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ORIGINAL ARTICLE

# Solving multi-criteria optimization problem in submerged arc welding consuming a mixture of fresh flux and fused slag

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**Abstract** In the present work, application of the Taguchi method in combination with grey relational analysis has been applied for solving multiple criteria (objective) optimization problem in submerged arc welding (SAW). A grey relational grade evaluated with grey relational analysis has been adopted to reveal an optimal parameter combination in order to obtain acceptable features of weld quality characteristics in submerged arc bead-on-plate welding. The idea of slag utilization, in subsequent runs, after mixing it with fresh unmelted flux, has been introduced. The parentage of slag in the mixture of fresh flux and fused flux (slag) has been denoted as slag-mix%. Apart from two conventional process parameters: welding current and flux basicity index, the study aimed at using varying percentages of slag-mix, treated as another process variable, to show the extent of acceptability of using slag-mix in conventional SAW processes, without sacrificing any characteristic features of weld bead geometry and HAZ, within the experimental domain. The quality characteristics associated with bead geometry and HAZ were bead width, reinforcement, depth of penetration and HAZ width. Using grey relational grade as performance index, we have performed parametric optimization yielding the desired features of bead geometry and HAZ. Predicted results have been verified with confirmatory experiments, showing good agreement. This proves the utility of the proposed method for quality improvement in SAW process and provides the maximum (optimum) amount of slag-mix that can be consumed in the SAW process without any negative effect on characteristic features of the quality of the weldment in terms of bead geometry.

**Keywords** submerged arc welding · Taguchi method · grey relational analysis · optimization

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## 1 Introduction

Submerged arc welding is a versatile metal joining process in industry. It is a multi-variable, multi-objective metal fabrication process characterized by the use of granulated fusible flux which covers the molten weld pool during operation. This arrangement facilitates slower cooling rate, prevents atmospheric contamination into the weld pool and improves both mechanical properties and metallurgical characteristics of the weld bead as well as heat affected zone (HAZ). The acceptable quality characteristics of a weldment, in relation to the geometry of a weld bead and HAZ include deeper penetration, minimum reinforcement, minimum bead width and minimum HAZ width. To reduce weld metal consumption and thereby lowering fabrication cost bead volume should be kept at a minimum. Moreover, desired mechanical properties of the weldment depend on to some extent the features of bead geometry and HAZ. By parametric optimization of welding phenomena, the desired weld quality characteristics can be achieved easily.

Much work in the literature has explored various aspects of modeling, simulation and process optimization in conventional submerged arc welding (using 0% slag-mix) [1, 2]. The idea of slag recycling and slag processing has been investigated by few researchers [3–6]. The common approaches to tackle the optimalization problem in welding include multiple regression analysis, response surface methodology (RSM) [1, 2], artificial neural network (ANN) modeling [17] and the Taguchi Method [7–14]. In most of the cases the optimization has been performed using single objective function. For a multi-response process, while applying the optimal setting of control factors, it can be observed that, an increase/ improvement of one response may cause change in another response, beyond the acceptable limit. Thus, for solving the multi-criteria optimization problem, one must convert all the objectives into an equivalent single objective function. This equivalent objective function, which is the representative of all the quality characteristics of the product, is to be optimized (maximized).

Optimization using the desirability function approach is very helpful in this context [18]. Research related to this technique can also be found in literature [19]. This approach converts each of the responses (objectives) into their individual desirability value (that varies from 0 to 1). These individual desirability values are then accumulated to compute overall desirability function, which is to be optimized (maximized) next. In desirability function optimization technique, it is required to develop a mathematical model between overall desirability and the process variables. Optimum parametric combination can be obtained by optimizing the aforesaid mathematical relation. But in doing so, it is frequently experienced that, within experimental domain, either the function becomes too insensitive or the optimum solution may occur at a saddle point. To overcome this, a trial-and-error technique is adopted to yield satisfactory results.

Taguchi's optimization approach is very efficient in this context. This method can evaluate the optimal setting by using a limited number of experimental runs. However, the general Taguchi method cannot solve multi-objective optimization problems; therefore, the Taguchi method coupled with grey relational analysis is the appropriate option.

Literature reveals that the use of recycled slag would be a new area of research because of less work is done so far in this respect. Beck, H. P. and Jackson, A. R. [3] concluded that according to code requirement, the properly processed slag could be reliable and could be used as an alternative for fresh flux. They further claimed a savings of 50% of the procured flux by recycled flux. Livshit, L. G. and Shiryaev, A. I. [4] demonstrated that it was possible to use pulverized slag crust mixed with iron fillings for hard facing applications. In the present context, research related to slag reconsumption in the conventional SAW process has been carried out by Moi, S. C. et al. [5], and Pal, P. K. et al. [6]. The study introduced the concept of using slag-mix% as a process variable. The main effect of using slag-mix and interactive effects of process parameters (including slag-mix%) on features of bead geometry and HAZ, in terms of bead height, depth of penetration, bead width and HAZ width has been evaluated through the analysis of variance (ANOVA) method. But their work did not provide the optimal factor combination to yield acceptable weldment and the maximum slag-mix% that can be used during SAW process without affecting bead geometry as well as HAZ dimension.

In order to complete their work, we made an attempt to search for an optimal parametric combination in order to obtain acceptable weld bead and HAZ quality, produced by non-conventional submerged arc bead-on-plate welding on mild steel. Experiments were conducted using Taguchi's $L_{16}$ orthogonal array (OA) design with two conventional process parameters: welding current and flux basicity index, varied at four different levels. Slag generated during submerged arc welding has been consumed in further runs after mixing it with fresh flux; the wt% of fused flux/ slag in the mixture of fresh flux and fused slag has been termed as slag-mix%; treated as another process variable [5, 6]. The study aimed at investigating whether use of slag-mix impose any negative/ adverse effect on bead quality with the goal of recommending this new method in practical applications in industry to create a "waste to wealth" concept . The selected bead quality features are bead width, reinforcement, depth of penetration and HAZ width. A grey relational grade evaluated from grey relational analysis was adopted to convert this multi-objective optimization problem into a single performance index. Based on grey relational analysis followed by the Taguchi method, the optimal welding parameter combination was determined. A confirmation experiment was conducted to verify the optimal parameter setting as predicted by grey relational analysis and the Taguchi method. Finally, analysis of variance (ANOVA) was carried out to observe the level of significance of factor(s) and/ or interaction of factors including slag-mix %, on the overall grey relational grade.

## 2 Overview of Taguchi method

Taguchi's philosophy is an efficient tool for the design of high quality manufacturing system. Dr. Genichi Taguchi, a Japanese quality management consultant, has developed a method based on orthogonal array experiments, which provide much reduced variance for the experiment with an optimum setting of process control parameters. Thus, the

Taguchi Method achieves the integration of design of experiments (DOE) with the parametric optimization of the process, yielding the desired results.. The orthogonal array (OA) provides a set of well balanced (minimum experimental runs) experiments and Taguchi's signal-to-noise ratios (S/N), which are logarithmic functions of desired output, serving as objective functions for optimization. This technique helps in data analysis and prediction of optimum results. In order to evaluate optimal parameter settings, the Taguchi Method uses a statistical measure of performance called signal-to-noise ratio. The S/ N ratio takes both the mean and the variability into account. The S/ N ratio is the ratio of the mean (signal) to the standard deviation (noise). The ratio depends on the quality characteristics of the product/ process to be optimized [9]. The standard S/ N ratios generally used are as follows: nominal is best (NB), lower the better (LB) and higher the better (HB). The optimal setting is the parameter combination which has the highest S/ N ratio.

## 3 Theory of grey relational analysis

In grey relational analysis, experimental data i.e., measured features of quality characteristics, are first normalized in a range from zero to one. This process is known as grey relational generation. Next, based on normalized experimental data, the grey relational coefficient is calculated to represent the correlation between the desired and actual experimental data. Then overall grey relational grade is determined by averaging the grey relational coefficient corresponding to selected responses. The overall performance characteristic of the multiple response process depends on the calculated grey relational grade. This approach converts a multiple response process optimization problem into a single response optimization situation with the objective function of overall grey relational grade. The optimal parametric combination is then evaluated which would result highest grey relational grade.

In grey relational generation, the normalized bead width, reinforcement and HAZ width, corresponding to lower-the-better (LB) criterion can be expressed as

$$x_i(k) = \frac{\max\ y_i(k) - y_i(k)}{\max\ y_i(k) - \min y_i(k)} \tag{i}$$

Bead penetration should follow larger-the-better criterion can be expressed as

$$x_i(k) = \frac{y_i(k) - \min y_i(k)}{\max y_i(k) - \min y_i(k)} \tag{ii}$$

where $x_i(k)$ is the value after the grey relational generation, min $y_i(k)$ is the smallest value of $y_i(k)$ for the *kth* response, and max $y_i(k)$ is the largest value of $y_i(k)$ for the *kth* response: bead width ($k$=1), reinforcement ($k$=2), bead penetration ($k$=3) and HAZ width ($k$=4). The normalized data after grey relational generation are tabulated in Table 6. An ideal sequence is $x_0(k)$ ($k$=1,2,3,4) for the responses. The definition of grey relational grade in the course of grey relational analysis is to reveal the degree of relation between the sixteen sequences [$x_0(k)$ *and* $x_i(k)$, $i$= 1,2,3.......,16]. The grey relational coefficient $\xi_i$ ($k$) can be calculated as

$$\xi_i(k) = \frac{\Delta_{\min} + \psi\Delta_{\max}}{\Delta_{0i}(k) + \psi\Delta_{\max}} \tag{iii}$$

where $\Delta_{0i} = \|x_0(k) - x_i(k)\| =$ difference of the absolute value $x_0(k)$ and $x_i$ ($k$); $\psi$ is the distinguishing coefficient $0 \leq \psi \leq 1$; $\Delta_{\min} = \forall j^{\min} \in i \forall k^{\min}\|x_0(k) - xj(k)\| =$ the smallest value of $\Delta_{0i}$; and $\Delta_{\max} = \forall j^{\max} \in i \forall k^{\max}\|x_0(k) - xj(k)\| =$ largest value of $\Delta_{0i}$. The grey relational coefficients for the present case are furnished in Table 8.

After averaging the grey relational coefficients, the grey relational grade $\gamma_i$ can be computed as

$$\gamma_i = \frac{1}{n}\sum_{k=1}^{n} \xi_i(k) \tag{iv}$$

where $n$ = number of process responses. Table 9 represents experimental results for the grey relational grade order by using the colleted experimental data (Table 4). The higher value of grey relational grade corresponds to intense relational degree between the ideal sequence $x_0(k)$ and the given sequence $x_i(k)$. The ideal sequence $x_0(k)$ represents the best process sequence, ($x_0(k)$=1, $k$=1,2,3,4); therefore, higher grey relational grade means that the corresponding parameter combination is closer to the optimal.

## 4 Experimentation and data analysis

Submerged arc welding is a multi-factor metal fabrication technique. Various process parameters influencing bead geometry, bead quality, and mechanical-metallurgical characteristics of the weldment include welding current, voltage, wire feed rate, traverse speed, electrode diameter, electrode stick-out, type of flux, height of the flux layer, etc. In full factorial design, the number of experimental runs exponentially increases as the number for factors as well as their level increases [15]. This results in huge experimentation cost and considerable time. So, in order to compromise these two adverse factors, the present study was planned to use only two conventional process parameters, along with a slag-mix%, treated as another process variable. It has been reported by previous researchers that among the conventional process parameters in SAW welding, current is the most significant factor [16] that influences different quality characteristics of sub-

**Table 1** Process parameters and their limits

| Serial No. | Parameter | Notation | Unit | Level 1 | Level 2 | Level 3 | Level 4 |
|---|---|---|---|---|---|---|---|
| 1 | Current | C | Ampere | 150 | 200 | 250 | 300 |
| 2 | % of slag-mix | S | – | 0 | 10 | 15 | 20 |
| 3 | Basisity index | B | – | 0.8 | 1.0 | 1.2 | 1.6 |

merged arc weldment. As, the flux basicity index is related to the chemical composition of flux, which in turn affects mechanical properties and metallurgical features of the weld, it is also an important factor. Therefore, in the present case, the welding current and flux basicity index were chosen as conventional process parameters along with the newly introduced parameter i.e., slag-mix%.

Experiments were conducted with four different levels of welding current, flux basicity index (type of flux) and slag-mix% to obtain bead-on-plate weldment on mild steel plates (100 × 40 × 12) by submerged arc welding. Process parameters with their notations, unit and values at different levels are listed in Table 1. Design matrix has been selected based on Taguchi's orthogonal array (OA) design of L16 (4**3), consisting of 16 sets of coded conditions (Table 4). The experiments have been performed in a submerged arc welding machine (Maker: IOL Ltd., India).

The required number of plates were kept aside for doing bead-on-plate welding with a mixture of fresh flux and fused slag, while the rest of the plates were utilized for collecting fused slag. For the latter purpose, welding was done on the plates with the conventional use of unmelted fresh flux only of four different types. The chemical composition of these four fluxes, along with basicity indices, is shown in Table 2.

A sufficient amount of welding was done to collect fused slag in desired quantities/ volumes for each of the above fluxes. The slag was then broken and finally crushed to the granular size almost as that of the original flux(es). Thus fused slag of four varieties was kept ready for subsequent welding. The parameters which have been kept invariant are listed in Table 3. Weld beads being done, the transverse sections of the weld beads were taken from the middle portions of the plates as specimens. These middle portions were polished by belt grinder, and subsequently by a series of finer grades of emery paper (grades 1G, 1, 1/0, 2/0, 3/0 and 4/0). Finally they were smoothened with a polishing cloth. The properly polished specimens have been etched with 2% Nital solution for about 30 sec duration, which has been followed by investigation and analysis. For each of the bead-on-plate specimens, the dimensions of the weld bead geometry, including the depth of penetration and HAZ width, were measured by Trinacular Metallurgical Microscope, Maker: Leica, INDIA (Table 5).

## 5 Evaluation of optimal process condition

Experimental data were normalized first (grey relational generation). The normalized data for each of the parameters of bead geometry and HAZ are furnished in Table 6. For bead width, reinforcement and width of HAZ, lower-the-better criterion was selected, and for depth of penetration, higher-the-better criterion was selected. Grey relational coefficients for each performance characteristics have been calculated using equation (iii) (Tables 7 and 8). These grey relational coefficients for each response were accumulated to evaluate grey relational grade (equation iv), that is, the overall representative of all the features of weld quality (Table 9). Thus, the the multi-criteria optimization problem has been transformed into a single equivalent objective function optimization problem using the combination of the Taguchi approach and grey relational analyses. The higher the value of the grey relational grade, the corresponding factor combination is said to be close to the optimal. The mean response table for the overall grey relational grade is shown in Table 10 and is represented graphically in Figure 1. The overall grey relational grade was calculated using equation (iv). The ordinate of Figure 1 represents the

**Table 2** Chemical composition of fluxes used

| Flux | Character | Type of manufacture | Chemical composition (%) | | | | Basicity index |
|---|---|---|---|---|---|---|---|
| | | | $Al_2O_3$+$MnO_2$ | CaO+MgO | $SiO_2$+$TiO_2$ | $CaF_2$ | |
| $B_1$ | Acid | Agglomerated | 55 | 5 | 30 | 5 | 0.8 |
| $B_2$ | Neutral | Fused | – | – | – | – | 1.0 |
| $B_3$ | Basic | Agglomerated | 25 | 35 | 35 | – | 1.2 |
| $B_4$ | Basic | Agglomerated | 35 | 25 | 20 | 15 | 1.6 |

**Table 3** Constant parameters in the experiment

| Parameters |
|---|
| Travel Speed : 20 cm/min |
| Nozzle Angle: $90^0$ |
| Voltage : 28 V |
| Electrode wire: 3.15 mm diameter copper-coated mild steel wire |
| Thickness of flux layer: fairly constant. |

S/N ratio for overall grey relational grade calculated using LB (larger-the-better) criteria which is shown below:-

$$SN(Larger - the - better) = -10 \log \left[ \frac{1}{n} \sum_{i=1}^{n} \frac{1}{y_i^2} \right] \quad \text{(v)}$$

where $n$ is the number of measurements, and $y_i$ the measured characteristic value.

For the orthogonal experimental design, it is possible to separate out the effect of each welding parameter at different levels. For example, the mean grey relational grade for the current at levels 1, 2, 3 and 4 can be calculated by averaging the grey relational grades for the experiments 1–4, 5–8, 9–12 and 13–16 respectively. The mean grey relational grade ratio for each level of the other parameters can be computed in the similar manner (Table 10). Total mean grey relational grade is the average of all entries in Table 9.

With the help of the figure, the optimal parametric combination was determined. The optimal factor setting becomes **C1S2B2**.

**Table 4** Taguchi's orthogonal array L16 (4**3)

| Serial number | Current C | Slag-mix% S | Flux basicity B |
|---|---|---|---|
| 1 | 1 | 1 | 1 |
| 2 | 1 | 2 | 2 |
| 3 | 1 | 3 | 3 |
| 4 | 1 | 4 | 4 |
| 5 | 2 | 1 | 2 |
| 6 | 2 | 2 | 1 |
| 7 | 2 | 3 | 4 |
| 8 | 2 | 4 | 3 |
| 9 | 3 | 1 | 3 |
| 10 | 3 | 2 | 4 |
| 11 | 3 | 3 | 1 |
| 12 | 3 | 4 | 2 |
| 13 | 4 | 1 | 4 |
| 14 | 4 | 2 | 3 |
| 15 | 4 | 3 | 2 |
| 16 | 4 | 4 | 1 |

**Table 5** Experimental data

| Serial number | Bead width (mm) | Reinforcement (mm) | Penetration (mm) | Width of HAZ (mm) |
|---|---|---|---|---|
| 1 | 9.36 | 3.28 | 1.58 | 2.11 |
| 2 | 9.04 | 3.14 | 1.84 | 1.62 |
| 3 | 10.72 | 3.75 | 1.91 | 1.98 |
| 4 | 13.12 | 3.94 | 1.98 | 2.31 |
| 5 | 11.65 | 3.43 | 2.37 | 3.65 |
| 6 | 12.47 | 4.16 | 1.88 | 2.59 |
| 7 | 13.75 | 4.32 | 2.26 | 3.10 |
| 8 | 10.11 | 3.71 | 2.30 | 2.25 |
| 9 | 16.09 | 4.30 | 2.80 | 4.41 |
| 10 | 15.55 | 4.41 | 2.51 | 4.10 |
| 11 | 13.18 | 4.60 | 2.40 | 3.84 |
| 12 | 15.36 | 4.06 | 3.40 | 4.02 |
| 13 | 16.25 | 4.68 | 3.01 | 4.91 |
| 14 | 16.25 | 4.62 | 3.30 | 4.30 |
| 15 | 15.73 | 4.50 | 3.90 | 4.02 |
| 16 | 13.61 | 4.96 | 3.05 | 4.16 |

## 6 Confirmatory experiment

After evaluating the optimal parameter settings, the next step is to predict and verify the enhancement of quality characteristics using the optimal parametric combination. The estimated grey relational grade $\hat{\gamma}$ using the optimal level of the design parameters can be calculated as:

$$\hat{\gamma} = \gamma_m + \sum_{i=1}^{o} (\overline{\gamma} - \gamma_m) \quad \text{(vi)}$$

**Table 6** Data preprocessing of each performance characteristic (grey relational generation)

| Experiment No. | Bead width | Reinforcement | Penetration | HAZ Width |
|---|---|---|---|---|
| Ideal sequence | 1 | 1 | 1 | 1 |
| 1 | 0.9556 | 0.9231 | 0.0000 | 0.8511 |
| 2 | 1.0000 | 1.0000 | 0.1121 | 1.0000 |
| 3 | 0.7670 | 0.6648 | 0.1422 | 0.8906 |
| 4 | 0.4341 | 0.5604 | 0.1724 | 0.7903 |
| 5 | 0.6380 | 0.8407 | 0.3405 | 0.3830 |
| 6 | 0.5243 | 0.4396 | 0.1293 | 0.7052 |
| 7 | 0.3467 | 0.3516 | 0.2931 | 0.5502 |
| 8 | 0.8516 | 0.6868 | 0.3103 | 0.8085 |
| 9 | 0.0222 | 0.3626 | 0.5259 | 0.1520 |
| 10 | 0.0971 | 0.3022 | 0.4009 | 0.2462 |
| 11 | 0.4258 | 0.1978 | 0.3534 | 0.3252 |
| 12 | 0.1234 | 0.4945 | 0.7845 | 0.2705 |
| 13 | 0.0000 | 0.1538 | 0.6164 | 0.0000 |
| 14 | 0.0000 | 0.1868 | 0.7414 | 0.1854 |
| 15 | 0.0721 | 0.2527 | 1.0000 | 0.2705 |
| 16 | 0.3662 | 0.0000 | 0.6336 | 0.2280 |

**Table 7** Evaluation of $\Delta_{0i}$ for each of the responses

| Experiment No. | Bead width | Reinforcement | Penetration | HAZ width |
|---|---|---|---|---|
| Ideal sequence | 1 | 1 | 1 | 1 |
| 1 | 0.0444 | 0.0769 | 1.0000 | 0.1489 |
| 2 | 0.0000 | 0.0000 | 0.8879 | 0.0000 |
| 3 | 0.2330 | 0.3352 | 0.8578 | 0.1094 |
| 4 | 0.5659 | 0.4396 | 0.8276 | 0.2097 |
| 5 | 0.3620 | 0.1593 | 0.6595 | 0.6170 |
| 6 | 0.4757 | 0.5604 | 0.8707 | 0.2948 |
| 7 | 0.6533 | 0.6484 | 0.7069 | 0.4498 |
| 8 | 0.1484 | 0.3132 | 0.6897 | 0.1915 |
| 9 | 0.9778 | 0.6374 | 0.4741 | 0.8480 |
| 10 | 0.9029 | 0.6978 | 0.5991 | 0.7538 |
| 11 | 0.5742 | 0.8022 | 0.6466 | 0.6748 |
| 12 | 0.8766 | 0.5055 | 0.2155 | 0.7295 |
| 13 | 1.0000 | 0.8462 | 0.3836 | 1.0000 |
| 14 | 1.0000 | 0.8132 | 0.2586 | 0.8146 |
| 15 | 0.9279 | 0.7473 | 0.0000 | 0.7295 |
| 16 | 0.6338 | 1.0000 | 0.3664 | 0.7720 |

where $\gamma_m$ is the total mean grey relational grade, $\gamma_i$ is the mean grey relational grade at the optimal level, and o is the number of the main design parameters that affect the quality characteristics. That means that the predicted or estimated grey relational grade (optimal) is equal to the mean grey relational grade plus the summation of the difference between the overall mean grey relational grade and the mean grey relational grade for each of the factors at optimal level. Table 11 represents the comparison of the predicted bead geometry parameters and HAZ dimension with that of the actual parameters by using the optimal welding conditions; good agreement between the two has been observed. This proves the utility of the proposed approach in relation to product/ process optimization, where more than one objective has to be fulfilled simultaneously.

**Table 8** Grey relational coefficient of each performance characteristics (with ψ=0.5)

| Experiment No. | Bead width | Reinforcement | Penetration | HAZ width |
|---|---|---|---|---|
| Ideal sequence | 1 | 1 | 1 | 1 |
| 1 | 0.9184 | 0.8667 | 0.3333 | 0.7705 |
| 2 | 1.0000 | 1.0000 | 0.3603 | 1.0000 |
| 3 | 0.6821 | 0.5987 | 0.3682 | 0.8205 |
| 4 | 0.4691 | 0.5321 | 0.3766 | 0.7045 |
| 5 | 0.5800 | 0.7584 | 0.4312 | 0.4476 |
| 6 | 0.5125 | 0.4715 | 0.3648 | 0.6291 |
| 7 | 0.4335 | 0.4354 | 0.4143 | 0.5264 |
| 8 | 0.7711 | 0.6149 | 0.4203 | 0.7231 |
| 9 | 0.3383 | 0.4396 | 0.5133 | 0.3709 |
| 10 | 0.3564 | 0.4174 | 0.4549 | 0.3988 |
| 11 | 0.4655 | 0.3840 | 0.4361 | 0.4256 |
| 12 | 0.3632 | 0.4973 | 0.6988 | 0.4067 |
| 13 | 0.3333 | 0.3714 | 0.5659 | 0.3333 |
| 14 | 0.3333 | 0.3807 | 0.6591 | 0.3803 |
| 15 | 0.3502 | 0.4009 | 1.0000 | 0.4067 |
| 16 | 0.4410 | 0.3333 | 0.5771 | 0.3931 |

**Table 9** Grey relational grade

| Experiment No. | Grey relational grade |
|---|---|
| 1 | 0.7222 |
| 2 | 0.8401 |
| 3 | 0.6174 |
| 4 | 0.5206 |
| 5 | 0.5543 |
| 6 | 0.4945 |
| 7 | 0.4524 |
| 8 | 0.6324 |
| 9 | 0.4155 |
| 10 | 0.4069 |
| 11 | 0.4278 |
| 12 | 0.4915 |
| 13 | 0.4010 |
| 14 | 0.4384 |
| 15 | 0.5394 |
| 16 | 0.4361 |

## 7 Analysis of variance (ANOVA)

ANOVA is a statistical technique which can infer some important conclusions based on analysis of the experimental data. The method is very useful for revealing the level of significance of influence of factor(s) or interaction of factors on a particular response. It separates the total variability of the response (sum of squared deviations about the grand mean) into contributions rendered by each of the parameter/ factor and the error. Thus

$$SS_T = SS_F + SS_e \tag{vii}$$

**Table 10** Response table (mean) for the grey relational grade

| Factor | Grey relational grade | | | | |
|---|---|---|---|---|---|
| | Level 1 | Level 2 | Level 3 | Level 4 | Delta |
| C | 0.6751 | 0.5334 | 0.4354 | 0.4537 | 0.2397 |
| S | 0.5233 | 0.5450 | 0.5093 | 0.5202 | 0.0357 |
| B | 0.5202 | 0.6063 | 0.5259 | 0.4452 | 0.1611 |
| Total mean grey relational grade=0.5244 | | | | | |

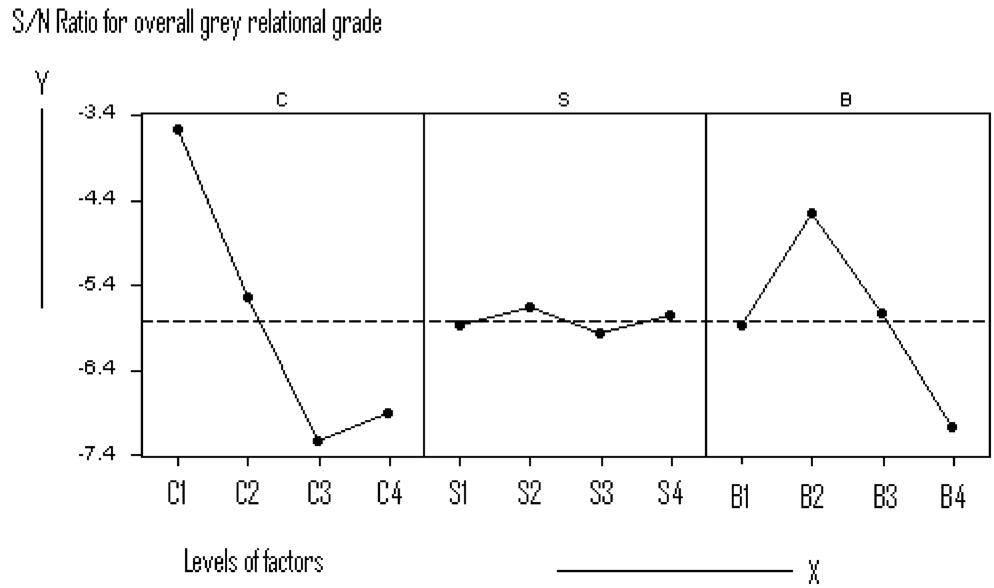


**Fig. 1** Grey relational grade graph

where $SS_T = \sum_{j=1}^{p} \left(\gamma_j - \gamma_m\right)^2$ (viii)

and

- $SS_T$ Total sum of squared deviations about the mean.
- $\gamma_j$ Mean response for *jth* experiment.
- $\gamma_m$ Grand mean of the response.
- $P$ Number of experiments in the orthogonal array.
- $SS_F$ Sum of squared deviations due to each factor
- $SS_e$ Sum of squared deviations due to error

In the ANOVA table mean square deviation is defined as:

$$MS = \frac{SS(Sum\ of\ squared\ deviation)}{DF(Degree\ of\ freedom)}$$

F-value of Fisher's F ratio (Variance ratio) is defined as:

$$F = \frac{MS\ for\ a\ term}{MS\ for\ the\ error\ term}$$

Depending on F-value, P-value (probability of significance) is then calculated. It P-value for a term appears less than 0.05 (For 95% confidence level) then it can be concluded that the effect of the factor(s)/ interaction of factors is significant on the selected response, MINITAB release 13.1 (user manual).

In the ANOVA table, the degrees of freedom are used to calculate the mean square (MS). In general, the degrees of freedom measure how much "independent" information is available to calculate each sum of squares (SS).

DF for total = DF for all factors + DF for all interactions + DF for errorDF total = n−1 where n is the total number of observations andDF for factor = k−1 where k is the number of the factor levels.

DF for Interaction = (k1−1) * (k2−1) where k1 is the number of levels of factor one, and k2 is the number of levels of factor two. The same rule applies to interactions of more than two factors. In the present study, the interaction of factors has been neglected.

The sequential sum of squares for each term in the model (factor or interaction) measures the amount of variation in the response that is explained by adding each term to the model sequentially in the order listed under source. Thus, the sequential sums of squares for terms are specific to the order of the terms specified in the linear model.

The adjusted sum of squares for a term in the model (factor or interaction) measures the amount of additional variation in the response that is explained by the term, given that all the other terms are already in the model.

**Table 11** Results of confirmatory experiment

| | Initial factor setting | Optimal condition | |
|---|---|---|---|
| | | Prediction | Experiment |
| Level | C1S1B1 | C1S2B2 | C1S2B2 |
| Bead width (mm) | 9.36 | | 9.04 |
| Reinforcement (mm) | 3.28 | | 3.14 |
| Penetration (mm) | 1.58 | | 1.84 |
| HAZ width (mm) | 2.11 | | 1.62 |
| Grey relational grade | 0.7222 | 0.7832 | 0.8401 |
| Improvement in grey relational grade=0.1179 | | | |

**Table 12** Analysis of variance using adjusted SS for tests

| Source | DF | Seq SS | Adj SS | Adj MS | F | P |
|---|---|---|---|---|---|---|
| C | 3 | 0.142782 | 0.142782 | 0.047594 | 8.08 | 0.016 |
| S | 3 | 0.002689 | 0.002689 | 0.000896 | 0.15 | 0.925 |
| B | 3 | 0.052003 | 0.052003 | 0.017334 | 2.94 | 0.121 |
| Error | 6 | 0.035342 | 0.035342 | 0.005890 | | |
| Total | 15 | 0.232816 | | | | |

Thus, the values for the adjusted sums of squares do not depend on the order of the terms listed under source.

The adjusted mean square for a term is simply the adjusted sum of squares (Adj SS) divided by the degrees of freedom.

The ANOVA for grey relational grade is shown in Table 12. It is observed that P-value for welding current is 0.016 (less than 0.05). So it is evident that current is the most significant factor. The effects of slag-mix% and flux basicity index on grey relational grade appear insignificant. As the statistical significance of slag-mix% appears insignificant on quality features of the weld, it indicates the application feasibility of using slag-mix as an alternative to fresh flux to yield a "waste-to-wealth" concept, the primary focus of the ongoing study.

## 8 Conclusion

In the present study, the methodology of Taguchi optimization technique with grey relational analysis are reported and applied for evaluating the optimal parametric combination to achieve acceptable features of weld bead geometry and HAZ in non-conventional submerged arc bead-on-plate welding, which consumes a fresh flux and fused slag mixture as a protective layer instead of only fresh flux as in the conventional SAW process. The method is very efficient for solving multiple-objective optimization problems that can be performed in a limited number of experimental runs. Apart from parametric optimization, the paper introduces the concept of utilizing a slag-mix in further runs of SAW process. It has been revealed that 10% slag-mix should be used (optimum amount) to achieve favorable quality weld in terms of bead geometry (within the experimental domain). ANOVA reveals that the use of a slag-mix (up to 20%) does not impose any adverse effect on weld quality characteristics associated with bead geometry and HAZ. Hence there exists a scope to examine the results, even at higher values of the slag-mix%, since the mixture between 0 to 20% does not statistically influence the quality of welding. It is also necessary to analyze the cost aspects of slag recycling and reconsumption in SA welding. If it becomes cost-effective, the new method can be recommended in practical applications to yield "waste to wealth" concepts.

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