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ORIGINAL ARTICLE

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The building of spindle thermal displacement model of high speed machine center

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Abstract This article mainly analyzes the issues of spindle thermal displacement upon the acceleration of spindle, and proposes a complex multivariable regression analytical method to build a spindle thermal displacement model, so as to predict the thermal displacement of spindle caused by the heat under every time slot during the practical high speed machining process. Upon using the compensation of spindle displacement model, the largest value of cutting depth of the hole error of 62 μm that is caused by the spindle thermal displacement not being compensated can be reduced to a maximum cutting depth of the hole of 4.62 μm, and its average error value of cutting depth is 1.58 μm. Therefore, upon compensating the spindle thermal displacement model proposed in this study, the machining precision can effectively be improved.

Keywords High speed machining · Multiple regression analysis · Spindle thermal displacement

1 Introduction

In today's prosperous industrial development, with the multifarious design of products and reduction of production cycle, high speed machining technology has been widely adopted by manufacturers. Nevertheless, researchers also need to overcome problems that arise during the high speed machining. Since high speed machining speed represents the high speed rotation of spindle, the phenomenon of spindle thermal displacement occurs more frequently than that in traditional spindle. The most significant parameter affecting the spindle thermal displacement is the friction heat in the front and back bearings of the spindle. When the spindle rotates, heat occurs at the front and back bearings because of friction, and the heat is then transmitted to the spindle head and tool header etc. Thermal displacement consequently occurred at the spindle because of temperature increase.

The objective of this article is to build a new complex multivariable regression model for the thermal displacement of high speed spindle, so as to accurately predict the spindle thermal displacement under different rotation conditions. Furthermore, online compensation is proceeded, so as to improve the machining precision of the highspeed machining center while further modification on the design of machine is not required. Chen [1] and Yang [2] have made related studies on using the neural network to construct the thermal displacement model of the spindle, and the lead screw of the feed spindle caused by heat. Their results indicated that neural network is not only suitable for the prediction of thermal displacement of non-linear changes, but these results are also very accurate. Tseng [3] and Chen [4] etc. used multivariable regression analysis model to build the thermal displacement model of machine, so as to be referential when undergoing immediate online compensation. They used temperature as regression parameters, and upon the completion of practical compensation, such model can effectively improve the machining precision of the machine tool.

To high speed spindle, spindle shows different thermal displacement properties under different rotational speed. It is impossible to indicate its total thermal displacement property only by simple regression model. This article proposes an analytical method of complex multivariable regression model, so as to predict the high speed spindle thermal displacement in every time slot when the rotational speed of the spindle keeps changing. Besides, the most common regression analysis of spindle thermal displacement is made on the structural elements of the machine tool, when many temperature sensors are installed to build a thermal displacement model by means of the temperature of each sensor. Since production cost of such method is high and the installation is difficult, it is not commonly adopted by most machine tool manufacturers. This article uses spindle rotational speed and the thermal displacement

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of every time slot to be the parameters, so as to accurately predict the thermal displacement changes of spindle without installing sensors on the machine tool.

2 Multivariable regression analysis

In this section, this article first introduces multivariable regression analysis and its equation. Multivariable regression is the extension of simple regression by increasing one variable into many, so as to study the relationship between a group of variables with a single reaction variable Y. The main purposes of such research are [5]:

(1) Reduce the random error. This result is related to the analysis of regression variables. By further increasing the numbers of interpellation variables, researchers can increase the changeable parts that can be explained by regression, and thus lower the error variance.
(2) Avoid any missing to the important interpellation variable which has a greater influence to the reaction variable Y.

2.1 Multivariable linear regression and the least square method [5]

When the number of p independent variables are put in, the definition of the generalized multivariable regression model is as follows:

$$y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \ldots\ldots + \beta_p X_p + \varepsilon. \tag{1}$$

Randomly pick n sets of sample observation value (n>p), and its individual measurement value can be written into:

$$\begin{aligned} y_i &= \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \ldots\ldots + \beta_p X_{ip} + \varepsilon_i \\ i &= 1, 2, \ldots, n \end{aligned} \tag{2}$$

To any sets of random $\left(X_{i1}, X_{i2}, \ldots, X_{ip}\right)$, the corresponding error of reaction variable Y_i is ε_i, and this set of n measurement value can be indicated by matrix form.

Use matrix form to write the general linear regression model as following equation:

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} = \begin{bmatrix} 1 & X_{11} & X_{12} \cdots X_{1p} \\ 1 & X_{21} & X_{22} \cdots X_{2p} \\ \vdots & \vdots & \vdots \quad \vdots \\ 1 & X_{n1} & X_{n2} \cdots X_{np} \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_p \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{bmatrix} \tag{3}$$

In short:

$$Y = X\beta + \varepsilon. \tag{4}$$

If matrix β is (p+1)×1 vector and it is assumed that a inverse matrix exists for matrix $X'X$, then we can acquired the least square normal equation matrix indication of multivariable linear regression model as follows:

$$(X'X)\beta = X'Y. \tag{5}$$

So the estimate matrix of the least square method is indicated as:

$$\beta = (X'X)^{-1} X'Y. \tag{6}$$

3 The establishment of the high speed spindle thermal displacement model

3.1 The establishment of the high speed spindle multivariable regression thermal displacement model

The method adopted in multivariable regression analysis is to use multivariable regression to mark the sampling points and thermal displacement to undergo statistical analysis. Most studies are implemented by experimental design to sample the temperature at the specific points of the machine tool and correspond to thermal displacement, so as to build the mathematical relationship of temperature and thermal displacement between specific points. This article observes the thermal displacement condition of the high speed spindle, and discovers that the spindle thermal displacement would change according to the rotational speed and its thermal displacement change of its rotational processes of the previous few times. Therefore, this article proposes to use spindle rotational speed and thermal displacement value of every time slot to be the parameters of multivariable regression model, so as to predict the thermal displacement of every time slot during the spindle rotational process. Since the heating value of the spindle bearing is in positive proportion to spindle temperature, and so as the spindle temperature to spindle thermal displacement value. Thus, spindle thermal displacement valueis in positive proportion to spindle bearing heating value. And spindle bearing heating value and spindle rotational speed is in square relationship with each other, as a result, it is known that the spindle thermal displacement value is in square relationship with the spindle rotational speed. Therefore, in this regression model, this article uses $N^2_{(t-j)}$ and $\Delta z_{(t\text{-}j)}$ to be its regression parameters, and its regression model is as follows:

$$\Delta z_{(t)} = a_0 + \sum_{i=1}^{n} a_i \Delta z_{(t-i)} + \sum_{i=0}^{m} b_i N^2_{(t-i)} \tag{7}$$

i	Unit time slot (in this article, one unit time slot is 5 min).
$\Delta z_{(t\text{-}j)}$	The spindle thermal displacement value when it is in the time slot of (t-i)
$N^2_{(t-j)}$	The square of spindle rotational speed when it is in the time slot of (t-i)
$a_0, a_1 \ldots a_n, b_0, b_1 \ldots .b_m$	Regression coefficient
n,m	The parameters of n sets of $\Delta z_{(t\text{-}i)}$ and m sets of $N^2_{(t-i)}$ that can affect the target value $\Delta z_{(t)}$.

In Eq. (7), if the values of n and m are too small, then the situation of missing would happen to the important influence parameters ($\Delta z_{(t\text{-}i)}$ and $N^2_{(t-i)}$), which have imposed greater influence to target values ($\Delta z_{(t)}$) and cannot accurately predict the thermal displacement of spindle. However, when the values of n and m are too great, then too many influence parameters ($\Delta z_{(t\text{-}i)}$ and $N^2_{(t-i)}$) would lead to too much meaningless calculation. This experiment first discusses the n and m that are most suitable for the spindle thermal displacement property of the machine being used, so as to accurately and rapidly predict the thermal displacement of its spindle. Table 1 shows the calculated regression coefficient ofdifferent n and m values, and the percentage of its coefficient vs. its total coefficient. Among which, a_0 is a constant, so it is not considered as a weight value. However, a_i is the function of $\Delta z_{(t\text{-}i)}$ and b_i is the function of spindle rotational speed $N^2_{(t-i)}$, so this article separately calculates its regression coefficient weight. If its weight percentage is larger than 5%, then that variable of a_i or b_i needs to be calculated. If it is less than 5%, then it is neglected. From this table, the following conditions are acquired:

1. When n=4 and m=4, in which the influence of a_4 and b_4 towards it are 3% and 3.08% respectively, which are both less than 5%, then they impose a minimal influence towards its thermal displacement. And when n=4 and m=3 or n=3 and m=4, when the parameters of a_4 or b_4 etc. are calculated, more calculation time are needed, whereas their influences are all less than 5%. Therefore, this article neglects to calculate a_4 and b_4.

From the above description and Table 1, it is founded that n=3 and m=3 is the most suitable regression coefficient order of spindle thermal displacement multivariable regession model to be induced by this article, because it can

Table 1 The multivariable regression coefficient value at different n and m

Value of a_i and b_i / Value of n and m, and weight Percentage	a_0	a_1	a_2	a_3	a_4	b_0	b_1	b_2	b_3	b_4
n=2,m=2	3.2864	1.0957	-0.3306	#	#	0.1345	-0.0666	0.0426	#	#
Percentage (%)	#	76.8	23.2	#	#	55.2	27.3	17.5	#	#
n=3,m=3	4.0136	1.1208	-0.6768	0.385	#	0.1286	-0.0707	0.0541	0.0316	#
Percentage (%)	#	51.4	31.0	17.6	#	45.1	24.8	19.0	11.1	#
n=4,m=4	2.2827	1.2516	-0.8881	0.6259	-0.0882	0.2670	-0.1664	0.1684	-0.0840	-0.0218
Percentage (%)	#	43.9	31.1	21.9	3.0	37.7	23.5	23.8	11.9	3.08
n=2,m=3	3.6750	1.1271	-0.4117	#	#	0.2604	-0.1447	0.0554	-0.0352	#
Percentage (%)	#	73.2	26.8	#	#	52.53	29.2	11.2	7.1	#
n=3,m=2	3.2927	1.2546	-0.5484	0.2778	#	0.2483	-0.1736	0.0632	#	#
Percentage (%)	#	60.3	26.4	13.4	#	51.2	35.8	13.0	#	#
n=4,m=3	2.4039	1.3242	-0.9843	0.7530	-0.1836	0.2638	-0.1852	0.1806	-0.1170	#
Percentage (%)	#	41.2	30.6	26.4	4.8	35.3	24.8	24.2	15.67	#
n=3,m=4	3.3277	1.2036	-0.8027	0.4895	#	0.2668	-0.1542	0.1553	-0.0555	-0.0325
Percentage (%)	#	48.2	32.2	19.6	#	40.2	23.2	23.4	8.4	4.89

rapidly and accurately predict the spindle thermal displacement value.

Upon setting n=3 and m=3, rewrite Eq. (7) as the following equation:

$$\Delta z_{(t)} = a_0 + a_1\Delta z_{(t-1)} + a_2\Delta z_{(t-2)} + a_3\Delta z_{(t-3)} + b_0 N^2_{(t)} + b_1 N^2_{(t-1)} + b_2 N^2_{(t-2)} + b_3 N^2_{(t-3)} \tag{8}$$

If we set the constant of the above Eq. (8) that $b_0 = a_4, \quad b_1 = a_5, \quad b_2 = a_6, \quad b_3 = a_7$ and rewrite it into the following equation:

$$\Delta z_{(t)} = a_0 + a_1\Delta z_{(t-1)} + a_2\Delta z_{(t-2)} + a_3\Delta z_{(t-3)} + a_4 N^2_{(t)} + a_5 N^2_{(t-1)} + a_6 N^2_{(t-2)} + a_7 N^2_{(t-3)} \tag{9}$$

Besides, if the parameters of Eq. (9) are assumed to be $\Delta z_{(t)} = Y$ and $\Delta z_{(t-1)} = X_1$, $\Delta z_{(t-2)} = X_2$ and $\Delta z_{(t-3)} = X_3$, $N^2_{(t)} = X_4, N^2_{(t-1)} = X_5, N^2_{(t-2)} = X_6$ and $N^2_{(t-3)} = X_7$,then it can be indicated as multivariable regression normal equation. Thereforethe least square matrix of the above multivariable regression analysis can beused to solve the regression coefficient as indicated in Eq. (6), so as to acquire the spindle multivariable regression thermal displacement model upon the calculation of regression coefficient.

3.2 The establishment of the spindle complex multivariable regression model of high speed spindle thermal displacement

When high speed machining is used, since the spindle rotational speed and the feed speed are increased, the material removal rate is greatly improved, and consequently reduces the time required for machining. Besides, the practical machining time of every tool during the machining process has also been reduced. Generally, for machining of every several ten minutes, the next cutting tool has to be replaced to proceed with the next machining path. As the material, size, type and machining path ofevery cutting tool is different, and also its corresponding machining conditions, such as feed speed and spindle rotation speed etc. are not exactly the same. It means that the spindle needs to change its rotational speed for every several ten minutes. Under this circumstance, spindle has to rotate under different rotational speed all the time, affecting that the above mentioned spindle multivariable regression model of Eq. (9) cannot predict the spindle thermal displacement value during the practical machining process. Equation (9) is only suitable for the prediction of thermal displacement under a fixed spindle rotational speed.

Here, this article proposes another complex multivariable regression model analytical method to predict the high speed spindle thermal displacement value when the spindle rotational speed keeps on changing. The theory of this method is tocombine different regression models that are built in this article together, and based on the spindle rotational speed of every time slot to select their corresponding regression coefficient. The building procedures and calculation method of this complex multivariable regression model are described as the following by means of the multivariable regression model of Eq. (7). From Eq. (7), it is known that the coefficient a_i of the multivariable regression model used in this article is the function of $\Delta z_{(t\text{-}1)}$, whereas b_i is the function of spindle rotational speed $N^2_{(t-i)}$ in different time slot. Under the situation that the rotation speed of spindle keeps on changing, a_i are b_i are uncertain values, but ever-changing along with the spindle rotational speed under different rotational time slot. Therefore, a_i should be indicated as $a_{i(t)}$, and b_i indicated as $b_{i(t\text{-}i)}$, so as to indicate the thermal displacement properties of spindle under the rotational speed change. As a result, substitute $a_{i(t)}$ and $b_{i(t\text{-}i)}$ into Eq. (7), and its complex multivariable regression model can be written into the following equation:

$$\Delta z_{(t)} = a_0(t) + \sum_{i=1}^{n} a_i(t)\Delta z_{(t-i)} + \sum_{i=0}^{m} b_{i_{(t-i)}} N^2_{(t-i)} \tag{10}$$

i	Unit timeslot
$\Delta z_{(t\text{-}i)}$	The spindle thermal displacement value when it is in the time slot of (t-i)
$N^2_{(t-i)}$	The square of spindle rotational speed when it is in the time slot of (t-i)
$a_{0(t)}, a_{1(t)} \ldots a_{n(t)}$	Regression coefficient related to the parameters of spindle thermal displacement value in the time slot of (t)
$b_{0(t-i)}, b_{1(t-i)} \ldots . b_{m(t-i)}$	The regression coefficient related to the parameters of spindle rotational speed in the time slot of (t-i).
n,m	Take the no. of n $\Delta z_{(t\text{-}i)}$ and no. of $mN^2_{(t-i)}$ parameters affecting the target value of $\Delta z_{(t)}$.

As in the above section, if it set n=3 and m=3, and rewrite the equation as following:

$$\Delta z_{(t)} = a_{0_{(t)}} + a_{1_{(t)}}\Delta z_{(t-1)} + a_{2_{(t)}}\Delta z_{(t-2)} + a_{3_{(t)}}\Delta z_{(t-3)} + b_{0_{(t)}} N^2_{(t)} + b_{1_{(t-1)}} N^2_{(t-1)} + b_{2_{(t-2)}} N^2_{(t-2)} + b_{3_{(t-3)}} N^2_{(t-3)} \tag{11}$$

set the above parameters as $b_{0(t)}=a_{4(t)}$, $b_{1(t-1)}=a_{5(t-1)}$, $b_{2(t-2)}$, and $b_{3(t-3)}=a_{7(t-3)}$, and rewrite these parameters into the following equation:

$$\Delta z_{(t)} = a_{0_{(t)}} + a_{1_{(t)}}\Delta z_{(t-1)} + a_{2_{(t)}}\Delta z_{(t-2)} + a_{3_{(t)}}\Delta z_{(t-3)} + a_{4_{(t)}}N^2_{(t)} + a_{5_{(t-1)}}N^2_{(t-1)} + a_{6_{(t-2)}}N^2_{(t-2)} + a_{7_{(t-3)}}N^2_{(t-3)} \quad (12)$$

The above Eq. (12) is the complex multivariable regression model of the thermal displacement of the highspeed spindle that is induced by this article, which is an extension of multivariable regression model. The prediction procedure is to substitute the regression coefficient of the multivariable regression model under three sets of different rotational speed that are solved by the above section into Eq. (12), and acquire the spindle thermal displacement value of every time slot when the rotational speed of the spindle keeps on changing.

4 The experiment and result analysis

4.1 Experimental equipment

(1) Highspeed machining center
The high speed machining center used in this experiment is produced by Taiwan Takumi Machinery Co., LTD, its model number is TAKUMI V10 and its specification is indicated in Table 2.

(2) Digital Indicator
The meter used in this experiment is the digital indicator produced by Japan Mitutoyo Company. Its measurement range is 50 mm and its measurement precision can reach 1 μm.

(3) High speed machining tool holder
The toolholder used in this experiment is high speed professional tool holder made in Taiwan. It has passed the dynamic balance test, with balance quality grade reaching G2.5, and rotation speed 15,000 rpm.

(4) High precision rod
This experiment uses a precision rod which the rod surface has been grinded smoothly and this rod can be used to contact directly with the digital indicator to measure the thermal displacement of the spindle.

(5) High speed machining tool
This experiment uses an end milling cutter specifically for high speed machining with diameter of ϕ6 mm. It is used as a machining tool for implement practical machining compensation experiment.

(6) Coordinate measurement machine (CMM)
This experiment uses manual CMM produced by Mitutoyo. Its measurement precision can reach 1 μm and is used for measuring the residual error upon the practical machining compensation.

(7) Aluminum alloy material
This experiment uses 300*300*50 aluminum alloy material to be the machining material for practical machining compensation experiment. The overall experimental structure is indicated in Fig. 1.

Table 2 Specification of high speed machining center [6]

	Item	Unit/ Type	Specification
Working Table	Area of the working table	mm	1060*510
	Maximum load of the working table	kg	600
Schedule	Schedule of the X axis	mm	1000
	Schedule of the Y axis	mm	510
	Schedule of the Z axis	mm	560
Spindle	Spindle taper	BT	40
	Spindle rotational speed	rpm	12000
Speed	Machining speed	mm/min	7000
	Fast-moving speed	m/min	20*20*14
Motor	Spindle Motor	HP	10
	X/Y/Z axis motor	HP	12/12/12

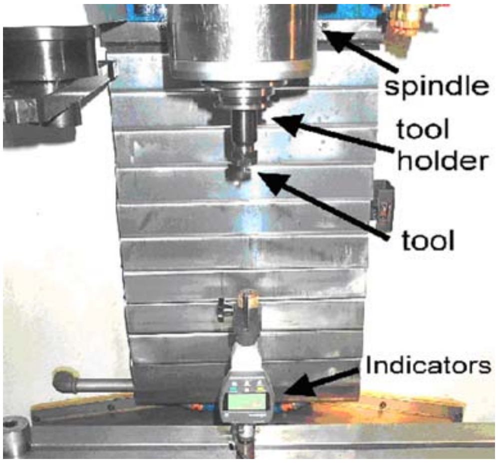

Fig. 1 Experimental allocation

4.2 Experimental steps

4.2.1 Experimental steps for the measurement of thermal displacement

The experiment of this article uses a grinded precision rod that is installed on the tool header of the spindle. It is used as a tool for measuring the spindle thermal displacement. Consequently, after the spindle has been rotated for a period of time, upon receiving the heat transmitted from the spindle, this precision rod would cause thermal displacement of thermal expansion. This paper assumes that the thermal displacement of the rod is equivalent to the thermal displacement of the tool caused by the rotation of spindle when the tool is clipped on the tool holder of the spindle. With the contact of indicator with the rod at this point, the value of thermal displacement of the spindle can be acquired.

The experimental steps taken are as follows:

1. As shown in Fig. 1, installed the measurement tool, tool holder and rod used for the experiment on the machine tool.
2. Key in the completed NC equation into the controller.
3. Execute the main program.
4. Spindle rotates at specified rotational speed. The spindle thermal displacement value is measured at a rotation of every 5 minutes. Record the respective thermal displacement value.
5. Upon the measurement of every spindle thermal displacement, repeat step 4 until all specified rotational time has been measured.
6. After reaching the specified rotational time, the spindle stops rotating. Measure its thermal displacement for every 5 minutes as before, and record the spindle thermal displacement value after it stops rotating.
7. Based on different rotational speed, repeat the above steps and record the spindle thermal displacement value under different rotational speed and timeslot.

4.2.2 Experimental steps for the practical spindle thermal displacement error compensation verification

In order to verify that the thermal displacement value predicted by the complex multivariable regression model

Table 3 The thermal displacement volume of every time slot when the spindle rotates at different rotational speed

Experiment 1	4000RPM																	
Experiment 2	7000RPM																	
Experiment 3	10000RPM																	
Rotational Time (Min)	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90
Experiment 1																		
Thermal Displacement (μm)	10	16	18	21	24	25	26	28	31	32	32	35	36	36	36	36	36	36
Experiment 2																		
Thermal Displacement (μm)	13	21	28	33	36	40	47	52	55	56	59	61	61	61	61	61	61	61
Experiment 3																		
Thermal Displacement (μm)	11	18	30	40	47	49	51	54	55	57	57	58	59	59	59	61	61	61

induced in this article can improve the machining precision of machine tool after experimental compensation, a verification experiment concerning the practical machining thermal displacement error compensation of the spindle has been executed, and the steps are as follows:

1. Install the tool, tool holder and material used for the compensation verification on the machine tool.
2. Write a machining equation of verification experiment, and subtract the predicted spindle thermal displacement value of every time slot from the depth of the practical machining of that time slot, so as to undergo compensation.
3. Key in the completed NC equation into the controller.
4. Execute that machining compensation equation to complete the machining of work piece material.
5. Use CMM to inspect the cutting depth value of the machining material after compensation.

4.3 Experiment result

In order to acquire the regression model of spindle thermal displacement, this experiment takes every 5 minutes as a time unit, and measure the spindle thermal displacement value once for every 5 minutes. Every set of rotational speed would rotate for 90 minutes respectively, and the spindle thermal displacement values of 3 sets of fixed rotational speed are measured. Its experimental result is indicated in Table 3.

In order to acquire an accurate spindle thermal displacement multivariable regression model, not only the properties of spindle thermal displacement during its rotation have to be considered, but also the spindle thermal displacement value during the cooling process has to be measured after it stops rotating at different rotational speed. When using the complex multivariable regression model, take the results into multivariable regression model, so that the properties of spindle thermal displacement can be accurately predicted when the spindle rotational speed keeps changing. The following Table 4 continuously measure its thermal displacement value of the above 3 sets of spindle thermal displacement experiments after the spindle stops rotating. The time units are all set at 5 minutes.

From the experiment result of Table 3, arrange the spindle thermal displacement value at every rotational speed as Fig. 2, so as to observe the properties of spindle thermal displacement. Among which, the spindle thermal displacement reaches its balance after spindle rotating for about 60 minutes, when it would keep on rotating without affecting its thermal displacement value. The spindle thermal displacement value at 4000 rpm of spindle rotating speed rises comparatively slowly, and its maximum thermal displacement value is 36 μm. When the spindle rotating speed is at 7000 rpm, the thermal displacement changes drastically, and its maximum thermal displacement value is 61 μm. The most drastic increase occurs for the thermal displacement value is at 10000 rpm of spindle rotating speed, and its maximum thermal displacement value is equal to that of 7000 rpm: 61 μm. This indicates that when the spindle has reached a certain rotating speed, and after its rotation has reached the rotational time of a steady maximum thermal displacement, its thermal displacement value would remain the same.

Table 4 The thermal displacement value of every time slot after the spindle stops rotating at different rotational speed

Experiment 1	4000RPM
Experiment 2	7000RPM
Experiment 3	10000RPM

Rotational Time (min)	0	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75
Experiment 1																
Thermal Displacement (μm)	36	33	30	26	24	20	18	16	14	12	11	9	8	6	5	4
Experiment 2																
Thermal Displacement (μm)	61	58	55	52	49	43	38	33	32	31	28	26	25	23	20	18
Experiment 3																
Thermal Displacement (μm)	61	59	57	53	49	46	43	39	36	33	31	28	26	25	23	20

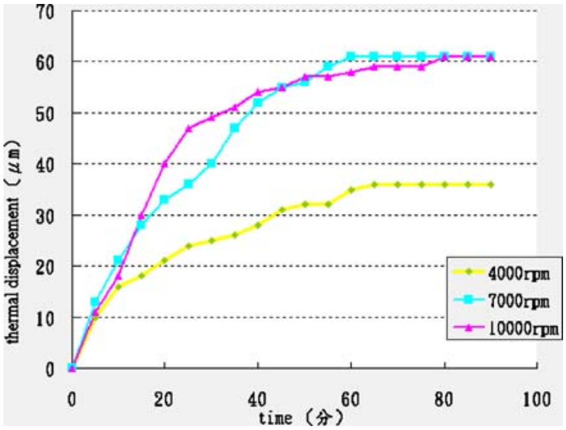

Fig. 2 The spindle thermal displacement value at all kinds of spindle fixed rotational speed

4.4 Experiment result analysis and verification of the thermal displacement prediction result

From the experiment result indicated in Fig. 2, it is found that when the high speed spindle works at different rotational speed, it has different thermal displacement properties. Based on a specific multivariable regression model, the thermal displacement value under all spindle rotational speeds can never be predicted accurately. Therefore, this article builds three multivariable regression thermal displacement models of different rotational ranges based onspindle rotational speed, and then respectively substitutes the experiment data into Eq. (12). Then, using the least square matrix equation of multivariable regression analysis introduced in this article to calculate the regression coefficient, and the results of a_0, $a_1 \ldots a_7$ are indicated in Table 5.

From the above regression coefficient, based on the experimental spindle rotation condition, the thermal displacement value of every time slot underfixed spindle rotation speed can be acquired. Moreover, the researcher can use the complex multivariable regression analytical method proposed in this article to predict the thermal displacement value of every time slot when the spindle is undergoing high speed machining with an ever-changing rotational speed.

In order to verify that the complex multivariable regression analytical method proposed in this article can accurately predict the thermal displacement value of every time slot when the spindle is undergoing high speed machining with an ever-changing rotational speed, this article undergoes one measurement and verification experiment of spindle thermal displacement value under an ever-changing rotational speed, and one verification experiment of practical machining compensation. In the measurement and verification experiment of spindle thermal displacement value, based on the experimental steps, the thermal displacement value of every time slot is measured when the spindle rotational speed keeps on changing. The results of the measurement and verification experiment is shown in Fig. 3. In the experiment of practical machining compensation, the rotational condition of the compensation verification experiment is taken to be the practical machining condition, and its regression predicted value is taken as the basis of compensation, so as to plan for the practical machining experiment. The results of the compensation experiment are shown in Fig. 4. Its experiment steps are indicated as in Section 4.2.2, so as to undergo compensation of the predicted thermal displacement value of every time unit in times of spindle machining process.

The assumption of this paper will not consider the influence of toolwear, tool vibration and run-out towards the precision of the cutting depth. From Fig. 3, it is known that the definition of residual error in this paper is: spindle thermal displacement value measured through a precision rod and a digital indicator. The difference between this value with the regression predicted spindle thermal displacement value that is acquired by the regression formula proposed in this paper will then be the residual error. Therefore, residual error is the calculated error caused by the influence of spindle thermal displacement. From the result obtained by comparing the result of compensation verification experiment and practical compensation result shown in Fig. 4, the error value of the cutting depth value of the tool practical machining after the compensated with the ideal cutting value planned by the experiment is very close to the order value of residual error in Fig. 3. Therefore, this paper can consider to neglect the influence of tool wear, tool vibration and run-out, as all the assumptions are within the experimental

Table 5 Result of the multivariable regression analysis at different spindle rotational speed

Regression Coefficient / Spindle Rotational Speed	a_0	a_1	a_2	a_3	a_4	a_5	a_6	a_7
2500~5500	1.3708	0.8925	-0.3778	0.2784	0.3809	-0.0813	0.09	0.0253
5500~8500	4.0136	1.1208	-0.4768	0.185	0.1286	-0.0707	0.0541	0.0316
8500~11500	5.1428	1.102	-0.2604	-0.0152	0.0453	-0.0334	0.0486	-0.0049

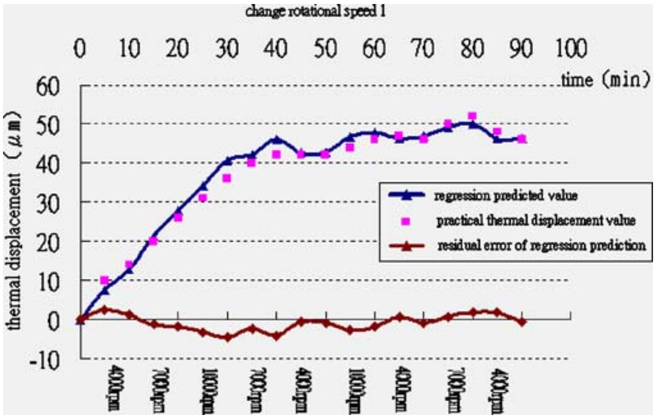

Fig. 3 Comparison of the result of the measurement and spindle thermal displacement verification experiment

scope. Furthermore, it is proven that the residual error value can only consider the error between the regression predicted spindle thermal displacement value and practical spindle thermal displacement value.

Besides, in order to ensure that the compensation experiment can easily measure the residual error after compensation, for every spindle rotation of 9 minutes, this machining compensation experiment would undergo approximately 1 minute of a hole machining by a end milling cutter with a diameter of ϕ6 mm. In other words, it operates a hole machining in every 10 minutes, and the diameter and depth ofthe hole are 10 mm respectively. The total machining time is 90 minutes, and 9 holes machining are completed. Furthermore, in order to prevent the wear of tool from affecting the result, the machining material used in this machining compensation is aluminum alloy, which is easier for machining. After the completion of machining, CMM is used to measure the cutting depth by compensation of every hole. Then the residual error value after compensation is calculated. The verification result is shown in Table 6. Figures 3 and 4 have regression

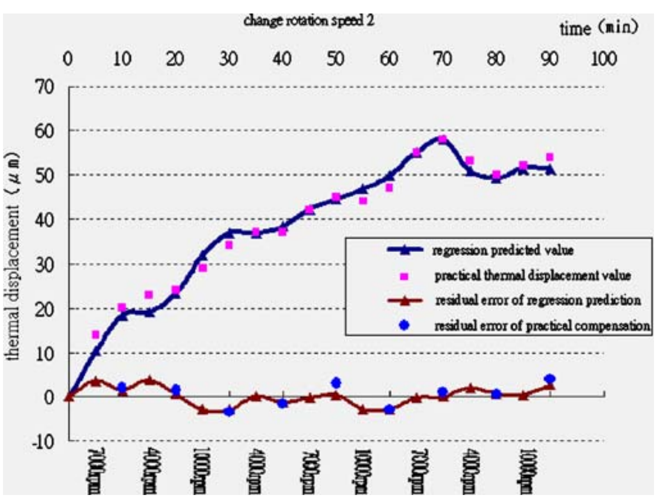

Fig. 4 Comparison of the result of the compensation verification experiment

predicted values and the comparison of practical thermal displacement value. From this prediction and verification result, it is found that the maximum difference between the prediction result of spindle thermal displacement value and practical thermal displacement value is 4.62 μm, and the average value is 1.58 μm. The maximum error of predicted thermal displacement value is 19% of the practical thermal displacement value, and the average error of predicted thermal value is 5.4% of the practical thermal displacement value. Here, it is known that the difference between the practical spindle thermal displacement value and the spindle thermal displacement value calculated by this complex multivariable regression model is less than 5 μm. From Table 6 and Fig. 4, it shows that the residual error value is less than 5 μm after compensation verification experiment. Therefore, the spindle thermal displacement value predicted by the complex multivariable regression model used in this article can act accurately as a basis of its thermal displacement compensation, and also suitable to be applied for measuring all kinds of rotational speed. This can improve the machining precision of the machine tool. Further explanation to the spindle thermal displacement properties indicated in the below Fig. 4, when the spindle reaches the highest point of thermal displacement value when it has rotated for 70 minutes, the thermal displacement value would drop when it keeps on rotating. This is because between 40 to 70 minutes, the spindle rotates at a high rotational speed of 7000 to 10000 rpm. Therefore, it reaches the maximum value of 57.9 μm at 70 minutes. The spindle rotates at a low speed of 4000 rpm between 70 to 80 minutes, because the maximum thermal displacement value of 4000 rpm that keeps on rotating continuously and steadily at 36 μm. Therefore, the spindle thermal displacement value rapidly falls at that time slot. At the last 10 minutes' rotation, although the spindle rotational speed is 10000 rpm, since the former two units of time slot is 4000 rpm, the thermal displacement would slowly rise at the final stage, but not increase considerably.

Since the spindle thermal displacement is mainly the spindle thermal expansion value in the spindle vertical direction, hence this paper only considers a single direction. Therefore, in machining a 3D surface milling, the direction of the spindle thermal expansion that is vertical to the spindle is only available. At this point, the value of NC code can undergo compensation adjustment based on the thermal expansion value of the spindle vertical direction.

5 Conclusion

From the building and analysis of the above high speed spindle thermal displacement model, the following conclusion can be acquired:

1. Different thermal displacement properties can be acquired when the spindle rotates at different rotational speed. The complex multivariable regression displacement model induced in this article includes the

Table 6 Verification result of the spindle rotational speed changes

Rotational Time (min)	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90
Measurement and Verification Experiment	4000RPM	4000RPM	7000RPM	7000RPM	10000RPM	10000RPM	7000RPM	7000RPM	4000RPM	4000RPM	10000RPM	10000RPM	4000RPM	4000RPM	7000RPM	7000RPM	4000RPM	4000RPM
Regression Predicted Value (μm)	8.1	12.8	21.3	27.8	34.2	40.6	42.2	46.1	42.6	42.8	46.7	47.8	46.4	46.8	49.2	50.1	46.2	46.4
Practical Thermal Displacement Volume (μm)	10	14	20	26	31	36	40	42	42	42	44	46	47	46	50	52	48	46
Predicted Error Rate (%)	19.0	8.6	6.5	6.9	10.3	12.8	5.5	9.8	1.4	1.9	6.1	3.9	1.3	1.7	1.6	3.7	3.8	0.9
Compensation Verification Experiment	7000RPM	7000RPM	4000RPM	4000RPM	10000RPM	10000RPM	4000RPM	4000RPM	7000RPM	7000RPM	10000RPM	10000RPM	7000RPM	7000RPM	4000RPM	4000RPM	10000RPM	10000RPM
regression Predicted Value (μm)	10.3	18.4	19.2	23.4	31.9	36.9	36.8	38.3	42.2	44.5	46.8	49.8	55.2	57.9	50.9	49.3	51.5	51.3
Practical Thermal Displacement Value (μm)	12	20	23	24	29	34	37	37	42	45	44	47	55	58	53	50	52	54
Predicted Error Rate (%)	14.2	8.0	16.5	2.5	10.0	8.5	0.5	3.5	0.5	1.1	6.4	6.0	0.4	0.2	4.0	1.4	1.0	5.0
Compensation Experiment results	7000RPM		4000RPM		10000RPM		4000RPM		7000RPM		10000RPM		7000RPM		4000RPM		10000RPM	
Residual Error (μm)	-2.0		-1.5		3.5		1.5		-3.0		3.0		-1.0		-0.5		-4.0	

multivariable regression models acquired at all kinds of rotational speeds. It can accurately predict the spindle thermal displacement condition at all kinds of fixed rotational speeds, and more, it can accurately predict the spindle thermal displacement value of every time slot during the practical high speed machining process when the spindle rotational speed keeps changing. After the compensation of its spindle thermal displacement model, the maximum value of spindle thermal displacement drops from 62 μm to 4.62 μm, and the average error value is 1.58 μm. It can effectively improve the machining precision of the high speed machine tool.

2. Since the spindle thermal displacement properties of the machine tool is very complicated, and the thermal displacement properties of every machine tool are not exactly the same, although this article has induced a complex spindle thermal displacement model that can accurately predict the thermal displacement of the spindle, the established model might not be exactly applicable to all types of machine tools. When building the spindle thermal displacement model of machine tool, it is necessary to look into different properties of every machine tool, and adjust its spindle thermal displacement model accordingly, so as to accurately predict the spindle thermal displacement value of every machine tool.
3. The spindle thermal displacement multivariable regression models used by most spindles take the temperature of the spindle as their regression parameter. It is difficult to decide where to install the temperature sensors, and the time spent on model

establishment is very lengthy. This article uses spindle rotational speed and spindle thermal displacement value as regression parameters. By doing so, not only the cost of temperature sensor installation can be cut, but the model can also be built speedily and accurately.

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