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ORIGINAL ARTICLE

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A CUSUM scheme with variable sample sizes for monitoring process shifts

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Abstract The cumulative sum scheme (CUSUM) and the adaptive control chart are two approaches to improve chart performance in detecting process shifts. A weighted loss function CUSUM scheme (*WLC*) is able to monitor both the mean shift and the increasing variance shift by manipulating a single chart. This paper investigates the *WLC* scheme with a variable sample sizes (VSS) feature. A design procedure is firstly proposed for the VSS *WLC* scheme. Then, the performance of the chart is compared with that of four other competitive control charts. The results show that the VSS *WLC* scheme is more powerful than the other charts from an overall viewpoint. More importantly, the VSS *WLC* scheme is simpler to design and operate. A case study in the manufacturing industry is used to illustrate the chart application. The proposed VSS *WLC* scheme suits the scenario where the strategy of varying sample sizes is feasible and preferable to pursue a high capability of detecting process variations.

Keywords Quality control · Control chart · Statistical process control · Variable sample sizes · Cumulative sum chart · Mean and variance shifts

Notation

CUSUM	Cumulative sum chart
WL	Weighted loss function chart or statistic used by this chart
WLC	Weighted loss function CUSUM scheme
CCC	Joint three one-sided CUSUM chart (*I*, *D* and *V* charts)
I	CUSUM chart for detecting increasing mean shifts
D	CUSUM chart for detecting decreasing mean shifts
V	CUSUM chart for detecting increasing variance shifts
VSS	Variable sample sizes
VSI	Variable sampling intervals
VSSI	Variable sample sizes and variable sampling intervals
A_t	Statistic used by a *WLC* scheme
λ	Weighting factor in a *WLC* scheme
w_A, H_A	Warning and control limits of a *WLC* scheme
k_A	Reference parameter of a *WLC* scheme
$LWL_{\overline{X}}$, $UWL_{\overline{X}}$	Lower and upper warning limits of an $\overline{X}$ chart
$LCL_{\overline{X}}$, $UCL_{\overline{X}}$	Lower and upper control limits of an $\overline{X}$ chart
UWL_{S1}, UCL_{S1}	Warning and control limits of an *S* chart when using relax samples
UWL_{S2}, UCL_{S2}	Warning and control limits of an *S* chart when using alert samples
w_I, H_I, k_I	Warning limit, control limit and reference parameter of an *I* chart
w_D, H_D, k_D	Warning limit, control limit and reference parameter of a *D* chart
w_V, H_V, k_V	Warning limit, control limit and reference parameter of a *V* chart
x	Quality parameter
μ, σ^2	Mean and variance of x
$\bar{x}$, s	Sample mean and standard deviation
μ_0, δ_0	In-control mean and standard deviation
μ_d, δ_d	Out-of-control mean and standard deviation
n_1, n_2	Relax and alert sample sizes
h	Sampling interval
B_2	Stabilised probability for a process being in a warning zone
ATS	Average time to signal
ATS_0	In-control average time to signal
$ATS_{\min}$	Used to store the minimum *ATS* during a chart design
ARATS	Average ratios of *ATS*s
$ARATS_s$	*ARATS* in a small shift region

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$ARATS_{\mathrm{m}}$	*ARATS* in a moderate shift region
$ARATS_{\mathrm{l}}$	*ARATS* in a large shift region
$ARATS_{\mathrm{o}}$	*ARATS* in an overall shift region
$\overline{ARATS}$	Average of *ARATS* values for a chart over all runs
τ	Allowed minimum in-control *ATS*
R	Allowed maximum in-control average inspection ratio
$n_{\max}$	Allowed maximum sample size
$\delta_{\mu d},\ \delta_{\sigma d}$	Selected mean shift and variance shift for chart design
$\widehat{\delta}_{\mu},\ \widehat{\delta}_{\sigma}$	Estimated mean shift and variance shift
m_1, m_2	Numbers of relax and alert samples taken after a change point
$\bar{x}_{ij},\ s_{ij}^2$	Sample mean and variance of the *j*th sample of size n_i after a change point

1 Introduction

Statistical process control (SPC) is an important technique for quality insurance. For a process characterised by a quality parameter x, the process variation can be indicated by the shift of the mean μ and the shift of the variance σ^2. In general, both the mean and variance shifts have to be monitored [1]. Two commonly used approaches are the Shewhart $\overline{X}$ & S (or $\overline{X}$ & R) control chart and the more advanced multiple cumulative sum scheme (CUSUM).

The static chart (i.e. using fixed sample size, sampling interval and control limits) has the advantage of easy design and operation. However, its effectiveness in detecting process shifts is generally less adequate than that of the charts with adaptive features. An adaptive control chart allows its parameters to be changed according to the values of sample statistics which indicate the current process state. Typical adaptive charts include the variable sample sizes (VSS) chart, the variable sampling intervals (VSI) chart and the variable sample sizes and variable sampling intervals (VSSI) chart [2, 3].

A VSS chart becomes favoured under certain circumstances where the sample size can be easily varied but the sampling interval is required to be fixed due to some managerial and/or operational reasons. Many research works have been published on the VSS schemes. For example, some researchers investigated the VSS $\overline{X}$ charts for detecting mean shifts [4, 5]. Costa studied the VSS $\overline{X}$ &R chart for monitoring both the mean and variance shifts [6]. A VSS CUSUM chart for monitoring the one-sided mean shift has also been reported [7, 8]. The VSS charts commonly utilise only two different sample sizes for the simplicity of operation [9–11].

Using the CUSUM scheme is another approach to improve the detection capability. Multiple CUSUM charts have been studied for detecting both mean shifts and variance shifts [12–14]. Usually, a CUSUM scheme consists of two one-sided CUSUM mean charts and two one-sided CUSUM variance charts. Since the mean shift and increasing variance shift reflect quality deterioration [15] and are usually the major concern, a CUSUM scheme (called the *CCC* scheme) can be used. A *CCC* scheme comprises three one-sided CUSUM charts (the *I*, *D* and *V* CUSUM charts), which monitor the increasing mean shift, the decreasing mean shift and the increasing variance shift, respectively.

The performance of a control chart is usually measured by the average time to signal (*ATS*), that is, the average time required to signal an out-of-control case after it occurs. A *CCC* scheme is quite difficult to design and operate. Especially, the interaction among the three CUSUM charts makes it difficult to express the *ATS* of a *CCC* or VSS *CCC* scheme by simply using the corresponding *ATS* values of the three individual CUSUM charts. A two-dimensional Markov chain procedure was suggested to evaluate the *ATS* of a VSS *CC* scheme for monitoring two-sided mean shifts [8], but considerable computational effort is required. For the design of a *CCC* or VSS *CCC* scheme, simulation may be the only option for evaluating the *ATS* [12]. When implementing a *CCC* scheme, the three CUSUM charts have to be handled concurrently. The operators have to update three statistics and plot three sample points on three individual CUSUM charts. A computer is almost a must to run the *CCC* scheme. The operational complexity may seriously hinder the application of the *CCC* (or VSS *CCC*) scheme.

To simplify the chart design and operation, a single CUSUM chart is desired for monitoring both the two-sided mean shifts and the increasing variance shift. Some progress has been reported in this area. These include the Omnibus EWMA chart [16], the *B* chart [17], the likelihood ratio test [18], the MaxMin EWMA chart [19] and the MaxEWMA chart [20]. The *L* chart [15, 21] and *WL* chart [22] are also the interesting charting methods in this line.

Both the *L* (loss function) chart and the modified *WL* (weighted loss function) chart adopt the loss function to measure the cost due to poor qualities [23, 24]. The *L* chart uses a fixed combination of the losses caused by the mean shifts and the losses caused by the increasing variance shift. The *WL* chart, on the other hand, uses a weighting factor λ ($0 \le \lambda \le 1$) to adjust the weights of these two types of losses in an optimal manner. It employs the following statistic:

$$WL = \lambda s^2 + (1-\lambda)(\bar{x}-\mu_0)^2 \tag{1}$$

where $\bar{x}$ and s are the sample mean and sample standard deviation, respectively, and μ_0 is the in-control process mean.

Further studies have incorporated the weighted loss function *WL* into a single CUSUM chart (the *WLC* scheme) [25]. The *WLC* scheme with a VSI feature has also been investigated [26]. The statistic used in the *WLC* scheme is A_t:

$$\begin{aligned} &A_0 = 0 \\ &A_t = \max(0,\ A_{t-1} + WL_t - k_A) \end{aligned} \tag{2}$$

where k_A is the reference parameter.

The *WLC* scheme is capable of monitoring both the two-sided mean shifts and the increasing variance shift. However, since the *WLC* scheme consists of only a single CUSUM chart, its design and operation is much simpler than that of the *CCC* scheme.

This work studies the *WLC* scheme with a VSS feature. A systematic procedure has been developed for the design and operation of this scheme. The VSS *WLC* scheme is also compared with some existing charts in terms of the effectiveness of detecting process shifts.

In this article, the quality parameter x is assumed to follow a normal probability distribution with known in-control mean μ_0 and standard deviation σ_0.

2 General features of the VSS *WLC* scheme

The VSS *WLC* scheme uses two different sample sizes and a fixed sampling interval. It has a warning limit w_A and a control limit H_A, which divide the chart into three zones: the central zone (0, w_A), the warning zone (w_A, H_A) and the action zone (H_A, ∞). The VSS *WLC* scheme can be implemented as any other VSS CUSUM charts. Namely, if the statistic A_t falls into the central zone, a *relax sample* with a smaller sample size n_1 is adopted for the next sample. If A_t falls into the warning zone, an *alert sample* with a larger sample size n_2 should be chosen. Both samples use the same sampling interval h. An out-of-control signal will be triggered if A_t falls into the action zone.

To design a VSS *WLC* scheme, four specifications need to be provided: τ (the allowed minimum in-control *ATS*), R (the allowed maximum in-control average inspection ratio), $n_{\max}$ (the allowed maximum sample size) and $\left(\delta_{\mu d},\ \delta_{\sigma d}\right)$ (the selected process shift point for chart design). The value of τ is decided with regards to the tolerable false alarm rate. The actual in-control ATS_0 must be no smaller than τ. The value of R depends on the available resources for inspection (e.g. manpower and instruments). The actual inspection rate r defined below must not exceed R:

$$r = \frac{(1 - B_2)n_1 + B_2 n_2}{h} \tag{3}$$

where B_2 is the stabilised probability for the process being in the warning zone. Under steady-state modes, the process usually runs in an in-control status for a long period and has entered the stabilised status before an out-of-control shift takes place at a random time. The value of $n_{\max}$ is decided by some practical considerations, such as the number of units that can be inspected in a short time period. A process shift point $\left(\delta_{\mu d},\ \delta_{\sigma d}\right)$ is selected for chart design, where $\delta_{\mu d} = \frac{(\mu_d - \mu_0)}{\sigma_0}$ and $\delta_{\sigma d} = \frac{\sigma_d}{\sigma_0}$, with μ_d and σ_d being the out-of-control values of mean and standard deviation that should be detected quickly [14]. The specified values of $\delta_{\mu d}$ and $\delta_{\sigma d}$ should provide a sound compromise for both the mean and increasing variance shifts, and also for both small and large shifts. This ensures that the chart is effective over a broad process shift domain.

3 Optimisation design of the VSS *WLC* scheme

The optimal values of seven parameters (λ, n_1, n_2, h, k_A, w_A and H_A) need to be determined for the VSS *WLC* scheme in order to minimise the out-of-control *ATS* at the predetermined process shift point $\left(\delta_{\mu d},\ \delta_{\sigma d}\right)$ [27]. Hence, the design of the VSS *WLC* scheme is formulated as:

$$\begin{aligned} &\min(ATS)|_{(\delta_{\mu d},\ \delta_{\sigma d})} \text{ subject to } ATS_0 \geq \tau, \\ &r \leq R,\ n_1 \leq n_2 \leq n_{\max} \end{aligned} \tag{4}$$

Like most design strategies for control charts, this work makes no attempt to secure a global optimal solution for the VSS *WLC* scheme design. Instead, it aims at deriving a convenient and systematic procedure to attain a satisfactory and workable solution that could be adopted in practice [28]. A six-level algorithm is proposed for the design of the VSS *WLC* scheme:

1. Initialise $ATS_{\min}$ as a very large number, say 10^7 ($ATS_{\min}$ is used to store the minimum *ATS*).
2. At level one, the value of the weighting factor λ in Eq. 1 is searched with a step size of 0.01 within the range $0 \leq \lambda \leq 1$.
3. At level two, the value of n_2 is searched from three to n_{max}.
4. At level three, the value of n_1 is searched from two to the current value of n_2.
5. At level four, the value of the sampling interval h is determined through the search of B_2 (i.e. the stabilised probability for the process being in the warning zone). The sampling interval h has a one-to-one relationship with the probability B_2 as follows:

   $$h = \frac{(1 - B_2)n_1 + B_2 n_2}{R}$$

   However, it is relatively easier to handle B_2, as it can be searched within a well defined range ($0.1 \leq B_2 \leq 0.7$) with a suitable increment of 0.1 [29]. When the optimisation search finds an optimal value of B_2 that minimises the objective function *ATS*, the corresponding value of h determined by the above equation will also result in the *ATS*.
6. At level five, the value of k_A is searched around a starting value (an estimated optimal value, say 0.6, of k_A for the general cases) with a step size of 0.05.
7. At level six, the values of H_A and w_A are adjusted alongside each other in a recursive manner. Adjusting H_A mainly makes ATS_0 equal to τ and adjusting w_A mainly makes r equal to R.

8. When all of the parameters are tentatively determined, the out-of-control *ATS* at $(\delta_{\mu d},\ \delta_{\sigma d})$ can be calculated (the calculation of ATS_0 and *ATS* is described in Appendix 2).
9. If the calculated *ATS* is smaller than the current $ATS_{\min}$, replace the latter by the former and the current values of the parameters λ, n_1, n_2, h, k_A, w_A and H_A are stored as a temporary optimal design.
10. At the end of the entire search, the VSS *WLC* scheme which results in a minimum *ATS* at $(\delta_{\mu d},\ \delta_{\sigma d})$ and which satisfies all design constraints can be identified.

4 Comparison studies

The VSS *WLC* scheme is compared with four other charts, that is, the static $\overline{X}$ &*S* chart, the VSS $\overline{X}$ &*S* chart, the VSS *CCC* scheme and the static *WLC* scheme. All of the charts are optimised in order to minimise *ATS* at the same process shift point $(\delta_{\mu d},\ \delta_{\sigma d})$. The static $\overline{X}$ &*S* and *WLC* charts use a fixed sample size *n* and sampling interval *h*, while both *n* and *h* will be optimised. The design of a VSS $\overline{X}$ &*S* chart needs to determine nine parameters (i.e. the sample sizes and sampling interval n_1, n_2, h, the warning and control limits $LWL_{\overline{X}}$, $LCL_{\overline{X}}$, $UWL_{\overline{X}}$, $UCL_{\overline{X}}$ for the $\overline{X}$ chart, the warning and control limits UWL_{S1}, UCL_{S1} for the *S* chart when n=n_1 and, finally, the warning and control limits UWL_{S2}, UCL_{S2} for the *S* chart when n=n_2. The design of the VSS *CCC* scheme is more complicated. Eleven parameters need to be determined, including the sample sizes and sampling interval n_1, n_2, h, the warning limits, control limits and reference parameters w_I, H_I and k_I for the *I* chart (for detecting increasing mean shifts), w_D, H_D and k_D for the *D* chart (for detecting decreasing mean shifts), w_V, H_V and k_v for the *V* chart (for detecting increasing variance shifts). For the static $\overline{X}$ &*S* chart, the VSS $\overline{X}$ chart and the VSS *CCC* scheme, the allocation of overall type I error between the joint charts, i.e. the $\overline{X}$ chart and the *S* chart, or the *I*, *D* and *V* charts, will also be optimised [22]. Due to the lack of an analytic expression for the *ATS* of the VSS *CCC* scheme, simulation is used to evaluate ATS_0 and *ATS*. However, it makes it extremely time consuming to search the optimal values of the eleven parameters of the VSS *CCC* scheme in multiple levels as for the VSS *WLC* scheme. As an alternative, the VSS *CCC* scheme may use the same sampling interval *h* and sample sizes n_1, n_2 as that used by the VSS *WLC* scheme. It will substantially reduce the computing time and result in a workable, if not optimal, design of the VSS *CCC* scheme.

Two studies are conducted for the performance comparison of the five charts. Study one examines the chart performance under a general design condition. Study two investigates the influence of design specifications on the chart performance.

4.1 Study one: comparison under a general condition

The first study is conducted under the following general conditions:

$$n_{\max} = 10,\ \tau = 400,\ R = 5,\ \delta_{\mu d} = 0.6,\ \delta_{\sigma d} = 1.3 \qquad (5)$$

The five charts are worked out and their parameters are listed in Table 1. All of the charts are then used to detect the shifts in a process shift domain of $(0 < \delta_\mu \le 2,\ 1 < \delta_\sigma \le 2)$. The resultant values of *ATS* are displayed in Table 2. Several findings are observed from Table 2:

1. All of the charts generate identical values of ATS_0 (around τ=400) when the process is in control $(\delta_\mu = 0,\ \delta_\sigma = 1)$. This ensures a fair comparison between the charts.
2. The VSS *WLC* scheme is more powerful than other charts in detecting shifts in moderate and large shift regions, while the VSS *CCC* scheme is most sensitive to small shifts. The $\overline{X}$ &*S* chart displays shift detection capabilities in-between.
3. The VSS feature makes the VSS *WLC* scheme and the VSS $\overline{X}$ &*S* chart more powerful than their static

Table 1 The design parameters of the five charts $(n_{\max} = 10,\ \tau = 400,\ R = 5,\ \delta_{\mu d} = 0.6,\ \delta_{\sigma d} = 1.3)$

Chart	Sample size	Sampling interval	Control and warning limits		Reference parameter	Weighting factor
Static $\overline{X}$&*S*	n=10	h=2.00	$LCL_{\overline{X}} = -3.0230$	$UCL_{\overline{X}} = 3.0230$		
				$UCL_S = 1.6819$		
VSS $\overline{X}$&*S*	n_1=3	h=1.02	$LWL_{\overline{X}} = -1.3939$	$LCL_{\overline{X}} = -3.2214$		
	n_2=10		$UWL_{\overline{X}} = 1.3939$	$UCL_{\overline{X}} = 3.2214$		
			$L_S = 1.2011$	$UCL_S = 1.7398$		
VSS *CCC*	n_1=5	h=1.06	w_I=2.9594	H_I=5.3370	k_I=0.5089	
	n_2=8		w_D=−2.9594	H_D=−5.3370	k_D=−0.5089	
			w_V=0.8454	H_V=1.7763	k_V=0.3044	
Static *WLC*	n=10	h=2.00		H_A=0.2941	k_A=0.6874	λ=0.30
VSS *WLC*	n_1=5	h=1.06	W_A=0.5758	H_A=1.4086	k_A=0.5590	λ=0.29
	n_2=8					

Table 2 The average time to signal (*ATS*) values of the five charts ($n_{\max} = 10,\ R = 5,\ \tau = 400,\ \delta_{\mu d} = 0.6,\ \delta_{\sigma d} = 1.3$)

δ_σ	Chart	δ_μ										
		0.0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0
1.0	Static $\overline{X}$&S	400.00	180.44	46.87	14.11	5.40	2.59	1.56	1.17	1.04	1.01	1.00
	VSS $\overline{X}$&S	400.00	233.25	64.86	16.00	5.31	2.59	1.66	1.26	1.05	0.91	0.80
	VSS *CCC*	401.63	43.14	10.48	5.64	3.78	2.78	2.28	1.92	1.65	1.46	1.34
	Static *WLC*	400.59	159.82	40.61	12.59	5.06	2.59	1.62	1.22	1.06	1.01	1.00
	VSS *WLC*	404.93	174.28	38.87	11.03	4.83	2.75	1.80	1.28	0.98	0.78	0.66
1.2	Static $\overline{X}$&S	38.70	31.02	17.53	8.69	4.46	2.55	1.67	1.26	1.09	1.03	1.01
	VSS $\overline{X}$&S	40.35	32.34	17.70	8.19	4.06	2.38	1.64	1.27	1.06	0.92	0.82
	VSS *CCC*	21.59	16.25	8.43	5.30	3.64	2.77	2.31	1.88	1.68	1.46	1.33
	Static *WLC*	33.39	23.71	11.91	6.01	3.37	2.13	1.52	1.21	1.08	1.02	1.01
	VSS *WLC*	22.60	16.38	9.23	5.25	3.26	2.20	1.59	1.20	0.95	0.79	0.68
1.4	Static $\overline{X}$&S	9.59	8.82	6.95	4.89	3.29	2.25	1.64	1.30	1.13	1.05	1.02
	VSS $\overline{X}$&S	8.65	7.91	6.13	4.26	2.88	2.04	1.54	1.24	1.05	0.92	0.83
	VSS *CCC*	6.55	6.30	5.08	3.99	3.22	2.59	2.16	1.86	1.65	1.45	1.31
	Static *WLC*	8.23	7.20	5.21	3.54	2.46	1.80	1.41	1.19	1.08	1.03	1.01
	VSS *WLC*	6.04	5.50	4.34	3.23	2.39	1.80	1.40	1.12	0.93	0.79	0.69
1.6	Static $\overline{X}$&S	4.09	3.93	3.49	2.91	2.34	1.86	1.51	1.28	1.14	1.06	1.03
	VSS $\overline{X}$&S	3.50	3.36	3.00	2.52	2.05	1.67	1.38	1.17	1.02	0.91	0.83
	VSS *CCC*	3.75	3.60	3.39	2.94	2.67	2.30	2.00	1.73	1.58	1.38	1.28
	Static *WLC*	3.70	3.48	2.96	2.39	1.90	1.55	1.31	1.17	1.08	1.03	1.01
	VSS *WLC*	3.20	3.06	2.70	2.26	1.85	1.51	1.25	1.05	0.89	0.78	0.69
1.8	Static $\overline{X}$&S	2.39	2.34	2.19	1.99	1.75	1.54	1.36	1.22	1.12	1.06	1.03
	VSS $\overline{X}$&S	2.08	2.03	1.92	1.75	1.56	1.38	1.22	1.08	0.97	0.89	0.82
	VSS *CCC*	2.60	2.60	2.48	2.32	2.16	1.98	1.73	1.63	1.46	1.33	1.21
	Static *WLC*	2.27	2.20	2.02	1.79	1.56	1.37	1.23	1.13	1.07	1.03	1.02
	VSS *WLC*	2.13	2.07	1.92	1.71	1.50	1.29	1.12	0.97	0.85	0.76	0.69
2.0	Static $\overline{X}$&S	1.70	1.68	1.62	1.53	1.43	1.33	1.23	1.15	1.10	1.06	1.03
	VSS $\overline{X}$&S	1.51	1.49	1.44	1.37	1.27	1.18	1.08	1.00	0.92	0.85	0.80
	VSS *CCC*	2.08	1.99	1.91	1.88	1.77	1.66	1.57	1.47	1.33	1.26	1.16
	Static *WLC*	1.66	1.63	1.56	1.45	1.34	1.24	1.16	1.10	1.06	1.03	1.02
	VSS *WLC*	1.58	1.56	1.48	1.37	1.25	1.12	1.01	0.90	0.82	0.74	0.69

counterparts, except in some cells where the variance shifts are zero or small.

An average of ratios of *ATS*s (*ARATS*) is defined as a general and quantitative performance measurement for a chart compared with the VSS *WLC* scheme in a shift region of interest:

$$ARATS = \frac{\sum_{i=1}^{m} \frac{ATS\left(\delta_{\mu i},\ \delta_{\sigma i}\right)}{ATS_{VSS\ WLC}\left(\delta_{\mu i},\ \delta_{\sigma i}\right)}}{m} \tag{6}$$

where $\left(\delta_{\mu i},\ \delta_{\sigma i}\right)$ are the mean and standard deviation shifts in the *i*th cell, *m* is the number of the out-of-control cells in a region, $ATS\left(\delta_{\mu i},\ \delta_{\sigma i}\right)$ is the out-of-control *ATS* produced by a chart and $ATS_{VSS\ WLC}\left(\delta_{\mu i},\ \delta_{\sigma i}\right)$ by the VSS *WLC* scheme. Apparently, if a chart has an *ARATS* value larger than one, it is generally less effective than the VSS *WLC* scheme in that region and vice versa. The values of the *ARATS* are investigated in the following four regions:

1. The overall shift region $\left(0 < \delta_\mu \le 2.0 \text{ and } 1 < \delta_\sigma \le 2.0\right)$
2. The small shift region $\left(0 < \delta_\mu \le 0.6 \text{ and } 1 < \delta_\sigma \le 1.2\right)$
3. The moderate shift region (the range of $\left(0 < \delta_\mu \le 1.6 \text{ and } 1.0 \le \delta_\sigma \le 1.6\right)$, excluding the small shift region)
4. The large shift region (the overall region, excluding the small and moderate shift regions)

The *ARATS* values related to these four regions are denoted by $ARATS_{\mathrm{o}}$, $ARATS_{\mathrm{s}}$, $ARATS_{\mathrm{m}}$ and $ARATS_{\mathrm{l}}$, respectively. They are calculated and listed under RUN 0 in Table 3, based on the data in Table 2. For this particular Run 0, it can be seen that the VSS *WLC* scheme is very effective in most of the cases, except for small process shifts, while the VSS *CCC* scheme is quite sensitive to small process shifts.

Table 3 The average of ratios of *ATS*s (ARATS) values of the five charts ($\delta_{\mu d} = 0.6,\ \delta_{\sigma d} = 1.3$)

RUN	τ	R	$n_{\max}$	Chart	*ARATS*			
					$ARATS_s$	$ARATS_m$	$ARATS_l$	$ARATS_o$
0	400	5	10	Static $\overline{X}$&S	1.5259	1.2323	1.2751	1.2837
				VSS $\overline{X}$&S	1.6708	1.1367	1.0944	1.1747
				VSS *CCC*	0.6998	1.3815	1.5980	1.4080
				Static *WLC*	1.2090	1.0756	1.2248	1.1588
				VSS *WLC*	1.0000	1.0000	1.0000	1.0000
1	200	3	6	Static $\overline{X}$&S	1.4365	1.2284	1.1488	1.2141
				VSS $\overline{X}$&S	1.4904	1.1994	1.0934	1.1818
				VSS *CCC*	0.7763	1.3028	1.5034	1.3387
				Static *WLC*	1.1173	1.0309	1.0648	1.0558
				VSS *WLC*	1.0000	1.0000	1.0000	1.0000
2			14	Static $\overline{X}$&S	1.5195	1.3441	1.2079	1.3001
				VSS $\overline{X}$&S	1.5766	1.2967	1.1565	1.2621
				VSS *CCC*	0.6521	1.2478	1.5639	1.3296
				Static *WLC*	1.0133	1.1088	1.0860	1.0880
				VSS *WLC*	1.0000	1.0000	1.0000	1.0000
3		7	6	Static $\overline{X}$&S	1.1301	1.1908	1.3081	1.2384
				VSS $\overline{X}$&S	1.2345	1.0754	0.9850	1.0508
				VSS *CCC*	0.8478	1.3077	1.1603	1.1901
				Static *WLC*	1.1166	1.1565	1.2915	1.2145
				VSS *WLC*	1.0000	1.0000	1.0000	1.0000
4			14	Static $\overline{X}$&S	1.2128	1.2059	1.3131	1.2561
				VSS $\overline{X}$&S	1.3983	1.0487	0.9462	1.0391
				VSS *CCC*	0.7574	1.4623	1.3114	1.3167
				Static *WLC*	1.0855	1.1316	1.2866	1.1982
				VSS *WLC*	1.0000	1.0000	1.0000	1.0000
5	600	3	6	Static $\overline{X}$&S	1.8792	1.4261	1.1722	1.3577
				VSS $\overline{X}$&S	1.9570	1.3717	1.1234	1.3201
				VSS *CCC*	0.7398	1.3488	1.6412	1.4182
				Static *WLC*	1.1971	1.0494	1.0423	1.0621
				VSS *WLC*	1.0000	1.0000	1.0000	1.0000
6			14	Static $\overline{X}$&S	2.3106	1.6108	1.1797	1.4872
				VSS $\overline{X}$&S	2.4229	1.5317	1.1293	1.4419
				VSS *CCC*	0.6792	1.4298	1.7562	1.4996
				Static *WLC*	1.2329	1.1491	1.0535	1.1140
				VSS *WLC*	1.0000	1.0000	1.0000	1.0000
7		7	6	Static $\overline{X}$&S	1.2707	1.2356	1.4472	1.3371
				VSS $\overline{X}$&S	1.4563	1.0701	1.0480	1.1015
				VSS *CCC*	0.7191	1.4636	1.5068	1.4034
				Static *WLC*	1.1366	1.1556	1.4172	1.2743
				VSS *WLC*	1.0000	1.0000	1.0000	1.0000
8			14	Static $\overline{X}$&S	1.5650	1.2543	1.2839	1.3014
				VSS $\overline{X}$&S	1.7930	1.0727	0.9690	1.1024
				VSS *CCC*	0.7118	1.5875	1.5552	1.4783
				Static *WLC*	1.1755	1.1046	1.2443	1.1767
				VSS *WLC*	1.0000	1.0000	1.0000	1.0000

4.2 Study two: influence of design specifications

This study examines the influence of the chart design specifications on the chart performance. The three parameters (i.e. $n_{\max}$, τ and R) are firstly examined using a 2^3 factorial experiment [1]. Each of the parameters varies at two levels, i.e. 6 and 14 for $n_{\max}$, 200 and 600 for τ, and 3 and 7 for R. This results in eight runs (eight combinations of the values of the three specifications), with RUN 0 being the centre of the 2^3 factorial experiment. In all of the runs, the control charts are designed at the same shift point $(\delta_{\mu d} = 0.6,\ \delta_{\sigma d} = 1.3)$.

A table (not shown in this article) displaying the *ATS* values of the five charts can be produced for each of the eight runs. The performance comparison among the charts in these eight runs is similar to that as shown in Table 2 for RUN 0. The *ARATS* values in the four shift regions for the eight runs are also calculated and displayed in Table 3. It shows that the VSS *WLC* scheme is usually more powerful than other charts in the whole shift domain, especially in the moderate and large shift regions. Also similar to the previous studies, the VSS *CCC* scheme is, again, most effective in the small shift region. The performance of the VSS $\overline{X}$ &S chart is generally inferior to that of the *WLC* schemes in a few cases for large process shifts.

A grand average $\overline{ARATS}$, the average of the *ARATS* values for a chart over all runs, can be further calculated to compare the five charts from a more general viewpoint. The values of the $\overline{ARATS}$ in Table 4 $(\delta_{\mu d} = 0.6,\ \delta_{\sigma d} = 1.3)$ suggest that, over the whole shift domain, the VSS *WLC* scheme is more effective than the static *WLC* scheme, the VSS $\overline{X}$ chart, the static $\overline{X}$ chart and the VSS *CCC* scheme by 14.92%, 18.60%, 30.84% and 37.58%, respectively.

Finally, the influence of the design shift point on the chart performance is studied by investigating two more shift points $(\delta_{\mu d} = 0.4,\ \delta_{\sigma d} = 1.1)$ and $(\delta_{\mu d} = 0.8,\ \delta_{\sigma d} = 1.5)$. A similar 2^3 factorial experiment is carried out for each design point and the values of the grand average $\overline{ARATS}$ are calculated and listed in Table 4. The values of the grand average $\overline{ARATS}$ indicate that the relative effectiveness of the five charts remains similar to that for the design shift point $(\delta_{\mu d} = 0.6,\ \delta_{\sigma d} = 1.3)$.

In summary, the VSS *WLC* scheme is generally most effective in the moderate and large shift regions. Particularly, it is always most effective in the overall shift region. This is a very valuable feature because, in most cases, the type and size of a process shift are unknown and cannot be predicted and, hence, the capability of detecting process shifts in a broad shift domain is desired. Furthermore, if a more important factor, the ease of design and operation, is taken into account, the VSS *WLC* scheme is more likely to be the best choice for most of the applications where the process mean and variance need to be monitored simultaneously. It may be even viable to run a VSS *WLC* scheme manually for some SPC applications where an on-site computer is not affordable or allowed, but high detection effectiveness is required.

However, if detection effectiveness for small process shifts is the main concern in a particular application, the VSS *CCC* scheme may be considered and selected. It is noticed that the design and implementation of this scheme is quite complicated.

5 Design table for the VSS *WLC* scheme

For the users' convenience, a design table (Table 5) is provided for the VSS *WLC* scheme. The design is set up according to the specifications of three parameters, each at three levels (i.e. 200, 400 and 600 for τ; 3, 5 and 7 for R; 6,

Table 4 The $\overline{ARATS}$ values of the five charts

Design shift point	Chart	$\overline{ARATS}$			
		$\overline{ARATS}_s$	$\overline{ARATS}_m$	$\overline{ARATS}_l$	$\overline{ARATS}_o$
$(\delta_{\mu d} = 0.4,\ \delta_{\sigma d} = 1.1)$	Static $\overline{X}$&S	1.8521	1.2386	1.1591	1.2680
	VSS $\overline{X}$&S	1.8470	1.2429	1.1512	1.2656
	VSS *CCC*	0.8789	1.3016	1.3676	1.2866
	Static *WLC*	1.2627	1.0325	1.1022	1.0895
	VSS *WLC*	1.0000	1.0000	1.0000	1.0000
$(\delta_{\mu d} = 0.6,\ \delta_{\sigma d} = 1.3)$	Static $\overline{X}$&S	1.5389	1.3031	1.2596	1.3084
	VSS $\overline{X}$&S	1.6666	1.2003	1.0606	1.1860
	VSS *CCC*	0.7315	1.3924	1.5107	1.3758
	Static *WLC*	1.1426	1.1069	1.1901	1.1492
	VSS *WLC*	1.0000	1.0000	1.0000	1.0000
$(\delta_{\mu d} = 0.8,\ \delta_{\sigma d} = 1.5)$	Static $\overline{X}$&S	1.3069	1.2794	1.2815	1.2833
	VSS $\overline{X}$&S	1.5663	1.1638	1.0073	1.1349
	VSS *CCC*	0.6290	1.3879	1.5177	1.3661
	Static *WLC*	1.0305	1.0669	1.1592	1.1056
	VSS *WLC*	1.0000	1.0000	1.0000	1.0000

10 and 14 for n_{max}). All of the charts are optimised at the medium design shift point $(\delta_{\mu d} = 0.6,\ \delta_{\sigma d} = 1.3)$. The design table presents the values of n_1, n_2, h, $\frac{w_A}{\sigma_0^2}$, $\frac{H_A}{\sigma_0^2}$ and $\frac{k_A}{\sigma_0^2}$ for each set of τ, R and n_{max}. Alternatively, the users can write a computer program to design a VSS *WLC* scheme for a specific set of design specifications.

6 Diagnosis of process shifts

The capability to diagnose process shifts is desired for SPC. Diagnosis means identifying the shift type, estimating the shift magnitude and locating the process change point (the time when the process shift occurs). An accurate diagnosis result facilitates the users to troubleshoot the problem and eliminate assignable causes [18, 29].

The CUSUM type charts (e.g. the *CCC* or *WLC* schemes) are relatively more capable than the $\overline{X}$ &S type charts for estimating the change point by inspecting patterns of sample points. The time from which onward sample points are invariably larger than zero can be taken as the change point for CUSUM type charts. For a VSS *WLC* scheme, the magnitude of the mean shift and variance shift can be estimated as [1]:

$$\widehat{\delta}_\mu = \frac{1}{\sigma_0}\left(\frac{n_1\sum_{j=1}^{m_1}\overline{x}_{1j} + n_2\sum_{j=1}^{m_2}\overline{x}_{2j}}{m_1 n_1 + m_2 n_2} - \mu_0\right) \tag{7}$$

$$\widehat{\delta}_\sigma = \frac{1}{\sigma_0}\sqrt{\frac{\frac{(n_1-1)\sum_{j=1}^{m_1}s_{1j}^2}{m_1} + \frac{(n_2-1)\sum_{j=1}^{m_2}s_{2j}^2}{m_2}}{n_1+n_2-2}} \tag{8}$$

where m_1 and m_2 are the numbers of the relax and alert samples taken after the estimated change point, $\overline{x}_{ij}$ and s_{ij}^2 are the sample mean and sample variance, respectively, of the jth sample of size n_i after the change point, which are available during the calculation of A_t at each sample point. The magnitudes of the estimated shifts may, in turn, help decide the type of shift.

Table 5 Design table of the VSS *WLC* scheme $(n_{max} = \{6,\ 10,\ 14\},\ \delta_{\mu d} = 0.6,\ \delta_{\sigma d} = 1.3)$

Input			Output						
τ	R	n_{max}	λ	n_1	n_2	h	k_A/σ_0^2	w_A/σ_0^2	H_A/σ_0^2
200	3	6	0.31	4	6	1.40	0.6383	0.7113	1.4216
		10	0.28	8	10	2.73	0.5525	0.1990	0.4974
		14	0.31	9	13	3.27	0.4962	0.1086	0.4481
	5	6	0.29	3	5	0.64	0.6983	1.1056	2.3600
		10	0.30	5	9	1.16	0.5434	0.3921	1.1651
		14	0.31	10	14	2.16	0.4993	0.0740	0.4420
	7	6	0.28	3	5	0.46	0.7302	0.9526	2.3586
		10	0.30	5	9	0.77	0.5841	0.5336	1.2501
		14	0.31	9	14	1.36	0.5338	0.1818	0.5335
400	3	6	0.31	3	5	1.07	0.7196	1.0214	2.3474
		10	0.29	5	8	1.87	0.5574	0.3628	1.2249
		14	0.30	9	13	3.13	0.5172	0.2194	0.5531
	5	6	0.31	5	6	1.02	0.6143	0.5092	1.4465
		10	0.29	5	8	1.06	0.5590	0.5758	1.4086
		14	0.31	9	14	1.90	0.5331	0.1870	0.5861
	7	6	0.31	4	6	0.60	0.6607	0.6456	1.8792
		10	0.30	5	9	0.77	0.5795	0.5751	1.4588
		14	0.31	8	14	1.23	0.4983	0.3543	0.8590
600	3	6	0.29	4	6	1.40	0.6342	0.7647	1.8505
		10	0.32	6	10	2.13	0.5828	0.3502	0.9775
		14	0.28	7	12	2.50	0.5374	0.3350	0.8535
	5	6	0.29	5	6	1.06	0.6278	0.0287	1.4196
		10	0.29	6	10	1.28	0.5663	0.3980	1.1385
		14	0.28	9	14	1.90	0.5152	0.2291	0.6867
	7	6	0.31	5	6	0.76	0.6331	0.0304	1.5036
		10	0.29	5	9	0.77	0.5468	0.7523	1.7528
		14	0.28	9	14	1.36	0.5215	0.2170	0.7255

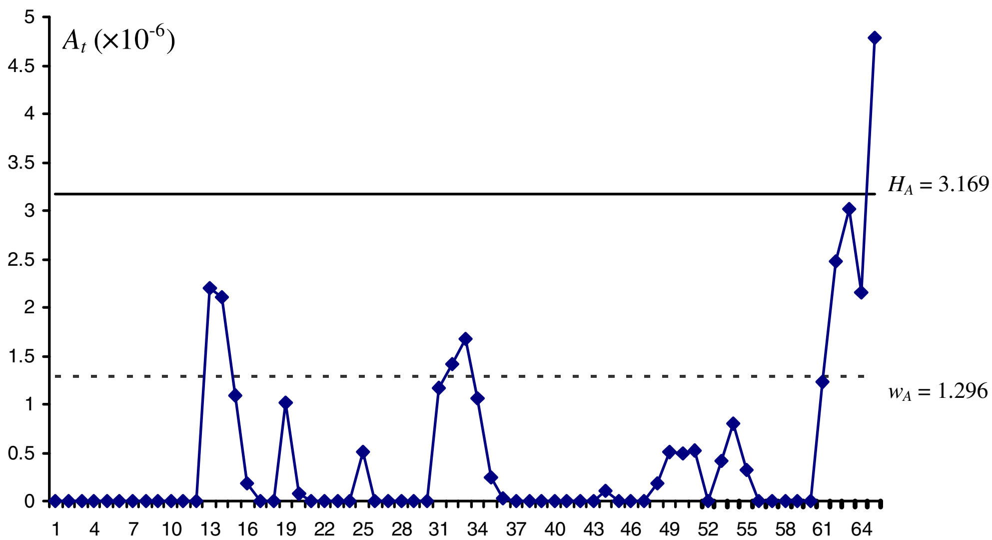

Fig. 1 The operation of a VSS *WLC* scheme

7 A case study

A manufacturing factory produces a shaft for an aero-engine. The diameter x of the shaft is a key dimension. Its nominal value and tolerance are specified as 74±0.009 mm. From the observations during the pilot run, the probability distribution of x is found to be approximating a normal distribution. The standard deviation σ_0 is estimated as 0.0015 mm. The process mean μ_0 can be easily adjusted to the nominal value of 74 mm.

A VSS *WLC* scheme is to be designed to monitor both the mean and variance shifts of x. The design specifications are decided as $n_{\max}$=10, τ= 400 h and R=5/h. From Table 5, the values of the charting parameters are found as n_1=5, n_2=8, h=1.06, w_A/σ_0^2=0.5758, H_A/σ_0^2=1.4086 and k_A/σ_0^2=0.5590. Thus, the parameters w_A, H_A and k_A are determined as 1.296×10^{-6} mm, 3.169×10^{-6} mm and 1.258×10^{-6} mm, respectively.

The values of *ATS* versus the process shift in the domain of $\left(0 < \delta_\mu \leq 2,\ 1 < \delta_\sigma \leq 2\right)$ for this VSS *WLC* scheme are enumerated in Table 2. A simulated charting process of this VSS *WLC* scheme is illustrated in Fig. 1. The process stays in control until t=60, when an out-of-control case takes place, in which $\delta_\mu = 0.6$ and $\delta_\sigma = 1.4$.

The problem is signalled by the VSS *WLC* scheme at t= 65. The pattern of the sample points in Fig. 1 suggests that the process become abnormal around t=60, since all of the subsequent sampling points (three alert samples) result in an A_t value greater than zero. The mean shift and standard deviation shift are estimated as $\widehat{\delta}_\mu = 0.5477$ and $\widehat{\delta}_\sigma = 1.3764$, which are quite close to the actual ones. Since $\widehat{\delta}_\mu > 0$ and $\widehat{\delta}_\sigma > 1$, it is diagnosed that both increasing mean shift and increasing variance shift have happened.

8 Conclusion

A VSS *WLC* (variable sample sizes weighted loss function cumulative sum) scheme for monitoring process shifts is proposed in this work. The scheme helps improve the effectiveness in detecting the mean shifts and increasing variance shift in a broad process shift domain. The improvement is achieved without increasing the false alarm rate and the inspection rate. While one competitor control chart (the VSS *CCC* scheme) is sensitive to small process shifts, the VSS *WLC* scheme averagely outperforms other charts when a broad process shift domain is taken into consideration.

More importantly, the VSS *WLC* scheme is much simpler in design and operation. The VSS *WLC* scheme consists of a single CUSUM scheme which does not interact with other charts. As a result, the design can be carried out in a well-formulated procedure and the operation is analogous to a CUSUM chart only monitoring a one-sided mean shift. The research also shows that the VSS *WLC* scheme has the capability of diagnosing mean and/or variance shifts.

The VSS *WLC* scheme suits the scenario where the strategy of varying sample sizes is feasible and preferable for achieving a higher capability of detecting process variations.

Appendix 1: calculation of the cumulative probability function $F(Y)$ of *WL* [22]

The weighted loss function *WL* (Eq. 1) is greater than Y when:

$$\bar{x} < \mu_0 - a \text{ or } \bar{x} > \mu_0 + a \text{ or}$$
$$\mu_0 - a < \bar{x} < \mu_0 + a \text{ and } s^2 > \frac{Y-(1-\lambda)(\bar{x}-\mu_0)^2}{\lambda} \quad \text{(A1)}$$

where:

$$a = \sqrt{\frac{Y}{1-\lambda}} \tag{A2}$$

Therefore, the probability that WL is greater than Y can be calculated by conditioning on the value of the sample mean $\bar{x}$:

$$\Pr(WL > Y) = P_1 + P_2 + P_3 \tag{A3}$$

$$P_1 = \Pr(\bar{x} < \mu_0 - a) = \Phi\left[\frac{-\sqrt{n}(a + \delta_\mu \sigma_0)}{\delta_\sigma \sigma_0}\right] \tag{A4}$$

$$P_2 = \Pr(\bar{x} > \mu_0 + a) = 1 - \Phi\left[\frac{\sqrt{n}(a - \delta_\mu \sigma_0)}{\delta_\sigma \sigma_0}\right] \tag{A5}$$

$$P_3 = \int_{\mu_0 - a}^{\mu_0 + a} p(\bar{x}) f(\bar{x}) d\bar{x} \tag{A6}$$

where $f(\bar{x})$ is the probability density function of $\bar{x}$, which has a normal distribution with mean equal to $\mu_0 + \delta_\mu \sigma_0$ and variance equal to $\frac{(\delta_\mu \sigma_0)^2}{n}$, and $p(\bar{x})$ is the probability that WL is greater than Y for a given $\bar{x}$. From Eq. A1:

$$\begin{aligned} p(\bar{x}) &= \Pr\left(s^2 > \frac{Y - (1-\lambda)(\bar{x} - \mu_0)^2}{\lambda}\right) \\ &= \Pr(Q > b(\bar{x})) \\ &= 1 - \chi^2_{n-1}(b(\bar{x})) \end{aligned} \tag{A7}$$

where:

$$b(\bar{x}) = \frac{(n-1)\left[Y - (1-\lambda)(\bar{x} - \mu_0)^2\right]}{\lambda(\delta_\sigma \sigma_o)^2} \tag{A8}$$

The random variable Q follows a Chi-square distribution with (n−1) degrees of freedom.

The cumulative probability function $F(Y)$ of WL is calculated by:

$$\begin{aligned} F(Y) &= \Pr(WL \le Y) \\ &= 1 - \Pr(WL > Y) \\ &= 1 - P_1 - P_2 - P_3 \end{aligned} \tag{A9}$$

Appendix 2: calculation of the in-control ATS_0 and out-of-control ATS of the VSS WLC scheme

The VSS WLC scheme can be described by a Markov chain 0 using M different transitional in-control states (from state zero to state (M−1)) with equal width d=H_A/M. The value of M is set as 100 in our computation. The centre, O_i, of state i is equal to $i \cdot d$. Furthermore, states 0 to g are in the central zone and states (g+1) to (M−1) are in the warning zone, where g is the integer closest to w_A/d. Let p_{ij} be the transition probability from state i to state j:

$$p_{ij} = \begin{cases} \Pr[O_i + WL - k_A \le 0.5d] = F[(0.5 - i)d + k_A] & \text{for } j = 0 \\ \Pr\left[O_j - 0.5d \le O_i + WL - k_A \le O_j + 0.5d\right] & \text{for } j > 0 \\ \quad = F[(j - i + 0.5)d + k_A] - F[(j - i - 0.5)d + k_A] & \end{cases} \tag{B1}$$

where $F(\cdot)$ is determined by Eq. A9 with $\delta_\mu = 0$ and $\delta_\sigma = 1$. Furthermore, the sample size n_1 should be used if $i \le g$ and n_2 should be used if $i > g$.

The in-control transition probability matrix $\mathbf{R}_0$ of size (M×M) can be established using p_{ij} as its elements. Then, the in-control ATS_0 is equal to the first element of vector $\mathbf{U}$ given by the following expression:

$$\mathbf{U} = (\mathbf{I} - \mathbf{R}_0)^{-1}\mathbf{H} \tag{B2}$$

where $\mathbf{I}$ is an identity matrix and $\mathbf{H}$ is the sampling interval vector with all elements being h.

The in-control stabilised probability vector $\mathbf{B}$=[b_0, b_1,..., b_{M-1}]^T is obtained by solving the following equation:

$$\mathbf{B} = \mathbf{R}_0^T \mathbf{B}, \text{ subject to } \mathbf{1}^T \mathbf{B} = 1 \tag{B3}$$

where **1** is a vector of ones. The stabilised probability associated with the alert samples is:

$$B_2 = \sum_{i=g+1}^{M-1} b_i \tag{B4}$$

B_2 is used to calculate the average inspection rate r in Eq. 3.

The out-of-control transition probability matrix $\mathbf{R}$ for calculating the out-of-control ATS can be derived similarly as $\mathbf{R}_0$, except that the elements p_{ij} of $\mathbf{R}$ must be evaluated under the out-of-control conditions $\delta_\mu \neq 0$ or/and $\delta_\sigma > 1$. It is assumed that the control statistic A_t has reached its stationary distribution at the time when the process shift occurs. It is also assumed that the random time of process shift has a uniform distribution within the sampling interval [13]. Thus, the out-of-control ATS is calculated by:

$$ATS = \mathbf{B}^T \left[(\mathbf{I} - \mathbf{R})^{-1} \mathbf{H} - \frac{\mathbf{H}}{2} \right] \tag{B5}$$

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