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ORIGINAL ARTICLE

Modeling and numerical simulation for the machining of helical surface profiles on cutting tools

Yuwen Sun · Jun Wang · Dongming Guo · Qiang Zhang

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Abstract The classical conjugation and envelope method is very accurate and effective for forward and inverse calculations of grinding helical surfaces. However, this method involves complicated mathematics and requires that the profiles be continuous. It can also result in undercutting or interference to the desired surface profiles. In this paper, a new approach is proposed to simulate the grinding process of helical surfaces on cutting tools. The paper begins with the reconstruction of cutter helicoids from sampled points. Using the recovered helical parameters from the sample points, the cross-sectional profile of the cutter surface is derived using a polynomial curve. A numerical method for calculating the profile of the grinding wheel required for the cutter surface profile is then provided. Finally, an optimization method is presented for solving the problem of inverse calculation to determine the helical surface profile for a given grinding wheel profile and setting parameters. The feasibility of the approach is tested by simulation results, which shows that the proposed approach can eliminate undesired tool-work interferences and undercutting.

Keywords Helical surface · Profiling · Numerical simulation · Grinding · Modeling

Y. Sun (✉) · D. Guo
School of Mechanical Engineering,
Dalian University of Technology,
Dalian 116024, People's Republic of China
e-mail: xiands@dlut.edu.cn

J. Wang · Q. Zhang
School of Mechanical and Manufacturing Engineering,
The University of New South Wales,
Sydney, NSW 2052, Australia

1 Introduction

Helical surfaces are often used as rake face or flute in various cutting tools such as twist drills, hob cutters, and milling cutters. During a cutter design process, some part of the cutter geometric surfaces may be determined to give high cutting performance. To produce such geometrical surfaces as helical profiles, it is often the case where reconstructing helical surface from a limited number of sampled points is made first, followed by the calculation of a suitable grinding wheel profile to generate the helical surface. Due to the positioning or setup errors of the grinding process and the geometrical errors of the dressed grinding wheel, deviation of the actual helical surface profile from the desired profile is a major concern. As a result, it is necessary to calculate the profile of the grinding wheel and simulate the grinding process to generate the desired helical surface on a cutter.

Extensive work on grinding helical cutter surfaces has been carried out [1–4]. Based on the principle of distance minimization, a numerical method is proposed to machine helical surfaces with cutting tools bounded by surfaces of revolution [5]. This method provides a simple, yet very accurate, means for determination of the tool profile for completely arbitrary helical surfaces of constant pitch. Ivanov and Nankov [6] present a generalized analytical method for forming helical surfaces on all types of rotational tools. The profile of these surfaces is specified discretely and there are no limits for its complexity, variety, and determination in a certain section. When the profile of the initial tool surface and the parameters determining the tool orientation towards the workpiece are specified, they developed a mathematical model for the determination of the profile of the helical surface [7]. Based on the clearance as a result of the rotation angle of the return stroke, Xiao et al. [8] studied the influence of the meshing clearance on the performance of conjugate helical surface, such as conjugation interaction, contact area, continuity of the contact line, noise level, and smooth running. Mohan and Shunmugam [9] propose a CAD approach for the simulation of generation machining and identification of contact lines between the grinding wheel (or cutter) and workpiece. For this purpose, the cutter and workpiece are taken as solid models and the simulation is performed using Boolean operation to remove unwanted material in an incremental manner by maintaining the kinematical relationship. Zhang et al. [10] present an adaptive and intuitive direct method for drill flute modeling. With this method, the grinding wheel, with analytically defined profile, is assumed to move helically around a stationary drill; the flute is generated by the overlapping part between the wheel locus and the workpiece.

Generally, mathematic modeling of helical surface grinding can be classified into two categories: one is to calculate the profile of the grinding wheel according to a known helical surface and setting parameters of the grinding process, and the other is to determine the helical surface for a given grinding wheel profile and the grinding setting parameters. As far as the grinding process is concerned, the latter is an inverse calculation able to consider the influence of the errors of setup parameters and grinding wheel profile on the cutter geometrical quality, and the former is a forward calculation. At this stage of development, most of the available methods for forward calculation are based on the theory of conjugation. By contrast, inverse calculation is often solved based on the theory of envelope or computer simulation of actual machining process for cutter helicoid. Although the conjugation method and envelope method can achieve accurate solutions, both need analytical descriptions of surfaces and can cause undercutting in addition to the complexity of the computation involved. In the computer simulation approaches, the wheel and the workpiece are often decomposed into small cubes, so that it is inconvenient and imprecise for error estimation in the process of inverse calculation. Hence, solving the forward and inverse calculation problems with high accuracy and efficiency is still a challenge, despite the reported investigations as reviewed above.

This paper aims at providing a digital method to deal with various issues associated with the forward and inverse calculations in grinding helical surfaces. The major difference of the proposed method from the conjugation or envelope method as well as other computer simulation methods is that a new and simple mathematic model is established based on constrained optimization. The solution strategy proposed in this paper is shown in Fig. 1. Since sampling from a given cutter helical surface is essential for

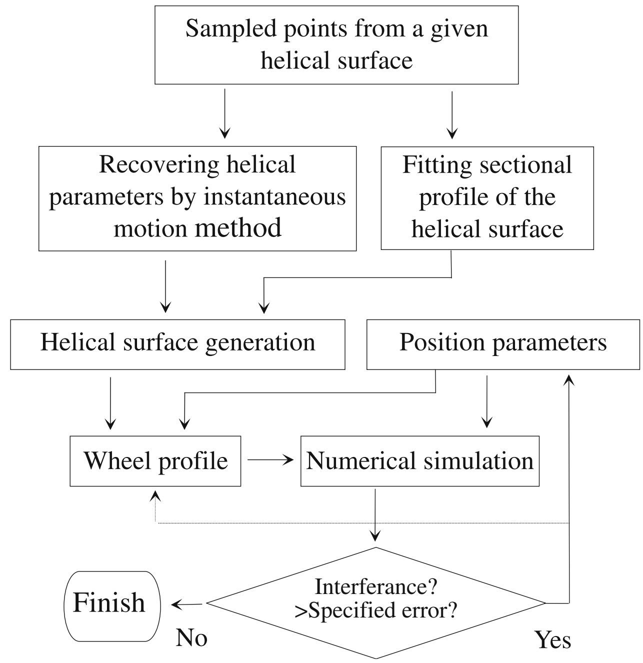

Fig. 1 Strategy of grinding helical surface

rapid cutter design and manufacturing, the reconstruction of the helical surface is discussed first. Then a method of calculating the profile of the grinding wheel is given. Subsequently, a mathematical model for geometric simulation of helical surface grinding is established. Finally, the feasibility of the proposed method is verified by simulation experiments with its advantages amply demonstrated.

Nomenclature

a	Unit vector of the cutter axis
A	Distance between rotational axis z_1 of the helical surface and axis z_2 of the grinding wheels
B	Rotation matrix
CCS	Cutter coordinate system
d	Distance from a point to a given surface
i,j,k	Unit vector attached to frame axes
L	Contact point between the helical surface and the grinding wheel
L*	Project point of the contact point ***L*** in the axis of the grinding wheel
n	Unit normal vector
$\boldsymbol{O}_1,\boldsymbol{O}_2$	Original points of coordinate systems
p	Position vector (non-parametric form)
p	Helical parameter, $p=H/(2\pi)$, H, is the pitch of the helical surface
R	Position vector (parametric form)
t	Reference parameter
V	Velocity vector
WCS	Wheel coordinate System

α,β	Grinding wheel installation angles in different coordinate directions
φ	Helical angle
Γ	Plane

2 Helical surface reconstruction

Compared to sculptured surface reconstruction, recovering a helical surface from sampled points is very complicated due to the difficulties in determining the helical surface parameters. A cylindrical helicoid surface with a constant pitch is often formed by some profile curve that rotates uniformly around the cylindrical axis and, at the same time, traverses in the direction parallel to the cylindrical axis. Therefore, in this paper the reconstruction of the cutter helicoid surface is implemented in two steps. The first step is to find the pitch and the screw axis represented by a unit direction vector together with its normal position vector. The second step involves recovering the cross-sectional profile of the helical surface.

The problem of effectively identifying helical surface from sampled points is often solved using the instantaneous motion method or differential motion analysis method. As shown in Fig. 2, for a given sampled point $\mathbf{P}_i, i = 1, 2, \cdots, n$, let n_i be the estimated normal vector at point $\boldsymbol{P}_i$ in the helical surface, respectively. Considering a uniform screw motion of the point $\boldsymbol{P}_i$ in the helical surface, let $\boldsymbol{v}(\boldsymbol{P}_i)$ denote the velocity vector of that point, then the uniform screw motion has a velocity vector field with the form

$$\mathbf{v}(\mathbf{P}_i) = \overline{c} + c \times \mathbf{P}_i \tag{1}$$

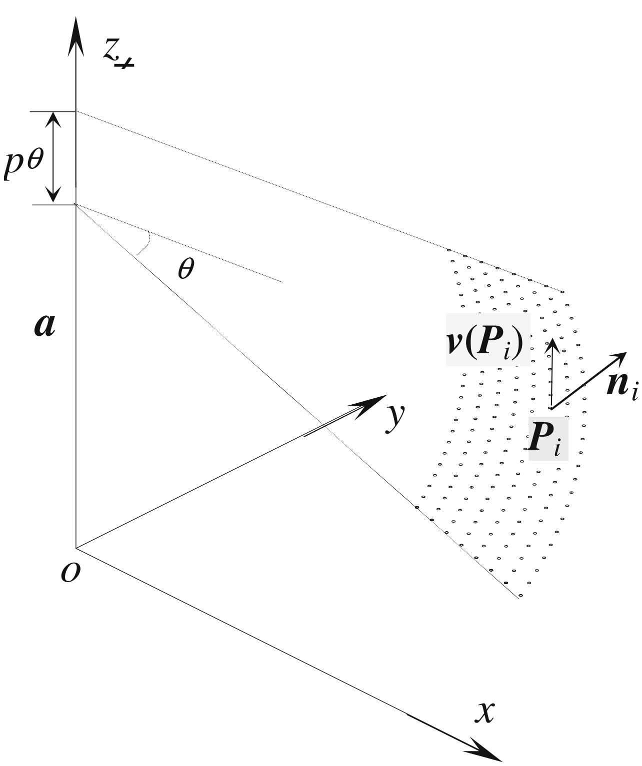

Fig. 2 Instantaneous motion of a point in a helical surface

The vector pair $(\overline{c}, c)$ means the rotational axis of the helical surface has a direction vector c and passes through point $\boldsymbol{P}$ with $\overline{c} = c \times P$. Thus, the issue of identifying screw parameters can be converted into finding a motion with velocity field characterized by $(\overline{c}, c)$ to ensure that the velocity vectors $\boldsymbol{v}(\boldsymbol{P}_i)$ at points $\boldsymbol{P}_i$ forms an angle α_i close to $\pi/2$ with the normal vector $\boldsymbol{n}_i$. According to [11], we can have

$$\min(\overline{c}, c) = \sum_{i=1}^{n} \left(\overline{c} \cdot n_i + c \cdot \overline{n}_i\right)^2 \tag{2}$$

with $\overline{n}_i = d_i \times n_i, \|c\|^2 = 1$. The solution of the above minimization equation is a generalized eigenvector corresponding to the smallest generalized eigenvalue. From the solution vector $(\overline{c}, c)$ of the minimization equation, the axis vector $(a, \overline{a})$ and the helical parameter p of the screw motion can be calculated as

$$a = \frac{c}{\|c\|}, \overline{a} = \frac{\overline{c} - pc}{\|c\|},\ p = \frac{c \cdot \overline{c}}{c^2} \tag{3}$$

Once the screw parameters are known, the subsequent task is to derive the equation of the helical surface. However, due to the non-coincidence of the coordinate axis of the measuring system and the screw axis, the equation of the helical surface will be rather complicated. In this case, a new coordinate system is established for the convenience of expressing the equation of the desired helical surface. By assuming that the original point of the coordinate system is $\boldsymbol{O}_0$, the z axis is along the direction of vector a, and the y axis is along the direction of the vector $(\mathbf{P}^* - \mathbf{O}_0) \times \mathbf{a}$, the coordinates of each sampled point can be given as

$$\begin{gathered} x_i = (\mathbf{P}_i - \mathbf{O}_0) \cdot \mathbf{h}_1, y_i = (\mathbf{P}_i - \mathbf{O}_0) \cdot \mathbf{g}_1, z_i = (\mathbf{P}_i - \mathbf{O}_0) \cdot \mathbf{a} \\ \mathbf{h}_1 = \frac{(\mathbf{P}_i - \mathbf{O}_0) \times \mathbf{a}}{|(\mathbf{P}_i - \mathbf{O}_0) \times \mathbf{a}|}, \mathbf{g} = \frac{\mathbf{h_1} \times \mathbf{a}}{|\mathbf{h_1} \times \mathbf{a}|} \end{gathered} \tag{4}$$

where point $\boldsymbol{P}^*$ is a sampled point. In the new coordinate system, the equation of the helical surface is defined as follows

$$\mathbf{R}(t, \varphi) = \mathbf{B}_1 \mathbf{r}(t) + p\varphi \mathbf{k} \tag{5}$$

where rotation matrix $\mathbf{B}_1 = \begin{bmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{bmatrix}$, k=[0 0 1]^T, and the cross-sectional profile of the helical surface $\mathbf{r}(t) = [x(t) y(t)]^T$.

In the process of designing and analyzing cutter helicoids such as drill flutes, the profile in a cross section normal to the cutter axis is often approximated as a polynomial curve. To fit the cross-sectional profile, surface points in the cross section and the location of the cross section plane must be gained in advance. However, sampled points from a given cutter helicoid are discrete; it is not sure that these points are exactly in the plane. Hence, to acquire these cross-sectional points, uniform screw motion analysis of a point will be used to deal with this situation. Without losing generality, let the cross-section plane pass through the point $\boldsymbol{P}_0$ in the screw axis, then the equation of the plane can be written as

$$\mathbf{a} \cdot (\mathbf{P} - \mathbf{P}_0) = 0 \tag{6}$$

Thus, the points used to fit the curve in the plane can be given as intersectional points of the plane and helices formed by the screw motion of the sampled points $\boldsymbol{P}_\mathrm{i}$. These points have the following expressions

$$\mathbf{r}_j(x_i(t), y_i(t)) = \mathbf{B}_1^{-1}(\mathbf{P}_i - p\varphi\mathbf{k}), j = 1, 2, \cdots, m \tag{7}$$

where $\mathbf{B}_1^{-1} = \begin{bmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{bmatrix}$, $\varphi = \mathbf{a} \cdot (\mathbf{P}_i - \mathbf{P}_0)/p$ Given a set of sectional points $P_i^c(x_i(t), y_i(t), 0)$ in the plane, an operation of coordination transformation has to be performed first for the simplification of fitting process. As shown in Fig. 3, the origin of the new coordinate system can be fixed at the point $\boldsymbol{P}_0$, the z axis aligns with the screw axis, and the x axis is along the direction of the vector $P_{\max}^c - P_0$. $\boldsymbol{P}_{\max}$ is the most outward point among sectional points. i,j and k are unit vectors along the positive x,y and z directions of the coordinate system, respectively. The equation of the cross-sectional profile can be assumed to take the form of

$$\mathbf{r}(x, y) = x\mathbf{i} + \sum_{i=1}^{q} b_i x^i \mathbf{j} \tag{8}$$

where the coefficient q is determined according to the geometric complexity of the cross-sectional profile of the cutting tool, and the coefficients b_i can be determined by solving over-determined linear equations, i.e.,

$$\sum_{i=1}^{q} b_i x_j^i = y_j, \ j = 1, 2, \cdots, m \tag{9}$$

Essentially, the fitting criterion of the above linear equations is to minimize the errors along the direction of y axis. Uniformly rotating the cross-sectional curve about the axis a and proportionally translating it parallel to a at the same time, the desired cutter helicoid can be derived as

$$\begin{aligned} \mathbf{R}(\mathbf{t}, \varphi) &= \mathbf{x}(\mathbf{t}, \varphi)\mathbf{i} + \mathbf{y}(\mathbf{t}, \varphi)\mathbf{j} + \mathbf{z}(\mathbf{t}, \varphi)\mathbf{k} \\ &= \left(t\cos\varphi - \sum_{i=1}^{n} b_i t^i \sin\varphi \right)\mathbf{i} + \left(t\sin\varphi + \sum_{i=1}^{n} b_i t^i \cos\varphi \right)\mathbf{j} + p\varphi\mathbf{k} \end{aligned} \tag{10}$$

In addition, if the cross-sectional profile of the cutter helicoid consists of many segments from different curves, piecewise polynomial curves or spline curve can be used to recovery the desired profile from the given cross-sectional data.

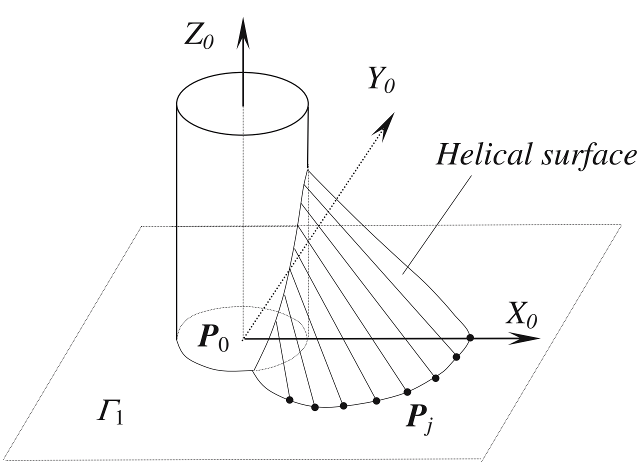

Fig. 3 Coordinate frame assignment

3 Calculation of grinding wheel profile

To calculate the profile of the grinding wheel required for the desired cutter helicoid surface, two coordinate systems are first established. As shown in Fig. 4, $\{0_1, x_1, y_1, z_1\}$ is the cutter coordinate system (CCS). The $z_1$axis is coincident with the given cutter axis. $\{0_2, x_2, y_2, z_2\}$ is a local coordinate system associated with the grinding wheel, or wheel coordinate system (WCS). The $z_2$axis is coincident with the axis of the grinding axis. α and β are the installation angles of the grinding wheel, or wheel position parameters, to specify the wheel position, where α is the angle of rotation from X_1 axis to X_2 axis about the Z_1 axis in the cutter frame CCS, and β is the angle of rotation from the Z_1 axis to Z_2 axis about the X_1 axis after the first rotation transformation. Both angles are considered positive if anticlockwise when looking down the rotation axis towards the origin. A is the distance between the original point O_1 and O_2. Axis y_1 and y_2 are determined according to the right-hand system.

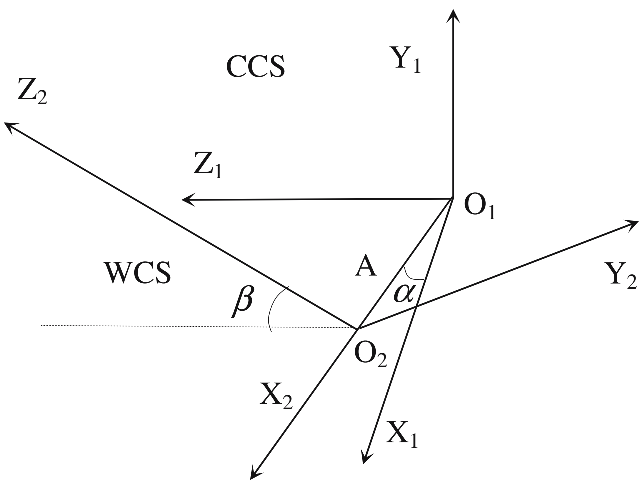

Fig. 4 Coordinate systems for machining: CCS denotes cutter frame, WCS denotes wheel frame

The profile of the grinding wheel for machining a cutter helicoid surface is usually derived using the theory of conjugation. The mathematics involved in this method is quite complex, resulting in difficulties in calculations, and to some extent the advantage of computer simulation is not taken into account in this approach. Hence, there is a need for a digital method that will be robust and easy in handling conjugate questions. In the process of grinding, cutter helicoid surfaces such as drill flutes, the geometric shape of the contact line between the grinding wheel, and the helical surface are constant. Likewise, the normal vector of the helical surface at the contact line must intersect with the axis of the grinding wheel. Using these geometric properties, the profile of the grinding wheel can be easily derived by a computer simulation method.

For a helix $R(t_0, \varphi)$ in the helical surface, the normal vector $n(t_0, \varphi)$ of the helical surface at the helical curve can be given as

$$n(t_0,\varphi) = \left(R_t \times R_\varphi\right)/\left|R_t \times R_\varphi\right| \tag{11}$$

Let a line denoted by L_1 be parallel to the normal vector $n(t_o,\varphi)$ and pass through the point $R(t_o,\varphi)$. Similarly, the line L_2 aligns with the wheel axis. If the distance of the two lines is equal to zero, the line L_1 must intersect with the line L_2. In this case, the exact angle φ corresponding to the contact point of the helix and the wheel surface can be derived.

In the grinding wheel coordinate system WCS, a point in the helical surface and its normal vector can be transformed as the following equation

$$\begin{aligned} S_w &= \mathbf{B}(\beta)\mathbf{B}(\alpha)\mathbf{R}(t_0,\varphi) - A\mathbf{i}, \\ \mathbf{n}_w &= \mathbf{B}(\beta)\mathbf{B}(\alpha)\mathbf{n}(t_0,\varphi) \end{aligned} \tag{12}$$

where

$$\mathbf{B}(\beta) = \begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \cos\beta & \sin\beta \\ \mathbf{0} & -\sin\beta & \cos\beta \end{bmatrix}. \mathbf{B}(\alpha) = \begin{bmatrix} \cos\alpha & \sin\alpha & \mathbf{0} \\ -\sin\alpha & \cos\alpha & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} \end{bmatrix}$$

If a plane passing through the grinding wheel axis is parallel to the normal vector, the vector n_w^p of the plane can be written as follows

$$\mathbf{n}_w^p = \mathbf{n}_w \times \mathbf{b}, \mathbf{b} = [001]^{\mathrm{T}} \tag{13}$$

Thus, the distance of the line L_1 and the wheel axis becomes the distance from a point in the line L_1 to the plane that passes through the wheel axis which can be written as

$$d = \mathbf{n}_w^p \cdot \mathbf{S}_w \tag{14}$$

Hence, when the distance d is equal to zero, the point S_w in the helix becomes one contact point between the helix and the wheel surface. The exact angle φ at the contact point can be gained by solving the following equation

$$\begin{aligned} d(\varphi) &= (x(t_0,\varphi)A)n_{y_u} \\ &\quad - (y(t_0,\varphi)\cos\beta - z(t_0,\varphi)\sin\beta)n_{x_u} = 0 \end{aligned} \tag{15}$$

By substituting the analytical solution φ^* into the above equation into Eq. (10), the contact point between the helix and the grinding wheel can be determined. However, the above equation cannot be easily solved and the following solution procedure is used. Since the distance d is a signed distance and changes continuously, the bisection method can be recursively used until the computing accuracy is controlled within a specified value. For a given set of parameters t_i of the cutter helicoid surface, a set of contact points q_i can be used to model the profile of the grinding wheel. Let $R(h)$ represent the distance from the contact point q_i to the axis of the grinding wheel, which can be written as

$$R(h_i) = |(\mathbf{q}_i - \mathbf{o}_1) \times \mathbf{g}| \tag{16}$$

where h_i is the z_2-axis coordinate of the contact point in the grinding wheel system, so that

$$h_i = (\mathbf{q}_i - \mathbf{o}_1) \cdot \mathbf{g}$$

Then a polynomial curve can be fitted to the points $(h_i, R(h_i))$, $i = 0, 1, \cdots, k$ to form the profile of the grinding wheel as

$$R(h) = \sum_{i=1}^{\gamma} d_i h^i \tag{17}$$

To represent the grinding wheel as a rotation surface profile, the following equation is used

$$x_1 = R(h)\cos\theta, y_1 = R(h)\sin\theta, z_1 = h \tag{18}$$

4 Geometric simulation model

The inverse calculation of conjugate motion for grinding helical surface is to derive the envelope surface of the moving wheel with a uniform screw motion relative to the tool blank (or workpiece). In this case, the motion parameters, and the profile of the grinding wheel are known. However, in most cases, calculating the envelope surface is not the final step, as the calculation may be unable to take into account all system parameters that affect the ground profile details due to the complexity of the problem. As a result, it is often necessary to undertake a geometric simulation to study the machined profile and its quality by incorporating such factors as grinding wheel installation or setup errors into the grinding process. Based on such a simulation, the validity of the parameter setting as well as the undercutting phenomenon that may exist in all enveloping processes can be detected and possible remedial measures can be taken. Thus, a good geometric simulation approach that is easy to perform and is able to handle such phenomena as undercutting is necessary in studying the machining of helical surfaces.

Aiming at establishing a model for predicting geometrical errors of the ground surface, a geometric simulation method is provided here so as to get direct evaluation about the status of the grinding process. The method based on constrained optimization can be given an intuitive interpretation. According to the motion relations between the grinding wheel and the cutter surface (or workpiece), the grinding process can be regarded as that the cutter is stationary while the grinding wheel helically moves itself about the cutter axis at a rotation angle of φ and a pitch of p. When φ=0, the grinding wheel is in the status of initial setting position of the wheel and the cutter. For a given point in the helical surface, a pole is assumed to pass through the point and align with the normal vector of the surface at that point. Accordingly, grinding is viewed as a process in which the pole is cut progressively by the cutter. At the time, when the cutter no longer contacts the pole, the length of the pole becomes smallest. Thus the proposed simulation model can go beyond the limit existing in the envelope method. It should be noted that the final surface maybe is not the resulted surface generated by a pure conjugate action. In order to find a proper position of the helically moving wheel so as to cut the pole to the shortest length, the problem can be treated as an unconstrained optimization process. However, directly calculating the length of the pole left involves the calculation of the intersectional point between a line and a rotation surface with a cross-sectional profile expressed by a polynomial curve. Such a calculation is very difficult to make. Owing to the complicity of calculation, the issue of line-surface intersection is converted into an optimization problem with constraints. Hence, the simulation model for error calculation of grinding helical surface is essentially a constrained optimization process.

As shown in Fig. 5, Γ is a plane that passes through the wheel axis. For a given line, L is a point in the line. The line segment LL^* is normal to the wheel axis. If the length of the line segment LL^* is equal to the wheel radius at that position, the point L must be at the intersection of the line and the wheel surface. In this case, the mathematic model for calculating the error reduces the complicity of computation. When the wheel helically moves around the cutter axis, the normal vector of the plane, the vector of the wheel axis and the original point O_1 can be transformed into

$$\begin{aligned} &\mathbf{O}_1^t = \mathbf{B}(\alpha)^{-1}\left(\mathbf{B}(\beta)^{-1}\mathbf{O}_1 + A\mathbf{i}\right), \mathbf{n}_p^t = \mathbf{B}(\alpha)^{-1}\mathbf{B}(\beta)^{-1}\mathbf{n}_p \\ &\mathbf{a}^t = \mathbf{B}(\alpha)^{-1}\mathbf{B}(\beta)^{-1}\mathbf{a} \end{aligned} \tag{19}$$

where I=$[1\ 0\ 0]^{\mathrm{T}}$. At this moment, for an arbitrary point P in the cutter surface and its associated normal vector n, the line equation can be expressed as

$$\mathbf{L} = \mathbf{P} + t\mathbf{n} \tag{20}$$

When the wheel moves to a proper position, the wheel surface will intersect with this line. Given a point L^* in the line with the following expression

$$\mathbf{L}^* = \mathbf{P} + t^*\mathbf{n} \tag{21}$$

the distance from the intersectional point to the wheel axis can be written as

$$d = \left|\left(\mathbf{L}^* - \mathbf{O}_2^t\right) \times \mathbf{a}\right| \tag{23}$$

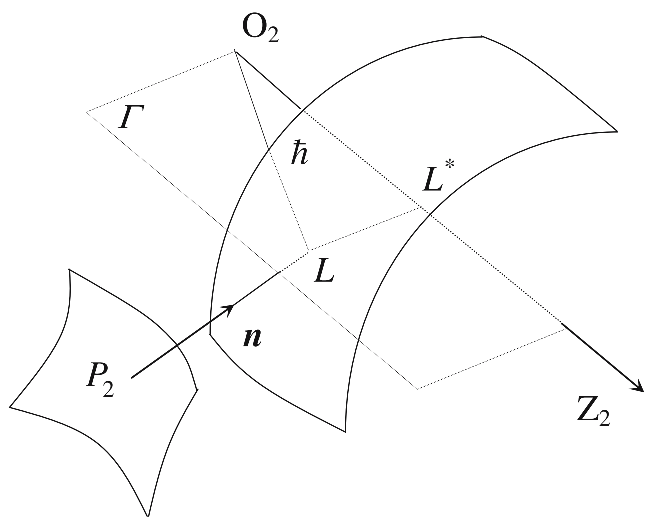

Fig. 5 Model for intersectional point calculation

It follows that if the point L^* is the intersectional point of the line and the wheel surface, the distance must be equal to the radius of the wheel at that position. Let $l = \left(\mathbf{L}^* - \mathbf{O}_2^t\right) \cdot \mathbf{a}$, the error model at the point can be established as follows

$$\min(\varphi, t^*) \quad d = t^* \, s.t. \quad \left|\left(\mathbf{L}^* - \mathbf{O}_2^t\right) \times \mathbf{a}\right| - \sum_i^n b_i\left(\left(\mathbf{L}^* - \mathbf{O}_2^t\right) \cdot \mathbf{a}\right)^i = 0 \tag{23}$$

This is a constrained optimization question that can be solved by various optimization algorithms. The normal vector used in Eq. (20) is the normal vector of the reference surface. The reference surface is usually the ideal cutter helicoid. Then the error model is established based on the reference surface. Therefore, once a reference surface point is given, the unit normal vector is determined in advance, so it has no relationship with the whole optimization process. The purpose of using constrained optimization is to avoid the computational complexity of directly using the envelope method.

The above-mentioned constrained optimization can be converted into an unconstrained optimization by introducing the Lagrange multiplier. Usually it is necessary to give the initial parameters for performing the unconstrained optimization procedure. The accuracy and efficiency of the optimization operation depend very much on the validity of the initial parameters. Since deviations between the real surface and the desired surface are very small, the initial parameters can be set at the position where the wheel axis intersects with the pole. In this case, the optimization will be convergent to a minimum at several iterations. By performing the optimization operation for each of the sampled points, a set of errors between the ground surface and the desired helical surface can be derived.

5 Numerical examples

To testify the mathematic model and numerical simulation method given in the previous section for helical surface profiling, an Archimedean helical surface is used as an example to demonstrate the process of helical surface reconstruction and grinding.

5.1 Helical surface reconstruction

Two hundred points are sampled from the Archimedean helical surface. To imitate the measuring behavior of a coordinate measuring machine (CMM), each point $\boldsymbol{P}$ is perturbed randomly by the following equation with ξ= 0.005 [12]

$$\mathbf{P}' = \mathbf{P} + \xi \frac{\left(e_x, e_y, e_z\right)^T}{\sqrt{e_x^2 + e_y^2 + e_z^2}} \tag{24}$$

where e_{x}, e_{y} and e_{z} are randomly chosen numbers that vary from −1 to 1. The normal vector at each point in the surface is calculated according to a local surface-fitting method. According to formula (2), the following matrix can be derived

$$\mathbf{B} = \mathbf{M}^T\mathbf{M}, \quad \mathbf{M} = \begin{bmatrix} \mathbf{n}_1 & \mathbf{n}_1 \times \mathbf{P}_1 \\ \mathbf{n}_2 & \mathbf{n}_2 \times \mathbf{P}_{2i} \\ \vdots & \vdots \\ \mathbf{n}_{200} & \mathbf{n}_{200} \times \mathbf{P}_{200} \end{bmatrix} \tag{25}$$

$\mathbf{B}$ is a symmetric and non-negative definite 6×6 matrix. From this matrix, the eigenvector corresponding to the smallest generalized eigenvalue can be calculated. Thus, according to Eq. (3), the screw parameters are subsequently

Table 1 Comparison of theoretical helicoids surface and reconstructed helical surface

Parameter	Value	Parameter	Value
Number of sampled points from helical surface	200	Simulated sampling error	±0.005 mm
Theoretic screw axis vector	(0, 0, 1)	Helical parameter	9.0
Recovered screw axis vector	(0.000417, −0.000002, 1.000000)	Recovered helical parameter	8.998746
Minimum diameter of cylindrical helicoid	50.0 mm	Maximum diameter of cylindrical helicoid	70.0 mm
Theoretical installation angle α	45°	Theoretical installation angle β	18°
Installation angle error $\Delta\alpha$	0.1146°	Installation angle error $\Delta\beta$	0.1146°
Minimum diameter of wheel	41.61 mm	Maximum diameter of wheel	58.54 mm
Fitting error of the wheel profile	0.02 μm		
Axial distance A	50 mm		
Axial distance error	0.03 mm		
Number of helix	10	The axis distance of the wheel start point from the origin	27.932269 mm

given. With regard to the cross-sectional profile of the helical surface, it can be reconstructed by helically moving the points into a fixed plane according to Eqs. (4)~(9), finally the reconstructed helical surface is expressed by Eq. (10). The test data and results of reconstructing the helical surface are illustrated in Table 1. It can be noticed that the recovered screw parameters are very close to their theoretical values within negligible errors. It may be deduced from this example that the reconstruction strategy proposed in this paper is feasible and accurate.

5.2 Forward and inverse calculation of grinding helical surface

The setting parameters for grinding the helical surface for this example are shown in Table 1. The theoretical installation angle α=45° and β=18°. The distance between the rotation axis of the helical surface and the axis of the grinding wheel is set A=50 mm. These theoretical installation parameters are often determined after several adjustments or trials to avoid interference between the helicoid surface and the grinding wheel. The forward calculation to derive the profile of the grinding wheel is performed first. According to Eq. (14), the problem of conjugate contact between the helical surface and the grinding wheel is converted into an intersectional problem of a family of helix and the wheel axis based on the foregoing analysis. In the calculation, the number of helix is set at ten, and the grinding wheel contact point in each helix is computed. Consequently, a third degree polynomial curve is fitted to these contact points. As shown in Table 1, the maximum fitting error is less than 0.02 μm. It can be seen that the grinding wheel has a complicated profile and its fitting accuracy is very high when using a third-degree polynomial curve. The resulted profile of the grinding wheel is represented graphically as shown in Fig. 6.

To testify the influence of position or setting parameters on the ground surface in the inverse calculation with a given grinding wheel profile, the deviations of installation angle is set $\Delta\alpha$=0.1146° and $\Delta\beta$=0.1146°, respectively. In the Archimedean helical surface, the contact line between the grinding wheel and the helical surface is used to examine the deviations of the ground surface from the ideal

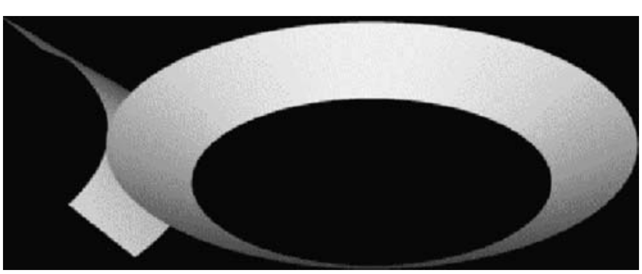

Fig. 6 Interference of helicoid and grinding wheel

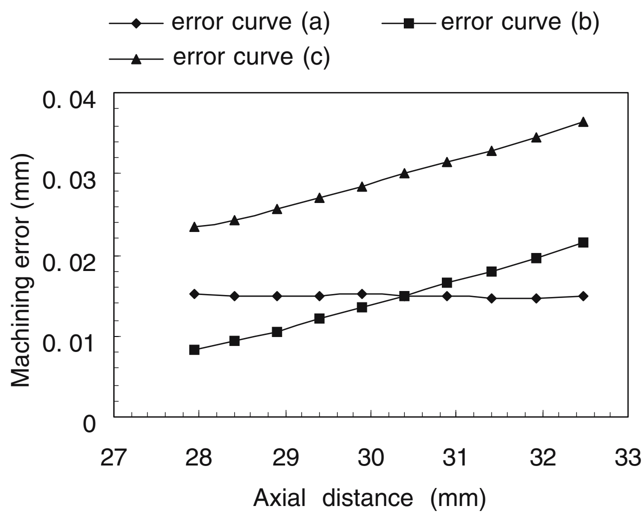

Fig. 7 Machining errors due to deviations of position parameters

surface as a result of the errors of position parameters. Ten points are sampled from the contact line to establish the error-calculation model. The axis distance of the wheel start point from the origin is shown in Table 1, and the origin is just the point O_1 in the coordinate system WCS. The individual and combined effects of the wheel setting angle errors on the grinding errors have been calculated and are shown in Fig. 7. From this figure, it can be seen that the deviations of two installation angles have a different influence on the accuracy of the ground surface, where error curve (a) shows the ground surface errors caused by the deviations of the installation angle α, which is almost constant as the axial distance of the grinding wheel changes, the error curve (b) presents the machined errors caused by the installation angle β, which shows that the error increases with the axial distance of grinding wheel, and the error curve (c) shows the combined effect of the two installation angle errors on the machined surface errors at selected points. It can be seen that the effects of the

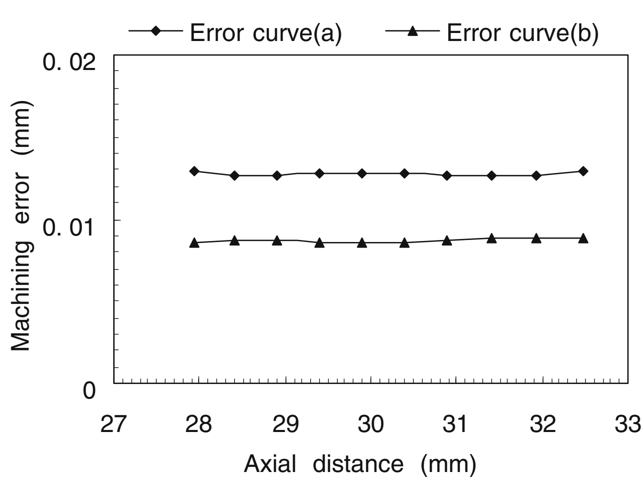

Fig. 8 Machining errors due to deviations of grinding wheel profile

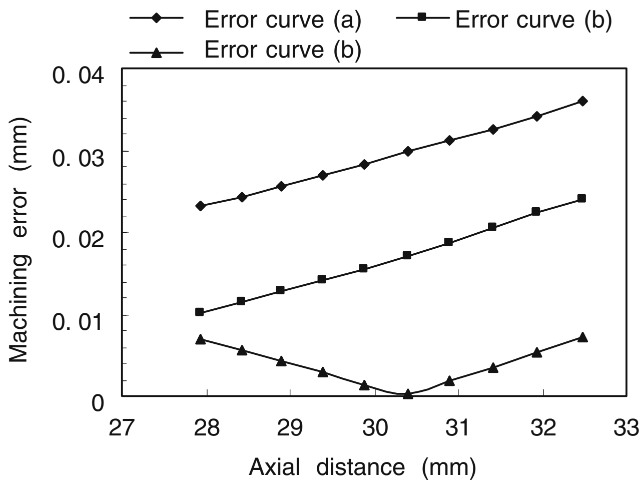

Fig. 9 Machining errors due to deviations of position parameters and wheel profile

wheel installation angles on the helical surface accuracy are significant, and proper measures must be taken to reduce these setup errors. Other major factors that affect the machined profile error are the dressed grinding wheel profile error and the distance error between the tool axis and the wheel axis. It is common that a complex profiled grinding wheel is dressed in such a way that the wheel profile is made of some circular or linear segments, rather than the desired complex profile. Such an approximation will result in geometrical errors in the helical surface grinding. As illustrated in Fig. 8, error curve (a) shows that when the distance error is 0.02 mm, the corresponding machining error is approximately 0.012 mm. On the other hand, if the wheel profile is reduced by 0.02 mm along the radial direction of the grinding wheel as a result of the dressing error, the resulting helical surface profile error is shown in error curve (b). Although this error is smaller than the wheel error, it is still significant. Figure 9 shows the combined effects of positioning parameter error and profile error on the machining error. From error curves (a), (b), and (c) it can be seen that although the value of each kind of error stays constant, the machining error varies dramatically due to the change of error direction. If these errors become significant in affecting the machining errors, an alternative strategy is to make machining errors caused by angle errors, distance error, and dressing error be counteracted as possible as we can. Error curve (c) shows that if suitable directions are chosen of position errors and profile errors the machining errors can be controlled within a specified value.

6 Conclusions

A new simulation model has been presented for the generation of helical surface profiles found in cutting tools, such as milling cutters, twist drills, and hob cutters. This model can be used to determine the required grinding wheel profile for a desired helical surface and a given set of grinding wheel setting parameters, as well as the helical surface profile for a given grinding wheel profile and setting parameters. Compared to the classical conjugation and envelope methods, the proposed simulation model can take into account the effects of the various errors of grinding wheel profile and wheel setting parameters on the accuracy of the helical surface profile found in cutting tools. In addition, the numerical simulation approach based on constrained optimization eliminates the need to evaluate the enveloped surface, therefore, greatly reducing the complexity of the calculation. On the other hand, although the conventional simulation process can be performed in the conventional software package, its accuracy depends on the incremental value or the number of sample points selected for the motion parameter and the decomposition size of the cutter surface and the grinding wheel surface. A large number of sample points will increase the computation accuracy and the time as well. Compared to the conventional simulation method, the proposed method does not have such limitations and provides some benefits to the cutter design and manufacture. Parameters needed for grinding can be selected and readjusted according to the simulation results. Besides, the model has the ability to deal with envelope case and non-envelope case. The numerical examples have proven that the provided simulation procedure can be successfully conducted on a PC with adequate calculation accuracies. Generally, the simulation model provided in this paper is closer to the real cutter surface machining. Also, it can be applied to other cases with complicated conjugation action. Relative motion between the cutter surface and the wheel is not only limited to helical motion. However, it still needs further research.

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