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ORIGINAL ARTICLE

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Three-half and half-axis patch-by-patch NC machining of sculptured surfaces

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Abstract Despite the inbuilt advantages offered by five-axis machining, the manufacturing industry has not widely adopted this technology due to the high cost of machines and insufficient support from CAD/CAM systems. Companies are used to three-axis machining and their shop floors are not yet ready for five-axis machining in terms of training and programming. The objective of this research is to develop and implement a machining technique that uses the simplicity of three-axis tool positioning and the flexibility of five-axis tool orientation, to machine sculptured surfaces. This technique, $3\frac{1}{2}\frac{1}{2}$-axis, divides a sculptured surface into patches and then machines each patch using a fixed tool orientation. This paper presents the surface partitioning scheme and the method of selecting an optimum number of sub-divisions along with actual machining experiments. For the example surface utilized in this study, the proposed hybrid method led to shorter machining time compared to traditional three-axis machining and comparable to simultaneous five-axis machining.

Keywords Five-axis machining · Surface partitioning sculptured surfaces · 3+2-axis machining

1 Introduction

Complex surfaces are conventionally machined on three-axis milling machines with ball nose end milling cutters. The trajectory of the tool is determined by offsetting the design surface. The tool centre moves from point to point along curves that lie on the offset surface. The spacing between the curves is called side step, while the spacing between subsequent points along the pass is called the feed forward step. The side step determines the number of passes required to machine a surface. The smaller the side step is, the larger the machining time will be.

Recently, five-axis machines have been used to machine complex surfaces. Five-axis machines change the tool or the workpiece orientation by using two additional axes to match the shape of the tool to the shape of the surface [1]. Since the shape of the tool and that of the surface match, the side step between passes would be much larger than in three-axis machining [2]. The reduced number of passes typically leads one to believe that five-axis machining would result in reduced machining time compared to three-axis machining. In reality, this is not always the case, because the rotary axes on a tilt/rotary table or on the headstock cannot turn fast enough to keep up with linear axes and thus slow down the actual feed rate of the tool. The slow down is further accentuated by the singularity point associated with the kinematics of these machines. Near the singularity point a small change in tool axis direction results in large rotation of the rotary axes.

Shape matching tool positioning methods on five-axis tool machining are relatively new and have not been adopted by CAM companies within their software. This increases the difficulty of programming five-axis machines. Furthermore, the kinematics complexity of five-axis machines introduces many questions regarding accuracy and accessibility for tool positioning [3]. Besides, five-axis machines are expensive to buy and require extensive training before they can be used effectively compared with the traditional three-axis machines.

Many industries have invested in machines that can change the tool orientation in discrete steps either manually or automatically. Such machines are used for five sided machining where orientation facilitates accessibility. The various sides of the part can be machined in one setup. Such machines are much less expensive than simultaneous five-axis machines and do not require excessive training because the tool trajectory on these machines is calculated using three-axis methods and software, which are well known on the shop floor. The availability of these machines provides the motivation of this work; to combine the flexibility of orientation offered by true five-axis machines,

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while maintaining low cost and ease of programming offered by three-axis machines.

2 Related work

Three-axis machining and five-axis machining have opposing characteristics as discussed above. Whereas the side step is larger in five-axis machining the sustainable feed rate is higher in three-axis machining. Many researchers have attempted to take advantage of five-axis methods without using the expensive five-axis machines. Suh et al. [4, 5] developed a CAM method by which five-axis machining can be carried out with a three-axis CNC machine together with a tilt/rotary table. In this method, the part surface and the machining environment are converted into a digitized workspace map. All the possible part setups that satisfy the machinability conditions are identified. The part surface is divided in a way that minimizes the number of part setups as well as the surface ridges where multiple tool paths join. While feasible, this approach requires extensive computation and does not guarantee optimality of the subdivision.

A surface division based on geometry interrogation was also developed by Lauwers et al. [6] to facilitate process planning for multi-axis machining. In this method the surface is divided into regions characterized by a preferred milling direction and tool diameter. Convex, concave, saddle and flat areas are identified based on curvature properties and they are milled separately. However, as demonstrated in [7] geometric parameters provide local estimations and are inferior to global information in complex settings.

Chen et al. [8] proposed a technique to bridge the gap between three- and five-axis machining. In their $3\frac{1}{2}\frac{1}{2}$-axis method, the surface is partitioned using clustering technique into a fixed number of patches. The partitioning is done on the basis of a collection of local geometric parameters such as the curvature, the normal and other surface parameters. The average normal for each patch is identified to determine the rotations of the part required to make the average normal vertical. The part is held in this orientation and the patch is machined using three-axis methods. Since simultaneous movement of five-axis is not required, this technique can be implemented on discrete machines used for five-sided machining. Chen et al. [8] partitioned an example surface into fourteen patches, but did not conduct machining tests or tool path calculations to validate the concept. Furthermore, they did not present any method for determining the tool orientation, and if the average normal is used to align the tool axis, one is limited to using a ball nose cutter since other efficient tools such as toroidal cutters would result in gouging. Actually, if the tool orientation proposed in this work is used, the 14 patch subdivision will increase the machining time in comparison to machining the surface as a single patch. As the number of patches increases the tool path length is reduced, but the machining time must also account for surface reorientation and air cutting which for a 14 patch subdivision can be significant and was not considered by the authors.

3 Proposed strategy

Both five-axis and three-axis machining offer advantages. In three-axis machining a higher feed rate can be achieved, and in five-axis machining a wider side step can be realized because of the better match between the tool and the surface shapes. The proposed strategy is designed to take advantage of these combined traits by dividing the surface into patches then determining an ideal tool orientation for each patch and followed by machining of each patch with a fixed tool orientation. The strategy ensures that within a patch the shape of the surface does not vary greatly from the shape of the tool in a particular orientation. This allows the method to have a larger side step compared to three-axis. At the same time the feed rate is high because the tool does not change its orientation within a patch. The method, however, introduces additional tool travel when the tool has to move from one patch to another and requires workpiece re-orientation which can take some time. On the one hand, if the number of patches is large, the overhead due to the movement between patches and due to re-orientation can be larger than the gains of the method. On the other hand, if the number of patches is small, the benefit of the method is not fully realized since the shape of the tool may vary greatly from that of the surface. Accordingly, a technique for selecting the optimum number of partition is also presented in this paper.

The proposed method begins by identifying a strategy that offers control on the number of patches the surface is divided into. The shape of the patch is influenced by the geometric properties used in the subdivision algorithm. This work presents a classification of these properties and proceeds to identify a set of groupings that results in good partitioning. Next, a method to identify the tool orientation to machine a patch is presented and, lastly, a method to determine the optimum number of patches to achieve minimum machining time is presented. The proposed strategy is implemented and tested on a sample surface and the test results are presented and discussed.

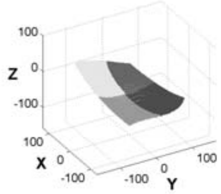
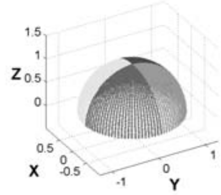
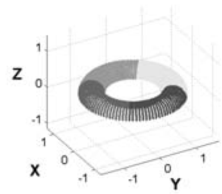
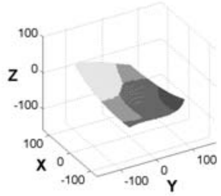
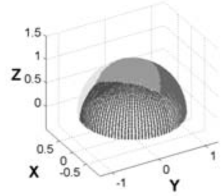
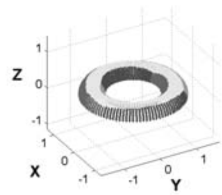
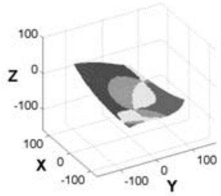
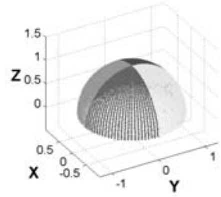
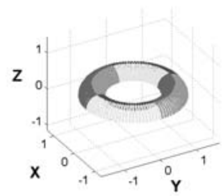
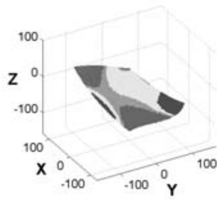
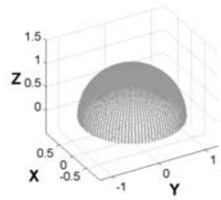
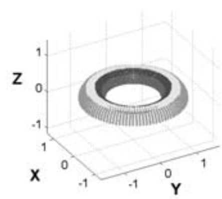
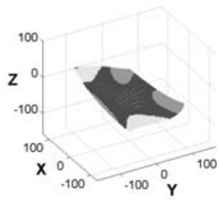
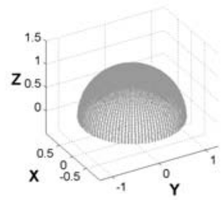
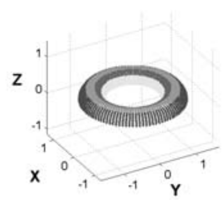
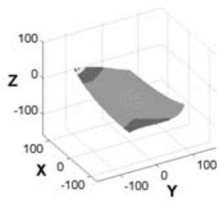
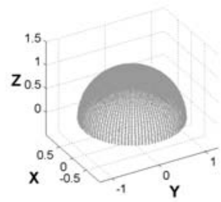
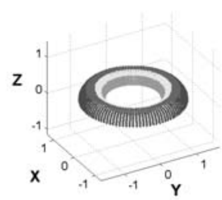
4 Surface partitioning analysis

Surface partitioning divides a surface into patches so that in each patch the surface variation is minimum. In the present work, the fuzzy c-means algorithm is selected to find the required patches. Fuzzy c-means is a technique where each data point belongs to a cluster to some degree that is specified by a membership grade [9]. Multidimensional space points can be grouped into a specific number of different clusters based on this grade. This algorithm is based on a generalization of the sum of square error function and divides the data set in a finite number of subsets [10]. Chen et al. [8] used the geometric properties of the surface to form a multidimensional vector that is utilized to partition the surface. In their work, all surface properties, namely: parametric coordinates, Gaussian and mean curvatures, maximum and minimum curvatures and the surface normal were lumped together into one vector for partitioning and were assigned the same weights. This vector included curvature in various forms that could result in redundancy that could be reflected in an inadequate partitioning of the surface.

4.1 Surface partitioning parameters

One of the main challenges in clustering is the selection of the properties that are used in the identification of the clusters. In this work, experiments were carried out with the c-means algorithm to identify the most relevant parameters. Different geometric properties which include the parametric duo (U, V), the normal vector (N), the tilt/rotary angles (A, C), the tool axis position (T), the Gaussian curvature (K), the mean curvature (H) and the principal curvatures ($K_{\max}$, $K_{\min}$) were used to test the partitioning of three known surfaces. The surfaces considered in this study were a half sphere, a half torus and a Bézier surface. The

Table 1. Clustering results of known surfaces using different properties

Clustering parameters	Surface	$\frac{1}{2}$ Sphere	$\frac{1}{2}$ Torus
U, V			
N			
A, C			
K			
H			
$K_{\max}$, $K_{\min}$			

half-sphere was selected since it is uni-curvatured and is a simple concave open surface. The half torus was selected because of its wide variation in curvature, and the Bézier surface was selected because it closely resembles surfaces in dies and moulds.

The effect of the different combination of parameters on surface partitioning was investigated. The surface was represented by a sample of points and the properties at these points were used as the input to the c-means algorithm. The output was a grouping of these points into clusters (patches), if however, the points did not lie in one closed region it would represent two or more disjointed sub-patches. An example of the partitioning is shown in Table 1 in which each surface was partitioned into four patches. This table lists the properties employed, in column one, and shows the partitioning results for the test Bézier surface, half-sphere and half-torus in columns two, three and four, respectively. If the number of disjointed sub-patches is large then the movement of the tool between patches will increase significantly and thus the partitioning is not considered useful. Such disjointed sub-patches occur for the Bézier surface in Table 1 when the tool axis inclination angles (A, C) and the curvature parameters were used to partition the surface.

The parameter duo (U, V) and the normal vector (N) are the most appropriate parameters to represent the proximity and orientation parameters, respectively. The curvature divided the half sphere and the half torus adequately but its partitioning of the Bézier surface was disjointed. Considering that the various curvature parameters lead to similar divisions, all curvature parameters should not be included in the partitioning group to prevent redundancy. The mean curvature (H) was found to be the most appropriate parameter to represent the curvature property.

To summarize, geometric properties can be classified into three groups, namely: proximity parameters, orientation parameters, and curvature parameters. It was determined that the multi-dimensional vector describing the geometry at a point on the surface must include information about the location of the point, the orientation of the normal and the curvature. The location of the point, defined by proximity parameters, helps to keep neighboring points in one cluster and in avoiding disjointed patches. The properties included in this paper are the parameter duo (U, V). The orientation parameters provide information on the orientation of the workpiece and the tool. Points that have similar orientation parameter can be machined with the same tool orientation and will result in similar surface finish. The orientation parameter is represented by the surface normal. The curvature parameter reflects the rate of change of the surface in the vicinity of the point and it is sufficient to represent it by the mean curvature.

5 Selecting the best partitioning of a surface

A Bézier surface was selected in the current investigation mainly for its mathematical simplicity. Other surfaces, however, can readily be used in its place. The parameters identified above can be grouped together in a variety of combinations. All the parameters have to be normalized for the clustering experiments. Different weights can be assigned to these parameters which further increase the possibilities. Figure 1 show four classification vectors, UVN, UVNH, UV2N and UV4N, utilized to divide the Bézier surface. The results obtained using the UVN and UVNH classification vectors, shown in Fig. 1a and b, yield similar partitionings. Utilizing the mean curvature does not reflect a considerable influence for this surface. Considering that the curvature of the surface is also implicit in the normal vector, the classification vector UVNH which includes both N and H will not be pursued any further. On the other hand, in the last two groups the normal is given a weight of two and four, respectively to emphasize its impact. Overemphasizing the weight of the normal, however, increases the probabilities of obtaining disjointed patches, as illustrated in Fig. 1d, which complicates the tool path generation.

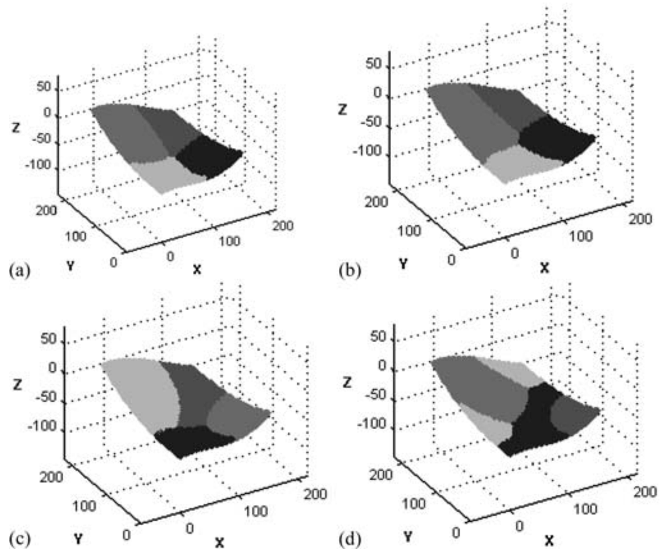


Fig. 1a–d. Clustering using weights and combined elements **a** UVN classification vector **b** UVNH classification vector **c** UV2N classification vector **d** UV4N classification vector

The classification vector UVN consistently resulted in large joined regions that were geometrically similar and were thus good classifications. Thus, this classification vector will be employed for the rest of the work. After a good classification vector is identified, one parameter still remains to be identified, namely, the number of sub-divisions or patches. In this work a partitioning is considered optimal if it requires the least amount of time to finish-machine the surface. This principle is used as the basis for developing a method to determine the optimal number of sub-division and is described later.

5.1 Tool orientation

A method for determining the tool orientation for each patch is developed here to prevent gouging. To calculate the tool axis, T, the normals, N_j's, at all points within the patch are moved to a common origin of a coordinate system as shown in Fig. 2a. In this figure the dimensions X, Y and Z are normalized with respect to the size of the patch. The N_j's form an irregular cone in this coordinate system. The N_j's are now projected onto a plane defined by the feed direction, Fy, and the vertical Fz axis. The

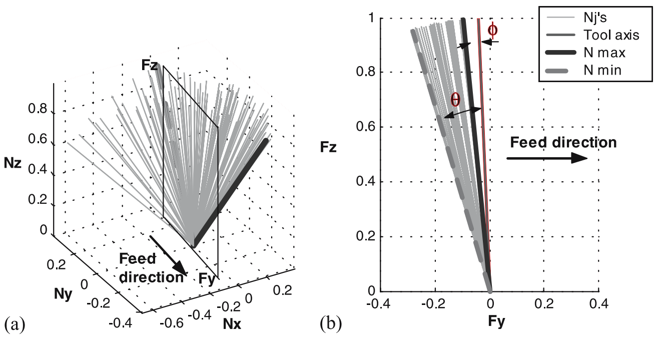


Fig. 2a,b. Tool axis inclination with respect to normal cone. **a** 3-D display of the normal vectors positioned in the origin **b** 2-D projection of the normal vector and tool axis with respect to the feed direction

projected normals are shown in Fig. 2b. T is determined from this figure. The projected N_j's are bounded in a small sector identified by the two bold lines. If T is selected to lie inside the bold lines then the tool can be imagined to be machining the bottom of a concave region as shown in Fig. 3a. At the bottom, the tool axis T is aligned with some normal N_a. In Fig. 3a the tool will gouge the surface. This can be avoided only if T is outside the bounded region as shown in Fig. 3b. If T is inclined by a large angle ϕ relative to the bold line then the benefit of inclining a tool are lost as the effective radius of the cutter becomes small. The effective radius is defined as the radius of curvature of the tool at the point of contact. The radius of curvature is infinity at the bottom of the insert (radius) and is equal to the radius of the insert at the side. The inclination angle, ϕ must be kept as small as possible provided it prevents gouging. In this work this angle is chosen to be 3 degrees; an inclination that avoids gouging into the selected surface in this study. If the feed direction is changed the tool inclination will change in this method. Furthermore, this method applies to concave surfaces. For convex surfaces ϕ is selected to be zero for improved efficiency. If a patch has concave and convex regions it is treated as a concave surface for the purpose of determining the tool axis T.

To prevent gouging, the tool axis, T, is calculated relative to $N_{\max}$, the most inclined normal vector with respect to the feed direction, F, by including the angle ϕ and is computed from

$$T = \frac{N_{\max}}{|N_{\max}|} \cdot \cos(\phi) - \frac{(N_{\max} \times F) \times N_{\max}}{|(N_{\max} \times F) \times N_{\max}|} \cdot \sin(\phi). \tag{1}$$

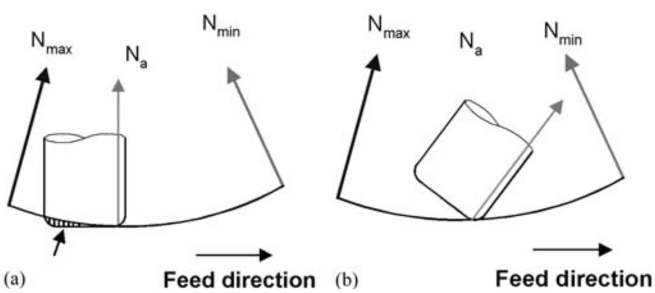
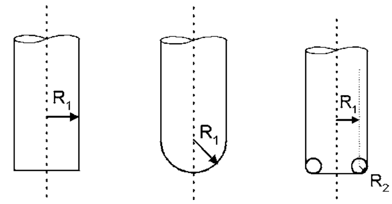


Fig. 3a,b. Using N_a to calculate the tool orientation **a** inside the cone, gouging, **b** outside the cone, no gouging

Fig. 4. **a** Flat end-mill, **b** ball nose end-mill and **c** toroidal end-mill



5.2 Tool position

Three different types of cutting tools, shown in Fig. 4 could be considered for this method: ball nose, toroidal and flat end-mills. A toroidal end-mill is selected for this surface since it helps achieve a better surface finish and higher efficiency. If the tool axis is along vector T and the normal at contact point C_j is N_j then the tool position P_j (bottom centre of the tool) is given by

$$C_j = P_j + R_2 N_j + \frac{N_j - (N_j \cdot T) \cdot T}{\left|N_j - (N_j \cdot T) \cdot T\right|} \cdot R_1 \tag{2}$$

where R_1 is the radius of the tool and R_2 is the radius of the insert. The toroidal tool can model both the ball nose ($R_1 = 0$, $R_2 = R$) and flat end milling cutter ($R_1 = R$, $R_2 = 0$).

5.3 Side step

Since the size and length of the patches are known, an estimate of the number of passes and the corresponding machining time can be obtained using the largest side step allowable. For typical surfaces encountered in industry the side step can be approximated using the surface tolerance, ε,

$$a \approx \sqrt{2 \cdot R_{eff} \cdot \varepsilon} \tag{3}$$

and the effective radius of the tool, R_{eff},

$$R_{eff} = R_2 + \frac{R_1}{\sin(\theta)} \approx \frac{R_1}{\sin(\theta)} \quad \text{for} \quad R_2 \ll R_1 \tag{4}$$

where θ is the angle between T and N_j.

5.4 Implementation

A new method for determining the best partitioning is developed in this work. Since the number of patches that results in the smallest machining time is not known a priori, the surface is partitioned into patches ranging from one to 16 (this range is user selected and can be modified within this method). Based on the clustering results, an estimated machining time for all the partitions is calculated and the one that results in the smallest time is chosen for machining.

The shape of the patches in any partitioning is complex and is outlined using a multi-segment boundary. The points belonging to a patch are processed in order to identify whether a point is inside or on the boundary using the nearest neighbour method. In this method, the points defining a patch are scanned and the patch boundary is extracted. To extract the patch boundary, the method uses the fact that the surface points used for clustering lie along a rectangular grid in parametric space. So if a point has four neighbours that all belong to the same cluster (patch), it is classified as lying inside the patch, otherwise it is classified as lying *on the boundary.* The boundary points are extracted and grouped into a boundary group, as illustrated in Fig. 5a. For a given surface there may be many boundary groups. Each point in the boundary group is unique. If a new point is to be classified, the Euclidean distance of the point is found for each point in a boundary group and the minimum distance is called the classification distance. The classification distance is obtained for all the boundary grouping and the boundary grouping that results in the smallest classification distance is used to identify the cluster (patch) that contains the new point.

Two types of tool paths can be selected by the user to estimate the machining time: zig-zag tool path or unidirectional cuts along parallel passes. The tool inclination is calculated for each patch and used to compute the side step. The tool path length for each patch is estimated using the side step to generate a tool path over the patch. The tool passes are defined using a set of curves corresponding to constant values of either parameter U or V, depending on the feed direction and limited by the boundaries of the patch as shown in Fig. 5b. The side step within a patch is kept constant and the tool moves at the cutting feed rate if either the start contact point or the end contact point lies on the surface; otherwise it moves at the rapid traverse rate. The time required to machine each patch is calculated by adding the retraction time estimate to the computed cutting time. From observations of the machine used in this study, five seconds is added each time to account for time consumed when a tool changes its orientation. The partitioning that results in the smallest estimated time is selected for the actual machining process.

5.5 Test case and cutting experiments

The method outlined above was applied to the Bézier surface shown in Fig. 6, and the control points for the surface are given in Table 2. Actual machining tests of the sculptured surface were conducted to validate the $3\frac{1}{2}\frac{1}{2}$-axis machining method. The cutting was done on a Deckel Maho five-axis machining centre. Although this machine could move the five-axis simultaneously, the program was designed to machine each patch in three axes, X, Y and Z. The axes A and C were only used to set the inclination of each patch. In this way the machine, in effect, becomes a three-axis with a tilt/rotary fixture. Every surface patch has a different tool orientation, side step, and is defined using a range of surface parameters U and V.

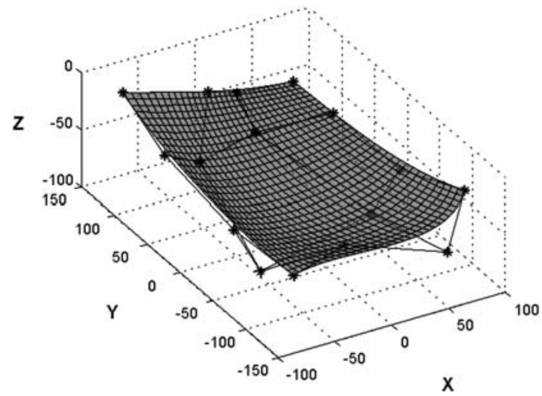


Fig. 6. Bézier surface and 16 control point polyhedron

Table 2. Control points for the Bézier surface

[0, 0, −47]	[0, 75, −52]	[0, 150, −42]	[0, 225, −5]
[50, 0, −35]	[50, 75, −99]	[50, 150, −56]	[50, 225, 0]
[100, 0, −65]	[100, 75, −79]	[100, 150, −28]	[100, 225, −37]
[150, 0, −17]	[150, 75, −49]	[150, 150, −50]	[150, 225, −53]

The proposed method was applied to the surface and the optimal partitioning for the surface was obtained from the estimated machining times shown in Fig. 7. Although the tool path length decreases continuously, the four-patch subdivision provides the minimum machining time for this surface and is illustrated in Fig. 8. A single patch configuration is also considered to provide a reference. Small marks within allowed tolerance in the final machined surface were generated between the patches' boundaries as shown in Fig. 9. In cases when the patch boundary is parallel to the feed direction the marks are almost negligible.

Table 3 presents a summary of the results obtained from the cutting tests including machining time and tool path length. The experiments confirmed that the surface divided into four patches has the shorter machining time. Comparing the estimated and the machining time, it can be seen that the time

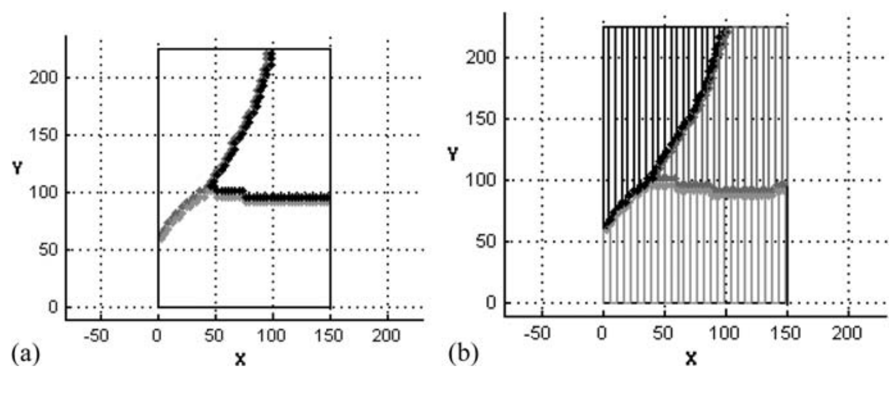


Fig. 5a,b. The patches are delimited using the nearest neighbor method. **a** Boundary points defining the limits of a three-patch surface. **b** Zig-zag tool path for a three-patch surface

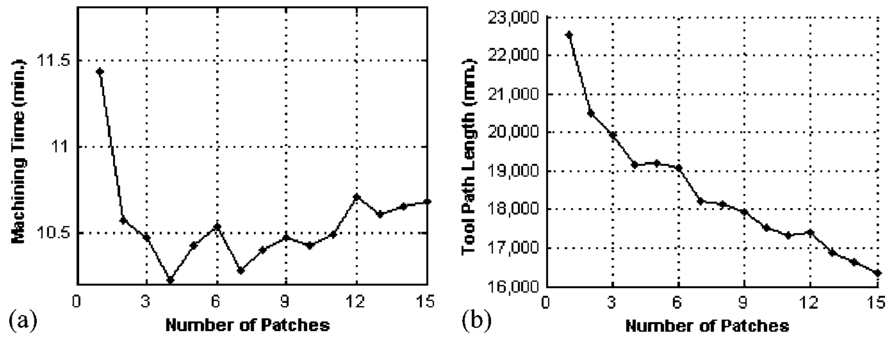


Fig. 7. Computed machining time for specific scallop height of 0.0254 mm

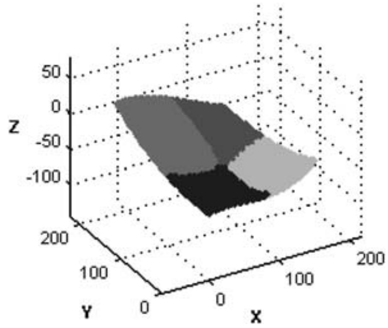


Fig. 8. Four-patch subdivision of the Bezier surface

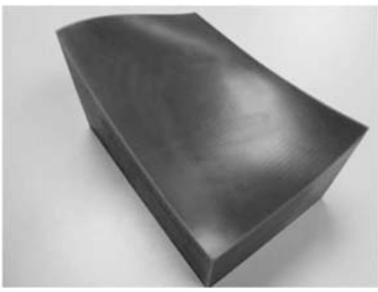


Fig. 9. Cutting test surface

Table 3. Estimated and machining time comparison

	Estimated time	Actual Machining time	Tool path length
$3\frac{1}{2}\frac{1}{2}$-axis (1 patch)	11.50 min	11.43 min	22535 mm
$3\frac{1}{2}\frac{1}{2}$-axis (4-patch)	10.57 min	10.23 min	19135 mm

estimate has a small difference from the actually measured machining time. The difference in the estimation can be attributed to neglecting acceleration and deceleration of the axes during tool movement and to the use of constant time penalty for part re-orientation.

These cutting experiments provide valuable information about patch-by-patch machining. The partitioning of a surface impacts machining in different ways. The transition from one patch to the next requires time for re-orientation. Additional tool movement is required between patches and the time spent by the tool in acceleration and deceleration increases. All these are offset by the increased feed rate. As the number of patches increases, the time spent between patches can have a negative impact on machining time. The cutting tests conducted have verified that the developed strategy can identify the optimal number of patches that provide the lowest machining time while satisfying the surface requirements. For completeness, the proposed $3\frac{1}{2}\frac{1}{2}$-axis machining is compared next with other multi-axis machining strategies.

6 Comparison between $3\frac{1}{2}\frac{1}{2}$-axis machining and other multi-axis machining methods

This work compares the results obtained with the $3\frac{1}{2}\frac{1}{2}$-axis machining strategy developed here with some of the methods described in literature for surface machining. The comparison is conducted using experimental cutting tests. The machining time obtained with the $3\frac{1}{2}\frac{1}{2}$-axis machining method is compared with those using other common techniques.

The machining data for this comparison is presented in Table 4. The first experiment is carried out using a three-axis machining strategy with a ball nose end mill. In the second experiment the same surface is machined using a simultaneous five-axis method known as the "Sturz" method, where a fixed inclination angle of the tool with respect to the surface normal is used for tool positioning. A three-degree angle was selected in this experiment.

The results obtained show that $3\frac{1}{2}\frac{1}{2}$-axis machining of the sample Bezier surface can be done in less time than the three-

Table 4. Machining data for the three multi-axis machining methods

	3-axis	5-axis	$3\frac{1}{2}\frac{1}{2}$-axis
Number of patches:	1	1	4
Tool type:	Ball nose	Toroidal	Toroidal
R_1 (tool size)	$R_1 = 12.7$ mm	$R_1 = 6.7$ mm	$R_1 = 6.7$ mm
R_2 (tool size)		$R_2 = 6.0$ mm	$R_2 = 6.0$ mm
Scallop height:	0.0254 mm	0.0254 mm	0.0254 mm
Feed rate:	2000 mm/min	2000 mm/min	2000 mm/min
Spindle speed:	6000 RPM	6000 RPM	6000 RPM

Table 5. Machining time and tool path length comparison

Configuration	Machining time	Tool path length
3-axis	12.01 min	23 801 mm
5-axis	9.71 min	7471 mm
$3\frac{1}{2}\frac{1}{2}$-axis (4 patches)	10.10 min	16 742 mm

axis. It however takes about the same time as the five-axis machining as illustrated in Table 5. The machining time for the proposed $3\frac{1}{2}\frac{1}{2}$-axis machining method is about 16% smaller than three-axis and 3% larger than five-axis machining. Although the five-axis tool path length is less than half the $3\frac{1}{2}\frac{1}{2}$-axis tool path yet, the machining time for the $3\frac{1}{2}\frac{1}{2}$-axis machining is longer by 23 s only. This difference can be attributed to the difference between the programmed and the actual feed rate.

7 Conclusions and future research

This work demonstrated that the fuzzy c-means method can successfully partition a surface. However, the partitioning depends on the geometric properties that form the multi-dimensional input to the algorithm. The effect of various geometric properties was studied on sample surfaces and a list of properties belonging to three categories namely proximity, orientation and curvature were identified. It was shown that although these properties can be grouped in various combinations and with varying weights, the combination of the parametric duo and the normal vector consistently results in good partitions. The number of partitions depends on the user and the surface at hand. The optimal number of partitions is difficult to determine because of two opposing effects; the large number of patches leads to a better match between the tool and the workpiece, but it also leads to many tool re-orientations between patches along with increased time for acceleration and deceleration.

The above method was used to machine a sample Bézier surface and identified four partitions as the optimal way to divide the surface. The boundaries for each patch were delimited using the nearest neighbor method in the u–v plane and used a constant side step to machine the surface. The machined surface had a good surface finish, although the boundaries could be easily identified because of different side steps. The unevenness was still within tolerance.

The paper also presented a method for determining the best tool orientation for each patch. The tool orientation is determined using the feed direction and lies outside the envelope of surface normal vectors. The inclination beyond the envelope is based on user input and can be further optimized. Reducing it can lead to gouging of the surface whereas increasing it leads to increased machining time.

The paper also compared the $3\frac{1}{2}\frac{1}{2}$-axis method with existing three- and five-axis techniques. Although this method showed improvements in machining time, the biggest advantage is the reduced investment in machine cost and operator training. The $3\frac{1}{2}\frac{1}{2}$-axis method presents a new alternative for surface machining.

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