## ORIGINAL ARTICLE

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# Maintenance scheduling to support the operation of manufacturing and production assets

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Abstract Manufacturing and production plants operate physical assets that deteriorate with usage and time, thus, maintenance actions are required to restore the assets back to their original predetermined operational conditions. In this paper, we extend previous work on maintenance scheduling to considering a multi-component system that optimises both cost and reliability simultaneously. The model uses the concept of imperfect maintenance and includes factors such as ageing due to the operation rate of the system, downtime for maintenance and lead time for spare parts. For each maintenance planning period, the model predicts which of the three possible actions (namely, maintaining the component, replacing the component or doing nothing) for each component should be taken, such that the reliability is at least at a required level and the net present cost for the entire planning period is minimised. The entire approach is illustrated through the use of a numerical example and the evaluation of the model is done by using an evolutionary algorithm.

Keywords Maintenance scheduling  $\cdot$  System reliability  $\cdot$  Management of multi-component asset  $\cdot$  Downtime for maintenance  $\cdot$  Operation rate

## **1** Introduction

Manufacturing and production plants operate physical assets that deteriorate with usage and time, thus, maintenance is required to restore the assets' original operational condition. Since unexpected breakdown outages normally incur a high penalty cost, maintenance is required to reduce the overall probability of such outages. This type of maintenance is called preventive maintenance (PM). Generally, PM includes actions such as inspection, cleaning, lubrication, alignment and adjustment and/or replacement. It is undesirable to have unexpected breakdown outages but, on the other hand, maintenance incurs a cost. It is left to the asset managers to define a maintenance policy such that the expected total cost of system failure and maintenance cost will be minimised. There are numerous models and methodologies developed to determine effective maintenance schedules and some works include that by Price [1] on the economics of tube thickness survey and Noori and Price [2] on inspection effectiveness.

Chaudhuri and Sahu [3] were some of the pioneers who introduced the concept of imperfect maintenance. The concept of imperfect maintenance considers that maintenance will, effectively, restore the system being maintained to a condition somewhere between "as good as new" to the age of the system before maintenance. Jayabalan and Chaudhuri [4] considered that maintenance on a system at a time t will effectively reduce the age of a system to a time  $t/\gamma$  ( $\gamma > 1$ ), which, as a result, reduces the system age. Jayabalan and Chaudhuri [4] calculated the total cost of both maintenance and replacement, and found that the optimal policy was that, when the cumulative maintenance cost exceeds the replacement cost of the system, replacement is carried out. Usher et al.'s work [5] was closely related to the work by Javabalan and Chaudhuri, which presented a straightforward model for determining a costoptimal maintenance and replacement schedule for a new system that deteriorates. However, they only considered a single-component system.

Wang [6] conducted a survey on various maintenance policies of deteriorating systems and stated that maintenance policies had become more and more general and complicated, which made implementation in practice very difficult. Wang's research pointed out that, for a multicomponent system, minimising cost did not necessarily imply maximising system reliability and achieving the best operating performance; an optimal maintenance policy needs to consider both cost and reliability simultaneously.

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Tsai et al. [7] developed a model for determining a periodical PM policy based on the availability of a multi-component system. Three maintenance actions, mechanical services, repair and replacement, were considered. Their research adopted the agree method (Rao [8]); however, the model has limited consideration in economics like breakdown outages cost, future worth and cost inflation of maintenance actions.

Recently, Cassady and Kutanoglu [9] developed a model that integrates maintenance planning and production scheduling; however, their approach only considered a single machine, in which the PM is assumed to be perfect, so that it can restore the system to "as good as new" condition. Their research suggested that "PM is a more comprehensive action than repair, perhaps corresponding to the replacement of several key components in the machine." This assumption of PM being perfect maintenance mixed up maintenance and replacement. The assumption of a single machine mixed up components and system, leading to randomising failure characteristics (Sherwin [10]).

The research work proposed in this paper is an extension to Usher et al.'s research [5] by taking into account a multicomponent case. The model utilises the concept of imperfect maintenance and the concept of inflation. For each maintenance planning period, the model plans which of the three possible actions, maintaining the component, replacing the component or doing nothing, for each component should be taken, such that the net prevention value of the asset life cycle is minimised. The definitions and notation used in this paper is given in Table 1.

Furthermore, a number of aspects which are critical in decision making but were not taken into account in previous research have been considered in the proposed work. These aspects include:

- 1. Asset deterioration subjected to utilisation rate of the system
- 2. Consideration of downtime for maintenance and lead time for spare parts

4. Maximisation of profit instead of minimisation of cost.

## 2 The model environment and derivation

A system with *i* components is considered, where each component failure characteristic is assumed to follow a twoparameter Weibull distribution. The life cycle is *T*, where it is divided into *j* planning periods. Profit is generated through operating the system. However, the availability of the system may not necessarily imply that the system is generating income, but, rather, it means that the system is available for operation. Most of the previous research considered that the systems age with increment of time (Usher et al. [5]; Jardine and Tsang [11]). This assumption is only valid in some assets, such as those that are in continuous demand, like the utility industry (power stations and water suppliers).

System availability means that the system is ready (or available) for operation, which does not necessarily mean that one needs to operate it. The proposed model considers a more general case where a system will age with operation, which depends on the operational tempo or the production requirement of the company. In this paper, it is defined as the operation rate, OR, which has to be smaller than the availability, A:

$$A = \frac{UpTime}{Total Time} \tag{1}$$

$$OR = \frac{OT}{Total \ Time(T)} \le A \tag{2}$$

At every planning period, three possible actions are planned so that the profit is maximised. Each of these actions, namely, doing nothing, replacement and maintenance, will

Т	Total time for the life cycle considered	$\overline{M}_i$	Maintenance cost for component <i>i</i> (\$/maintenance)
OT	Actual operation time in a given period	$F_{i, i}$	Failure cost per incident for component <i>i</i> at period <i>j</i>
DTC	Downtime cost (\$/unit time)	$R_{i, j}$	Replacement cost for component $i$ at period $j$
OTP	Operation time profit (\$/unit time)	$M_{i, j}$	Maintenance cost for component $i$ at period $j$
MOT	Maximum operation time per period	k	Discount factor (per period)
$TTM_i$	Time to maintenance	r	Inflation factor for replacement cost (per period)
$TTR_i$	Time to repair	т	Inflation factor for maintenance cost (per period)
$LT_i$	Lead time	$\alpha$	Maintenance improvement factor $(0 \le \alpha \le 1)$
v(t)	Failure rate (failure/period)	$C_{i, j}$	Total cost for component $i$ at period $j$
<i>t</i> <sub><i>i</i>, <i>j</i></sub>	Age of component at end of period j	<i>x</i> <sub><i>i</i>, <i>j</i></sub>	Binary number, 0 represents no maintenance and 1 represents maintenance for component $i$ at period $j$
n	Number of maintenance periods	Yi, j	Binary number, 0 represents no replacement and 1 represents replacement for component $i$ at period $j$
$\beta_i$	Weibull shape parameter for component <i>i</i>		
$\eta_i$	Weibull characteristic parameter for component i		
$\overline{R}_i$	Replacement cost for component <i>i</i> (\$/replacement)		

Table 1 Definitions and notation used

have an effect on the age of the component, which will affect the failure rate. These three actions will have different effects on the age of the individual component, i.e. affecting the failure rate as well as the reliability.

#### 2.1 Age of the components

The age of the component at period *j* is the age of the component at period j-1 plus the actual operation time  $(OT_j)$  at that period, which can be expressed as:

$$t_{i,j} = t_{i,j-1} + OT_j \tag{3}$$

If a replacement is performed, then the component is returned to an "as good as new" state, where the component will have an effective age of zero. Hence,  $t_{i,j}=0$ .

When maintenance is carried out at the end of the period, this maintenance activity effectively reduces the age of the system at the start of the next period. Implementing the imperfect maintenance concept, maintaining a component will return the component age to a state somewhere between replacing the component and doing nothing. If maintaining the component, the age of the component will be:

$$t_{i,j} = \alpha \times t_{i,j-1} \tag{4}$$

where  $\alpha$  is an "improvement factor" between 0 to 1. This factor effectively allows for a variable effect of maintenance on the aging of the component. When  $\alpha=0$ , the effect of maintenance is to return the component to an "as good as new" state. When  $\alpha=1$ , maintenance has no effect and the component remains in the sate of "as bad as old."

#### 2.2 Cost of preventive maintenance activities

There are four costs considered in this paper, namely, replacement cost, maintenance cost, maintenance down-time cost and the failure cost. At the end of period j, the total cost,  $C_j$ , associated with the PM activities will be made up of different combinations of the following four cost categories.

#### 2.2.1 Replacement cost

If the component is replaced at the end of period *j*, assuming that the initial cost of the component is  $\overline{R}_i$ , inflated at a rate of r% per period, then the incurred cost for this replacement action at period *j* will be:

$$R_{i,j} = \overline{R}_i (1+r)^j \tag{5}$$

#### 2.2.2 Maintenance cost

When a maintenance action is performed on the component in period j, assuming that the maintenance cost for component *i* is  $\overline{M}_i$ , inflated at a rate of m% per period, then the resulting maintenance cost for the component at the end of that period will be:

$$M_{i,j} = \overline{M}_i (1+m)^j \tag{6}$$

#### 2.2.3 Maintenance downtime cost

Most maintenance actions for a system require the system to be shutdown. In the case of a multi-component system, it is preferable to perform as much maintenance as possible at a predetermined shutdown period, such that the downtime for maintenance is minimised. At a given instance, if two components of a series system are being maintained, the downtime will be the longer time of the two. Thus, in this paper, the maintenance downtime cost (*MDTC*) will be:

$$MDTC_{j} = \max\left(x_{i, j}TTM_{i}, y_{i, j}TTR_{i}\right) \cdot DTC$$
(7)

where  $x_{i, j}$  and  $y_{i, j}$  are binary numbers. Taking an example, if, at period *j*, component 3 is replaced and component 4 is maintained, and component  $TTR_3 > TTM_4$ , then:

$$MDTC_j = \max(TTM_3, \ TTR_4) \cdot DTC$$
$$= TTR_3 \cdot DTC$$

## 2.2.4 Failure cost

The failure rate of the component, a conditional probability (Jardine and Tsang [11]), is also known as the hazard rate or the rate of occurrence of failure (*ROCOF*). At the vantage point t=0, it is impossible to know what the actual system failure will be. However, one can predict that, if a component carries a high *ROCOF* through a given period, this will add on the overall risk of the system experiencing a failure, hence, a high cost of failure. Accounting for this failure cost, the *ROCOF* is multiplied by the cost of failure, which is a summation of the replacement cost at that period plus the downtime cost. The downtime cost is defined as the amount of money spent, even when the asset is not operating.

The instantaneous failure rate is used in this paper to calculate the failure cost of system at a given period, which is given as:

$$\nu(t)_{i,j} = \frac{\beta_i}{\eta_i} \left(\frac{t_{i,j}}{\eta_i}\right)^{\beta_i - 1} \tag{8}$$

Then, the failure cost can be expressed as:

$$F_{i,j} = \left(R_{i,j} + \left(LT_i + TTR_i\right) \times DTC\right) \times v(t)_{i,j} \tag{9}$$

## 2.3 Total preventive maintenance cost

By combining all of the cost functions mentioned in the previous sections, the resulting total cost in a period is the sum of the maintenance, replacement, failure and maintenance downtime costs during that period, which is given as:

$$C_{j} = \left(\sum_{i} \left(x_{i,j}M_{i,j} + y_{i,j}R_{i,j}\right) \times \left(\left|x_{i,j} - y_{i,j}\right|\right) + F_{i,j}\right) + MDTC_{j}$$
(10)

#### 2.4 The profit function

As discussed in previous sections, the asset being available does not necessarily mean that it is being operated. In this paper, it is assumed that income is generated through operating the asset and the asset is considered to be ageing with operation, not with time. This profit-optimal maintenance scheduling model will be able to advise management on both the optimal maintenance schedule and the optimal operation rate. The total income generated by the system can be expressed as:

$$P_i = OR_i \times MOT \times OTP \tag{11}$$

Hence, the total net present value of profit for the entire period is given as:

$$NPV = \sum_{j} \left\{ OR_{j} \times MOT \times OTP - \left( \sum_{i} \left( x_{i, j}M_{i, j} + y_{i, j}R_{i, j} \right) \times \left( \left| x_{i, j} - y_{i, j} \right| \right) + F_{i, j} \right) - MDTC_{j} \right\}$$

$$\times (1 + k)^{-j}$$
(12)

This can be simplified as:

$$NPV = \sum_{j} \left( P_j - C_j \right) \times \left( 1 + k \right)^{-j}$$
(13)

### **3 Model optimisation**

This paper modelled a multi-component system with different failure and cost functions. However, simply optimising the cost/profit without taking system reliability into account will result in low reliability measures that are not accepted in industry. This is because different components in the system will have different maintenance costs, failure characteristics and reliability importance in the system, as suggested by Wang and Pham [12].

The objective optimisation is to obtain the best maintenance schedule and the relative operating rate with maximum profit subject to a minimum reliability requirement at any period *j*. Hence, the objective function is to:

- Maximise Eq. 13:

$$NPV = \sum_{j} (P_j - C_j) \times (1+k)^{-j}$$

subject to:

$$R(t)_{Sys, j} = \prod_{i} R(t)_{i, j} \ge R(t)_{REQ}$$
(14)

## 3.1 Size of the optimisation problem

Table 2 illustrates the exhaustive case for a two-component system with one period,  $3^{2(1)}=9$  possible cases (where 0 denotes no action and 1 denotes action taken at that period). For two components and considering only two periods, the possible combinations will be  $3^{2(2)}=81$ :

- M=0 and R=0 denotes "Do Nothing," equivalent to " $x_{i,j}=0, y_{i,j}=0$ "
- M=1 and R=0 denotes "Maintenance," equivalent to " $x_{i, j}=1, y_{i, j}=0$ "
- M = 0 and R=1 denotes "Replacement," equivalent to " $x_{i, j} = 0, y_{i, j} = 1$ "

Taking a closer look at the model proposed, the possible combination is  $3^{i \times n}$ , as there are 3 possible actions (doing

t	M1	R1	M2	R2	t	M1	R1	M2	R2	t	M1	R1	M2	R2
1	0	0	0	0	1	0	0	1	0	1	1	0	0	1
t	M1	R1	M2	R2	t	M1	R1	M2	R2	t	M1	R1	M2	R2
1	1	0	0	0	1	0	0	0	1	1	0	1	1	0
t	M1	R1	M2	R2	t	M1	R1	M2	R2	t	M1	R1	M2	R2
1	0	1	0	0	1	1	0	1	0	1	0	1	0	1

 Table 2 Exhaustive case for a two-component system

Table 3 Numerical example input parameters

	Component 1	Component 2	Component 3
	(C1)	(C2)	(C3)
Replacement cost (\$)	15,000	50,000	80,000
Maintenance cost (\$)	5,000	8,000	10,000
$\beta$	2	3	3.5
$\eta$	20,000	15,000	20,000
TTM (hours)	600	900	1,000
TTR (hours)	500	400	600
LT (hours)	240	360	480



Fig. 1 Reliability plot of optimisation case without reliability constraints

nothing, maintenance and replacement), *i* components and *n* planning periods.

Three methods for solving the problem were examined, namely, random search, genetic algorithm and the branchand-bound approach. Usher et al. [5] were aware of the size of the problem and proposed that, although the branch-andbound approach gives the optimal solution, it is only applicable for small-scale problems. For larger-scale problems, the evolutionary algorithm (genetic algorithm) is more suitable, even though it does not guarantee a global optimal solution (Robert and Shahabudeen [13]; Prins [14]; Goldberg [15]).



Fig. 2 Reliability plot of optimisation case with system reliability requirement of 80%

In this paper, the evolutionary algorithm is used to solve the optimisation problem for multi-component system. The algorithm operates by changing decision variables (doing nothing, maintenance or replacement) to minimise or maximise the objective functions (cost) while meeting the constraints (reliability).

## 4 Numerical example

An *i*-component system is considered, where each component is assumed to have a failure characteristic that follows a Weibull distribution. The failure cost, maintenance cost and replacement cost for each component are known. The inflation factors for maintenance and replacement are 3% and 5% per period, respectively. The maintenance improvement factor ( $\alpha$ =0.6) is assumed to be constant for all components in the system. The discount factor *k* is assumed to be 10% per period. This example considers *n*=30 periods (15 years with bi-annual maintenance). It is assumed that the initial purchasing cost of the system is \$200,000, the downtime cost (*DTC*) and the operation time profit (*OTP*) are \$1,000 per hour and \$1,500 per hour, respectively. The other input parameters are summarised in Table 3. If using the exhaustive method, there will be 3<sup>3(30)</sup> combinations.

Planning period	C1	C2	C3	Total	Planning period	C1	C2	C3	Total
0	0	0	0	-\$200,000	16	0	0	0	\$2,382,886
1	0	0	0	\$2,383,473	17	0	0	0	\$2,354,770
2	0	0	0	\$2,356,765	18	0	0	0	\$2,314,121
3	0	0	0	\$2,318,565	19	0	0	0	\$2,259,678
4	0	0	0	\$2,267,839	20	0	0	0	\$2,190,282
5	0	0	0	\$2,203,670	21	0	0	0	\$2,104,817
6	2	2	2	\$1,305,686	22	0	0	0	\$2,002,175
7	0	0	0	\$2,383,288	23	2	2	2	\$1,054,629
8	0	0	0	\$2,356,136	24	0	0	0	\$2,382,346
9	0	0	0	\$2,317,164	25	0	0	0	\$2,352,935
10	0	2	0	\$1,845,093	26	0	0	0	\$2,310,032
11	0	0	0	\$2,291,803	27	0	0	0	\$2,252,170
12	0	0	0	\$2,242,539	28	0	0	0	\$2,177,965
13	0	0	0	\$2,177,862	29	0	0	0	\$2,086,059
14	0	0	0	\$2,096,899	30	2	0	0	\$2,000,390
15	2	2	2	\$1,198,555				NPV	\$20,280,494

Table 4Optimisationwithout reliability constraint(0 represents "Do Nothing,"1 represents "Maintaining" thecomponent and 2 represents"Replacing" the component)

Table 5Optimisation resultswith minimum system reliabili-ty of 80% (0 represents "DoNothing," 1 represents "Main-taining" the component and2 represents "Replacing" thecomponent)

Planning period	C1	C2	C3	Total	Planning period	C1	C2	C3	Total
0	0	0	0	-\$200,000	16	2	0	0	\$1,640,064
1	0	0	0	\$2,383,473	17	0	2	0	\$1,804,250
2	0	0	0	\$2,356,765	18	1	0	2	\$1,288,440
3	0	2	0	\$1,894,184	19	0	0	0	\$2,365,092
4	2	0	0	\$1,705,538	20	0	0	0	\$2,329,349
5	0	0	0	\$2,333,208	21	2	2	0	\$1,556,705
6	0	0	0	\$2,278,585	22	0	0	0	\$2,357,721
7	0	2	0	\$1,797,330	23	0	0	0	\$2,312,714
8	2	0	2	\$1,356,736	24	0	0	0	\$2,251,602
9	0	0	0	\$2,373,624	25	2	2	0	\$1,423,815
10	0	0	0	\$2,339,547	26	0	0	2	\$1,201,172
11	2	2	2	\$1,252,001	27	0	0	0	\$2,360,100
12	0	0	0	\$2,383,086	28	1	0	0	\$1,437,028
13	0	2	0	\$1,876,721	29	0	0	0	\$2,298,415
14	0	0	0	\$2,346,914	30	0	0	0	\$2,232,828
15	0	0	0	\$2,309,649				NPV	\$19,064,252

Without taking reliability into consideration, the optimal schedule gives a total net present profit of \$2,028,494, see Table 4. However, the lowest resulting reliability is as low as 40%, refer to Fig 1. The results of this numerical example agree with the suggestion pointed out by Wang [6] that minimising cost rate may not imply maximising system reliability, as is sometimes the case when the cost is minimised such that the resulting reliability will be too low, which is not accepted in practice.

By considering reliability and profit simultaneously (i.e. solving the numerical example by using Eq. 14), the overall net present profit is reduced to \$19,064,252, where the system reliability is maintained at 80%, as shown in Fig 2. Table 5 provides a summary of the maintenance schedule.

It can be observed that replacement seems to be preferred in this numerical example. This is because the objective is to maximise profit, and since profit is driven by operating the system, it is required to minimise downtime. As replacement returns the component life to "as good as new," the component can be used for longer after a replacement when compared to maintenance and, therefore, minimises the overall downtime.

## **5** Conclusion

This paper expands previous work by presenting an approach for determining an optimal preventive maintenance (PM) schedule for a multi-component system. The model can be applied to schedule maintenance activities in support of the operation of manufacturing and production plants. Several critical aspects in decision making not addressed in previous researches are considered. They include:

- 1. Time step increment subjected to utilisation rate of the system
- 2. Considered downtime for maintenance and lead time for spare parts
- 3. Considered cost and reliability simultaneously

4. Maximising profit instead of minimising cost

This paper presented a modelling approach to assist decision making in maintenance and replacement scheduling for a manufacturing and production plant.

Much future work is required, especially in the case of estimating the maintenance improvement factor ( $\alpha$ ). Furthermore, future work should incorporate decisions such as purchasing a new system at certain stages, which may achieve a lower life-cycle cost. However, such decisions are only made at the senior management level. In order to enable them to make an informed decision, one must take into consideration managerial, strategic, tactical and operational problems. Further work is required to examine the possible ways to integrate these four dimensions in a mathematical manner.

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