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ORIGINAL ARTICLE

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Numerical computing method of noncircular gear tooth profiles generated by shaper cutters

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Abstract A simple and accurate numerical method was proposed for calculating the tooth profile of a noncircular gear. This method is directly based on the real gear shaping process, rather than deducing and solving complicated meshing equations used in the traditional method. The tooth profile is gradually obtained from the boundary produced by continuously plotting the cutter profile on the gear transverse plane. The key point of the method is picking up the graph boundaries. The relative position of the cutter profile on the gear transverse plane is determined by the given pitch line of the noncircular gear, parameters of the shaper cutter, and the shaping process data. In comparison with the traditional method, it is universal and is much more efficient and accurate, especially for noncircular gears, which have nontrivial pitch lines. Special problems in gear design and manufacturing, such as tooth pointing, undercut, and fillet interference, are included in the process. As an application example of the numerical method, a square internal gear is chosen from a new type of hydraulic motor with noncircular planetary gears, and the tooth profile of that gear is computed. The gear is successfully machined by electromagnetic discharge (EMD) using the resulting data.

Keywords Noncircular gears · Tooth profile · Numerical computation · EMD

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Nomenclature

o_g	Shaper cutter's center
O	Noncircular gear's rotation center
a_i	Pitch point of the generated noncircular gear
r_i	Polar radii of the pitch line at the point a_i
θ_i	Polar angle of the pitch line at the point a_i
φ_i	Shaper cutter's rotation angle, corresponding to the instantaneous pitch point a_i
$(X_{o,\ i}, Y_{o,\ i})$	Coordinates of the shaper cutter's center O_g in coordinate system S, corresponding to the instantaneous pitch point a_i
R_o	Position vector of the generated noncircular gear profile
R_g	Position vector of the shaper cutter surface
h_a	Addendum of the noncircular gear
h_{a0}	Addendum of the shaper cutter
h	Whole depth of the noncircular gear
K	Number of normal isometric family curves of the noncircular gear pitch in the whole depth
h_t	Distance from the tth normal isometric family curve to the pitch line of the noncircular gear
z_o	Number of teeth of the shaper cutter
Z	Number of teeth of the noncircular gear
R_a	Outside radii of the shaper cutter
R	Pitch radii of the shaper cutter
R_b	Base radii of the shaper cutter
R_f	Root radii of the shaper cutter
R_m	Radius of the concentric circles of the shaper cutter
$\eta_{m,\ k,\ S}$	Start polar angle of the concentric circles R_m and the kth tooth of the shaper cutter
$\eta_{m,\ k,\ E}$	End polar angle of the concentric circles R_m and the kth tooth of the shaper cutter
S_m	Tooth thickness of the concentric circles R_m
m	Module
α	Nominal pressure angle
α_m	Pressure angle in the concentric circles R_m
n	Number of tip curve scopes of the non-

	circular gear according to an equal polar angle
T_j	The jth (j=1, 2,..., n) points on the tip curve of the noncircular gear
$(X_{T_j},\ Y_{T_j})$	Coordinates of the point T_j in the coordinate system S
N	Number of generating process periods
$P_{J,\ 1}$	Start point of the Jth key line which lies in the space of the noncircular gear
$P_{J,\ 2}$	End point of the Jth key line which lies in the tooth of the noncircular gear
P_J	Tooth profile point in the Jth key line

1 Introduction

Up to now, although circular gears have been widely used in industry for power transmission, noncircular gears are not so popular, mainly due to the lack of understanding on their properties and manufacture processes.

In fact, noncircular gears can meet different specific functional requirements and achieve any change in displacement or velocity of the driven member. In comparison with cams and linkages, for a similar application, noncircular gears are more compact and dynamically balanced, give kinematically exact solutions, and produce continuous unidirectional cyclic motion [1–4]. Hence, noncircular gears have been successfully used in quick-return mechanisms, automatic-feed machines, shears, packing machines, and pumps [5–7].

Currently, elliptical gears are still the most popular noncircular gears in industry, and most research on noncircular gears has focused on elliptical gears [8]. Kuczewski [7] used a spur gear to approximate the profile of an elliptical gear. Litvin [9] adopted the concept of evolute curves to form the tooth profile, and also derived the tooth evolute of an ellipse. Chang and Tsay [10] used a shaper cutter and applied the inverse mechanism relationship and the equation of meshing to produce the mathematical model of elliptical gears, the rotation center of which rotates around one of its foci. Chang et al. [11] used a rack cutter and the same method to produce the mathematical model and undercutting conditions of the same type of elliptical gear. Bair [12] used a rack cutter with circular-arc teeth to generate the tooth profile of circular-arc elliptical gears, which have a convex–concave tooth profile contact. Bair [13] also simulates an elliptical gear drive, the axis of rotation of which is coincident with its geometric center, manufactured by shaper cutters. Although much research has been devoted to the tooth profile of elliptical gears generated by rack cutters or shaper cutters, little research has been done on the tooth profile of universal noncircular gears generated by a shaper cutter. Meanwhile, the method on calculating the tooth profile of elliptical gears generated by rack cutters or shaper cutters in all of the researches previously mentioned had to deduce and solve systems of meshing equations.

In this paper, a numerical model of noncircular gear shaping has been firstly proposed and established. In the model, the continuous process in which the shaper cuts away material from a noncircular gear blank and forms the tooth space has been simply described. The tooth profile of the noncircular gear was computed accurately and quickly. Moreover, combining the theory of conjugate mating tooth profiles with the calculating process of the model, the special problems in gear design and manufacturing, such as tooth pointing, undercut, etc., have been solved. This is a complete solution of the computing the tooth profile of noncircular gears. It is universal and applicable for any pitch lines. Elliptical gears is only a very simple special case for the method. As an application example of the method, a square internal gear is chosen from a new type of hydraulic motor with noncircular planetary gears, and the tooth profile of that gear is computed. Using the resulting data, the gear is successfully machined by electromagnetic discharge (EMD). So, the method has unlocked a very bright future for noncircular gears manufacturing and their applications.

2 Numerical description of the generation of noncircular gear tooth profiles

The kinematic relationship between the generated noncircular gear and the shaper cutter is illustrated in Fig. 1.

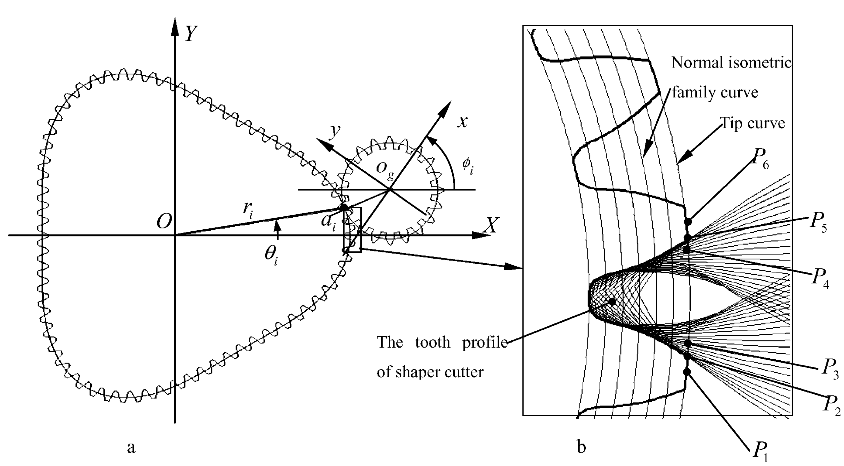

Fig. 1 The process of a noncircular gear generated by a shaper cutter

The shaper cutter rotates about its center o_g and translates along a curve that keeps the shaper centrode and pitch line of the generated noncircular gear in tangency at their instantaneous pitch point a_i. The coordinate systems in Fig. 1 are Cartesian coordinate systems with the right-handed two mutual perpendicular axes. Coordinate system $s(o_g–x, y)$ is attached to the shaper cutter, whose rotation center is o_g. Coordinate system $S(O–X, Y)$ is attached to the generated noncircular gear, whose rotation center is O. Parameter θ_i is the polar angle of the pitch line at the point a_i. Parameter ϕ_i, measured from the X axis of the noncircular gear to the x axis of the shaper cutter, represents the rotational angle of the shaper. Meanwhile, parameter $(X_{o,\ i}, Y_{o,\ i})$, which corresponds to the instantaneous pitch point a_i, is the coordination of the shaper center o_g in coordinate system $S(O–X, Y)$. Let R_o denote the position vector of the generated noncircular gear profile and R_g represent the position vector of the shaper cutter surface. By applying the following homogenous coordinate transformation matrix equation, the point of the shaper cutter represented in coordinate system $S(O–X, Y)$ can be obtained as follows:

$$R_O = \left[M_{O,\ g}\right] R_g \tag{1}$$

where $\left[M_{O,\ g}\right] = \begin{bmatrix} \cos\phi_i & -\sin\phi_i & X_{o,\ i} \\ \sin\phi_i & \cos\phi_i & Y_{o,\ i} \\ 0 & 0 & 1 \end{bmatrix}$

Meanwhile, the point of the noncircular gear represented in coordinate system $s(o_g–x, y)$ can be determined as:

$$R_g = \left[M_{g,\ O}\right] R_O \tag{2}$$

where $[M_{g,\ O}]$ is the inverse matrix of the matrix $[M_{O,\ g}]$.

Figure 1b is the zoomed-in image of the noncircular gear tooth in Fig. 1a. It shows the first tooth of the shaper cutter generating the first space of the generated noncircular gear. According to the zoomed-in image in Fig. 1b, all of the objectionable material is removed from the space of the noncircular gear. The tooth profile of the generated noncircular gear is the graph boundary. So, the tooth profile of the generated noncircular gear can be obtained through picking up the graph boundary. There are two intersect points on one normal isometric family curve of noncircular gear pitch lines between one space of generated noncircular gears. The two intersect points are on the two sides of the space respectively. If the normal isometric family curve of noncircular gear pitch lines are given from the gear root to the tip, all intersect points between the two sides of the space on all normal isometric family curves of the noncircular gear pitch line form the tooth profile of the noncircular gear. So, the process of computing the tooth profile of noncircular gears can be replaced by obtaining all of these intersect points on the normal isometric family curves of the noncircular gear pitch line.

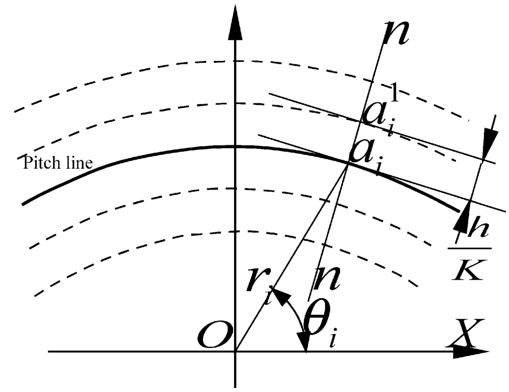

Fig. 2 Normal isometric family curves of the noncircular gear pitch line

3 Expressing the noncircular gear and the shaper cutter

3.1 Normal isometric family curves of the noncircular gear pitch line

Figure 2 shows the normal isometric family curves of the noncircular gear pitch line. The thick curve in Fig. 2 is the pitch line of the noncircular gear. Its equation in polar coordinates is expressed as Eq. 3:

$$r = r(\theta) \tag{3}$$

where r is the radius of polar and θ is the angle of polar.

In Fig. 2, point $a_i(r_i, \theta_i)$ is one arbitrary point that lies in the pitch line of the noncircular gear, and nn is the normal line on the pitch line of the noncircular gear through a_i. a_i^1, which corresponds to a_i and lies on the normal line nn, is one point of the normal isometric family curves of the noncircular gear pitch line.

Let h_a and h_{a0} be the addendum of the noncircular gear and the shaper cutter, then the whole depth h of the noncircular gear can be obtained as follows:

$$h = h_a + h_{a0} \tag{4}$$

Suppose that the number of normal isometric family curves of the noncircular gear pitch in the whole depth of the noncircular gear is K and the distance between each adjacent normal isometric family curve is equal, then the distance from the tth normal isometric family curve to the pitch line of the noncircular gear h_t can be obtained by the following equation:

$$h_t = \mp\left(h_a - t\frac{h}{K}\right) \tag{5}$$

where t=0, 1,..., K. We use the plus sign (+) in the external gear and the minus sign (−) in the internal gear.

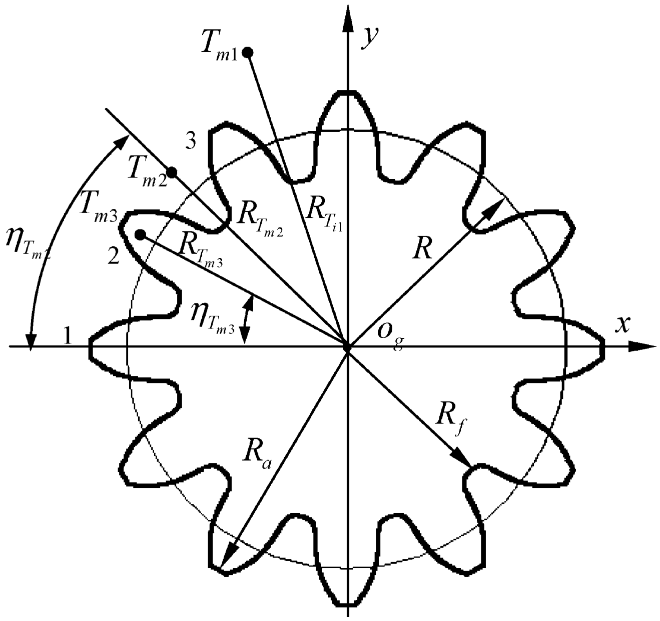

Fig. 3 Tooth of profile of the shaper cutter

The tth normal isometric family curve is expressed in coordinate system S by the following equation:

$$\begin{cases} X_t(\theta) = r(\theta)\cos\theta + \dfrac{h_t(r'(\theta)\sin\theta + r(\theta)\cos(\theta))}{\sqrt{r^2(\theta) + (r'(\theta))^2}} \\ Y_t(\theta) = -r(\theta)\sin\theta + \dfrac{h_t(r'(\theta)\cos\theta - r(\theta)\sin(\theta))}{\sqrt{r^2(\theta) + (r'(\theta))^2}} \end{cases} \tag{6}$$

where $r(\theta)$ is the equation of the noncircular gear pitch line and $r'(\theta)$ is the first derivative of function $r(\theta)$.

3.2 Shaper cutter profile

Shaper cutters are used to generate noncircular gears, and the profiles of the shaper cutters are the same as those of spur gears. Hence, the mathematical model of a shaper cutter can be expressed as that of a spur gear with an involute tooth profile. Figure 3 shows a shaper cutter and its profile expressed by the polar angle of concentric circles with its pitch circle in the coordinate system $s(o_g$–$x, y)$. Parameter z_0 is the number of teeth of the shaper cutter. R_a, R, R_b, and R_f denote the outside, pitch, base, and root radii of the shaper cutter, respectively.

The numbering of the teeth in the shaper cutter is shown as in Fig. 3. The first tooth profile of the shaper cutter is axially symmetric according to the negative half of the x axis. Parameter R_m is the radius of the concentric circles. The two polar angles of the concentric circles and the kth tooth of the shaper cutter are $\eta_{m,\ k,\ S}$ and $\eta_{m,\ k,\ E}$:

$$\begin{cases} \eta_{m,\ k,\ S} = -\dfrac{s_m}{2R_m} + \dfrac{2\pi}{z_0}(k-1) \\ \eta_{m,\ k,\ E} = \dfrac{s_m}{2R_m} + \dfrac{2\pi}{z_0}(k-1) \end{cases} \tag{7}$$

where s_m is the tooth thickness.

For $R_m \geq R_b$:

$$s_m = R_m\left(\frac{\pi}{z_0} - 2((\tan\alpha_m - \alpha_i) - (\tan\alpha - \alpha))\right)$$

For $R_m < R_b$: $s_m = R_m\left(\frac{\pi}{z_0} + 2inv(\alpha)\right)$ where α is the nominal pressure angle, $\alpha = \arccos\frac{R_b}{R}$, and α_m is the pressure angle in the concentric circles. Its radius is R_m, $\alpha_m = \arccos\frac{R_b}{R_m}$.

For a point $T(x, y)$ in the coordinate system $s(o_g$–$x, y)$, the polar angle η_T and polar radius R_T of point T can be calculated by following equation:

$$\begin{cases} R_T = \sqrt{x^2 + y^2} \\ \eta_T = \pi - \arctan\dfrac{x}{y} \end{cases} \tag{8}$$

If $R_T > R_a$, then point T lies on the outside of the shaper cutter, such as point T_{m1} in Fig. 3.

If $R_a \geq R_T \geq R_f$ and $\eta_T \in (\eta_{m,\ k,\ E}, \eta_{m,\ k+1,\ E})$ (k =1, 2,..., z_0−1) or $\eta_T \in \left(\eta_{m,\ z_0,\ E}, \eta_{m,\ 1,\ S}\right)$, the point T lies in the outside of the shaper cutter, such as point T_{m2} in Fig. 3.

If $R_a \geq R_T \geq R_f$ and $\eta_T \in (\eta_{m,\ k,\ S}, \eta_{m,\ k,\ E})$ (k =1, 2,..., z_0), the point T lies in the body of the shaper cutter, such as point T_{m3} in Fig. 3.

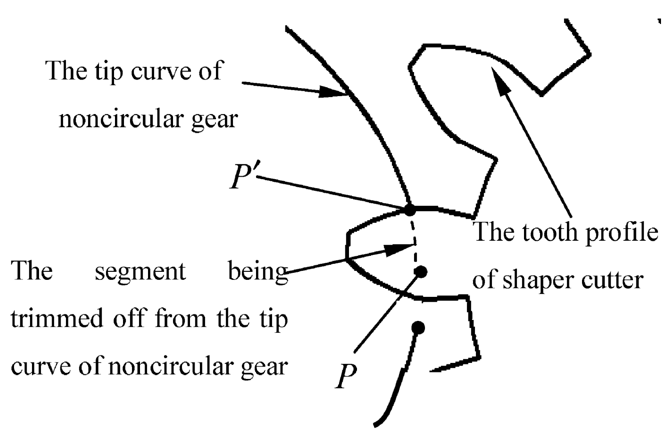

Fig. 4 One instantaneous scene of the cutting process

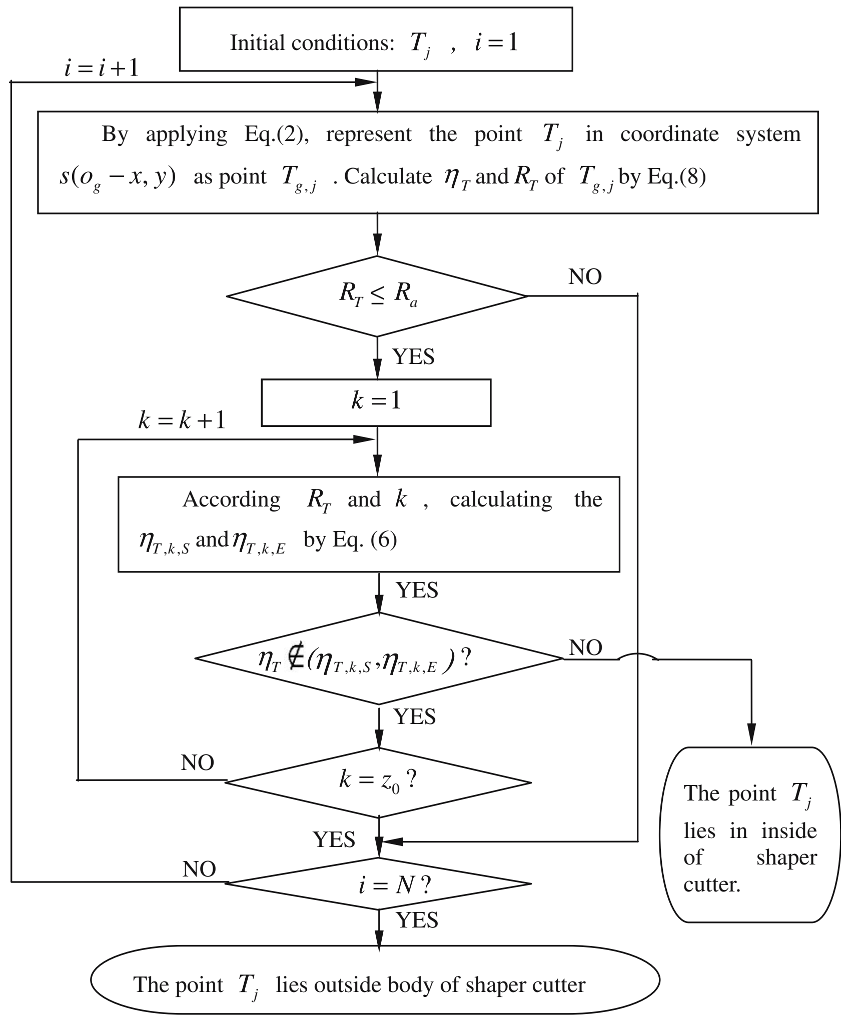

Fig. 5 Flow chart of the judgment procedure

4 Picking up the graph boundary

Because the tip curve of a noncircular gear is one piece of the normal isometric family curves of the noncircular gear pitch line, the method for picking up the graph boundary will be started from calculating the intersect points between the tip curves of the noncircular gear and shaper cutter in the process of generation.

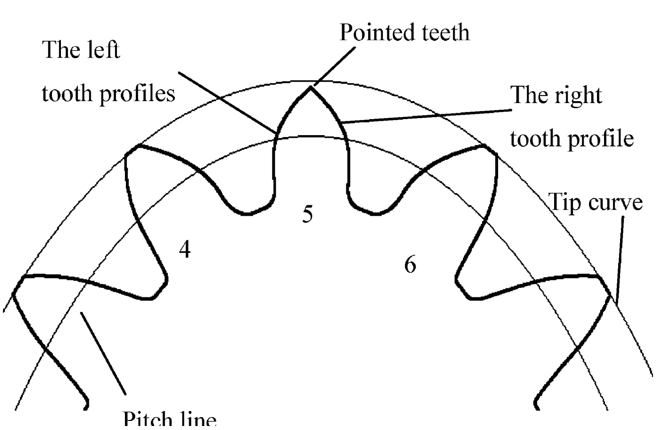

Fig. 6 Pointed teeth

In the process of generation, the tooth profile of the shaper cutter trims the tip curve of the noncircular gear and the intersect points will be obtained ultimately, such as points P_2 and P_5 in Fig. 1. Figure 4 displays one instantaneous scene of the generating process. The shaper cutter resects the section of curve in the tip curve of the noncircular gear PP' in Fig. 4, and P' is the instantaneous intersect point. Because the equation of the tip curve of the noncircular gear and the tooth profile of the shaper cutter are complex, the process for obtaining P_2 and P_5 through solving equations is very difficult. So, a more simple method is obtaining the scope of the intersect points firstly and then calculating the intersect points in the scope finally.

4.1 Getting the key scope with the intersect points

There are three steps for confirming the scopes of the process.

First, divide the tip curve of the noncircular gear into n scopes from equal polar angles θ. The jth scope is from the points T_{j-1} to T_j (j=1, 2,..., n) on the tip curve of the noncircular gear. On substituting t=0 into Eq. 6, the coordinates of the T_j

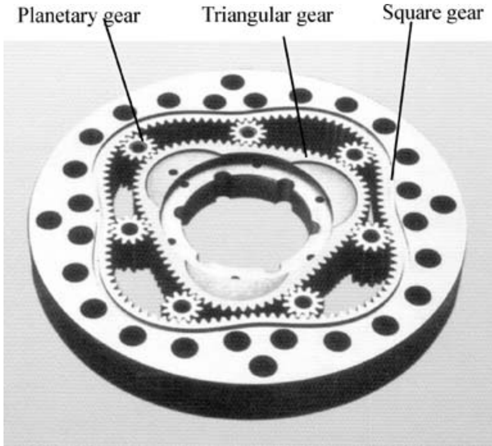

Fig. 7 Hydraulic motor with noncircular planetary gears

point $\left(X_{T_j}, Y_{T_j}\right)$ is expressed in the coordinate system $S(O\text{–}X, Y)$ by the following equation:

$$\begin{cases} X_{T_j} = X_t(\theta_j) \\ Y_{T_j} = Y_t(\theta_j) \end{cases} \quad (9)$$

where $\theta_j = j\frac{2\pi}{n}$ is the polar angle.

Second, divide the continuous generating process into N periods of time. The position of the shaper cutter in the ith period of time can be displayed by (ϕ_i, $X_{o,i}$, $Y_{o,i}$).

Then, a flow chart of the judgment procedure of the relationship between the start or end points T_j in the tip curves of the noncircular gear and the shaper cutter in period N of time is shown in Fig. 5.

According to the flow chart shown in Fig. 5, the point T_j can be distinguished into two types:

1. The point T_j in the tip curve of the noncircular gear lies in the body of the shaper cutter in one cutting process. The gear blank in the point T_j will be cut off by the shaper cutter and the point T_j is in the space of the finished gear, such as points P_3 and P_4 in Fig. 1.
2. The point T_j in the tip curve of the noncircular gear does not lie in the body of the shaper cutter in the entire generating process. The gear blank in the point T_j will not be cut off by shaper cutter, and the point T_j belongs to the teeth of the finishing gear, such as points P_1 and P_6 in Fig. 1.

Finally, if the start and end points of the scope in the tip curves of the noncircular gear belong to the different types mentioned above, there is one point in the scope which lies in the tooth profile of the tip curve. This scope is called the key scope, such as point P_2, which belongs to the tooth profile of the noncircular gear, and lies in the scope of the start and end points, of which, are P_1 and P_3 in Fig. 1. If the number of teeth of the noncircular gear is Z, the number of key scopes is $2Z$.

4.2 Accurate calculation of the tooth profile in key scopes

If the number of scopes n that the tip curves of the noncircular gear has been divided into is big enough (in practice, n=3,600), the segment of tip curves in those scopes can be replaced by the lines that the start and end points are homogenous with the corresponding scope. The line replacing the key scopes is called the key line. As the number of key scopes is $2Z$, the number of key lines is $2Z$. Let $P_{J,1}$ denote the start point of the Jth (J=1, 2,..., $2Z$) key line which lies in the space of the gear, $P_{J,2}$ denotes the end point of the Jth key line which lies in the tooth of the gear, and P_J denotes the tooth profile point in the J^{th} key line.

The tooth profile point P_J in the key line can be obtained by calculating the intersect point between the key line $\overline{P_{J,1}P_{J,2}}$ and the shaper cutter in the cutting process. The calculating method will be expressed in detail below.

For the ith period of time of the generating process, the intersect point between the key line $\overline{P_{J,1}P_{J,2}}$ and the kth

Fig. 8 Tooth profile of the square gear

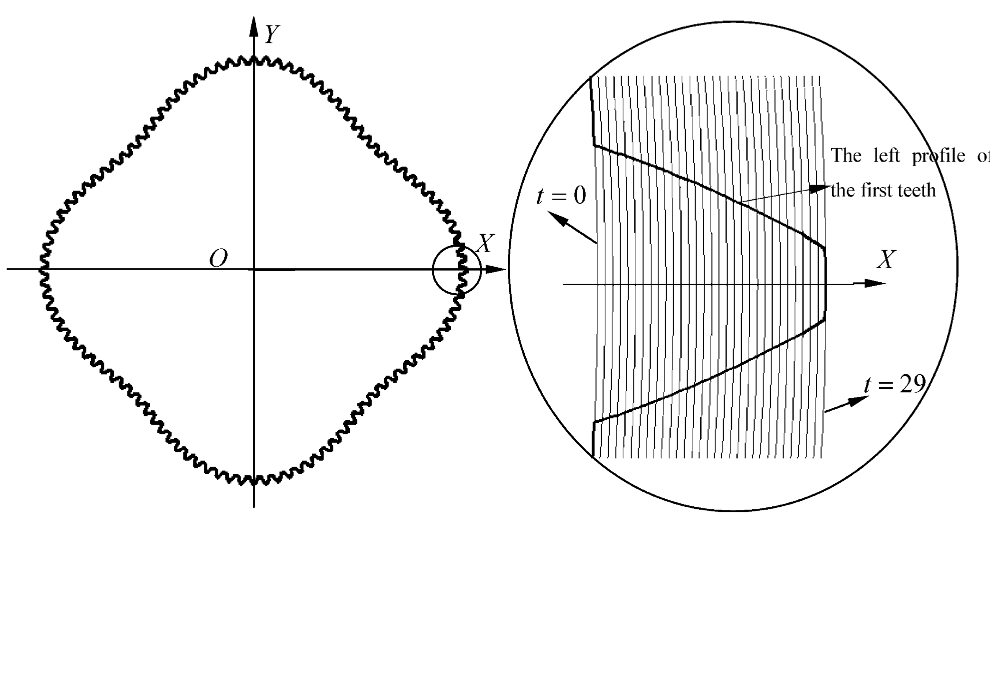

Fig. 9 Machining the square gear by electron discharge machining (EDM)

tooth of the shaper cutter can be computed by following three steps:

(a) According Eq. 2, the points $P_{J,1}\left(X_{i,P_{J,1}},\ Y_{i,P_{J,1}}\right)$ and $P_{J,2}\left(X_{i,P_{J,2}},\ Y_{i,P_{J,2}}\right)$ are represented in the coordinate system $s(o_g\text{–}x, y)$ as the points $P_{g,J,1}\left(x_{i,P_{J,1}},\ y_{i,P_{J,1}}\right)$ and $P_{g,J,2}\left(x_{i,P_{J,2}},\ y_{i,P_{J,2}}\right)$.

(b) The polar equation of the key line can be expressed as the following equations:

$$\begin{cases} y = \dfrac{y_{i,P_{J,2}} - y_{i,P_{J,1}}}{x_{i,P_{J,2}} - x_{i,P_{J,1}}}\left(x - x_{i,P_{J,1}}\right) + y_{i,P_{J,1}} \\ R_i = \sqrt{x^2 + y^2} \\ \eta_i = \arctan\dfrac{-x}{y} \\ x \in \left(x_{i,P_{J,1}},\ x_{i,P_{J,2}}\right) \end{cases} \tag{10}$$

$$\begin{cases} x = x_{i,P_{J,1}} \\ R_i = \sqrt{x^2 + y^2} \\ \eta_i = \arctan\dfrac{-x}{y} \\ y \in \left(y_{i,P_{J,1}},\ y_{i,P_{J,2}}\right) \end{cases} \tag{11}$$

Equation 10 is apt for the term that $x_{i,P_{J,1}} \neq x_{i,P_{J,2}}$. Equation 11 is apt for the term that $x_{i,P_{J,1}} = x_{i,P_{J,2}}$.

(c) Substitute Eq. 10 or Eq. 11 into Eq. 7 and Eq. 1. The intersect point $P_{i,J}$ between the key line $\overline{P_{J,1}P_{J,2}}$ and the tooth process of the shaper cutter can be obtained in the coordinate system $S(O_g\text{–}X, Y)$. Then, replace the $P_{J,1}$ by $P_{i,J}$. When i=1, 2,..., N, the tooth profile point P_J of the key line $\overline{P_{J,1}P_{J,2}}$ is to the right of $P_{J,1}$. Using the same procedure, all points in the tooth profile P_J (J=1, 2,..., 2Z) of the tip curve of the noncircular gear can be calculated.

At the same time, replacing the tip curve of the noncircular gear by another normal isometric family curve of the noncircular gear pitch line from tip to root, all of the points on the tooth profile can be obtained. Joining these points on the same tooth together with a line, the full tooth profile of the noncircular gear is obtained.

5 Treatment of special problems

In gear design and manufacturing, there are some special problems which have to be considered, such as tooth pointing, undercut, and fillet interference, etc. If the traditional method is used, that is, deducing and solving the meshing equations, sometimes, these problems are difficult to solve, especially for noncircular gears with complicated pitch lines. But it is quite easy and clear in the numerical method proposed in this paper. Tooth pointing is discussed below. Undercut and fillet interference will be discussed in another paper.

Table 1 Tooth profile points in the normal isometric family curve of the square gear pitch line

t	X mm	Y mm	t	X mm	Y mm	t	X mm	Y mm
0	135.0071	2.7260	10	136.4260	2.2152	20	137.8425	1.6033
1	135.1491	2.6792	11	136.5678	2.1591	21	137.9838	1.5369
2	135.2910	2.6323	12	136.7096	2.1011	22	138.1251	1.4695
3	135.4329	2.5838	13	136.8514	2.0423	23	138.2664	1.4009
4	135.5747	2.5350	14	136.9930	1.9827	24	138.4076	1.3315
5	135.7167	2.4843	15	137.1347	1.9214	25	138.5487	1.2611
6	135.8586	2.4325	16	137.2763	1.8601	26	138.6898	1.1906
7	136.0004	2.3797	17	137.4179	1.7978	27	138.8308	1.1179
8	136.1423	2.3269	18	137.5594	1.7340	28	138.9717	1.0454
9	136.2842	2.2719	19	137.7009	1.6696	29	139.1126	0.9713

A pointed tooth is a gear tooth of which the right and left-side tooth profiles intersect, and the intersection point lies on the inside (for external gears) or the outer side (for internal gears) of the tip curve of the gear shown in Fig. 6. Pointed teeth reduce the addendum, instantaneous contact teeth, average contact ratio, and gear strength. Gears with pointed teeth cannot be used in practice, and tooth pointing must be avoided in the design process of noncircular gears. Tooth pointing takes place usually for the shaper cutter with a larger positive-shifted modification or the part of pitch lines of a gear whose curvature radius is small.

As tooth pointing appears on the 5th tooth of the gear in Fig. 6, there is no intersect point between the tip curve of the gear and the tooth profile. In the process of obtaining the key scope described above, there is a lack of two key scopes. If the number of key scopes is $2Z{-}2U$ (U is a natural number), it is obvious that the number of pointed teeth in the gear is U. So, the judgment of pointed teeth can be obtained during the computing process.

6 Application example

Using the above computing procedure, the tooth profile of all types of noncircular gear can be obtained by the same computer program.

The inputs of the program are: parameters of the generated gear, parameters of the shaper cutter, and the control parameters. The parameters of the generated gear include the equation of the pitch line of the noncircular gear, the number of teeth Z, pressure angle α, module m, and addendum h_a. The parameters of the shaper cutter include the number of teeth z_0, tooth thickness, and addendum h_{a0}. The parameters of control are: the number of scopes of the pitch line n, the number of periods of time of generating N, and the number of normal isometric family curves of the noncircular gear pitch line K.

Computer programs are developed to implement the numerical method on the tooth profile of the noncircular gear generated by the shaper cutter.

The square gear used in the hydraulic motor is taken as the example for the computation. The layout of the hydraulic motor is shown in Fig. 7. The triangular gear (sun gear), square gear (annual gear), and the seven planetary gears constitute the work cavity of the hydraulic motor. Hydraulic oil is fed into the work cavity that drives the triangular gear rotation, and the triangular gear rotation drives the planetary gear rotation and revolution.

The square and triangle gears belong to the internal noncircular gears. The pitch line of the square gear is expressed by Eq. 3.

The input parameters are: m=2.5 mm, α=25°, Z=104, h_{ap}=2 mm, z_0=20, h_{a0}=2.5 mm, N=3,600, n=3,600, K=29.

Other parameters of the shaper cutter mentioned above can be obtained from the input parameters: $R = \frac{mz_0}{2} = 25mm$, $R_a = R + h_{a0} = 27.5\ mm$, $R_f = R - h_{a0} = 22.5\ mm$, and $R_b = R\cos\alpha = 22.658\ mm$.

According to the computing result of the program, there is no tooth pointing or undercut in the square gear. Figure 8 shows the computed graph of the tooth profile of the square gear; the right figure in Fig. 8 is the zoomed-in graph of the first tooth and the normal isometric family curves of the pitch line. Table 1 gives the data of the left tooth profile of the first tooth of the square gear in the normal isometric family curves of the pitch line.

The data computed were successfully used in machining the square gear by electron discharge machining (EDM), see Fig. 9, and the data can also be used in a coordinate measurement machine (CMM) to test the accuracy of the machined tooth profile.

7 Conclusions

The proposed method is a complete solution of the tooth profile of a noncircular gear. It is universal, complete, effective, and accurate. It is the foundation of the noncircular gear computer-aided manufacturing (CAM) system.

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