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## Optimum manufacturing tolerance to selective assembly technique for different assembly specifications by using genetic algorithm

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**Abstract** Tolerance on parts dimension plays a vital role as the quality of the product depends on sub components tolerance. Thus, precision products that are manufactured reflect at high manufacturing cost. To overcome this situation, sub components of an assembly may be manufactured with wider tolerance, measured (using latest technologies like image processing) and grouped in partition and corresponding group components may be mated randomly. This present work is to obtain an optimum manufacturing tolerance to selective assembly technique using GA and to obtain maximum number of closer assembly specification products from wider tolerance sub components. A two components product (fan shaft assembly) is considered as an example problem, in which the subcomponents are manufactured with wide tolerance and partitioned into three to ten groups. A combination of best groups is obtained for the various assembly specifications with different manufacturing tolerances. The proposed method resulted nearly 965 assemblies produced out of one thousand parts with 15.86% of savings in manufacturing cost.

**Keywords** Manufacturing cost · Manufacturing tolerance · Selective assembly · Tolerance synthesis

### 1 Introduction

Functioning of a product depends on the assembly clearance / tolerance between its subcomponents. Customers want quality and trouble free product at an attractive price. Most of the previous research in selective assembly focuses on the partitioning methods to minimize the surplus parts meeting the assembly specification for the given similar or dissimilar variance of the components distribution. Allen Pugh [1] proposed and developed a method for partitioning the mating parts for a limited specified number of groups. Shan [2] described and developed software for one-to-one pairing of components to be assembled. Fang [3] suggested making groups with equal probabilities and the groups are planned after manufacturing. Chan [4] discussed a grouping method based on the cumulative probability of the mating parts with dissimilar distribution. Thesen [5] introduced and evaluated a high-speed station for selective assembly of high precision automotive components by maintaining small buffer storage of parts. Kannan [6] introduced uniform grouping method for assemblies when the clearance range is greater than the difference in the standard deviation of the mating parts and proposed equal probability method for assemblies in which the clearance range is smaller than the difference in the standard deviation of the mating parts. Kern [7] introduced a general approach to selective assembly applicable when the distribution variations are different and also developed closed form equation for various selective assembly techniques. Mease [8] described the statistical formulation of the problem and developed optimal binning strategies under obsolete and squared error loss condition. And also, the authors compared the results with two commonly used heuristic methods (equal width and equal area method). Chen [9] developed a simplified algorithm applying Lagrange multiplier (LM) method to evaluate the optimal tolerances efficiently for enlarged tolerated mechanical components. Chase [10] described a detailed algorithm for performing tolerance allocation (loosening tolerance on costlier process and tightening tolerance on less costlier process) automatically based on optimization technique.

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Diplaris [11] formulated a new analytical cost tolerance model which produces results more closely to industrial practice based on available industrial knowledge and earlier published data. Carfagni [12] developed a methodology to allow an automatic tolerance allocation capable to minimize manufacturing cost based on Monte Carlo method to compute the statistical distribution of the control measurements and GA is adopted as optimized tool. Singh [13] introduced genetic algorithm (GA) to obtain global optimal tolerances by considering continuous cost function and compared GA results with exhaustive search Lagrange method values.

## 2 Problem definition

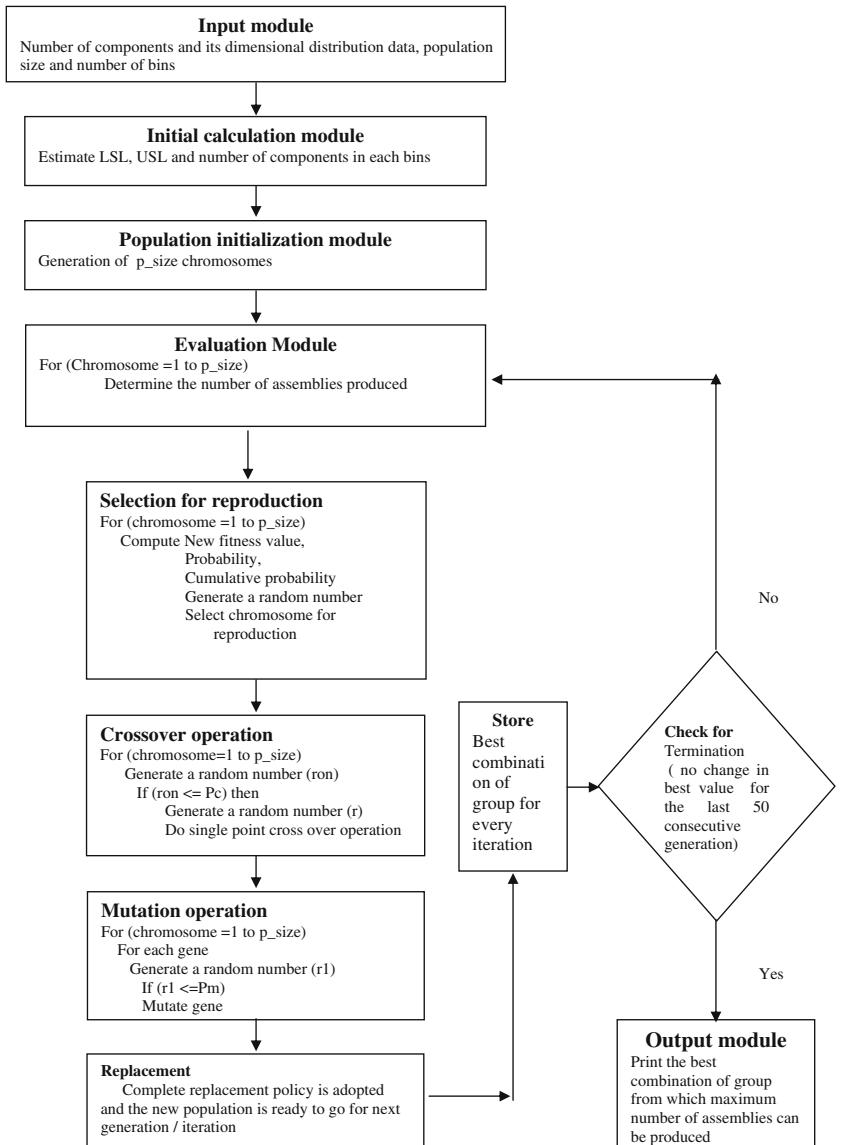
The manufacturing tolerance/allocated tolerance and number of groups are the two main factors that play an important role in controlling surplus parts. After parts are

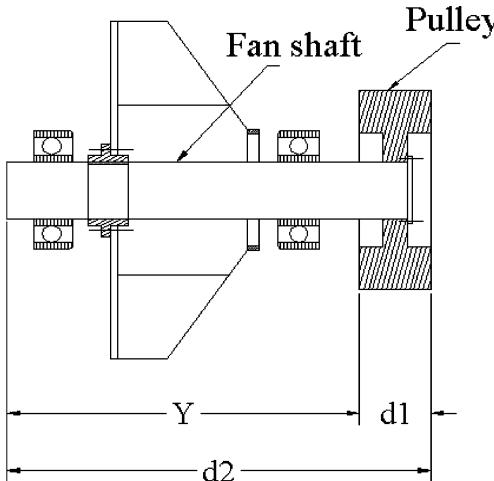
manufactured, it is tedious to obtain a best combination of groups for different assembly specification. The foresaid problem is challenging to process/design engineers and reduces the implementation in real situation. The present work is to design a methodology using GA to overcome the problem.

## 3 Methodology

For the example considered the simulated data of the subcomponents were obtained from MATLAB for 100%, 90% and 80% of the maximum process tolerance ( $T_{max}$ ). The number of components that lie between upper and lower specification limits of the groups was computed based on the given number of groups/partitions and others were omitted. Combination of best groups that produced minimum surplus parts were obtained for the different assembly tolerance ( $T_{asm}$ ) specifications based on genetic

**Fig. 1** Scheme of the present work





**Fig. 2** Fan shaft assembly

principles. Random assembly and the present method assembly cost were compared for the optimum manufacturing tolerance.

#### 4 Genetic algorithm

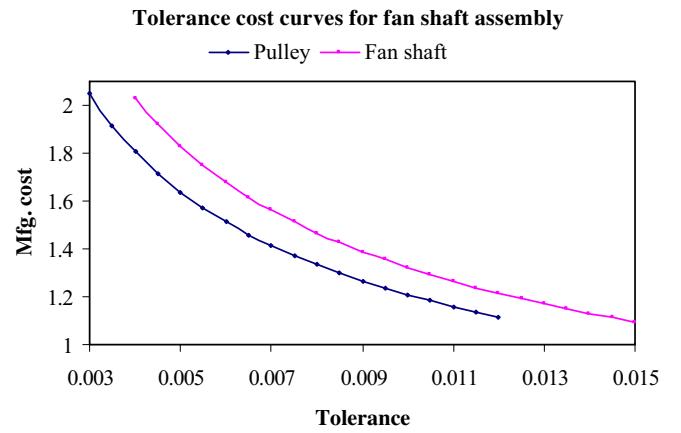
GA is based on the mechanics of natural genetics and natural selection and it combines the characteristics of direct search and probabilistic selection methods. It is very simple and a powerful tool for obtaining global optimum values for multi-modal and combinatorial problems. The new population is obtained after each iteration, by applying GA operators like reproduction, cross over and mutation. The flow chart is represented in Fig. 1. The description of the implementation of GA for the solution of the present work is presented below.

##### 4.1 Representation of variables

In this present work, group number is considered as gene, the length of the sub-string is equal to the number of groups and the total length of the chromosome is the product of number of components and the group numbers.

##### 4.2 Fitness function

The group widths of the individual components are estimated from manufacturing tolerance by using Eq. 1.



**Fig. 3** Tolerance cost curves

The upper and lower specification limits of each component are computed based on Eqs. 2 and 3.

$$gw_i = \frac{T_i}{N} \quad (1)$$

where,

$gw$  -group width  
 $T$  -manufacturing/allocated tolerance  
 $N$  -number of groups  
 $i$  -ith component

$$LSL_{ij} = \mu_i - 0.5*T_i + (j-1)*gw_i \quad (2)$$

$$USL_{ij} = LSL_{ij} + j^*gw_i \quad (3)$$

where,

$LSL$  -lower specification limit  
 $USL$  -upper specification limit  
 $\mu$  -mean value  
 $j$  -jth group (1,2,3,...N)

The number of assemblies may be produced from each gene of the sub string is estimated from Eq. 4. The total assemblies produced from the chromosome are computed by Eq. 5 based on the constraint expressed in Eq. 6.

$$nos_k = \min \left[ \frac{i=nc}{nos_{ki}} \right] \quad k = 1, 2, 3 \dots N \quad (4)$$

**Table 1** Chase's cost model constant values for the fan shaft assembly

Part name	Nominal size	B Value	K Value	$T_{\min}$	$T_{\max}$
Pulley ( $d_1$ )	7.80	0.15997	0.43899	0.00300	0.01200
Fan shaft ( $d_2$ )	13.70	0.15301	0.46823	0.00400	0.01500

Dimensions & tolerances – inches  
Cost – American dollars

**Table 2** Manufacturing specification of pulley

Pulley		$\mu$	$\sigma$	Tolerance	Lower limit	Upper limit
Mfg. tol. (%T <sub>max</sub> )						
100			0.0020	0.0120	7.7940	7.8060
90	7.8		0.0018	0.0108	7.7946	7.8054
80			0.0016	0.0096	7.7952	7.8048

**Table 3** Manufacturing specification of fan shaft

Fan shaft		$\mu$	$\sigma$	Tolerance	Lower limit	Upper limit
Mfg. tol. (%T <sub>max</sub> )						
100			0.00250	0.0150	13.69250	13.70750
90	13.7		0.00225	0.0135	13.69325	13.70675
80			0.00200	0.0120	13.69400	13.70600

where,

$nos_k$  -number of assembled products from the combination of kth gene group number of every component

$nc$  -number of components

The objective function  $f(x_q)$  for the present problem is maximizing the number of product assembled from the sub component groups. The exponential fitness function used is expressed in Eq. 7.

$$f(nos) = E^{-c^{*}nos_q} \quad (7)$$

where,

c-constant selected appropriately.

The cost of the product is estimated based on the Chase's cost model and the percentage deviation of the manufacturing cost are computed based on expression below.

$$f(x_q) = nos_q = \sum_{k=1}^{k=N} nos_k \quad (5)$$

where,

$nos_q$ -number of assembled products from the qth chromosome

$$\%Saving = \frac{100 * ((nos_q * C_{tLM}) - C_{mfg})}{C_{mfg}} \quad (8)$$

$$T_{asm} \geq \sum_{i=1}^{i=nc} (k_i * gw_i) \quad (6)$$

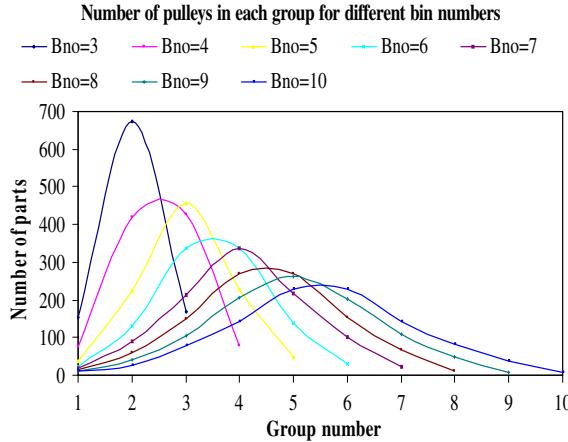
**Table 4** Optimum allocated tolerance and its manufacturing cost

Assembly requirement	Pulley		Fan shaft		Total mfg. cost
	T <sub>alo</sub>	C <sub>tLM</sub>	T <sub>alo</sub>	C <sub>tLM</sub>	
0.01800	0.00851	1.29637	0.00949	1.35473	2.65110
0.01900	0.00899	1.26553	0.01001	1.32105	2.58658
0.02000	0.00947	1.23714	0.01053	1.29010	2.52724
0.02100	0.00995	1.21057	0.01106	1.26116	2.47173

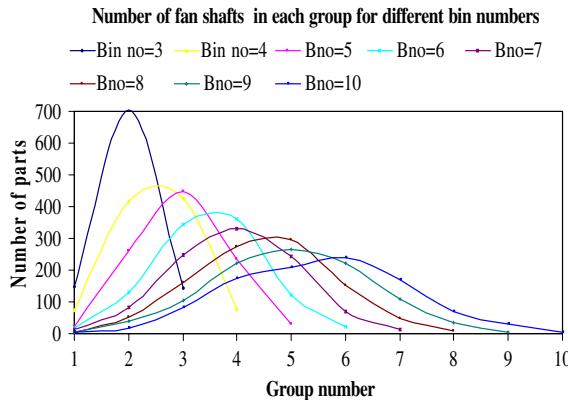
T<sub>alo</sub> – allocated tolerance based on LM method

**Table 5** Manufacturing tolerance and its cost

T <sub>mfg</sub> (% of T <sub>max</sub> )	Pulley		Fan shaft		Total mfg. cost
	Tolerance	Mfg. cost	Tolerance	Mfg. cost	
100.0	0.01200	1.11495	0.01500	1.09323	2.20819
90.0	0.01080	1.16773	0.01350	1.14852	2.31625
80.0	0.00960	1.22970	0.01200	1.21364	2.44334



**Fig. 4** Parts distribution of pulley



**Fig. 5** Parts distribution of shaft

$$C_{tLM} = \sum_{i=1}^{nc} A_i + \frac{B_i}{T_{alo}^{K_i}} \quad (9)$$

where,

B & K- Chase's cost model constants

$$C_{mfg} = 1000 * \sum_{i=1}^{nc} A_i + \frac{B_i}{T_{mfg}^{K_i}} \quad (10)$$

**Fig. 6** Number of assemblies produced for  $T_{max}$

where,

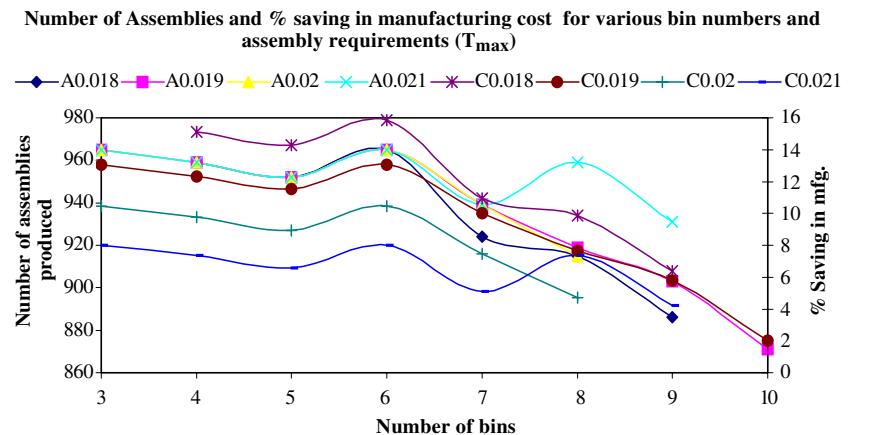
- $C_{tLM}$  -manufacturing cost for the specification based on LM allocation
- $C_{mfg}$  -production cost for the manufacturing tolerance

The roulette wheel selection method is used in this present work for the selection of reproduction. Single point cross over technique is adopted for the search of new strings in the search space. The cross over and mutation probability is considered as 0.62 and 0.05 respectively. The complete replacement policy has been implemented, since it yields better results. The C program developed for this purpose will stop automatically as there is no further change (up to 50 iterations) in the previous best solutions.

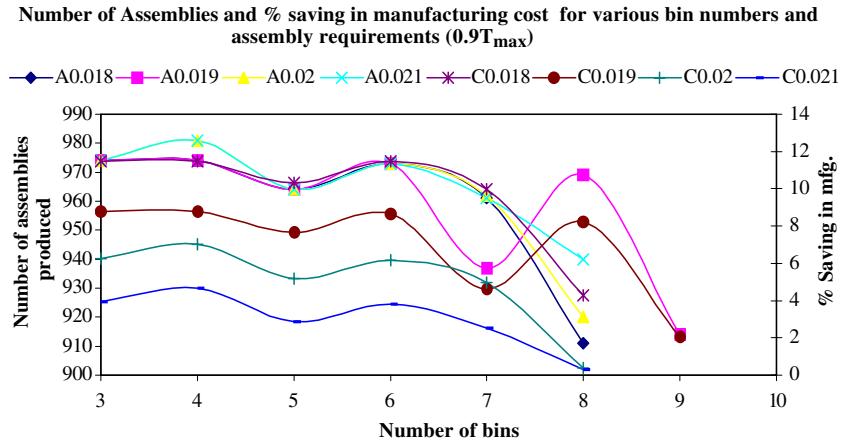
## 5 Example

A centrifugal fan shaft assembly consisting of fan shaft, bearing assembly, impeller, and pulley is considered as an example problem. The present work focuses on pulley ( $d_1$ ) and fan shaft ( $d_2$ ) as shown in Fig. 2. Both the components are produced in turning process. The Chase's cost model constants for the example problem are given in Table 1. The tolerance cost curves for the manufacturing process of the assembly is represented in Fig. 3.

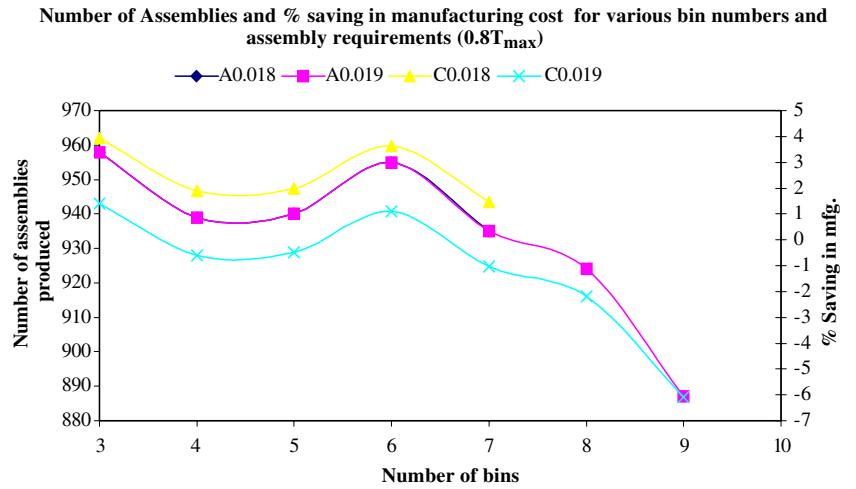
The manufacturing specifications like mean, standard deviation, tolerance, upper and lower limit of the pulley and fan shaft are represented in Tables 2 and 3. For random assembly, the allocated tolerance and its cost ( $C_{tLM}$ ) for the specified assembly requirements based on LM method are given in Table 4. The different manufacturing tolerances and its cost of the subcomponents are tabulated in Table 5. The graphical representation shown in Figs. 4 and 5 indicates number of parts fall in each group for various bin numbers of  $T_{max}$ . For the four different assembly specifications and three manufacturing tolerances, the maximum number of assemblies produced and its manufacturing cost for various partition numbers are estimated.



**Fig. 7** Number of assemblies produced for  $0.9T_{\max}$



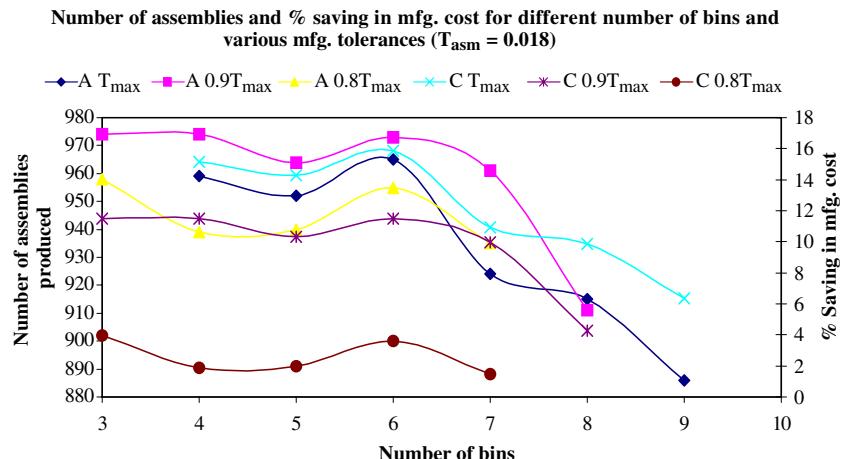
**Fig. 8** Number of assemblies produced for  $0.8T_{\max}$



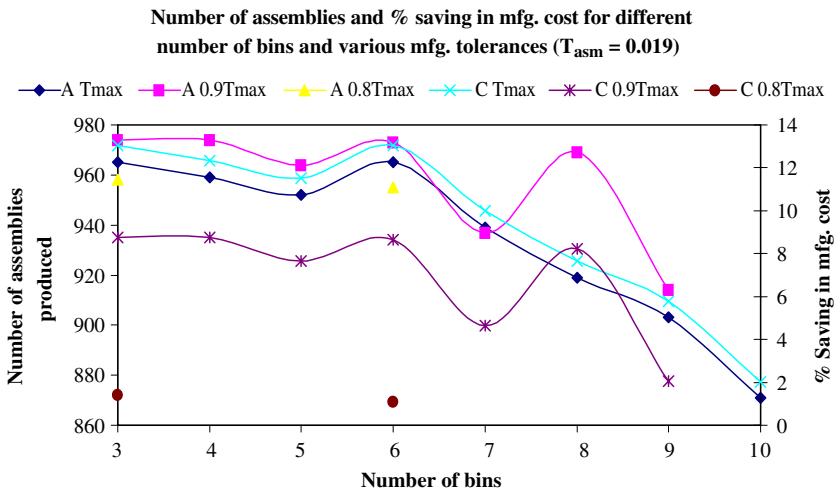
## 6 Results

- From the Fig. 6, it is observed that the maximum number of assemblies produced and percentage of savings in manufacturing cost for different bin numbers of assembly specification of  $T_{\max}$  amounts to considerable. If number of bins is equal to four, then it possible to produce 959 assemblies out of 1000 components with 15.13%, 12.33%, 9.75% and 7.34% of savings in manufacturing cost for 0.018, 0.019, 0.02 and 0.021 assembly specifications respectively from  $T_{\max}$  is considered as the manufacturing tolerance.
- Similar representation for 0.9 and 0.8  $T_{\max}$  are shown in Figs. 7 and 8 wherein it is noticed that even for a

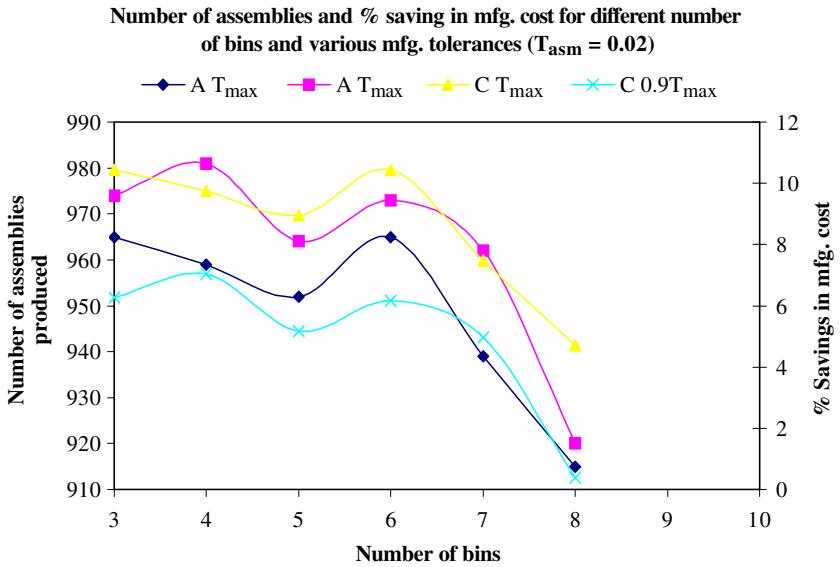
**Fig. 9** Number of assemblies produced for 0.018



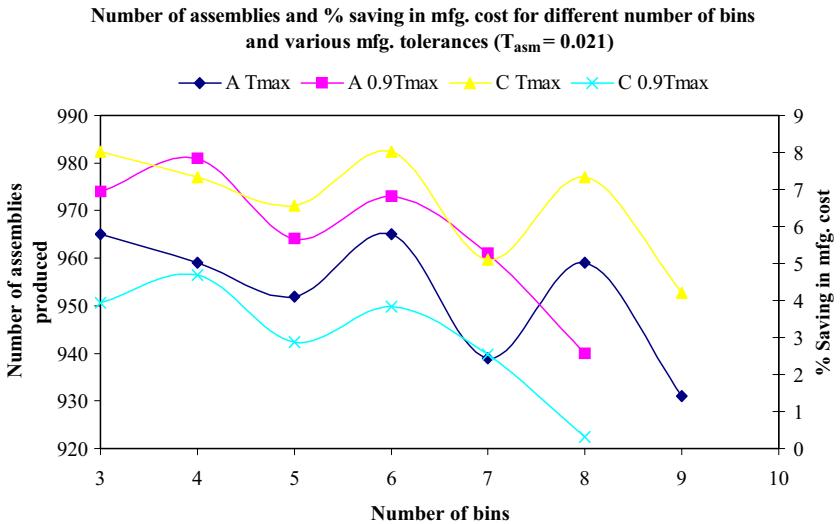
**Fig. 10** Number of assemblies produced for 0.019



**Fig. 11** Number of assemblies produced for 0.020



**Fig. 12** Number of assemblies produced for 0.021



closer manufacturing tolerance level considerable manufacturing cost savings is obtained.

- Figures 9, 10, 11, 12<sup>1</sup> shows the number of assemblies produced out of 1000 parts for various manufacturing tolerances respectively  $T_{\max}$ , 0.9 and  $0.8T_{\max}$  and number of bins when assembly specification is 0.018, 0.019, 0.02, and 0.021. If number of bins is equal to four, then it possible to produce 959, 974, and 934 assemblies out of 1000 parts from 100%, 90% and 80% of maximum manufacturing tolerance conditions for the assembly specification of 0.018 and it will produce 12.33%, 11.48% and 1.89% savings in manufacturing cost.

## 7 Conclusion

- It is possible to produce 965, 981 and 958 assemblies out of 1000 parts with approximately 4% to 15% savings in manufacturing cost for the different manufacturing tolerance, assembly specification and number of bins.
- For the example problem, a six bin system produced good number of assemblies (965) with countable savings (15.86%) in manufacturing cost for wider manufacturing tolerance ( $T_{\max}$ ) and closer assembly specification ( $T_{asm}=0.018$ ).
- The present work will help the manufacturing engineers to decide how close an assembly tolerance can be assigned for the product from the manufacturing process tolerance with maximum percentage of savings in manufacturing cost.

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<sup>1</sup> From Figs. 6, 7, 8, 9, 10, 11, 12, A indicates number of assemblies produced and C indicates the cost of manufacturing of it.