71.7	27.9	12.4	6.3	3.6	2.4	1.7
5	0.950	0.900	370.4	151.4	41.9	14.2	5.9	3.1	1.9	1.4	1.2
		0.450	370.4	269.1	138.8	69.0	36.0	19.9	11.7	7.3	4.9
		0.000	370.4	317.7	218.4	137.7	85.9	54.5	35.4	23.7	16.3
		−0.450	370.4	339.0	268.4	195.5	137.9	96.7	68.3	48.9	35.5
		−0.900	370.4	349.8	298.8	238.3	182.9	138.2	104.0	78.6	59.8
	0.475	0.450	370.4	138.9	36.0	11.8	4.9	2.6	1.7	1.3	1.1
		0.000	370.4	226.1	92.6	39.2	18.3	9.5	5.4	3.4	2.4
		−0.450	370.4	278.4	151.4	78.3	41.9	23.7	14.1	8.9	5.9
		−0.900	370.4	308.5	200.2	119.8	71.6	43.9	27.9	18.3	12.4
	0.000	0.000	370.4	133.1	33.4	10.8	4.5	2.4	1.6	1.2	1.1
		−0.450	370.4	196.4	68.9	26.5	11.7	5.9	3.4	2.3	1.6
		−0.900	370.4	243.1	108.9	48.9	23.8	12.6	7.3	4.5	3.1
	−0.475	−0.900	370.4	178.7	57.2	20.8	9.0	4.6	2.7	1.8	1.4

set up all of the charts for the same false alarm rate and the same average sample size per sampling when the process mean is on target, we could conclude that the DS control chart is substantially better than both the $\overline{X}$ control chart and the variable sample size (VSS) control chart in detecting mean shifts when there is moderate within-subgroup correlation. For processes with high autocorrelation, the advantage of using the DS chart is reduced if compared with the competing schemes.

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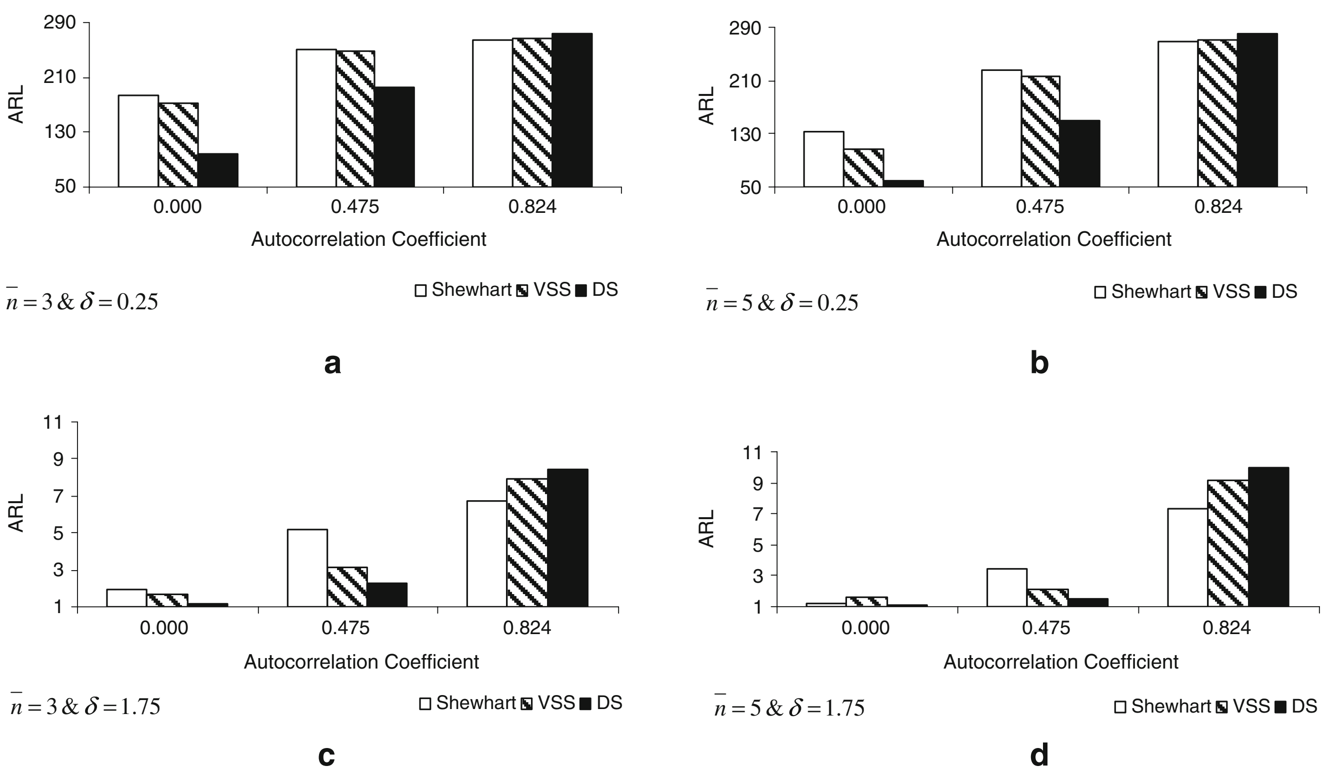

Fig. 2a–b Effect of the autocorrelation on the ARL of the Shewhart, VSS, and DS control charts at selected sample sizes and mean shift levels. **a** and δ=0.25. **b** and δ=0.25. **c** and δ=1.75. **d** and δ=1.75

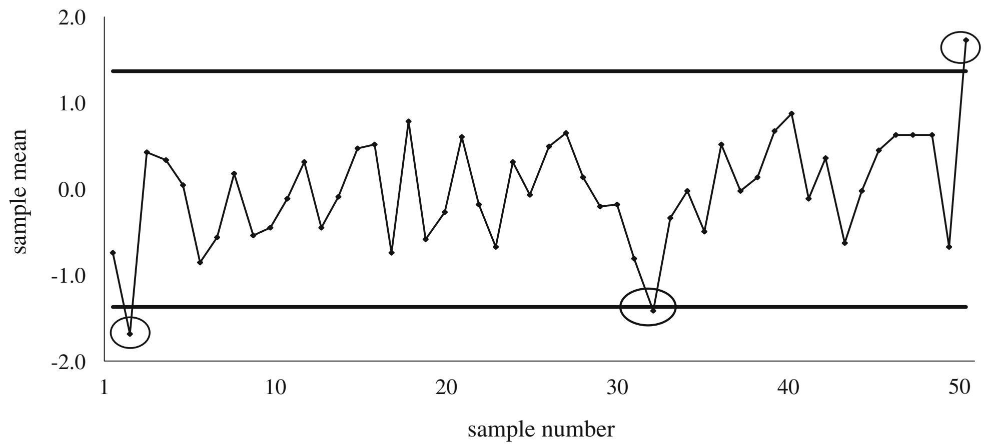

Fig. 3 Shewhart control chart (n=4) with unrecognized correlation

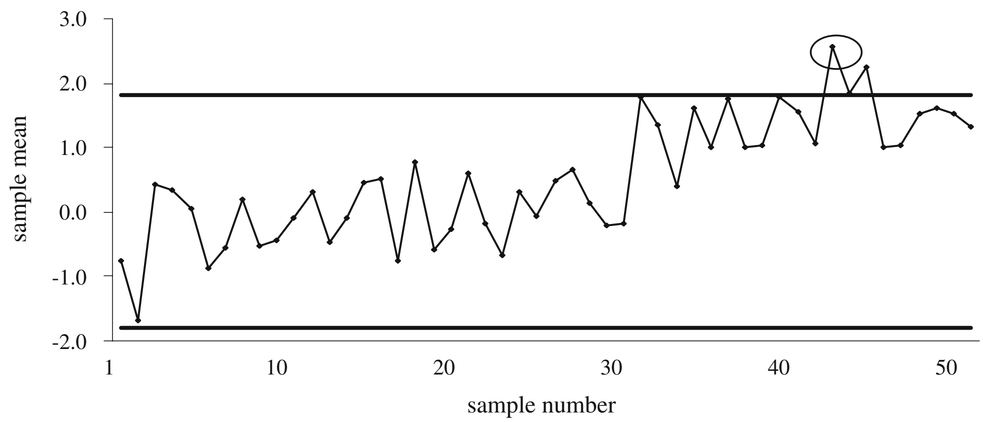

Fig. 4 Shewhart control chart (n=4) modified to account for the correlation

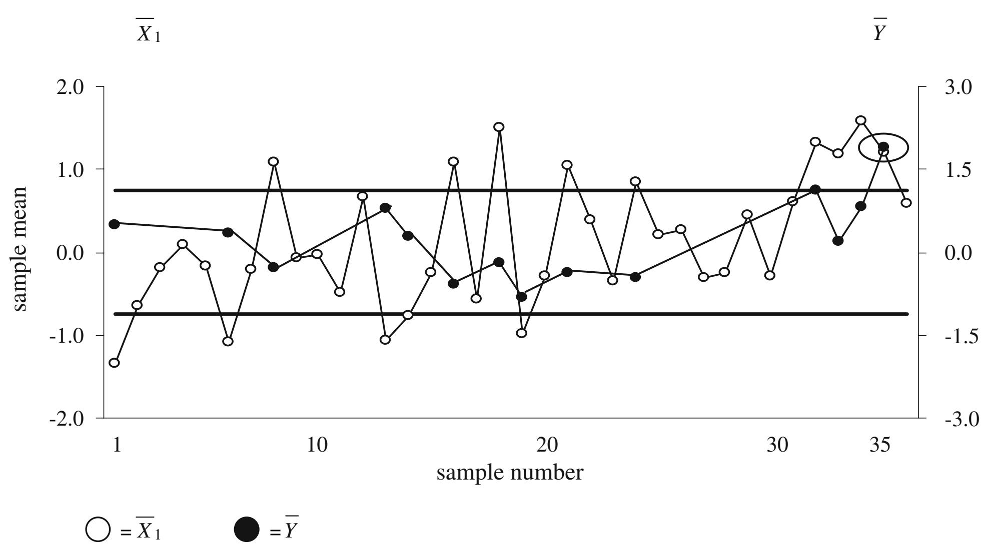

Fig. 5 DS control chart (n_1=2 and n_2=6) for the autocorrelated process

Appendix: expressions for finding P_1 and P_2

The observations X_t for a first-order autoregressive process ARMA(1, 1) are given by:

$$X_t = (1-\phi)\mu + \phi X_{t-1} - \theta\varepsilon_{t-1} + \varepsilon_t \qquad t = 1,\ 2,\ldots,\ n$$

where ϕ is the autoregressive parameter of the AR(1) process, μ is the process mean, θ is the moving average parameter, and the ε_ts are the random shock components of the ARMA(1, 1) process, which are i.i.d. normal random variables with mean zero and variance one. Without loss of generality, we take $\mu=\mu_0=0$ when the process is in control.

The expression for consecutive X_t observations forming a subgroup of size n, derived through recursive substitutions, is generically represented by:

$$X_t = \sum_{i=1}^{t}\phi^{t-1}\varepsilon_i - d_t\left(\sum_{i=1}^{t-1}\phi^{t-1}\varepsilon_i\right) \qquad t = 1,\ 2,\ldots,\ n \tag{6}$$

with $d_1=\theta$ for $t\neq 1$ and $d_t=0$ for $t=1$.

The expression in Eq. 6 holds as long as the observations within the subgroup fit an ARMA(1, 1) process. Based on the double sampling procedure, a master sample of size n can be partitioned into two subsamples of sizes n_1 and n_2 units, with $n=n_1+n_2$. The mean of the first subsample n_1 is given by:

$$\overline{X}_1 = \frac{\sum_{i=1}^{n_1} X_i}{n_1} = \frac{\sum_{i=1}^{n_1}\left[\phi^{n_1-i} + \sum_{i=1}^{n_1-1}(1-\theta)\phi^{i-1}\right]\varepsilon_i}{n_1}$$

$$= \sum_{i=1}^{n_1} A_i\varepsilon_i = A_1\varepsilon_1 + A_2\varepsilon_2 + \ldots + A_{n_1}\varepsilon_{n_1} \tag{7}$$

where $A_i = \frac{\alpha^{n_1-i} + \sum_{i=1}^{n_1-1}(1-\theta)\alpha^{i-1}}{n_1}$.

The variance $\sigma^2_{\overline{X}_1}$ is given by:

$$\sigma^2_{\overline{X}_1} = \left(A_1^2 + A_2^2 + \ldots + A_{n_1}^2\right) \tag{8}$$

so that:

$$\overline{X}_1 = Z\sqrt{A_1^2 + A_2^2 + \ldots + A_{n_1}^2} \quad \text{with } Z\sim\mathrm{N}(0,\ 1) \tag{9}$$

Given the fixed correlation structure within the subgroup, the expression of the second subsample mean $\overline{X}_2$ comprises two terms, $X_{i,\ \mathrm{k}}$ and $X_{i,\ \mathrm{u}}$, to distinguish, respectively, the known and unknown information ($\varepsilon_t$values) after the measurement of the first subsample:

$$\overline{X}_2 = \frac{\sum_{i=n_1+1}^{n} X_{i,\ \mathrm{k}} + \sum_{i=n_1+1}^{n} X_{i,\ \mathrm{u}}}{n_2} \tag{10}$$

with:

$$X_{i,\ \mathrm{k}} = W\sum_{t=0}^{n_1-1}\phi^{i-t-n_1}\varepsilon_{t+1} \tag{11}$$

where $W=\phi-\theta$ and $i=n_1+1,\ldots,\ n$

and:

$$X_{i,\ \mathrm{u}} = \varepsilon_i + W\sum_{t=1}^{i-(n_1+1)}\phi^{t-1}\varepsilon_{i-t} \qquad i=n_1+1,\ldots,\ n \tag{12}$$

The combined sample mean $\overline{Y}$ is expressed by:

$$\overline{Y} = \frac{n_1}{n}\overline{X}_1 + \frac{n_2}{n}\overline{X}_2 \tag{13}$$

thus:

$$\begin{gathered}\overline{Y} = \tfrac{1}{n}\left(\sum_{i=1}^{n_1}\left[\phi^{n_1-i} + \sum_{i=1}^{n_1-1}(1-\theta)\phi^{i-1}\right] + \sum_{i=n_1+1}^{n} X_{i,\ \mathrm{k}}\right) \\ + \tfrac{1}{n}\left(\sum_{i=n_1+1}^{n} X_{i,\ \mathrm{u}}\right)\end{gathered}$$

$$\begin{gathered}\overline{Y} = \tfrac{1}{n}\left[\sum_{i=1}^{n_1}\left(\phi^{n_1-i} + \sum_{i=1}^{n_1-1}(1-\theta)\phi^{i-1}\right)\varepsilon_i \right. \\ \left. + W\left(\sum_{i=n_{1+1}}^{n}\sum_{t=0}^{n_1-1}\phi^{i-t-n_1}\varepsilon_{t+1}\right)\right] \\ + \tfrac{n_2}{n}\left[\sum_{i=n_1+1}^{n}\left(\varepsilon_i + W\sum_{t=1}^{i-(n_1+1)}\phi^{t-1}\varepsilon_{i-t}\right)\right]\end{gathered} \tag{14}$$

and finally:

$$\overline{Y} = \Delta + \frac{n_2}{n}\overline{X}_2^* \tag{15}$$

where:

$$\begin{aligned}\Delta &= \tfrac{1}{n}\left[\sum_{i=1}^{n_1}\left(1 + W\sum_{t=1}^{n_1-1}\phi^{t-1}\right)\varepsilon_i\right] \\ &= \tfrac{1+W}{n}\left(\sum_{t=1}^{n-1}\phi^{t-1}\varepsilon_1 + \sum_{t=1}^{n-2}\phi^{t-1}\varepsilon_2 + \ldots + \sum_{t=1}^{n-n_1}\phi^{t-1}\varepsilon_{n_1}\right) \\ &= B_1\varepsilon_1 + B_2\varepsilon_2 + \ldots + B_{n_1}\varepsilon_{n_1}\end{aligned} \tag{16}$$

with $\Delta\sim\mathrm{N}(0,\ \sigma_\Delta)$, where $\sigma_\Delta = \sqrt{B_1^2 + B_2^2 + \ldots + B_{n_1}^2}$, so that:

$$\Delta = Z\sqrt{B_1^2 + B_2^2 + \ldots + B_{n_1}^2} \qquad \text{with } Z\sim\mathrm{N}(0,\ 1) \tag{17}$$

The variable $\overline{X}_2^*$ introduced in Eq. 15 is given by:

$$\overline{X}_2^* = \frac{\varepsilon_n + \sum_{t=n_1+1}^{n-1}\left(\varepsilon_t + W\sum_{i=t}^{n-1}\phi^{i-t}\varepsilon_t\right)}{n_2} \tag{18}$$

whose variance is:

$$\sigma^2_{\overline{X}_2^*} = C_1^2\sigma^2_{\varepsilon_{n_1+1}} + C_2^2\sigma^2_{\varepsilon_{n_1+2}} + \ldots + C_{n_2}^2\sigma^2_{\varepsilon_n} \tag{19}$$

where $C_j = \frac{\varepsilon_j + \sum_{i=j}^{n-1}\phi^{i-j}\varepsilon_j}{n_2}$ and $C_n = \frac{1}{n_2}$, with $j=n_1+1,\ldots, n-1$.

Based on the double sampling procedure, one has:

$$P_1 = \Pr\left[\overline{X}_1 \in I\right] = \Pr\left[-L_1\sigma_{\overline{X}_1} - \delta < \overline{X}_1 < L_1\sigma_{\overline{X}_1} - \delta\right]$$

which can be rewritten as:

$$P_1 = \Pr\left[\frac{-L_1\sigma_{\overline{X}_1} - \delta}{\sqrt{A_1^2 + A_2^2 + \ldots + A_{n_1}^2}} < Z < \frac{L_1\sigma_{\overline{X}_1} - \delta}{\sqrt{A_1^2 + A_2^2 + \ldots + A_{n_1}^2}}\right] \tag{20}$$

The probability P_2 is given by:

$$P_2 = \Pr\left[\left(\overline{Y} \in I_4\right) \cap \left(\overline{X}_1 \in I_2\right)\right]$$

which can be rewritten as:

$$P_2 = \int_{\frac{-L\sigma_{\overline{X}_1} - \delta}{\sqrt{A_1^2+A_2^2+\ldots+A_{n_1}^2}}}^{\frac{-L_1\sigma_{\overline{X}_1} - \delta}{\sqrt{A_1^2+A_2^2+\ldots+A_{n_1}^2}}} \left\{\Phi\left(\frac{n}{n_2}\right)\left(\frac{L_2\sigma_{\overline{Y}} - \Delta - \delta}{\sigma_{\overline{X}_2^*}}\right) - \Phi\left(\frac{n}{n_2}\right)\left(\frac{-L_2\sigma_{\overline{Y}} - \Delta - \delta}{\sigma_{\overline{X}_2^*}}\right)\right\} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}Z^2} dz$$
$$+ \int_{\frac{L_1\sigma_{\overline{X}_1} - \delta}{\sqrt{A_1^2+A_2^2+\ldots+A_{n_1}^2}}}^{\frac{L\sigma_{\overline{X}_1} - \delta}{\sqrt{A_1^2+A_2^2+\ldots+A_{n_1}^2}}} \left\{\Phi\left(\frac{n}{n_2}\right)\left(\frac{L_2\sigma_{\overline{Y}} - \Delta - \delta}{\sigma_{\overline{X}_2^*}}\right) - \Phi\left(\frac{n}{n_2}\right)\left(\frac{-L_2\sigma_{\overline{Y}} - \Delta - \delta}{\sigma_{\overline{X}_2^*}}\right)\right\} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}Z^2} dz \tag{21}$$

where Φ is the standard normal distribution function.

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