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ORIGINAL ARTICLE

A 3D curvature gouge detection and elimination method for 5-axis CNC milling of curved surfaces

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Abstract Based on the concept of *Eular-Meusnier Spheres* (EMS) that portrays the generic curvature model of surfaces, a new three-dimensional (3D) method for curvature gouge detection and elimination in sculpture surface machining is presented. The new method is superior for presenting one-dimensional (1D) and two-dimensional (2D) approaches for curvature gouge detection and curvature gouge-free machining due to its capability to consider both *normal* and *osculate curvatures* of the cutter and machined surfaces and their interactions. The method can be applied to all three types of commonly used milling cutters (end, torus and spherical mills) and all concave curved surfaces. Test results from machining simulations are presented to demonstrate the new method and its advantages. The work forms the foundation for further research on the automated generation of highly efficient and high quality 5-axis CNC tool paths for machining curved surfaces.

Keywords 5-axis CNC machining · Gouge-free tool path · Cutter orientation · Gouge detection · Avoidance

1 Introduction

Production of mechanical parts with complex curved surfaces using 5-axis CNC machining becomes more common today, due to the higher efficiency and greater flexibility of the technique. Five-axis CNC machining provides greater accessibility for the cutter to reach its material removing positions/orientations and offers better chances for gouge avoidance when more productive cutting tools, such as the end and torus mill cutters, are used. More efficient material removal and higher quality surface finish can be achieved. Meanwhile, the addition of two extra degrees of freedom to traditional 3-axis CNC machining has also introduced considerable new challenges. In order to take the full advantages that a 5-axis CNC machine can offer in machining sculptured parts with complex shape, the optimal orientation of the cutter at each instance of the cutting process needs to be determined, along with the optimal trajectories of the cutter – the conventional 3-axis CNC tool paths.

The focus of automated 5-axis CNC tool path generation is to create the most efficient cutter motion trajectory accompanied with gouge-avoidance cutter-axis orientation to machine high quality surfaces derived from the CAD model of the machined part. Identifying the appropriate and optimal orientations of the cutter axis is the most challenging task in 5-axis CNC tool path programming due to the lack of deep understanding and generic guidelines in this area. To facilitate the automated generation of 5-axis CNC tool path, the introduction of generic principles that guide the determination for cutter-axis orientation becomes critical.

One of the major considerations in determining cutter-axis orientation in machining sculpture surface is to avoid curvature gouge between the cutter and the workpiece. Curvature gouge occurs when the cutter surface conflicts with the desired shape of the workpiece surface at the cutter contact (CC) point and the cutter overcuts beyond the desired shape. The traditional, quick and easy approach to avoid curvature gouges for curved surface machining is to use a small diameter ball mill, with a cutter curvature that is larger or equal to the maximum curvature of the machined surface. However, the drawbacks of this easy approach

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include low machining productivity and poor surface quality due to the poor cutter-surface curvature match for the relatively smoother parts of the machined surface. Prolonged machining time is needed with denser tool paths and small side steps to reduce the resulting high cusps on the machined surface, unless poor surface quality with large cusps is accepted. Nevertheless, as a solution to no solution, this approach is widely used in industry.

Curvature gouge problem can be avoided by properly orienting the axis of end and/or torus mill cutters without the need of using small ball mill cutters. In recent years, extensive studies have been carried out on the development of methods for determining the gouge-free cutter orientations in 5-axis CNC machining [3–6, 8, 9]; no generic solution, however, has been established. A new method that can effectively identify curvature gouges in all occasions so as to facilitate the automatic generation of optimal curvature gouge-free tool path to machine curved surfaces is strongly demanded.

Recently, the authors have introduced a new mathematical model that represents the geometric interaction between the cutter and machined surfaces. Derived from the theory of differential geometry, this parametric model, named as the *Eular-Meusnier Spheres* (EMS), serves as a generic tool in analyzing the interactions between the cutter and machined surfaces. In this work, application of the *Eular-Meusnier Spheres* concept to form new three-dimensional (3D) gouge detection and elimination solution for 5-axis CNC machining is presented. Comparisons of this new 3D curvature gouge detection method to existing one-dimensional (1D) and two-dimensional (2D) methods are made. Applications of this new 3D curvature gouge detection and elimination method to carry out curvature gouge free machining are demonstrated using representative curved surfaces and the machining simulation tests supported by the Pro/ENGINEER integrated CAD/CAM system.

2 Related works

Studies on curvature gouge-free machining of sculpture surfaces have been carried out over the past several years. Many commendable methods for curvature gouge detection and prevention have been introduced and further developed. Based on the number of primary variables considered in corresponding methods, these existing approaches can be classified into either 1D or 2D solution schemes.

At the early stage of curvature gouge investigations, studies mainly focus on 1D or one-control variable solutions, mostly concerned with cutter-feed direction. These 1D solution methods are unable to provide satisfactory resolutions for curvature gouge-free machining because of their inability to identify gouges in many cases. More advanced techniques in curvature gouge detection, the 2D solution methods, were thus introduced. The 2D solution methods also examine potential curvature gouge problem on the tangent plane of the machined surface at the cutter contact (CC) point, leading to more rigorous resolutions for curvature gouge detection and avoidance. However, these relatively advanced 2D approaches still have considerable limitations since curvature gouge problem is actually an issue in the 3D space.

2.1 One-dimensional curvature gouge detection methods

The 1D solution methods for curvature gouge detection and avoidance investigate curvatures of cutter and workpiece surface on one or two normal planes; mostly on the plane perpendicular to cutter feed direction. Curvature gouge is only detected and corrected on the concerned plane(s). Representatives of the 1D solution method are *effective cutting shape* method and *active cutting edge* method.

The concept of *effective cutting shape* is among the first apparatuses to deal with curvature gouge problems. *Effective cutting shape* represents the projection of an end mill cutter's cutting edge on the plane perpendicular to cutter feed direction at the CC point. Vickers and Quan [7] introduced the concept of *effective cutter radius* to represent the projection of end mill cutter's cutting edge on the plane perpendicular to feed direction for analyzing machining conditions of end mill cutter. Yu et al. [8] and Lee [3] applied this concept of *effective cutting shape* to the estimation of cutter surface curvature on the normal plane perpendicular to feed direction for gouge detection. The *effective cutting shape* method works well in detecting curvature gouge for end mill cutter when tool paths are straight lines and cutter orientation is fixed. These studies introduced the early solutions for curvature gouge detection and avoidance. However, for surfaces that need curved tool paths and for machining in which cutter orientation varies with surface curvatures, the *effective cutting shape* method is limited as the projected cutting edge can no longer reflect the true geometry of the cutter due to the misalignment of the projection plane. This type of errors was analyzed in detail by Li and Chen [4]. Furthermore, the *effective cutting shape* method cannot detect curvature gouge for torus mill cutter where the cutter's flat end face is not used in material removal.

Choi and Park [1] and Park and Chung [5] proposed to use the *active cutting edge*, which is the intersection of the cutter and workpiece surfaces on the normal plane perpendicular to the cutter feed direction at the CC point, for curvature gouge detection. Lee [3] also used this concept of *active cutting edge* on two planes, i.e., planes perpendicular to and aligned with the cutter feed direction, for curvature gouge detection. This *active cutting edge* method works well when the cutter and workpiece share the same tangent plane at the CC point, although gouge detection is still limited on the selected normal planes. However, when the cutter and workpiece surfaces do not share a tangent plane at the CC point, such as the case of an end mill cutter, the active cutting edge method may fail since it overlooks the trailing effect of the cutter's end face.

Rao et al. [6] proposed the principal axis method (PAM). As an application and extension of the active cutting edge method, PAM employs the concept of curvature match introduced by Jensen [2]. Pursuing cutter-surface curvature match to achieve better surface quality and more efficient material removal in milling, PAM is not a direct strategy for curvature gouge detection and prevention. However, in a special case, when the machined surface has uniform curvature and on the normal plane where curvature match is obtained the machined surface has the maximum curvature, PAM can avoid curvature gouge. Generally, PAM cannot be used as a curvature gouge detection tool. In addition, the method cannot be used for an end mill cutter of which the principal curvatures cannot be calculated as ball and torus mill cutters.

Generally, the 1D solution methods for curvature gouge detection provide an excellent start point in solving gouge problems; they are, though, restricted on one or two normal planes at the CC point and overlook the possibilities of surface curvature interference at locations in any other directions. These solutions do not guarantee curvature gouge-free machining.

2.2 Two-dimensional curvature gouge detection method

The 2D solution methods for curvature gouge detection examine *normal curvatures* represented on the common tangent plane shared by the cutter and workpiece surfaces. These methods provide general *normal curvature* gouge avoidance solutions. The foundation of these 2D methods is the Dupin's indicatrix, a planar expression of the *Eular* theorem from differential geometry.

The *Dupin's indicatrix* method for curvature gouge detection was introduced in Yoon's early work [9]. For a given CC point, the *Dupin's indicatrix* represents the *normal curvatures* in all directions of the cutter and machined surfaces by projecting them onto the tangent plane as an ellipse or parallel lines. By comparing the *Dupin's indicatrices* of cutter and the machined surface, *normal curvature* gouge problems for the concerned point can be revealed. However, curvature gouge problems may still arise since these problems are introduced not only by normal curvature conflict, but also by osculate curvature interference.

An example of this 2D *normal curvature* gouge detection method is illustrated in Fig. 1. The *Dupin's indicatrices* of torus mill cutter and the cylindrical machined surfaces (in Fig. 1a) at the CC point are shown in Fig. 1b.

In this machining example, the torus mill cutter has a center radius R_T (R_T=0.25 unit) and corner radius r_T (r_T= 0.25 unit). The machined surface is a half cylinder with a radius R (R=1 unit). The cutter feed is in a direction that the minimal principal curvatures of the cutter and workpiece surfaces forming an angle θ of 61.78°. The cutter inclination angle Γ, i.e., the angle between cutter axis and the surface normal, is 15°. Curvature match on the normal plane perpendicular to cutter feed direction is perfect, i.e., the normal curvatures of cutter and the machined surface on the normal plane are equal, as defined in Eq. (1). The curvature matching normal plane is represented by the line through the center and CC point (C).

$$K_{Tc} = K_{Wc} \tag{1}$$

where K_{Tc} represents the *normal curvature* of cutter at point C; K_{Wc} represents the *normal curvature* of the machined surface at point C.

In Fig. 1b, the two horizontal lines represent the *Dupin's indicatrix* of the cylinder surface. The ellipse represents the *Dupin's indicatrix* of the cutter. *Dupin's indicatrices* of cutter and the cylinder are tangent at CC point (C), so that the perfect *normal curvature* match is accomplished. The cutter feed direction is determined so that the curvature match of cutter and workpiece surface is optimal, i.e., the cutter minimal principal curvature is the largest possible.

Normal curvatures conflicts between the cutter and cylinder surface is completely eliminated because the

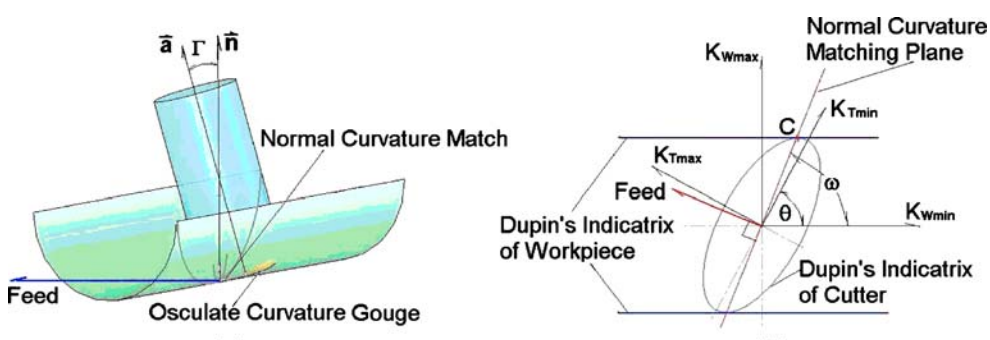


Fig. 1 2D solution method (**a**) machining set up; (**b**) Dupin's indicatrices

Dupin's indicatrix of the cutter locates completely inside the *Dupin's indicatrix* of the workpiece Fig. 1b; there exists no *normal curvature* conflict. However, Fig. 1a shows that the cutter over cuts the cylinder surface. This over cut comes from the *osculate curvature* interference between the cutter and workpiece surface. This example indicates that *osculate curvature* conflicts exist independently to *normal curvatures,* and they cannot be overlooked.

The example illustrates that the 2D curvature gouge detection methods cannot detect all potential curvature gouge problems at the CC point and ensure curvature gouge-free machining, since curvature gouge may also be introduced from *osculate curvature* interference.

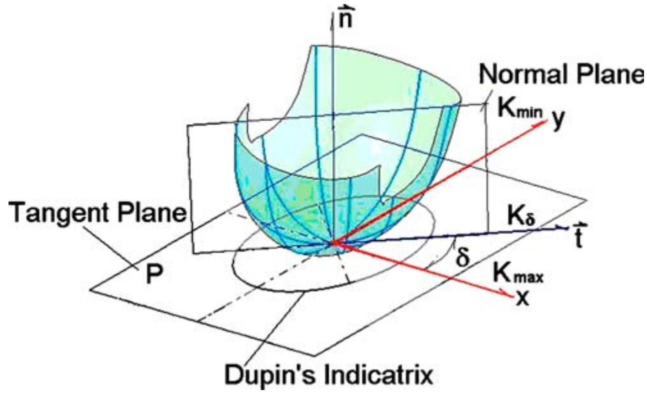


Fig. 2 Normal curvature representation

3 Proposed 3D curvature gouge detection and elimination solution

3.1 Eular-Meusnier Spheres (EMS)

To introduce a new 3D curvature gouge detection and elimination method for 5-axis CNC machining, the authors have recently introduced a new 3D geometry interaction model between the cutter and the machined surfaces. This parametric mathematical model, named as the *Eular-Meusnier Spheres* (EMS) is derived from the theory of differential geometry, serving as a generic tool for analyzing the interactions between cutter and machined surfaces.

Detailed discussions on the *Eular-Meusnier Spheres* are presented in the authors' other publications. The concept of the *Eular-Meusnier Spheres* can be briefly introduced below. Given a point on a surface with second order of continuity, there exists a surface normal vector $\overrightarrow{n}$ and a tangent plane **P**, as shown in Fig. 2.

A *normal* plane that goes through the surface *normal* $\overrightarrow{n}$ intersects the surface, forming a *normal curve* (or *normal* section). For any given point on the surface, there exist unlimited numbers of *normal curves* (around 360°). The *normal curves* with the maximum and minimum curvatures form the *principal curvatures* of the surface at the point, represented by the maximum and minimum *principal curvatures* ($\boldsymbol{K_{max}}$ and $\boldsymbol{K_{min}}$) or the minimum and maximum curvature circle radii ($\boldsymbol{\rho_{min}}$ and $\boldsymbol{\rho_{max}}$). The *Eular* theorem is primarily concerned with these surface properties, so that an arbitrary *normal s*ection curvature can be determined with respect to the *principal curvatures*, using Eq. (2). Most of the existing curvature gouge detection methods focus on the relations between the *normal curvatures* of the machined surface and the cutter.

$$K_\delta = K_{\max}\cos^2\delta + K_{\min}\sin^2\delta \tag{2}$$

where K_δ represents normal curvature in the direction of angle δ to the maximal *principal curvature* K_{max}.

Dupin's indicatrix is an application of the *Eular* equation expressed in Eq. (2). By introducing the expression of x and y

$$\begin{cases} x = \frac{\cos\delta}{\sqrt{K_\delta}} \\ y = \frac{\sin\delta}{\sqrt{K_\delta}} \end{cases} \tag{3}$$

into Eq. (2), the *Dupin's indicatrix* is obtained as:

$$\frac{x^2}{r_{\max}^2} + \frac{y^2}{r_{\min}^2} = 1 \tag{4}$$

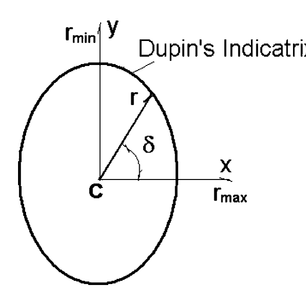
where r_{min} and r_{max} are axes of the *Dupin's indicatrix.*

$$\begin{cases} r_{\max} = \frac{1}{\sqrt{K_{\max}}} \\ r_{\min} = \frac{1}{\sqrt{K_{\min}}} \end{cases} \tag{5}$$

The Dupin's indicatrix in Fig. 3 represents the surface normal curvature distributions on the tangent plane at the concerned point.

At any given point on the surface, the normal plane intersects with the tangent plane, forming a tangent vector $\overrightarrow{t}$. Any plane that passes through the tangent vector $\overrightarrow{t}$ is an osculating plane of the concerned surface. There exist also unlimited numbers of osculating planes (within±90°). These osculating planes intersect with the surface, forming various osculate curves. Different from the normal curves, the osculate curves on the surface are generated in the planes that are not normal, but with a varying angle to the tangent plane of the surface. For a given point, a Meusnier

Fig. 3 Dupin's indicatrix



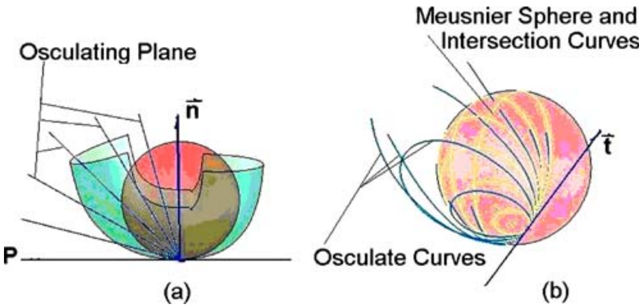


Fig. 4 Osculate curvature representation

sphere is formed by the curvature circles of the osculate curves on different osculating planes for the same tangent vector $\vec{t}$, as illustrated in Fig. 4.

The radius of the Meusnier sphere is:

$$\rho_\delta = \frac{1}{K_\delta} \tag{6}$$

where ρ_δ is the normal curvature circle radius corresponding to the tangent $\vec{t}$; K_δ is the normal curvature in tangent $\vec{t}$. When the tangent vector $\vec{t}$ rotates about the surface normal vector $\vec{n}$ at the point of interest, the Meusnier sphere will change its size due to the varied normal curvature circle radius ρ_δ. According to the Eular theorem of Eq. (2), the variation of the normal curvature circle is confined within the range of

$$\frac{1}{K_{\max}} \le \rho_\delta \le \frac{1}{K_{\min}} \tag{7}$$

When the tangent vector $\vec{t}$ rotates 360° about the surface normal vector $\vec{n}$, *Meusnier* spheres of varying sizes are formed. These *Meusnier* spheres are cotangent with each other at the concerned point. The largest and smallest *Meusnier* spheres of all *Meusnier* spheres formed can be determined by the *principal curvatures* of the surface at the concerned point. These two extreme size spheres define the scope of variation of all *Meusnier* spheres. The collection of all *Meusnier* spheres, as shown in Fig. 5, is named as *Eular-Meusnier Spheres* (EMS), which is generated by combining the *Eular* and *Meusnier* theorems. The concept of the *Eular-Meusnier Spheres* extends the representation of surface curvatures to include both normal and osculate curvatures in 3D space. Curvature gouge detection and elimination method based on the concept of EMS is, thus, three dimensional by nature. The smallest and largest *Eular-Meusnier Spheres* of a surface at a given point are illustrated in Fig. 5. The new method for detecting all curvature gouges by interpreting the relative sizes of these extreme EMS of the workpiece and cutter surfaces is discussed in the following section.

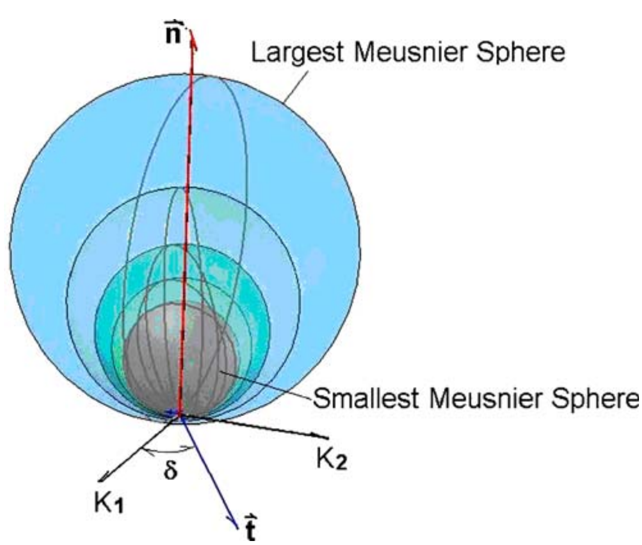


Fig. 5 Eular-meusnier sphere

3.2 EMS based 3D curvature gouge detection method

For any set of the *EMS,* there exist the largest and smallest *Meusnier* spheres. These extreme *Meusnier* spheres define the possible volume span of surface curvatures at the interested point. The extreme *Meusnier* spheres of the EMS from cutter and machined surface determine the interaction characteristics between cutter and the machined surface. Any volume overlap between the largest *Meusnier* sphere of the cutter surface and the smallest *Meusnier* sphere of the machined surface represents curvature interference, or gouging. The total elimination of curvature gouges can only be accomplished by ensuring that there is no overlap between the volumes defined by the largest and smallest *Meusnier* spheres of the cutter and the machined surface, as shown in Fig. 6.

The EMS based condition for curvature gouge elimination and avoidance is thus stated as: the largest *Meusnier* sphere of the cutter surface must be equal to or smaller than the smallest *Meusnier* sphere of the machined surface. At the extreme case, where a curvature gouge is just about to happen, the *Eular-Meusnier Spheres* of the cutter and the machined surfaces form three nested spheres, a large, a medium and a small one, as illustrated in Fig. 6. These three

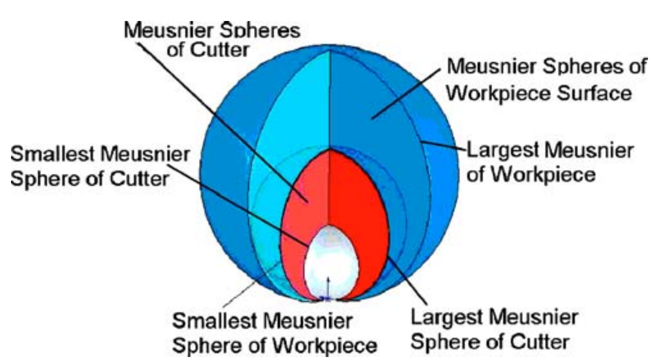


Fig. 6 Curvature gouge-free condition

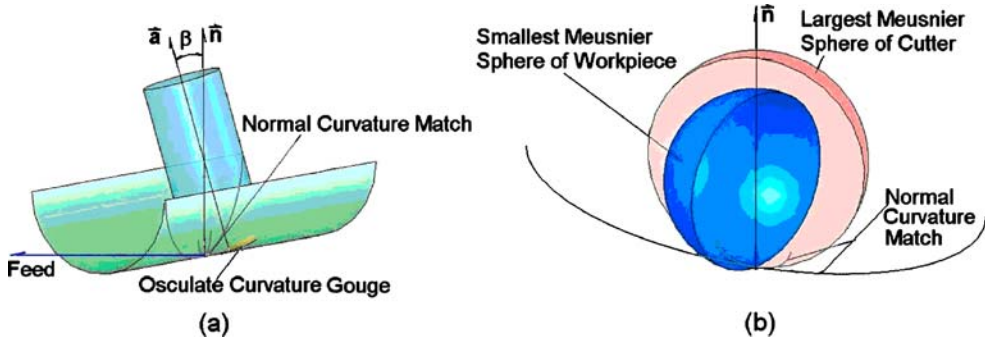


Fig. 7 3D solution (**a**) machining set up; (**b**) meusnier spheres

nested spheres define two different regions: a) the region defined by the large and medium spheres or the region defined by all *Meusnier* spheres of the workpiece surface; and b) the region defined by the medium and small spheres or the region defined by all *Meusnier* spheres of the cutter surface. To eliminate curvature gouge and to obtain best curvature match, the largest *Meusnier* sphere of cutter surface should be equal to the smallest *Meusnier* sphere of workpiece surface.

The volume space confined by the largest and the smallest *Meusnier* spheres of workpiece surface represents the curvature variation range of the machined surface. For a given surface, the variation at each point is determined. The volume space defined by the largest and the smallest *Meusnier* spheres of cutter represents the curvature variation range of the cutter surface. The exact curvature of the cutter surface at the CC point, as further discussed below, is a function of the cutter radius and the inclination angle of the end and/or torus mill cutters at the CC point.

Under the EMS based curvature gouge free condition, no volume overlap between the *Meusnier* spheres of the cutter and workpiece surfaces will occur, thus no curvature gouge between the cutter and workpiece surfaces will appear at the given CC point. If the workpiece surface has uniform curvatures, i.e., the *principal curvatures* at every point remain constant, the local curvature gouge free condition can be extended globally across the entire surface.

This new 3D curvature gouge detection and elimination method is applied to machined surfaces with either uniform or varying curvatures. Uniform curvature surfaces possess constant principal curvatures; cylinder and sphere surface are examples of these uniform curvature surfaces. Curvature gouge elimination solution for uniform curvature surfaces is globally effective and unique. Varying curvature surfaces have varied *principal curvatures*. The proposed curvature gouge elimination condition for varying curvature surfaces provides a safer solution when the overall maximum principal curvature of workpiece surface is used.

The detailed methods for determining the EMS for various milling cutters are presented in the authors' other publications. It should be noted, however, that the largest EMS for end and/or torus mill cutters also depends upon the orientation of the cutter axis. By adjusting cutter axis orientation or inclination angle, the largest EMS for end and/or torus mill cutters can be changed to be equal to the smallest EMS of the machined surface at every point. This can be done to avoid gouge and to achieve ideal curvature match without changing the size of the cutter. Both end and torus mill cutters can be used to maintain the ideal curvature match by adjusting the cutter inclination angle. To achieve the same effect using a ball mill cutter in the present practice, one has to continuously alter the radius of the cutter, which is infeasible.

Since the radius of the largest *Meusnier* sphere of the end and/or torus mill cutters will normally be larger than the radius of a ball cutter with the same cutter radius (unless the inclination angle is 90 degree), the end and/or torus mill cutters used in machining will have a smaller cutter radius. However, in the finish machining of complex curved surfaces, geometric accuracy of the surface is the major concern and the amount of material to be removed is vary

Table 1 Curvature gouge detection methods

	1D solution method		2D solution method	3D solution method	
Method	Effective cutting shape	Active cutting shape	Dupin's indicatrix	Numerical simulation	Eular-Meusnier spheres
Gouge detection	No	No	No	Possible	Yes
Gouge problem	Possible	Possible	Possible	Possible	Eliminated
Human intervention	Required	Required	Required	May Be	No
Calculation	Simple	Simple	Intensive	Intensive	Simple

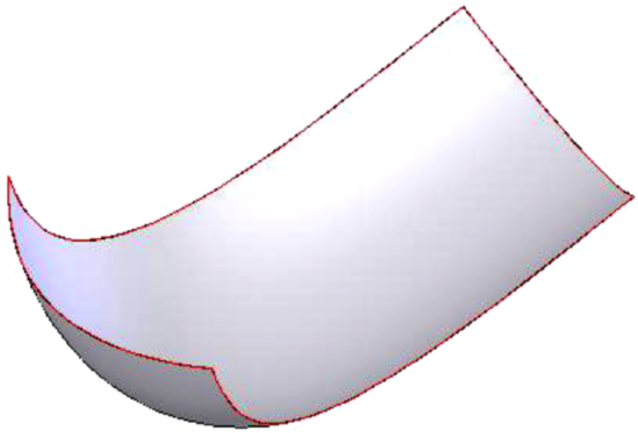


Fig. 8 Free-form surface

small. Machine dynamics with a slightly less rigid cutter is a less concern under these light machining conditions.

Also to be noted, the largest *Meusnier* sphere of the end and/or torus mill cutters enwraps the cutter's cutting edge completely; as a result, the rear gouge problem of the cutter is eliminated when the proposed 3D curvature gouge elimination condition at the CC point is satisfied. The proposed 3D curvature gouge elimination solution has a global effect.

3.3 Use of EMS based curvature calculation method in curvature match and gouge detection/elimination

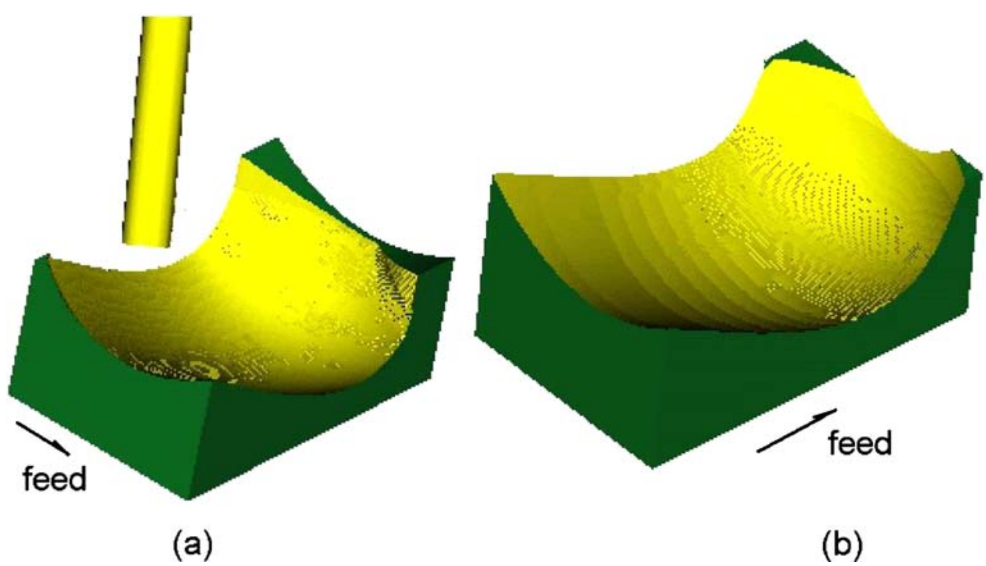
The *Eular-Meusnier Spheres* concept based 3D curvature gouge elimination method takes both *normal* and *osculate curvatures* into consideration. As a 3D curvature match and curvature gouge detection method, it can ensure best curvature match on the entire surface for improved surface quality and eliminates all *normal* and *osculate* curvature conflicts on the surface.

The curvature of a complex curved surface varies from point to point. By adjusting the cutter inclination angle, the radius of the largest *Meusnier* sphere of the end or torus cutter used can be adjusted to match the radius of the smallest *Meusnier* sphere or the minimum curvature radius of the machined surface at the CC point to achieve best possible curvature match without gouge. This 3D curvature match and curvature gouge detection method is independent to the direction of cutter feed, and is applicable to all three types of mill cutters, including ball, torus and end mills. In planning the extended CNC tool paths with cutter orientation (or inclination angle) at every point on the surface, the EMS based, 3D curvature match and curvature gouge detection method introduced in this work can serve as a constraint together with the physical constraints on cutter inclination and sizes, while the use of the largest possible cutter size and the minimum cutter orientation angle can serve as the objective of the optimization to obtain the best surface quality and machining productivity. Specific work in this area is to be reported in the authors' following work.

The use of this new 3D curvature gouge elimination method to reveal and eliminate curvature gouge problem can be illustrated using the machining example discussed previously in Fig. 1. The torus cutter and the cylindrical machined surface as well as the relation of their EMS are shown in Fig. 7. The *osculate curvature* gouge problem is revealed in Fig. 7b where the largest *Meusnier* sphere of cutter is larger than the smallest *Meusnier* sphere of the cylinder surface. This leads to the noticeable curvature gouge shown in Fig. 7a.

Once identified the curvature gouge can be easily eliminated or avoided by further incline the cutter to reduce the largest *Meusnier* sphere of cutter until it match the smallest *Meusnier* sphere of the cylinder surface at the CC

Fig. 9 End mill cutter simulation test result



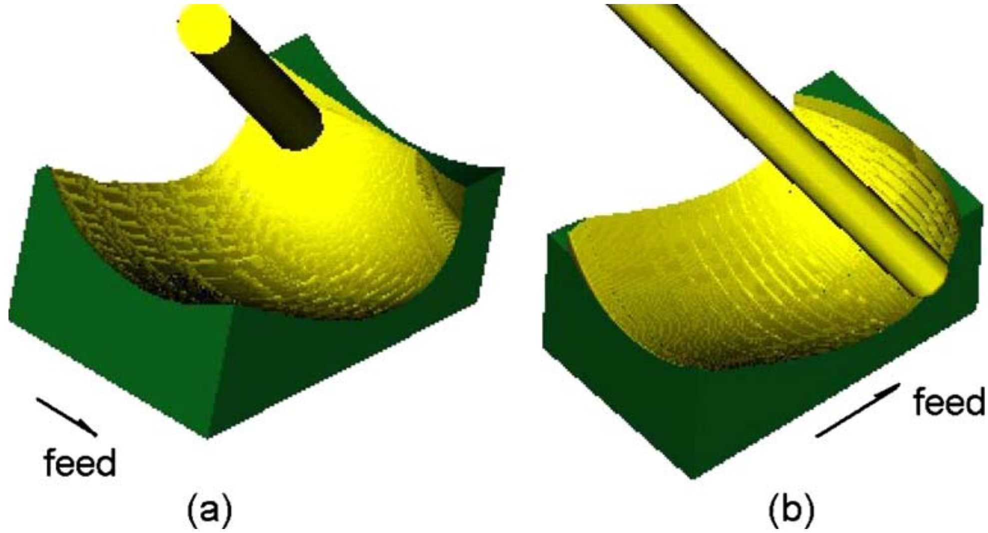


Fig. 10 Torus mill cutter simulation result

point. A scan on the smallest *Meusnie*r sphere of the machined surface and the maximum physically allowed cutter inclination angle allows one to select the maximum cutter radius that will ensure curvature gouge free machining for the entire surface without the need of changing the cutter.

In traditional curvature gouge detection and avoidance methods, the *normal curvature* match is pursued, while *osculate curvature* is overlooked. Potential osculate curvature gouge is later identified through computational intensive and sometimes unreliable cutting simulations. Corrections are then made to the original tool paths and cutter orientations to avoid the curvature gouge identified in the simulation. The newly introduced 3D curvature gouge detection and avoidance method identifies *normal* and *osculate curvature* interferences jointly, leading to much more efficient operation than the present approach, and provides a definite resolution for complete curvature interference avoidance. Curvature gouge-free machining can be ensured. No other tests, such as numerical simulations, are needed to assist the implementation for this 3D solution method. An overview of curvature gouge detection methods for curved surfaces is shown in Table 1. Uniform curvature surface is assumed for the ease of illustration. More complex surfaces share similar conditions and results. Extension of the 3D curvature gouge detection and avoidance method to a more generic concave surface is to be presented in the authors' following publications.

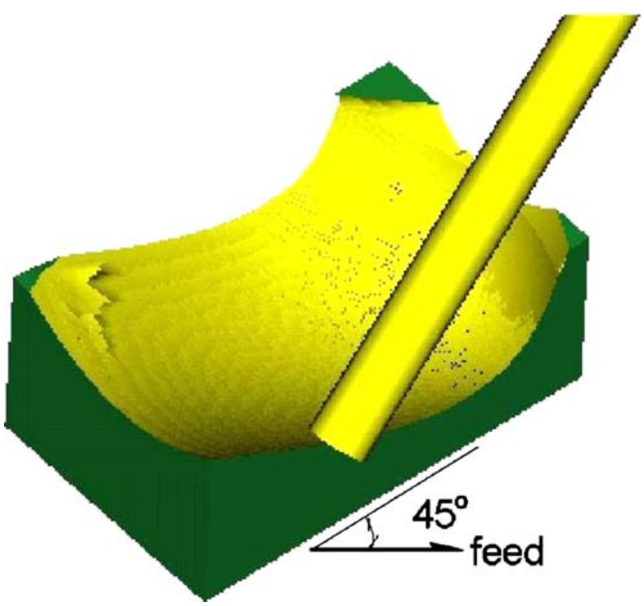


Fig. 11 End mill cutter simulation result

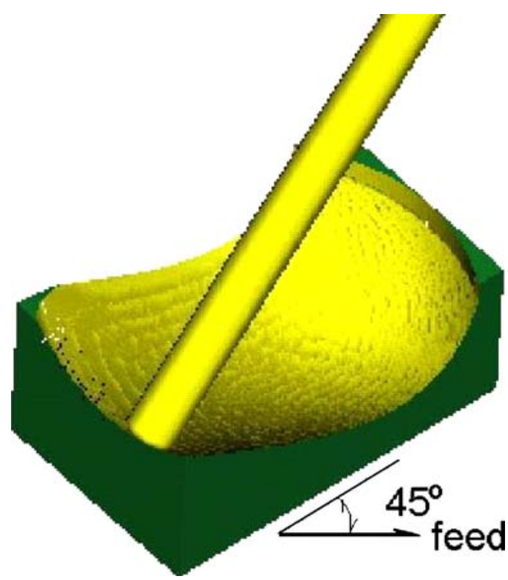


Fig. 12 Torus mill cutter simulation test result

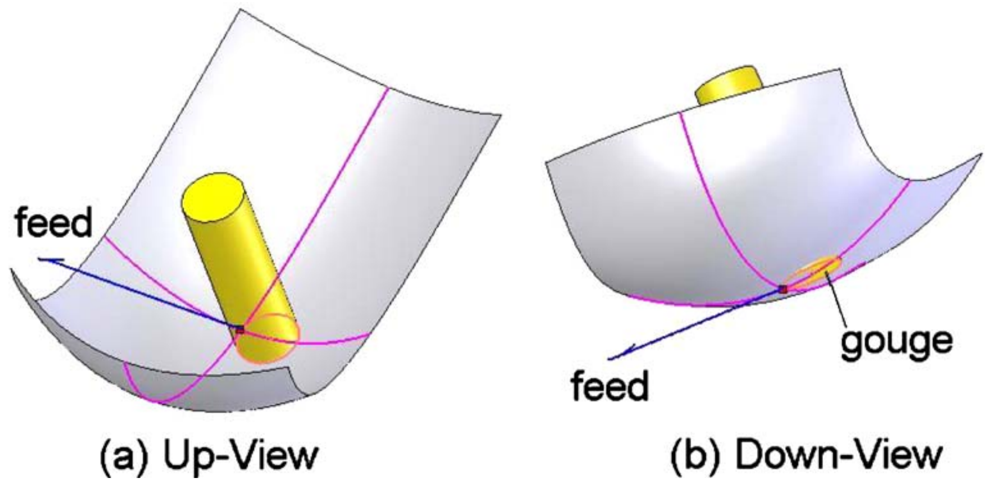


Fig. 13 1D solution of end mill cutter

4 Machining simulation tests

A number of representative machining simulation tests are carried out in this section to demonstrate the use of the proposed 3D curvature gouge detection method, to validate this new method and to illustrate its advantage over the traditional 1D and 2D gouge detection methods. The curved surface that serves the machined surface under the tests is of free-form, as shown in Fig. 8. The test surface has varying principal curvatures, with overall maximum and minimum principal curvatures as 3.4388 (k_{max}) and 0.0046 (k_{min}). Both end and torus mill cutters are used in these tests. The end mill cutter has a radius of 0.15 (unit); the torus mill cutter has a center radius 0.1 (unit) and corner radius also 0.05 (unit). Machining simulation tests are carried out using the Pro/ENGINEER (Pro/E) integrated CAD/CAM system.

To eliminate curvature gouge, the largest *Meusnier* sphere of the mill cutter should fit into the smallest *Meusnier* sphere of the machined surface. The cutter inclination angle Γ_E (for end mill) and Γ_T (for torus mill) are determined based on overall *maximum principal curvature* k_{max}, of the machined surface to obtain a safe, gouge-free solution. Calculations for obtaining angles Γ_E and Γ_T are based on the EMS based principle discussed in the previous section, and detailed in the authors' following publications. The results of these inclination angles are

$$\Gamma_E = 32^\circ \tag{8}$$

$$\Gamma_T = 25^\circ \tag{9}$$

4.1 Machining setup one: cutter feed direction aligned with test surface principal curvature direction

In this setup, cutter feed direction coincides with the principal curvature directions of the machined surface, simulation results of machining using end and torus mill cutters are shown in Figs. 9 and 10, respectively.

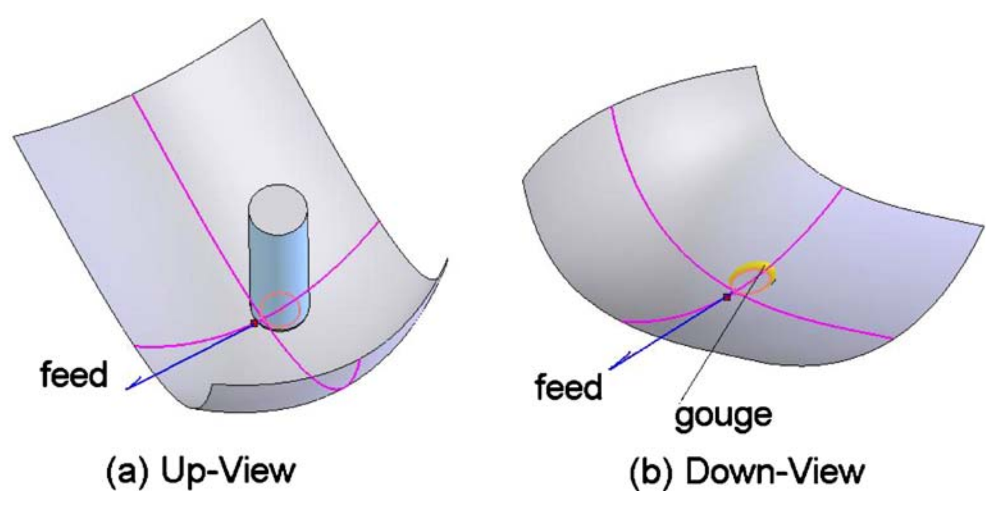


Fig. 14 1D solution of torus mill cutter

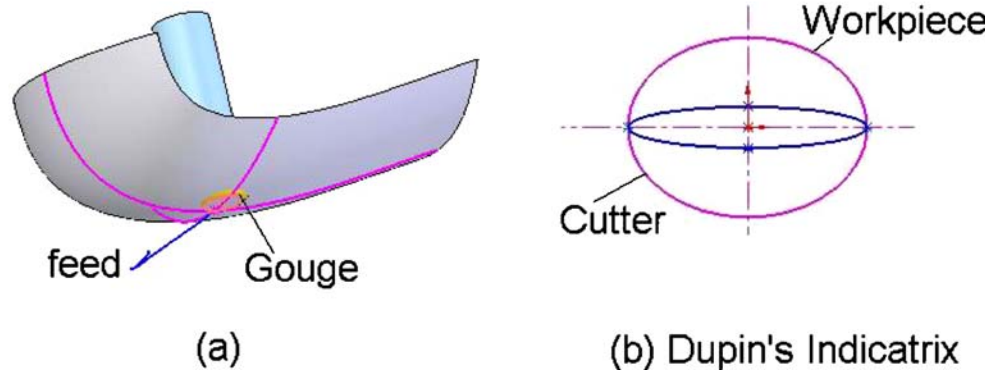


Fig. 15 2D solution of torus mill cutter

The simulation results present gouge-free machining and demonstrate that the proposed 3D curvature gouge elimination method has succeeded in avoiding curvature gouge, as verified by the Pro/E gouge checking mechanism.

4.2 Machining setup two: cutter feed direction in angle with test surface principal curvature direction

In this setup, the cutter feed direction is set at an angle of 45° away from the principal curvature direction of the surface, simulation results using end and torus mill cutters are shown in Figs. 11 and 12, respectively.

The simulation results also present gouge-free machining and demonstrate that the proposed 3D curvature gouge elimination method has succeeded in avoiding curvature gouge, as verified by the Pro/E gouge checking mechanism.

For the same machining setups, simulation tests have also been carried out using the traditional 1D and 2D curvature gouge detection methods to show their limitations. The 1D solution methods focus on the normal curvature match of the cutter and machined surfaces on one normal plane, and generate various cutter orientation angles along the tool path. These local *normal curvature* match principle based cutter orientations, however, incurs global curvature gouge problems, as detected by the Pro/E gouge check mechanism, and shown in Figs. 13 and 14. Similarly, the 2D solution method also incurs global curvature gouge due to its pursing for local *normal curvature* fit at the CC point, as detected by the Pro/E gouge check mechanism and shown Fig. 15. In these illustrations, the surface principal curvatures at the arbitrarily selected point are 0.591 and 1.0355, respectively.

In Fig. 13, the inclination angle Γ_E of the end mill cutter is 4.94° so that the cutter matches the *normal curvature* (0.591) of the surface at the CC point on the plane perpendicular to cutter feed direction. In Figs. 14 and 15, the inclination angle Γ_T of the torus mill cutter is 3.49° for the cutter to match the *normal curvature* (0.591) at the CC point.

Results from the completed Pro/E simulation tests showed that the new 3D curvature gouge elimination method led to curvature gouge-free CNC machining for the experimented surface. Once the curvature gouge-free criterion is satisfied, cutter feed direction does not affect the gouge elimination solution. As a result, cutter feed direction can be determined freely, according to other CNC tool path considerations. Traditional 1D and 2D gouge detection methods could not provide an easy solution to the given machining task. The presented test results are also summarized in Table 2.

5 Conclusions

In this work, a new 3D curvature match and curvature gouge detection and elimination method for 5-axis CNC machining of sculpture surface is presented. The method is based upon the newly introduced *Eular-Meusnier Spheres* concept, a generic mathematical and geometric model of surface curvature geometry. This *Eular-Meusnier Spheres* based 3D curvature gouge detection and elimination method serves as an easy to calculate and easy to implement curvature gouge-free constraint on the determination of cutter size and cutter orientation angle of end and torus mill cutters for machining any given curved surfaces

Table 2 Summary of tests results

	1D method		2D method	3D method
	Effective cutting shape method	Active cutting shape method	Dupin's indicatrix method	Eular-Meusnier Spheres
Setup 1	No	No	No	Yes
Setup 2	No	No	No	Yes

with known *principle curvatures*. The method provides a generic global solution for gouge detection and elimination in machining sculpture surfaces, ensure best possible curvature match for better surface quality, and possibly enable higher machining efficiency with less tightly packed tool paths. Computer simulation tests and machining examples have shown that the newly introduced 3D curvature gouge-free solutions can guarantee the elimination of curvature gouges.

The most important advantage of this 3D solution method is its ability to provide a definite resolution for complete curvature gouge elimination in curved surface machining. No other demanding tests, such as numerical simulation and cumbersome calculations that usually accompany all other traditional curvature gouge detection methods to obtain approximations for cutter-surface interference, are needed. This 3D curvature detection and elimination method applies to all three types of commonly used mill cutters: end, torus and ball, and all concave curved surfaces. The introduced *Eular-Meusnier Spheres* concept and the 3D curvature gouge detection/elimination method form the foundation for generating highly efficient and high quality surface producing 5-axis CNC machining tool path and provide the basis for cutter orientation planning, programming and optimization in machining complex sculptured parts.

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