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ORIGINAL ARTICLE

Simulation of gear shaving with considerations of cutter assembly errors and machine setting parameters

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Abstract Gear shaving is commonly used as a finishing process for gear manufacturing. The precision of the shaved gear tooth profile is highly dependent on both the parameter adjustments of the shaving machine and the cutter assembly errors. This paper develops the mathematical model of the shaved gear with crowning taking into account the setting parameters of the gear shaving machine and cutter assembly errors. The effects of the parameters on the work gear surface and the tooth lead crowning are also investigated. The results are thought to be relevant to the design, assembly, calibration, and control of a gear shaving machine.

Keywords Gear shaving · Gear shaving machine · Cutter assembly error · Gear tooth crowning

1 Introduction

Gear shaving is one of the most efficient and economical processes for gear finishing after the rough cuttings of hobbing or shaping. Through this process, the highest gear precision that can be achieved is DIN 6 or DIN 7. Lead crowning of gear tooth can also be accomplished by gear shaving. This is a type of tooth modification process that eliminates edge contact of the teeth, and hence significantly increases operation cycles. There are four basic shaving methods: axial shaving, tangential shaving, diagonal shaving, and plunge shaving. To induce crowning in axial (or diagonal) shaving, only rocking of the machine work table is required by using the built-in crowning mechanism. In tangential or plunge shaving, however, the cutter has to be modified several times to meet the demand resulting in much consumed time and inevitable higher cost. The results of this research can be further applied to gear shaving so that high precision gears can be produced by simply adjusting machine settings and assembly errors.

The meshing between cutter and work gear in shaving can be viewed as a meshing pair of gears in a 3D crossed-axis manner. The rotary shaving of an involute pinion was studied by Dugas [1]. Litvin [2] proposed the basic meshing conditions of a 3D crossed-axis helical gear pair. Tsay [3] studied tooth contact and stress analysis of helical gears. Miao and Koga [4] and Koga et al. [5] calculated the crossed-axis angle at operating pitch circle, and developed a mathematical model of a plunge shaving cutter. In addition to these researches on fundamental theories of gear shaving, software for designing shaving cutters has been provided by Kim et al. [6], where meshing between cutter and work gear was approximated as an equivalent 2D spur gear model. Moriwaki et al. [7, 8] proposed a stochastic model to predict the effect of shaving cutter performance on the finished tooth form. Hsu and Fong [9] proposed a mathematical model of a serration cutting edge of a plunge shaving cutter taking into account necessary hollow tooth form. The majority of literature focussed on studying the shaving cutter itself and the corresponding effects on the shaved gears.

However, high quality shaved gear can also be made through research on the gear shaving machine. Lin [10], one of the authors of this paper, constructed a mathematical model of a gear shaving machine by first deriving an

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analytical representation of an additional angle of rotation in shaving. The model can be extended in the future into the field of controller program design in power shaving which is a newly arising technology of gear shaving for better performance and efficiency. Based on this model, this paper investigates the important influences of machine setting parameters and cutter assembly errors that would enhance the quality of shaved gears.

2 Construction of coordinate systems and shaved gear tooth profile

To derive the locus equations of the shaved gear, the coordinate system has to be constructed first. The crowning mechanism of the gear shaving machine, shown in Fig. 1, can induce lead crowning on shaved gear by rocking the work table. In the motion, the pivot can be fed horizontally only, and the pin will move along the guideway. Once the angle θ between the guideway and the horizontal is specified ($\neq 0$) in the shaving process, the rocking motion of the work table can be achieved. When $\theta=0$, the work table will move horizontally without rocking and will therefore not produce any crowning effect.

The crowning mechanism can be further parameterized as shown in Fig. 2, where d_v and d_h are the vertical and horizontal distances between the pin and pivot at the initial position, respectively. While the pivot (work table) moves horizontally (z_t) in shaving from position I to position II, the pin will move a distance d_p along the guideway. The rotating angle of the work table ψ_t can be derived as shown in Eq. 1 [10].

The coordinate system of the shaving process can be simplified and illustrated as shown in Fig. 3, where the cutter assembly errors including horizontal, vertical, and centre distance errors are considered. The coordinate systems S_s and S_2' are connected to the shaving cutter and the work gear, respectively, while S_d is the fixed coordinate

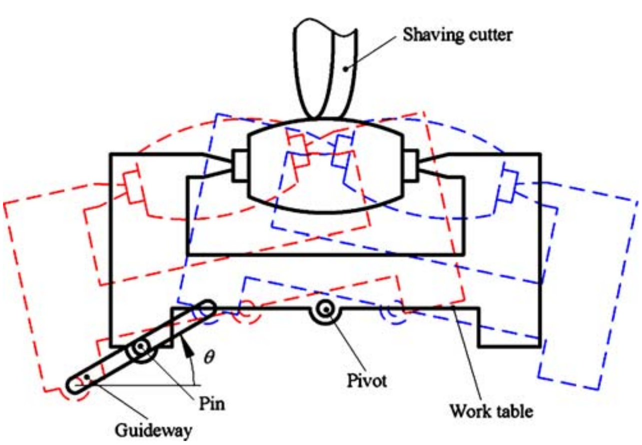

Fig. 1 Crowning mechanism of the gear shaving machine

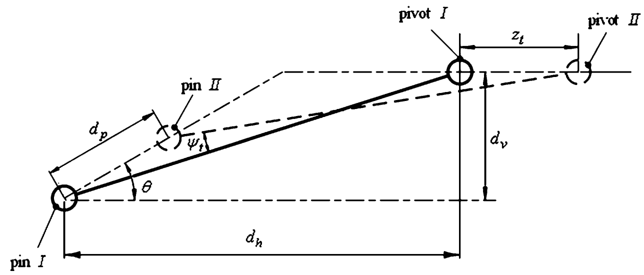

Fig. 2 Parametric representation of the crowning mechanism

system; S_h' and $S_\mathrm{v}'$are auxiliary coordinate systems for importing assembly errors into the horizontal and vertical directions; the angle Δh denotes the horizontal assembly error, the angle Δv denotes the vertical assembly error, and ΔE_0 indicates the error in the center distance. Other parameters in Fig. 3 are also described as follows: Z_t denotes the travelling distance of the shaving cutter along the axial direction of the work gear; C denotes the distance between the pivot and centre of the work gear; γ denotes the angle between the two crossed axes; E_0 represents the centre distance; ϕ_s and ϕ_2 represent the angles of rotation of the cutter and the gear, respectively, which are related to each other in the shaving operation.

$$\psi_t = \sin^{-1}\left(\frac{d_v}{\sqrt{d_h^2+d_v^2}}\right) + \sin^{-1}\left(\frac{d_h\sin\theta - d_v\cos\theta + z_t\sin\theta}{\sqrt{d_h^2+d_v^2}}\right) - \theta \qquad (1)$$

If the shaving cutter is assumed to be a helical involute gear, the surface profile and its unit normal can be

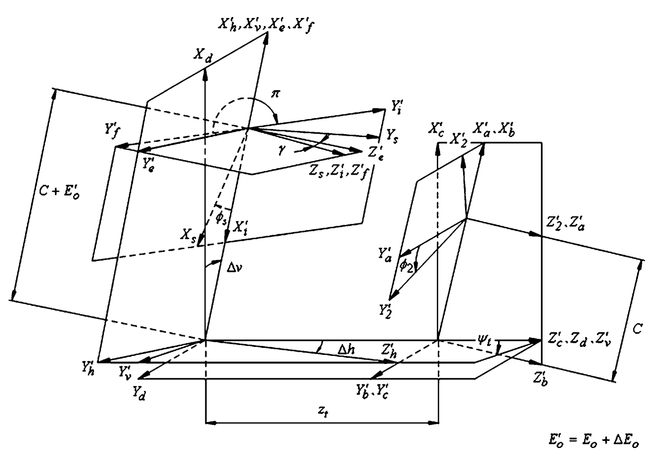

Fig. 3 Coordinate system of the gear shaving machine with considerations of cutter assembly errors

represented by Eqs. 2 and 3, derived by Litvin [2], where u_s and v_s are surface parameters.

$$\boldsymbol{r}_s(u_s, v_s) = [x_s \ y_s \ z_s]^T \tag{2}$$

$$\boldsymbol{n}_s = [n_{sx} \ n_{sy} \ n_{sz}]^T \tag{3}$$

Tooth profile $\boldsymbol{r}_s$ and surface unit normal $\boldsymbol{n}_s$ of the shaving cutter represented in the coordinate system S_s can be transformed into the coordinate system of work gear S_2' constructed in Fig. 3 by Eq. 4:

$$\boldsymbol{r}_2'(u_s, v_s, \phi_s, z_t) = M_{2's}(\phi_s, z_t) \cdot \boldsymbol{r}_s(u_s, v_s) = \begin{bmatrix} x_2' & y_2' & z_2' & 1 \end{bmatrix}^T \tag{4}$$

, in which

$$M_{2's} = M_{2'd} \cdot M_{ds}$$
$$M_{ds} = M_{dv'} \cdot M_{v'h'} \cdot M_{h'e'} \cdot M_{e'f'} \cdot M_{f'i'} \cdot M_{i's}$$
$$M_{2'd} = M_{2'a'} \cdot M_{a'b'} \cdot M_{b'c'} \cdot M_{c'd}$$

There are two major kinematic parameters ϕ_s and z_t in axial shaving gear with lead crowning so that two meshing equations (Eqs. 5 and 6) are required to calculate the enveloping surface of the work gear, where n_2' is derived from n_s through the same coordinate transformation mentioned above.

$$f_1(u_s, v_s, \phi_s, z_t) = \boldsymbol{n}_2' \cdot \frac{\partial \, r_2'}{\partial \, \phi_s} = 0 \, (z_t \ \text{constant}) \tag{5}$$

$$f_2(u_s, v_s, \phi_s, z_t) = \boldsymbol{n}_2' \cdot \frac{\partial \, r_2'}{\partial \, z_t} = 0 \, (\phi_s \ \text{constant}) \tag{6}$$

Table 1 Parameters of work gear and shaving cutter

Parameter description	Parameter Value
Work gear	
Number of teeth (T_2)	36
Normal module on pitch circle (m_{pm2})	2.65
Normal circular tooth thickness on pitch circle (S_{pn2})	4.858 mm
Normal pressure angle on pitch circle (α_{pn2})	20°
Helical angle on pitch circle (β_{pn2})	10°L.H.
Outer diameter	105.9 mm
Face width	28.4 mm
Shaving cutter	
Number of teeth (T_2)	73
Helical angle on pitch circle (β_{ps})	22°R.H.
Face width	25.4 mm
Normal circular tooth thickness on pitch circle (S_{pns})	3.348 mm

Table 2 Machine setting parameters in gear shaving

Machine setting parameters	
Angle between guideway and horizontal (θ)	2°50′
Vertical distance between pivot and pin (d_v)	188 mm
Horizontal distance between pivot and pin (d_h)	545 mm
Distance between pivot and centre of work (C)	385 mm

The parameters ϕ_s and ϕ_2 denote the angles of rotation of the cutter and gear, respectively, which are related to each other in the shaving operation by Eq. 7 when the gear ratio is assumed to be a fixed value, and where T_s and T_2 denote the numbers of shaving cutter and gear, respectively. Equation 7 needs to be modified as shown in Eq. 8 if feeding exists for the shaving cutter along the axis of the work gear, where ϕ_{2s} denotes the additional rotation angle of the work gear in axial shaving and its representation is shown in Eq. 9 [10]. $\boldsymbol{r}_{p2}$ and β_{p2} denote the pitch radius and the helical angle on the pitch circle, respectively.

$$\frac{\phi_2}{\phi_s} = m_{2s} = \frac{T_s}{T_2} \tag{7}$$

$$\phi_2 = \frac{T_s}{T_2}\phi_s \pm \phi_{2s} \tag{8}$$

$$\phi_{2s} = \frac{[(C + \boldsymbol{r}_{p2}) \sin \psi_t + z_t \cos \psi_t] \tan \beta_{p2}}{(C + \boldsymbol{r}_{p2}) \cos \psi_t + z_t \sin \psi_t - C} \tag{9}$$

Considering Eqs. 4, 5 and 6 simultaneously, the tooth surface of the shaved gear with lead crowning can be obtained under the ideal conditions ($\Delta h = \Delta v = 0°$ and $\Delta E_0 = 0$ mm). Simulation of gear shaving is carried out with properties of the work gear, the shaving cutter, and machine settings listed in Tables 1 and 2.

As shown in Fig. 4, the dashed line represents the original standard tooth surface, while the solid line represents the tooth profile with lead crowning. In the following sections, gear shaving under various working conditions is simulated for the purpose of studying the

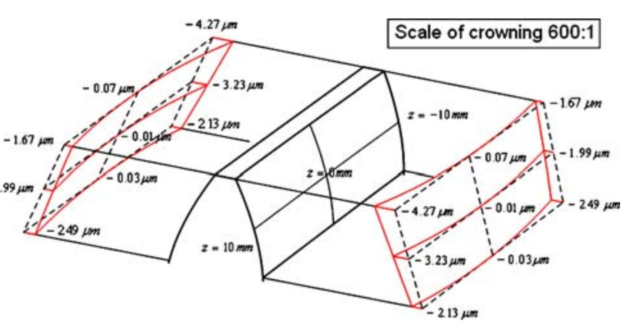

Fig. 4 Lead crowning of the shaved gear

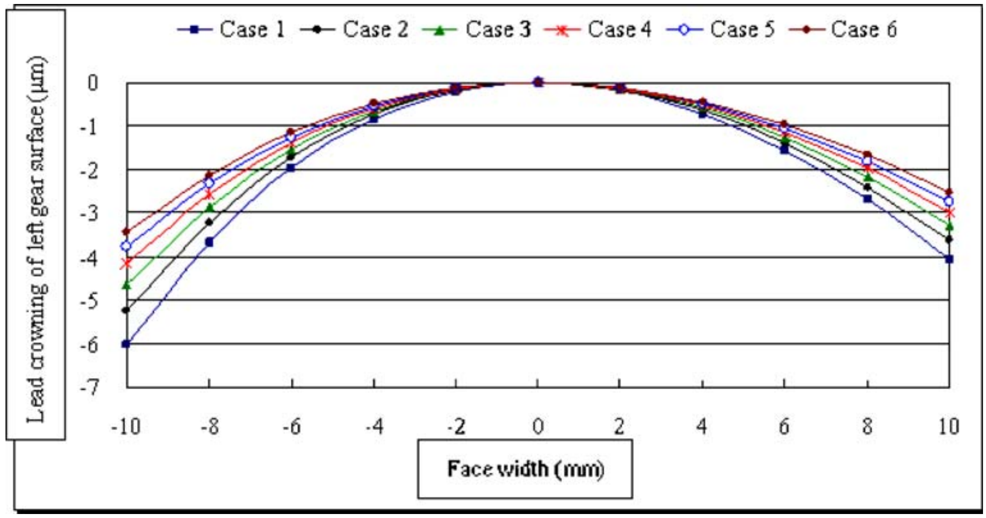

Fig. 5 Lead crowning under different conditions (d_v=188 mm, C=385 mm, θ=2°50′, d_h=350–600 mm, Δd_h=50 mm)

effects of machine setting parameters and cutter assembly errors on the shaved gears.

3 Effects of machine setting parameters on tooth crowning

From the mathematical model developed, in this section, four important machine setting parameters are investigated including the angle between the guideway and the horizontal θ, the vertical and horizontal distances between the pivot and the pin d_v and d_h, and the distance between the pivot and the center of the work gear C. The effects of these parameters are illustrated through the following examples.

3.1 Example 1

The fundamental properties of the shaving cutter and the work gear are listed in Table 1. Parameters d_v and d_h are changed to investigate the variations of lead crowning of the left surface on the operating pitch circle within the tooth face width (−10 mm to 10 mm) as shown in Figs. 5 and 6. For cases 1–6: d_v=188 mm, C=385 mm, θ=2°50′, and d_h varies from 350–600 mm in steps of 50 mm; and for cases 7–12: d_h=545 mm, C=385 mm, θ=2°50′, and d_v varies from 50–300 mm in steps of 50 mm.

It is found that the effects of parameter d_v on crowning are extremely small compared to that induced by d_h, and the variation of the parameter d_h is in inverse proportion to the amount of lead crowning. It is thought to be more effective to control this parameter in the design or assembly of the crowning mechanism.

3.2 Example 2

The properties of the shaving cutter and the work gear are listed in Table 1, and the machine parameters d_v, d_h and θ are listed in Table 2. With variations of C (case a: C=200 mm;

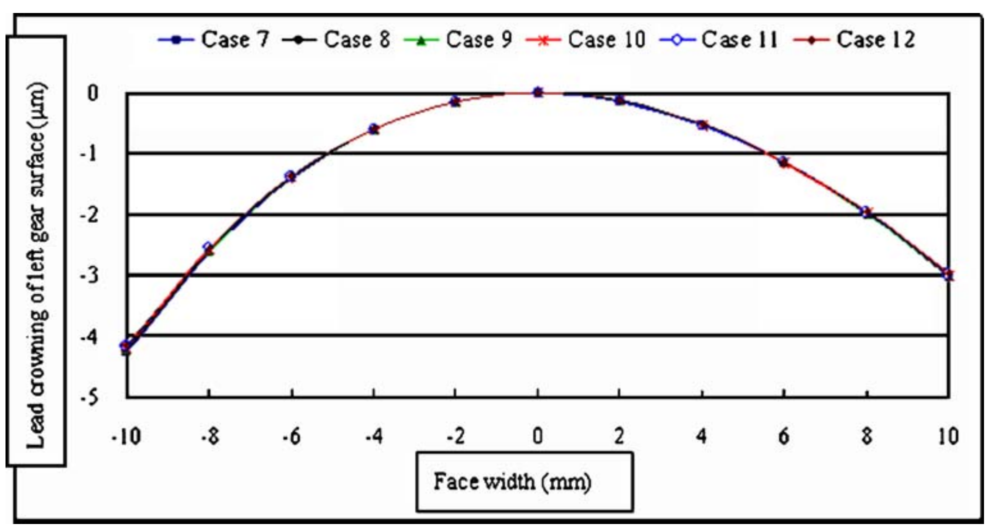

Fig. 6 Lead crowning under different conditions (d_h=545 mm, C=385 mm, θ=2°50′, d_v=50–300 mm, Δd_v=50 mm)

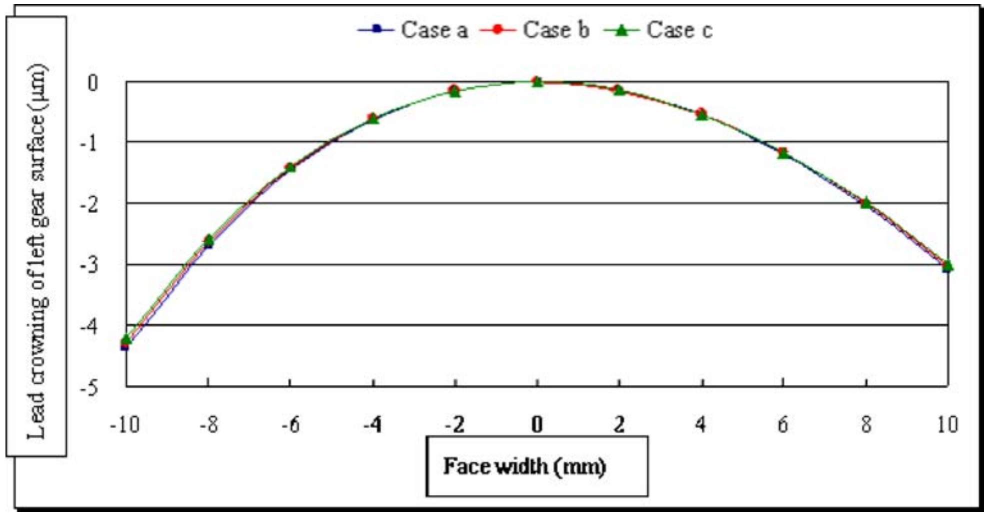

Fig. 7 Lead crowning under different conditions (d_v=188 mm, d_h=545 mm, θ=2°50′, C=200–400 mm, ΔC=100 mm)

case b: C=300 mm; case c: C=400 mm), as shown in Fig. 7, the results shows that the amount of lead crowning is in inverse proportion to the value of the variation of C but with a low sensitivity.

3.3 Example 3

The properties of the shaving cutter and the work gear are listed in Table 1, while the machine parameters d_v, d_h and C are listed in Table 2. With variations of θ (case A: θ=1°50′; case B: θ=2°50; case C: θ=3°50; case D: θ=4°50), as shown in Fig. 8, it is observed that the parameter θ indeed has great influence on the amount of tooth lead crowning.

From the above three examples, it can be summarized that:

1. The amount of gear tooth lead crowning is moderately sensitive to machine setting parameter d_h in the inverse proportion sense.
2. The amount of gear tooth lead crowning is very sensitive and proportional to the machine setting parameter θ.

In gear shaving, parameters θ and d_h can be adjusted to approach the desired amount of lead crowning. Results of this section can be further utilized with the technique of optimization to determine the best setting of the shaving machine for different work gears and shaving cutters.

4 Simulation of gear shaving with cutter assembly errors

Two examples are provided in this section to show the influences of cutter assembly errors on the quality of shaved gear. In example 5, cutter assembly errors including horizontal (Δh), vertical (Δv) and center distance (ΔE_0) are investigated by comparing them with the perfect work gear (shaved without assembly errors). However, the cutter assembly errors

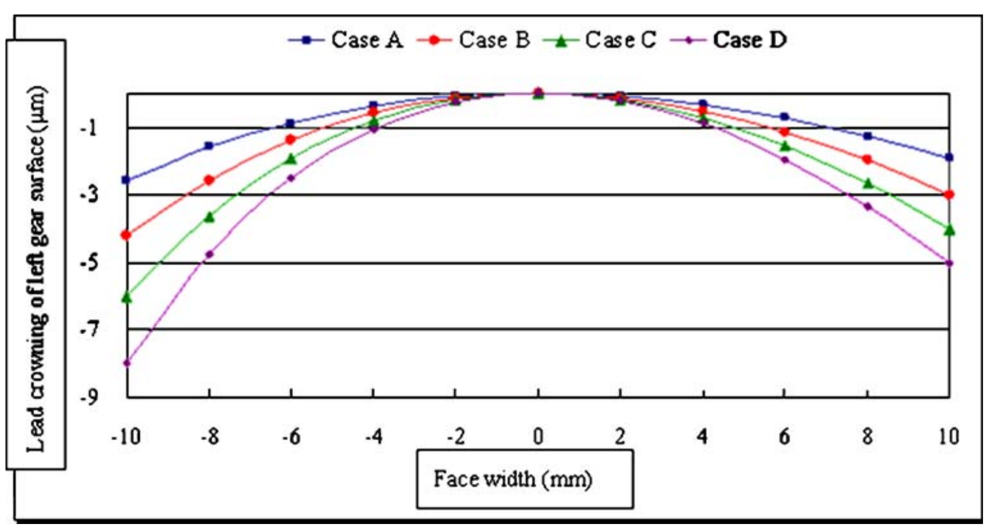

Fig. 8 Lead crowning under different conditions (d_v=188 mm, d_h=545 mm, C=385 mm, θ=1°50′–4°50′, $\Delta\theta$=1°)

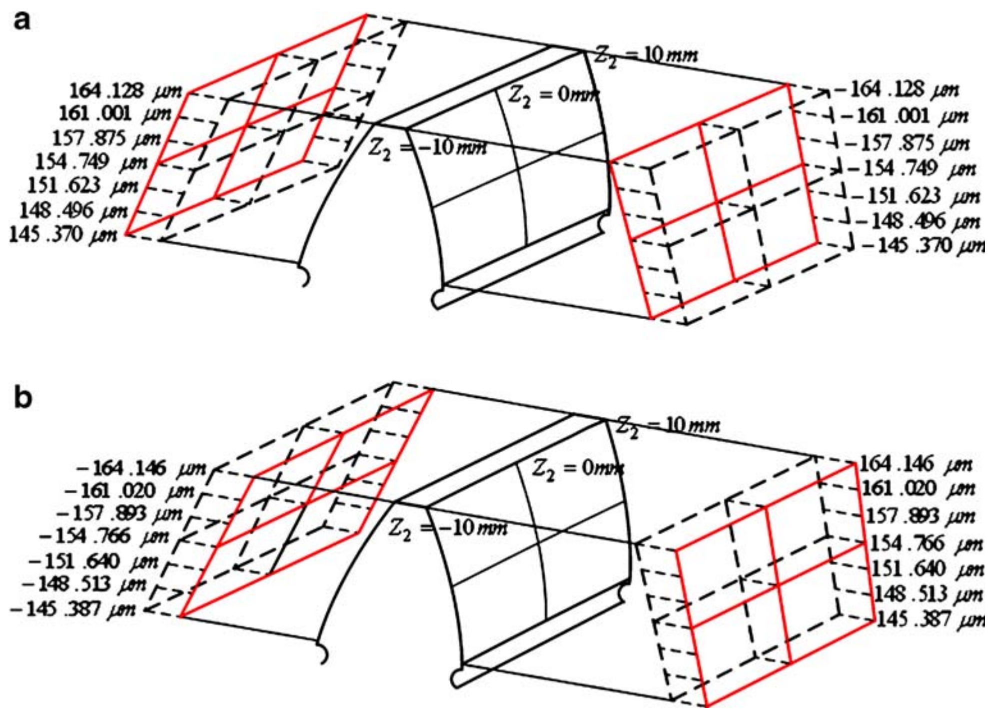

Fig. 9 Variations of circular tooth thickness from vertical error. **a** Δv=0.02°. **b** Δv=−0.02°

affect not only lead crowning, but also circular tooth thickness. Consequently, the theoretical work gear must be derived beforehand (without lead crowning, but still with cutter assembly errors). This is demonstrated in example 4. The influences of cutter assembly errors on crowning can then be studied by subtracting the tooth surface derived in example 4 from that obtained considering crowning and assembly errors.

4.1 Example 4

The properties of the shaving cutter and the work gear are listed in Table 1, and the machine setting parameters are: θ=0° (no lead crowning of gear tooth), d_v=188 mm, d_h=545 mm, and C=385 mm. Cutter assembly errors are considered in the following cases.

(i) Vertical error Δv ±0.02°
(ii) Horizontal error Δv=±0.02°
(iii) Error of center distance ΔE_0=±1 mm

Taking condition (i) as an example, as shown in Fig. 9a, the pressure angle of the right tooth surface is increased and it is decreased on the left tooth surface. Simulations of the other two conditions are completed in the same way, and demonstrated in the next example.

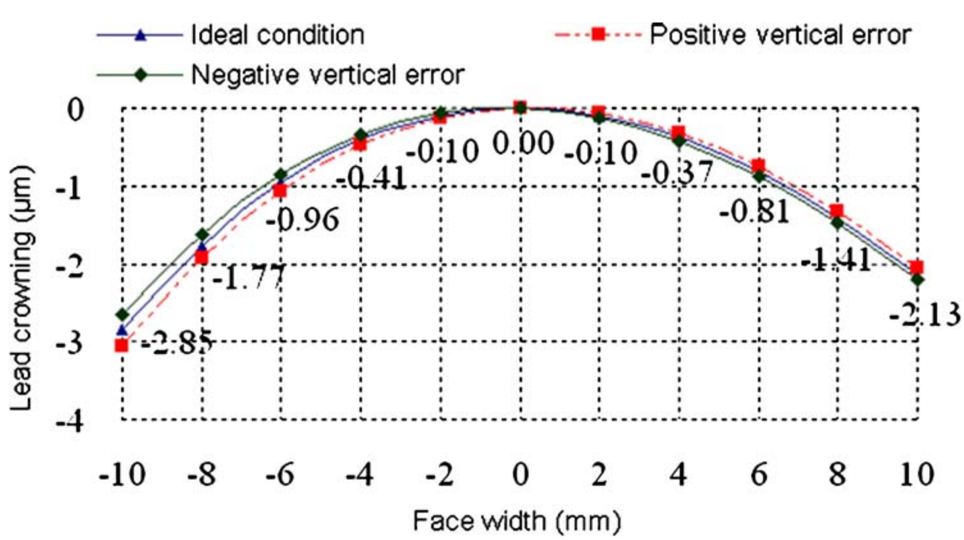

Fig. 10 Tooth lead crowning affected by vertical cutter assembly error

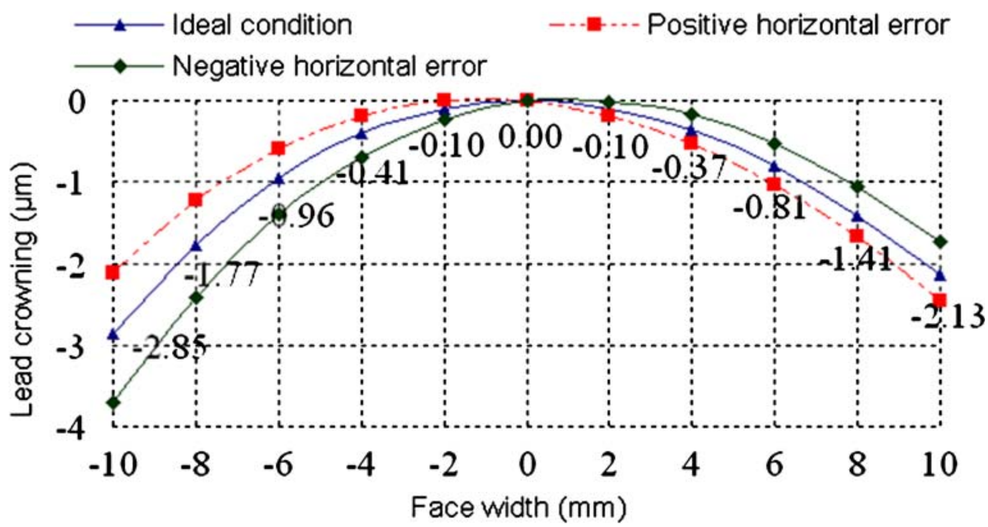

Fig. 11 Tooth lead crowning affected by horizontal cutter assembly error

4.2 Example 5

The properties of the shaving cutter and the work gear are listed in Table 1, and the machine setting parameters are listed in Table 2. The same cutter assembly errors as those in example 4 are considered. Variations of lead crowning are shown in Figs. 10, 11 and 12, in which the values of ideal condition are included for comparison. It can be seen that the horizontal and the center distance errors Δh and ΔE_0 have more significant effects on lead crowning than those caused by the vertical error Δv. From the results shown in examples 4 and 5, it can be concluded that the cutter assembly errors have significant effects on both the circular tooth thickness and tooth crowning.

5 Conclusion

In this paper, the axial shaving of the gear shaving machine has been simulated, and the gear tooth surface of the shaved gear has also been constructed. The effects of machine setting parameters and cutter assembly errors on the work gear surface and the tooth lead crowning have been investigated through numerical examples. For the crowning mechanism, the crowning effect is shown to be sensitive to the angle θ between guideway and horizontal as well as the horizontal distance d_h between pivot and pin. The horizontal and the center distance errors Δh and ΔE_0 are also proved significant to gear tooth crowning. The results in this paper can be used as instructions for design, assembly, and calibration of gear shaving machines and shaving cutters.

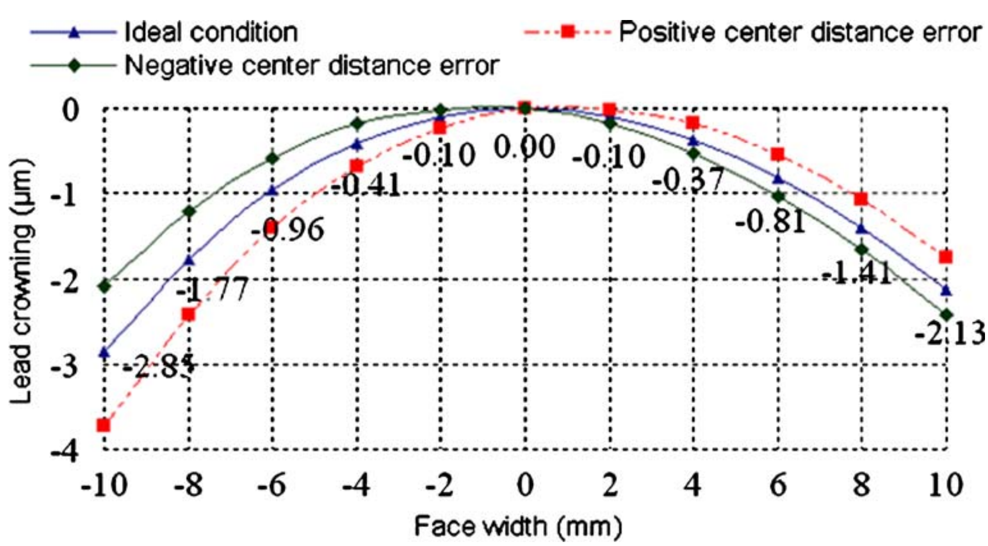

Fig. 12 Tooth lead crowning affected by cutter assembly error of the center distance

References

1. Dugas JP (1996) Gear shaving basics. Part I. Gear Technol 20: 24–31
2. Litvin FL (1994) Gear geometry and applied theory. PTR Prentice Hall, New Jersey
3. Tsay CB (1988) Helical gears with involute shaped teeth: geometry, computer simulation, tooth contact analysis, stress analysis. J Mech Trans Autom Des 110:482–491
4. Miao HC, Koga H (1996) Design and analysis of plunge shaving for finishing gears with tooth profile modifications. ASME Power Transmission and Gearing Conference, Design Engineer Division, San Diego, California, vol 88, pp 275–281
5. Koga H, Umezawa K, Miao HC (1996) Analysis of shaving processing for helical gears with tooth modification. ASME, Power Transmission and Gearing Conference, Design Engineer Division, San Diego, California, vol 88, pp 265–273
6. Kim JD, Kim DS (1996) The development of software for shaving cutter design. J Mater Process Technol 59:359–366
7. Moriwaki I, Okamoto T, Fujita M, Yanagimoto T (1990) Numerical analysis of tooth forms of shaved gears. JSME Int J Ser III 33(4):608–613
8. Moriwaki I, Fujita M (1994) Effect of cutter performance on finished tooth form in gear shaving. J Mech Des–T ASME 116(3):701–705
9. Hsu RH, Fong ZH (2006) Theoretical and practical investigations regarding the influence of the serration’s geometry and position on the tooth surface roughness by shaving with plunge gear cutter. J Mech Eng Sci 220(2):223–242
10. Lin HJ (2006) Simulation of gear shaving machines and gear tooth contact analysis. Masters Thesis, National Formosa University, Taiwan, China