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ORIGINAL ARTICLE

Optimization of measuring points for machine tool thermal error based on grey system theory

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Abstract Optimization of thermal sensors' placement on machine tools based on grey correlation model of grey system theory is studied. After optimization, the temperature variables in the thermal error' model are reduced from 16 to 4. It greatly reduces the time for variable searching and modelling and meanwhile it eliminates the coupling problems among temperature variables, so the robustness of the model could be increased and the predicting precision of the model is enhanced. Consequently, the real-time error compensation would be more effective and convenient.

Keywords Grey system theory · Machine tool · Modelling · Thermal error

1 Introduction

During mechanical processing, thermal errors cause a relative displacement between the workpiece and the tool on account of deformation or expansion of the machine elements due to an increase in their temperature and thus have an influence on the accuracy of the workpiece produced on metal-cutting machine tools [1]. Thermal error of a machine tool is one of the important factors which influence the processing precision of machine tool and will become critical in its contribution to the quality of the workpiece. Reducing the thermal errors will be beneficial to the precision of the manufacturing system [2–4]. Although these errors could be decreased by amending machine tools' structure through advanced design and manufacture technique, error compensation technique is regarded as a more effective and economical way to improve manufacturing precision in machine tools [5, 6]. Error compensation technique is based on measuring the temperature of a few points on the machine and then obtaining the empirical relationships between the temperature and the thermal deformation of the machine. Compensation was realized by coordinate system correction according to the value predicted by an off-line model [4]. Consuently, compensation technique has already been widely used in mechanical processing industry.

In the course of processing, CNC machine tool is always under the effect of many kinds of thermal sources. Furthermore, different machining conditions, as well as the different change extent, have made machine tool a complicated temperature field [7]. Generally speaking, there are six sources of thermal influences for a machine-tool system: (a) heat generated in the cutting process; (b) heat generated by the machine; (c) heating or cooling influence provided by various cooling systems; (d) heating or cooling influence provided by the room; (e) the effect of people; and (f) thermal memory from any previous environment. These error sources can interact with each other through conduction, convection and radiation to create a non-uniform temperature field on the machine structure, causing the thermal displacement of different parts of the machine, thus resulting in a loss of accuracy in the manufacturing process [3, 4].

Establishing a thermal error predicting model for thermal error compensation depends on distribution of the temperature field of machine tools, and the distribution is a major factor to effect thermal errors. Hence, in order to get a

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temperature field, a large number of temperature sensors would be required to be placed on the machine tool. Nevertheless, too many temperature sensors would increase the workload of calculation and measurement, and affect normal operation of machine tools. Therefore, selecting the most pivotal temperature measuring points for thermal errors is indispensable [8]. According to grey system theory founded by Professor Deng Julong [9, 10] of Huazhong University of Science and Technology, there are many complex factors contributing to thermal errors of mechanical processing, so in this way thermal errors show evident grey property. Therefore, in this paper, an analysis model of grey correlation, based on spot surveyed statistical data sequence, is established, and all influencing factors of temperature field for machine tools and their contribution to thermal errors of mechanical processing is analyzed. Then a synthetical appraisal of ability about these factors are given, and then the most pivotal points from numerous temperature measuring points are selected, so that the numbers of measuring points can be reduced, and the placement of thermal sensors on machine tools is optimized. Meanwhile, the repeated co-linearity in the model of thermal errors caused by excessive temperature variables would be avoided, and the robustness of thermal error modelling would be increased.

2 Measurement of temperature field and machine tool thermal error [8]

2.1 Setting of temperature sensors

As shown in Fig. 1, 16 temperature sensors are installed on the turning centre in order to measure the temperature field of the machine tool. The sensors are divided into five groups according to their places on the turning centre.

(1) two sensors (No. 0~1) are used to measure temperature of headstock;
(2) four sensors (2~5) are used to measure temperature of transmission nut;
(3) two sensors (6~7) are used to measure temperature of coolant;
(4) one sensor (8) is used to measure room temperature;
(5) seven sensors (9~15) are used to measure temperature of lathe bed.

Sketch map of measurement for thermal errors is shown in Fig. 2. Two displacement sensors, fixed on knife rest, measure thermal errors between principal axis of X direction (17) and Z direction (16) and knife rest. Since the workpiece is very short, the errors of the leaning angle are neglected.

2.2 Measuring test of thermal errors in the cycle of turning process

First, a simulation test of thermal errors in the cycle of turning process is carried out, that is: only the principal axis of the machine tool is rotating with no real cutting process, carriage is moving, and coolant flowing. This is known as empty turning. The machine tool keeps running for 3.25 hours, then pauses for one hour to simulate as nooning, then runs again for 3.5 hours, lastly, it stops for one hour. Changes of temperature and machine tool thermal errors are illustrated in Fig. 3 (a) and (b).

3 Grey correlation analysis model

Grey systems analysis is a kind of systemic analysis applying mathematics theory, according to comparability among systemic characteristic parameter series [11]. In the experimental data processing, grey system theory has the advantage over traditional statistic theory, namely: its results sufficiently embody the inherent properties of a system with less experimental data and unknown systemic probability [12]. The aim of the test is to adopt grey correlation analysis when experimental data of temperature measuring points are analyzed, and to seek the relationships

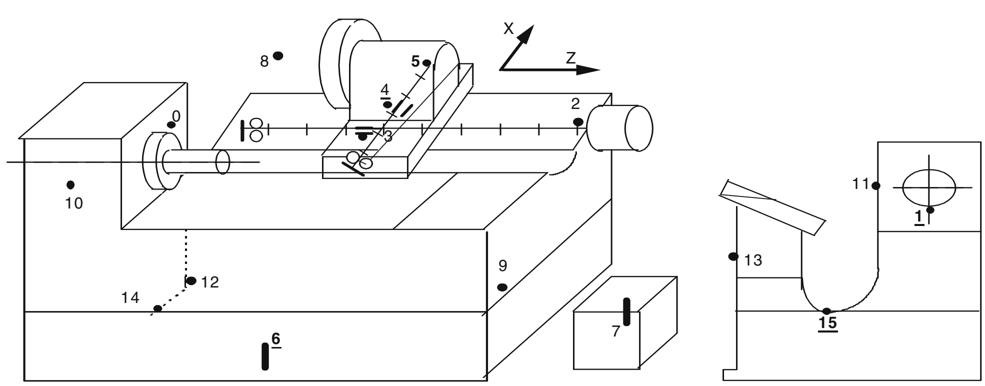


Fig. 1 Sketch map of turning centre and test disposal of temperature sensors

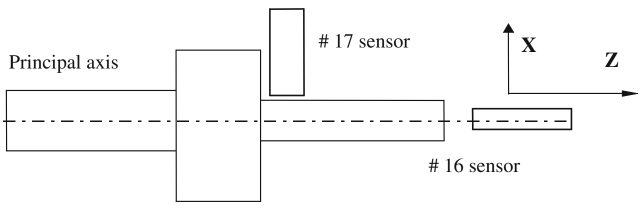


Fig. 2 Measurement of principal axis thermal errors

among all factors in the system, and finds out the key influencing factors to machine tool thermal errors.

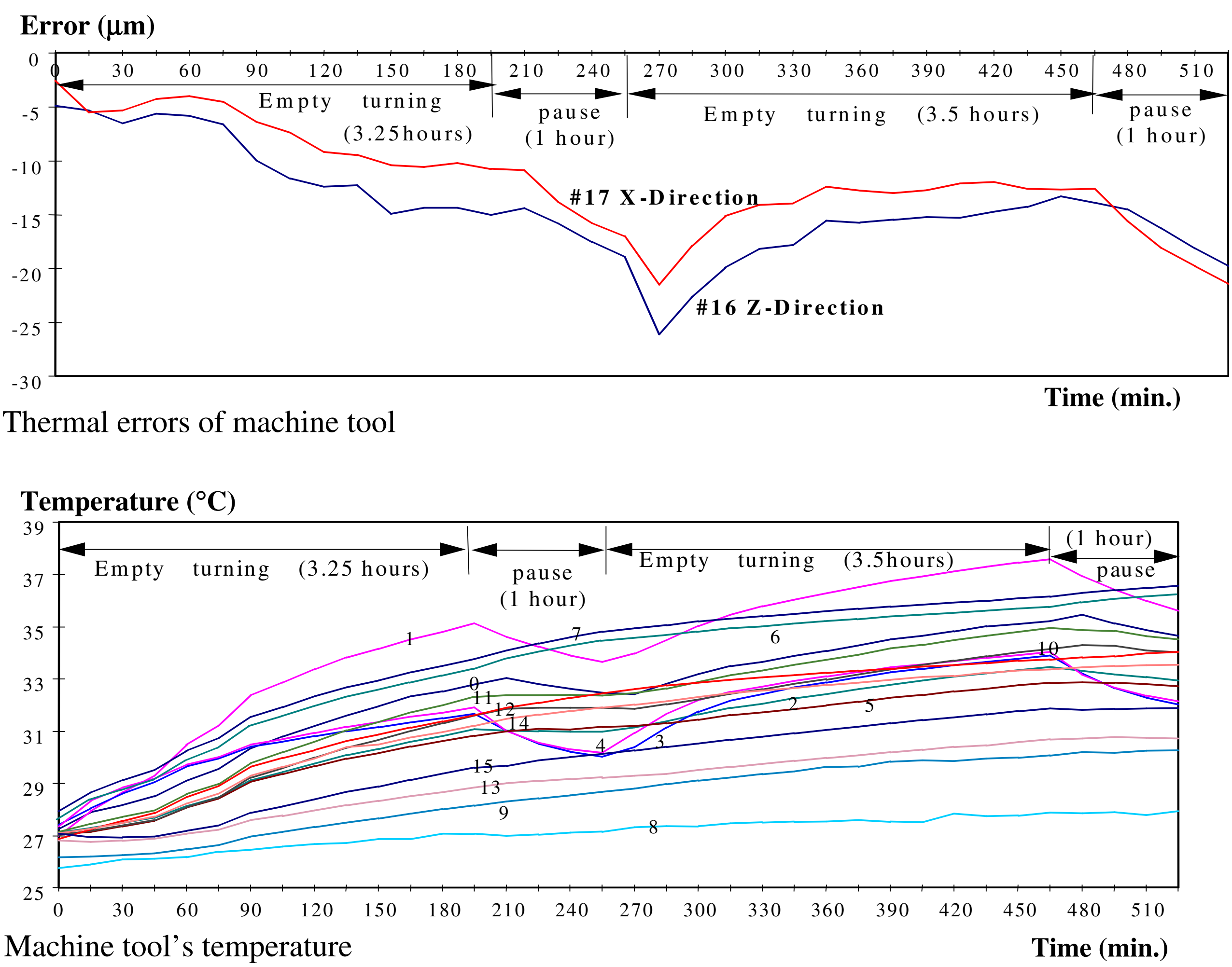
3.1 Grey relational algebra [13]

Grey relational algebra [14, 15] is based on the grey system theory [9]. The central idea in grey relational algebra is to analyze the uncertain relationship between two tuples by looking at the information flow from one tuple to another under the influence of all tuples in the relation. The term grey tuple dependency $\left(\xi_j^i\right)$ is used to specify the closeness of one tuple (t_j) to another tuple (t_i). The grey attribute dependency $\left(\xi_j^i(k)\right)$ of attribute a_k between tuples (t_i) and (t_j) is a value between 0 and 1 which measures the impact of the attribute (t_j, a_k) on the attribute (t_i, a_k) under the influence of the $a_k s$ of all other tuples in the relation. The basic element to compute $\xi_j^i(k)$ is the attribute difference $\left(\Delta_j^i(k)\right)$. Influence by the system can be introduced into the computation by means of $\Delta_{\max}^i$ and $\Delta_{\min}^i$ which are defined as:

$$\Delta_{\max}^i = \max_{1\le j\le m} \max_{1\le k\le n} \left(\Delta_j^i(k)\right) \tag{1}$$

$$\Delta_{\min}^i = \min_{1\le j\le m} \min_{1\le k\le n} \left(\Delta_j^i(k)\right). \tag{2}$$

$\Delta_{\max}^i$ is the maximum attribute difference of all tuples to tuple t_i and $\Delta_{\min}^i$ is the minimum attribute difference of all tuples to tuple t_i with respect to their attributes. Once all the



(a) Thermal errors of machine tool

(b) Machine tool's temperature

Fig. 3 Data curve of empty turning, (**a**) Thermal errors of machine tool, (**b**) Machine tool's temperature attribute differences are obtained, $\xi_j^i(k)$ can be computed by the following formula:

$$\xi_j^i(k) = \frac{\Delta_{\min}^i + \zeta\Delta_{\max}^i}{\Delta_j^i(k) + \zeta\Delta_{\max}^i} \tag{3}$$

In the above formula, ζ is a distinguishing coefficient between 0 and 1 which is used to weaken the effect of $\Delta_{\max}^i$ when it gets too big, and thus enlarges the difference significance of $\Delta_j^i(k)$. Finally, the grey tuple dependency $\left(\xi_j^i\right)$ is the mean of the grey attribute dependencies $\left(\xi_j^i(k)\right)$ of all attributes. That is,

$$\xi_j^i = \frac{1}{n}\sum_{k=1}^{n}\xi_j^i(k) \tag{4}$$

As for weighted grey tuple dependency, it allows us to put more emphasis on some particular attributes when calculating the grey tuple dependencies. The attribute difference of the emphasized attribute is multiplied by a constant to enlarge its effect on the calculation of grey tuple dependency. $\xi_j^i(k)$ can be computed by the following formula:

$$\xi_j^i(k) = \frac{\zeta\Delta_{\max}^i}{\lambda_1\cdot\Delta_j^i(k) + \lambda_2\cdot\Delta_j^i(k)' + \zeta\Delta_{\max}^i} \tag{5}$$

In the above formula, ζ is a distinguishing coefficient between 0 and 1 which is used to weaken the effect of $\Delta_{\max}^i$ when it gets too big, and thus enlarges the difference significance of $\Delta_j^i(k)$; λ_1 and λ_2 are weighted coefficients of displacement and changing rate respectively, they reflect changing trends among all sequences. $\Delta_j^i(k)'$ is the inverse accumulating generation of the attribute difference $\Delta_j^i(k)$.

Finally, the correlation degree between each son sequence and mother sequence is arranged according to the size to form a correlation sequence, which directly reflects the essentiality of each son sequence to the same mother sequence [16].

3.2 Standardization disposal of data sequence

Because the meanings and purpose of each influencing factor is different, the target value generally has different dimension and quantitative levels. If the data of two sequences differ greatly in size, the effect of the sequence of small numerical value is easily covered up by the sequence of big numerical value. So primary data must be disposed to assure the equivalence of all factors [11]. In the test, the following three ways are adopted to dispose primary data, initial value transform, average value transform and polar difference transform.

Initial value transform is that all data are divided by first data, then get a new data sequence, which has a percentage of the value of different times compared to that of first time; average value transform is that all data are divided by average value and get a sequence which is a certain percent of the average value; polar difference transform is that the larger the number is, the smaller the efficiency, as given in Eq. 4.

$$x(k) = \frac{\max\limits_{k\in n} x^{(0)}(k) - x^{(0)}(k)}{\max\limits_{k\in n} x^{(0)}(k) - \min\limits_{k\in n} x^{(0)}(k)}\,(k = 1, 2, \cdots\cdots, \mathrm{n}) \tag{6}$$

4 Application analysis of grey correlation model

The measuring values of seven key time nodes in the process of turning for the model is chosen in the paper. Radial thermal error sequence of the machine tool is regarded as the mother sequence x_{16} (for workpieces' precision of radial dimension has a rather high request, radial thermal error of the machine tool is only considered in this paper). In practice, researched objects are workpieces' errors of radial dimension which changed along with the temperature of the machine tool or the time. The son sequence x_i is the measuring value sequence of 16 temperature sensors

$$x_{16}(1),\ \mathrm{x}_{16}(2), \cdots,\ \mathrm{x}_{16}(\mathrm{k}), \cdots,\ \mathrm{x}_{16}(\mathrm{n})$$
$$x_i(1),\ \mathrm{x_i}(2), \cdots,\ \mathrm{x_i}(\mathrm{k}), \cdots,\ \mathrm{x_i}(\mathrm{n})$$
$$k = 1, 2, \cdots, \mathrm{n}\,; \mathrm{i} = 0, 2, \cdots, 15$$

Primary data would be transformed to eliminate dimension firstly and become comparable data, then gain the correlation coefficient and degree of correlation of the mother sequence x_{16} to each son sequence x_i at the point of each moment, the results are as follows:

(1) Degree of correlation of mother sequence to each son sequence with initial value transform

$\gamma_{16,0} = 0.5289$	$\gamma_{16,1} = 0.5341$	$\gamma_{16,2} = 0.52452$	$\gamma_{16,3} = 0.5255$
$\gamma_{16,4} = 0.5269$	$\gamma_{16,5} = 0.5241$	$\gamma_{16,6} = 0.53039$	$\gamma_{16,7} = 0.53036$
$\gamma_{16,8} = 0.5201$	$\gamma_{16,9} = 0.5213$	$\gamma_{16,10} = 0.5275$	$\gamma_{16,11} = 0.5280$
$\gamma_{16,12} = 0.5283$	$\gamma_{16,13} = 0.5219$	$\gamma_{16,14} = 0.5244$	$\gamma_{16,15} = 0.52455$

(2) Degree of correlation of mother sequence to each son sequence with average value transform

$\gamma_{16,0} = 0.6246$	$\gamma_{16,1} = 0.6315$	$\gamma_{16,2} = 0.6191$	$\gamma_{16,3} = 0.6122$
$\gamma_{16,4} = 0.6138$	$\gamma_{16,5} = 0.6137$	$\gamma_{16,6} = 0.6283$	$\gamma_{16,7} = 0.6290$
$\gamma_{16,8} = 0.5916$	$\gamma_{16,9} = 0.61378$	$\gamma_{16,10} = 0.6274$	$\gamma_{16,11} = 0.6277$
$\gamma_{16,12} = 0.6245$	$\gamma_{16,13} = 0.6104$	$\gamma_{16,14} = 0.6063$	$\gamma_{16,15} = 0.6230$

(3) Degree of correlation of mother sequence to each son sequence with polar difference transform

$\gamma_{16,0} = 0.4797$	$\gamma_{16,1} = 0.4881$	$\gamma_{16,2} = 0.4926$	$\gamma_{16,3} = 0.5262$
$\gamma_{16,4} = 0.5166$	$\gamma_{16,5} = 0.4803$	$\gamma_{16,6} = 0.4810$	$\gamma_{16,7} = 0.4717$
$\gamma_{16,8} = 0.4832$	$\gamma_{16,9} = 0.5100$	$\gamma_{16,10} = 0.4730$	$\gamma_{16,11} = 0.4795$
$\gamma_{16,12} = 0.4705$	$\gamma_{16,13} = 0.4922$	$\gamma_{16,14} = 0.4750$	$\gamma_{16,15} = 0.5041$

According to the above analysis, three sequences of grey correlation are established. For each sequence, the previous

Table 1 Correlation sequence

No.	Analyzing method	Correlation sequence (former 10)
1	Initial value transform	$x_1 > x_6 > x_7 > x_0 > x_{12} > x_{11} > x_{10} > x_4 > x_3 > x_{15}$
2	Average value transform	$x_1 > x_7 > x_6 > x_{11} > x_{10} > x_0 > x_{12} > x_{15} > x_2 > x_4$
3	Polar difference transform	$x_3 > x_4 > x_9 > x_{15} > x_2 > x_{13} > x_1 > x_8 > x_6 > x_5$

10 factors would be taken out to form a new sequence, then conclusive factors are chosen from those which appear in each of the three new sequences.

From Table 1, main factors are $\gamma_{16,\,1}$, $\gamma_{16,\,4}$, $\gamma_{16,\,6}$, $\gamma_{16,\,15}$, four corresponding temperature sensors are as follows:

(1) Temperature sensor No. 6 measuring the temperature of coolant (underside of lathe bed);
(2) Temperature sensor No. 15 measuring the temperature of the lathe bed (upside of lathe bed);
(3) Temperature sensor No. 4 measuring the temperature of the nut on the X-axis;
(4) Temperature sensor No. 1 measuring the temperature of the headstock.

The results are the same as in reference [17], in which modal analysis of thermal errors on the turning centre was applied. The previous 16 temperature sensors are decreased to 4, which could simplify the model of thermal error and then easily analyze machine tool thermal errors.

By the analyzing theory of weighted correlation [11], the degree of correlation between machine tool thermal errors and four key temperature points from above is deeply analyzed , the sequence is: $\gamma_{16,1} > \gamma_{16,6} > \gamma_{16,15} > \gamma_{16,4}$. The results show that the principle axis' temperature is the most important influencing factor to machine tool thermal errors, which are the same as grouping optimization modelling by selection of temperature variables [18]. It has been seen that temperature rising of different points in CNC machine tools has a different influence on thermal errors, and a rational model of thermal error could be established through discriminating key temperature points from a great deal of temperature points.

5 Conclusions

A grey system is a system with partial information known. According to grey system theory, any random process is changeful in a certain range of amplitude within a certain time zone, any random process can be regarded as a grey process. Although the appearance of the objective system is complicated and its data is discrete, it is sequential and possesses some certain whole function, so it must potentially contain inherent rule. Grey correlation method is the basic concept of grey system theory. Grey correlation refers to uncertain correlation among different things, or among different factors of a system, or between genes and main behaviour. The basic role of grey correlation analysis is to analyze and ascertain the degree of influence among the genes or the degree of contribution between genes and main behaviour based on the behaviour's microcosmic or macroscopical geometrical similar relation. The calculation of correlation coefficient is a kind of quantitative analysis of the correlation degree among the factors [11, 19]. Compared with other analyzing methods, the analyzing method of grey correlation has a small computing quantity, computes easily and requires no classic distribution of large stylebook of data sequence.

This paper has the same results as that of modal analysis of thermal errors which used grey system theory to optimize the distribution of temperature field of machine tools. Furthermore, it gains key temperature variables required by thermal error's model of machine tool, and decreases the number of temperature variable in thermal error's model. Hence it greatly reduces variable searching and modelling time. In addition, the optimization of measuring points eliminates the coupling problems, so the robustness of the model can be increased and the predicting precision of the model with the optimal combination of the temperature variables is enhanced. Consequently, the real-time compensation becomes a more effective tool.

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