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ORIGINAL ARTICLE

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An orthogonal array based optimization algorithm for computer-aided measurement of worm surface

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Abstract Being versatile and fast, a co-ordinate measuring machine is used for the measurement of worm. A best-fit surface is obtained from the measured points by a surface fitting method, which minimizes the root mean square of normal deviations. For this problem in discrete space, an iterative optimization algorithm based on an orthogonal array is developed. On minimizing the objective function, the deviations of worm parameters from the specified values are obtained. The algorithm is validated using input data points generated from a straight-sided in axial section worm (ZA worm) with known errors. The proposed algorithm requires fewer objective function evaluations and the result is highly repeatable as there are no random operations involved.

Keywords Worm measurement · Optimization algorithm · Orthogonal array

Nomenclature

b	half bottom width of worm groove
L	lead
L_m	range reduction factor
M	number of measured points
m	module
nd	normal deviation
n_x, n_y, n_z	unit normal vector along coordinate directions
N	normal vector
N_x, N_y, N_z	normal vector along coordinate directions
Nc	number of chromosomes
Ng	number of genes per chromosomes
Nd	number of binary digits per gene
p	screw parameter (lead per radian)
p_a	axial pitch
$Pcross$	cross over probability
$Pmute$	mutation probability
r_b	root radius of worm
r_p	probe radius
S_r	range of search
x_c,y_c, z_c	real surface coordinates (contact point)
x_m, y_m, z_m	measured coordinates (probe center point)
x_w,y_w, z_w	worm surface coordinates (ideal)
z_f	form number of worm
ΔL	error in lead
$\Delta\ \alpha$	error in pressure angle
α	axial pressure angle
γ	lead angle at reference diameter
u,θ	surface parameters
μ	root mean square of normal deviations (objective function)

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1 Introduction

Worms used in worm gear boxes are cut in a lathe using form tools and milled using a disc type cutter. They can also be machined using a special whirling attachment mounted on a lathe [1, 2]. Heat-treated worms are ground for removing decarburized layers and improving accuracy and surface finish. Manufactured worms are inspected to ascertain conformance to dimensional specifications and also to correct the machine setting based on the deviation in the worm parameters. The conventional method of inspection of worms is measurement over balls or rollers to obtain the tooth thickness at different diameters [3, 4]. Such measurements are not comprehensive to determine all the worm parameters as the measurements are made at two points contacted by ball or roller. For proper evaluation of geometrical parameters of the worm surface, measurement at many different points on the real surface is to be made.

Co-ordinate measuring machines (CMM) are versatile and fast in measurement of worm surfaces compared to conventional methods. The probe used in a CMM has a spherical-tip with definite radius and it makes tangential contact with the surface being measured. A best-fit surface is obtained from the measured coordinate data by suitable

optimization algorithm and corresponding worm parameters are obtained. The deviation of worm parameters is obtained as the difference between the specified values and values obtained from best-fit surface. Different surface fitting techniques based on the minimum zone principle, maximum material condition, etc., are reported in the literature [5–8]. However, determination of parameters of the real surface requires multi-variable optimization algorithms, such as direct-search and gradient-based methods [9]. For a problem in discrete space, gradient information is not readily available and hence the direct-search method is to be used. In the direct-search method, evolutionary optimization methods are also used. Attempts have been made by researchers to solve problems in discrete design space using an iterative optimization algorithm based on Taguchi's orthogonal arrays and design of experiments [10].

In this paper, an iterative algorithm based on an orthogonal array has been developed to obtain the best-fit surface by minimizing the root mean square of normal deviations. The details of the orthogonal array, its selection, setting of parameter levels [11] and steps involved in the optimization algorithm are explained. A case study involving measurement of ZA worm is also given. Validation of the algorithm is done by introducing known error in the worm parameters and predicting the parameters of the best-fit surface by the least squares method.

2 Worm measurement using CMM

In the measurement of worms using CMM, the probe touches the worm surface tangentially and the probe center lies along the surface normal emerging from the contact point. The co-ordinate data is obtained for the probe center location and the trace of many such data points is an offset surface of the real surface at a distance equal to the probe radius. Normal deviation of a point is the distance between it and the ideal or reference surface measured along the normal to the surface. As direct determination of the actual point of contact of the probe with the worm surface is not possible, normal deviation arrived by subtracting the probe radius from the normal deviation of the measured point (probe center point) is used to obtain a best-fit surface and its parameters by an optimization process. In the optimization process, a trial surface is defined using a set of parameter values, and normal deviation of the measured points from this surface is calculated. The trial surface is varied by changing the parameter values until it merges with the best-fit surface by minimization of root mean square of normal deviations. Figure 1 depicts a two-dimensional case where the difference between the actual normal deviation nd_c of the contact point P_c and that obtained by subtracting the probe radius r_p from the normal deviation nd of the measured point P_m from a trial surface is shown. The error involved tends to zero or a minimum as the trial surface reaches the best-fit surface in the optimization process. The objective function for minimization is defined as follows.

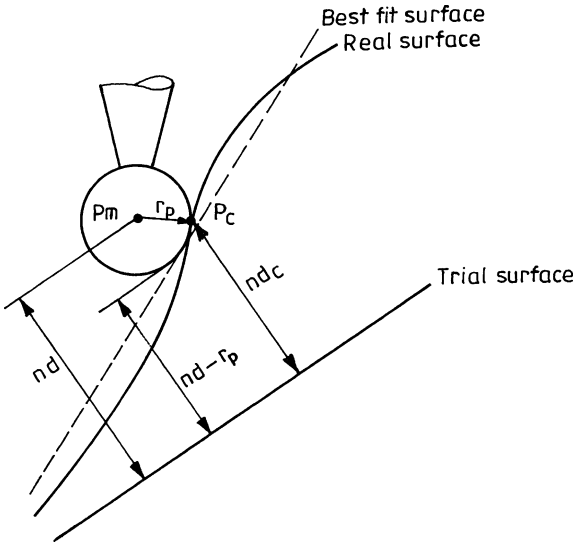


Fig. 1 Normal deviation and best-fit surface

Objective function (minimize)

$$= \sqrt{\frac{1}{M}\sum_{j=1}^{j=M}\left(nd_j - r_p\right)^2} \quad (1)$$

where M is the number of measured points, and nd_j is the normal deviation of the *jth* measured point from a trial surface. It is assumed that the reference coordinate frame used in CMM measurement is the same as that used in machining. To achieve this initial setting, the CMM probe is brought into the axial plane and centered inside the toothspace of the worm. In this piece of work, an orthogonal array based iterative algorithm is developed to optimize the objective function. Details of the optimization process are explained below.

3 Optimization algorithm based on an orthogonal array

3.1 Orthogonal array and ANOM

Orthogonal arrays (OA) are used in the design of experiments (DOE) to minimize the number of experiments compared to full factorial design. Depending on the number of factors and its setting levels, a suitable orthogonal array is selected. While selecting an OA, interaction between factors is also to be considered. In this piece of work, interaction effects are not considered for simplicity. Each column of the OA designates a factor and its setting levels in each experiment and each row designates an experiment with the level of different factors in that experiment. For an optimization problem with four factors A, B, C, and D with each factor at two levels, the minimum orthogonal array to be selected is L_8 as shown in Table 1. Columns 3, 5 and 6 assigned for interaction are not considered. The quality characteristic μ or

Table 1 L_8 orthogonal array

Experiment	Factors/parameters							Results
	A	B	CxD	C	BxD	BxC	D	
	1	2	3	4	5	6	7	
1	1	1	1	1	1	1	1	μ_1
2	1	1	1	2	2	2	2	μ_2
3	1	2	2	1	1	2	2	μ_3
4	1	2	2	2	2	1	1	μ_4
5	2	1	2	1	2	1	2	μ_5
6	2	1	2	2	1	2	1	μ_6
7	2	2	1	1	2	2	1	μ_7
8	2	2	1	2	1	1	2	μ_8

the objective function is evaluated in each experiment. The analysis of mean (ANOM) is carried out to find the optimum setting of factors. The optimum setting level of factor A is arrived as follows.

$$\begin{aligned} \mu_{A1} &= (\mu_1 + \mu_2 + \mu_3 + \mu_4)/4 \\ \mu_{A2} &= (\mu_5 + \mu_6 + \mu_7 + \mu_8)/4 \\ \mu_M &= \frac{1}{8}\sum_{i=1}^{8} \mu_i \end{aligned} \tag{2}$$

where μ_{A1} is the mean of all objective function values of factor A at level 1, μ_{A2} is the mean of all objective function values of factor A at level 2 and μ_M is the mean of all objective function values for the whole set of experiments. If the aim of optimization is to minimize the objective function, then level of a factor, which gives the minimum mean of objective function values, is selected as the optimum setting level, hence arriving at the optimum level of the factors. Table 2 shows the calculation of mean value of μ for each level of individual factor.

3.2 Optimization algorithm

The algorithm proposed in this paper arrives at the parameters of the best-fit surface from the measured points by considering the effect of each parameter on the objective function. The experiments in the orthogonal array provide initial possible solutions with parameter levels defined by range of search (S_r) from initial value. Objective functions are evaluated with parameter values as per the designed experiments and ANOM is carried out to obtain the optimum parameter levels. This completes an iteration of the search process. Then the non-optimum levels of each parameter is shifted towards the optimum level so that the range after modification is reduced by a factor L_m and the next iteration is carried out with modified parameter levels. This process continues until the termination criterion is met. Termination criterion is specified as number of iterations over which there is no improvement in the objective function value. Figure 2 shows the general flow chart of the algorithm proposed. Execution of the algorithm is explained step-by-step below.

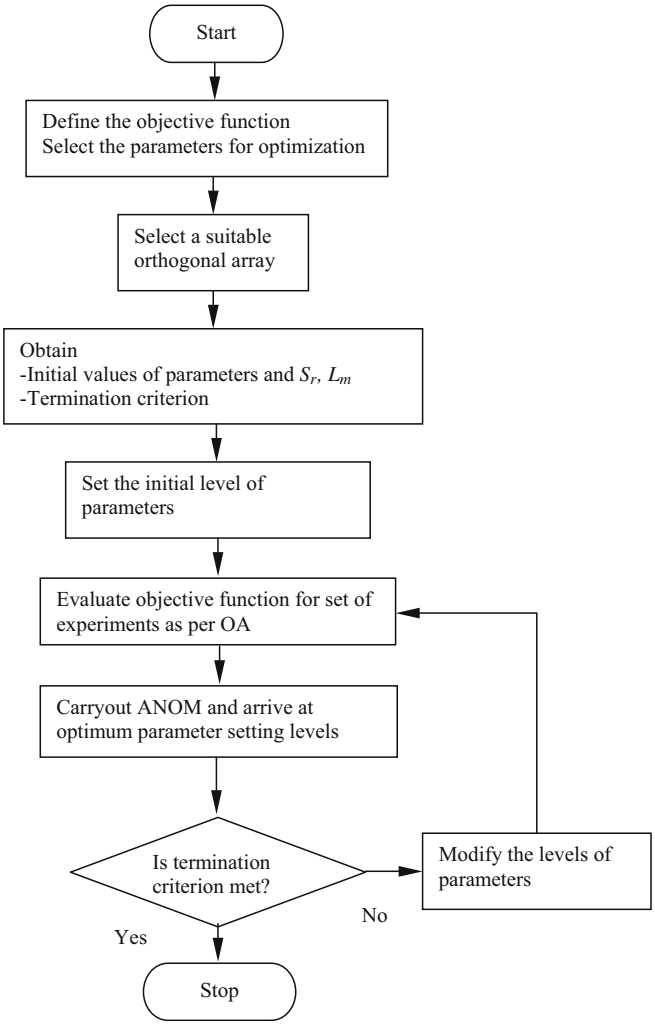


Fig. 2 Flow chart of the optimization algorithm

Step1

The objective function is to be defined and parameters to be optimized are selected.

Table 2 Mean of μ for each level of individual factor

Factors	Levels	
	1	2
A	$\mu_{A1} = (\mu_1 + \mu_2 + \mu_3 + \mu_4)/4$	$\mu_{A2} = (\mu_5 + \mu_6 + \mu_7 + \mu_8)/4$
B	$\mu_{B1} = (\mu_1 + \mu_2 + \mu_5 + \mu_6)/4$	$\mu_{B2} = (\mu_3 + \mu_4 + \mu_7 + \mu_8)/4$
C	$\mu_{C1} = (\mu_1 + \mu_3 + \mu_5 + \mu_7)/4$	$\mu_{C2} = (\mu_2 + \mu_4 + \mu_6 + \mu_8)/4$
D	$\mu_{D1} = (\mu_1 + \mu_4 + \mu_6 + \mu_7)/4$	$\mu_{D2} = (\mu_2 + \mu_3 + \mu_5 + \mu_8)/4$

Step2

Based on the number of parameters, a suitable orthogonal array is selected. The array selected must be small so that the number of objective function evaluations can be minimized.

Step3

Initial values of parameters to be optimized are obtained. These values are given as roughly measured values using simple measuring instruments like micrometers, calipers, profile projectors, etc. The range of search (S_r), range reduction factor (L_m) and termination criteria are specified at this stage.

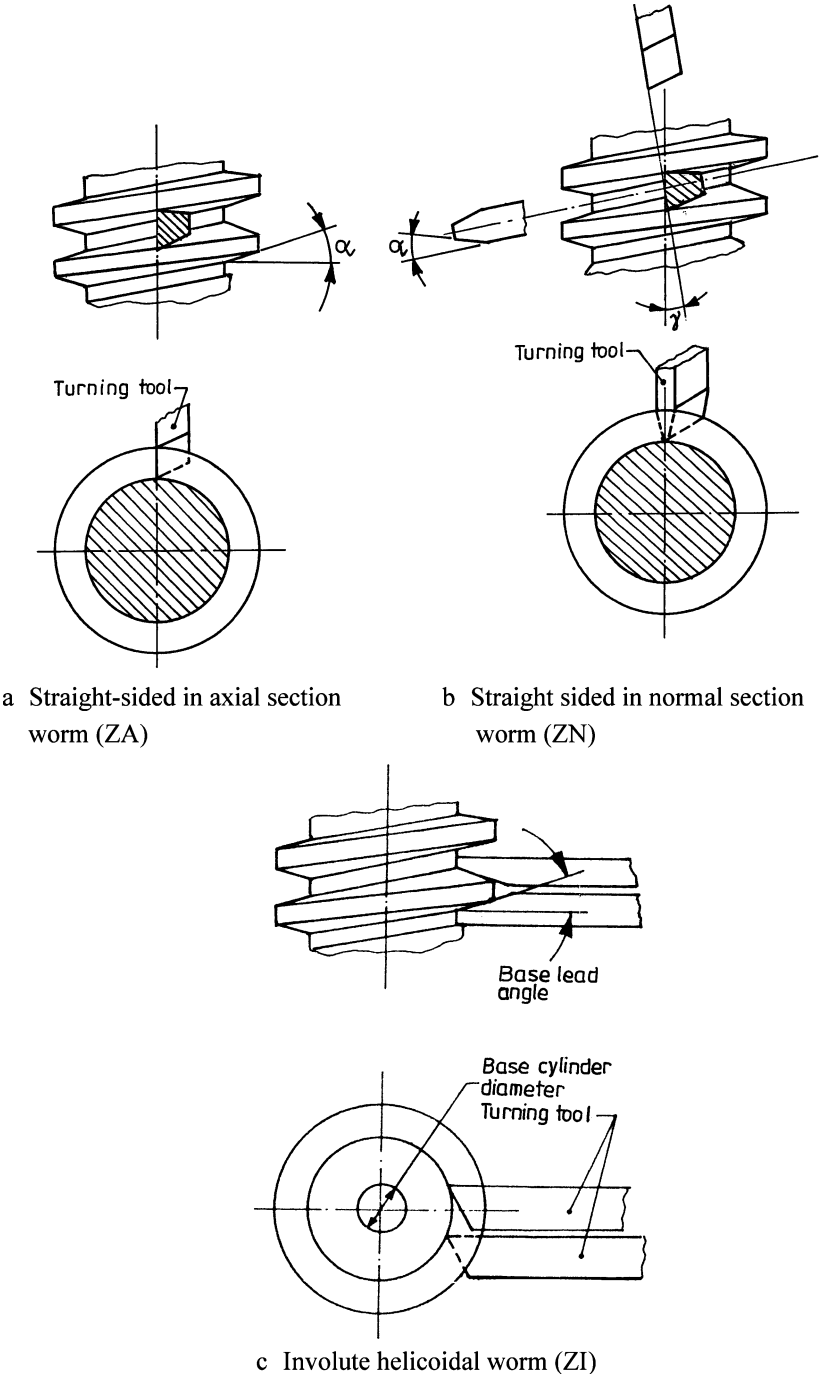
Step4

The level 1 and level 2 settings of each parameter are obtained from the range of search (S_r) from the initial value as given below.

Level 1 initial setting=initial value $x(1-S_r)$
Level 2 initial setting=initial value $x(1+S_r)$

Step5

The objective function is evaluated for each row of OA with the parameter setting levels defined in the array. Then the ANOM is carried out according to Eq. 2 to arrive at the optimum parameter setting level.



a Straight-sided in axial section worm (ZA)

b Straight sided in normal section worm (ZN)

c Involute helicoidal worm (ZI)

Fig. 3 Different types of ruled surface worms. **a** Straight-sided in axial section worm (ZA). **b** Straight-sided in normal section worm (ZN). **c** Involute helicoidal worm (ZI)

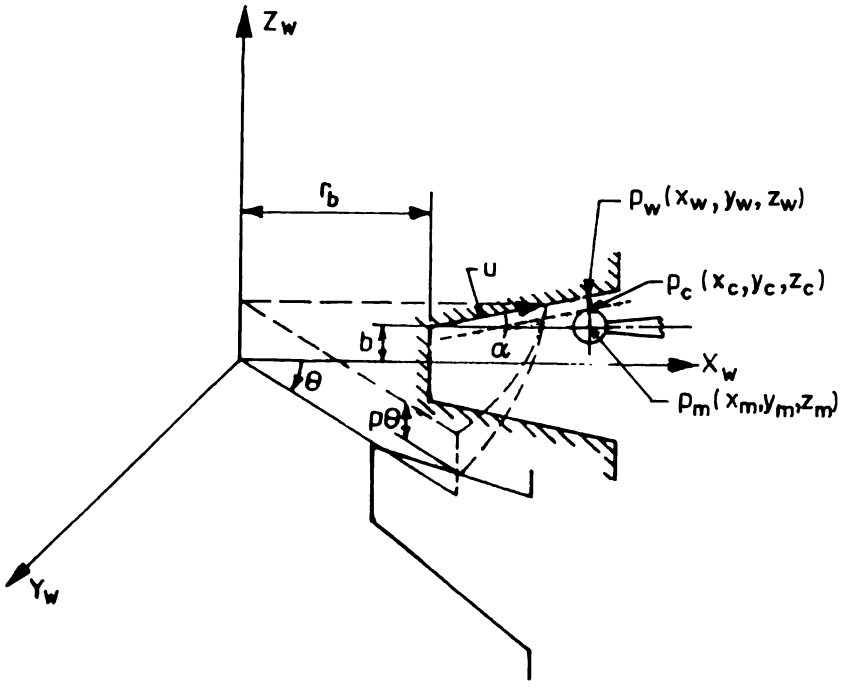


Fig. 4 ZA-worm geometry

Step6

The algorithm checks at this stage for the termination condition. If the termination condition is met, then the algorithm terminates. Otherwise, a non-optimum level of each parameter is shifted towards the optimum level so that the range after modification is reduced by the factor L_m as given below

New level 1=level 1+L_m (level 2−level 1); when level 2 is optimum
New level 2=level 2-L_m (level 2−level 1); when level 1 is optimum

The search process continues with the modified parameter levels until the termination condition is met. Computer coding has been developed in C for the above sequence of steps. The objective function values are evaluated by separate subroutine call inside the algorithm.

4 Case study and validation of algorithm

4.1 Surface geometry of worm

Ruled surface cylindrical worms are generally classified into three different types: straight-sided in axial section (ZA), straight-sided in normal section (ZN), and involute helicoid (ZI). Figure 3 shows the different types of ruled surface worms. In this paper, a right hand ZA worm is considered with four worm parameters: (1) root radius r_b; (2) half bottom width of worm groove b; (3) lead L; (4) axial pressure angle α. ZA worm can be cut on a lathe using a trapezoidal form tool whose rake face is set parallel to the worm axis and whose tool tip is set at a distance from the axis of the worm equal to the root radius of the worm. The geometric details are shown in Fig. 4. Coordinate frame S_w (O_w, X_w, Y_w, Z_w) is associated with the worm. The geometry of the worm surface is given by Eq. 3 for a left side flank of a right hand worm.

$$p_w = \begin{bmatrix} x_w \\ y_w \\ z_w \end{bmatrix} = \begin{bmatrix} (r_b + u\ \cos(\alpha))\cos(\theta) \\ (r_b + u\ \cos(\alpha))\sin(\theta) \\ b + u\ \sin(\alpha) - p\theta \end{bmatrix} \tag{3}$$

Table 3 Worm parameters of ZA right hand worm

Design details	
Module	4 mm
Axial pitch p_a	12.566 mm
Form number z_f	7.875
Number of starts	Single
Axial pressure angle	20°
Outer diameter of worm	39.5 mm
Reference diameter of worm	31.5 mm
Root diameter of worm	21.9 mm
Addendum	1 module
Dedendum	1.2 module

Table 4 Generated probe center coordinates with 0.5 mm probe radius

With systematic error: ΔL=+0.434 mm, $\Delta\alpha$=+0.8°											
Sl .no	U(mm)	Θ(rad)	Ideal surface coordinates, mm			Surface coordinates with systematic error, mm			Probe center points of surface with systematic error, mm		
			Xw	Yw	Zw	X	Y	Z	X	Y	Z
1	0.0000	0.5236	9.4830	5.4750	0.3471	9.4830	5.4730	0.3111	9.6779	5.4871	−0.1492
2	1.0000	0.5236	10.2330	5.9080	0.6891	10.2926	5.9424	0.6662	10.4845	5.9605	0.2049
3	2.0000	0.5236	10.9830	6.3410	1.0313	11.1021	6.4098	1.0213	11.2915	6.4330	0.5591
4	3.0000	0.5236	11.7330	6.7740	1.3733	11.9117	6.8772	1.3764	12.0988	6.9049	0.9136
5	4.0000	0.5236	12.4830	7.2071	1.7154	12.7213	7.3447	1.7315	12.9064	7.3761	1.2681
6	5.0000	0.5236	13.2330	7.6401	2.0571	13.5309	7.8121	2.0866	13.7142	7.8470	1.6228
7	6.0000	0.5236	13.9830	8.0731	2.3994	14.3405	8.2795	2.4418	14.5222	8.3174	1.9775
8	7.0000	0.5236	14.7330	8.5061	2.7414	15.1501	8.7469	2.7969	15.3304	8.7875	2.3323
9	8.0000	0.5236	15.4830	8.9391	3.0834	15.9596	9.2143	3.1520	16.1387	9.2574	2.6871
10	9.0000	0.5236	16.2330	9.3721	3.4255	16.7692	9.6817	3.5071	16.9471	9.7270	3.0420
11	0.0000	0.0000	10.9500	0.0000	1.3944	10.9500	0.0000	1.3944	11.1248	−0.0870	0.9342
12	1.0000	0.0000	11.8160	0.0000	1.7365	11.8848	0.0000	1.7495	12.0601	−0.0803	1.2882
13	2.0000	0.0000	12.6821	0.0000	2.0785	12.8197	0.0000	2.1047	12.9952	−0.0746	1.6425
14	3.0000	0.0000	13.5481	0.0000	2.4205	13.7545	0.0000	2.4598	13.9303	−0.0696	1.9969
15	4.0000	0.0000	14.4141	0.0000	2.7625	14.6893	0.0000	2.8149	14.8653	−0.0653	2.3515
16	5.0000	0.0000	15.2801	0.0000	3.1045	15.6241	0.0000	3.1700	15.8003	−0.0614	2.7061
17	6.0000	0.0000	16.1462	0.0000	3.4466	16.5590	0.0000	3.5251	16.7353	−0.0580	3.0608
18	7.0000	0.0000	17.0122	0.0000	3.7886	17.4938	0.0000	3.8802	17.6703	−0.0549	3.4156
19	8.0000	0.0000	17.8782	0.0000	4.1306	18.4286	0.0000	4.2353	18.6052	−0.0522	3.7704
20	9.0000	0.0000	18.7442	0.0000	4.4726	19.3634	0.0000	4.5904	19.5401	−0.0497	4.1253
21	0.0000	−0.5236	9.4830	−5.4750	2.4416	9.4830	−5.4750	2.4778	9.5909	−5.6377	2.0175
22	1.0000	−0.5236	10.2330	−5.9080	2.7836	10.2926	−5.9424	2.8329	10.4042	−6.0996	2.3715
23	2.0000	−0.5236	10.9830	−6.3410	3.1256	11.1021	−6.4098	3.1880	11.2169	−6.5622	2.7258
24	3.0000	−0.5236	11.7330	−6.7740	3.4677	11.9117	−6.8772	3.5431	12.0292	−7.0254	3.0802
25	4.0000	−0.5236	12.4830	−7.2071	3.8097	12.7213	−7.3447	3.8982	12.8411	−7.4892	3.4348
26	5.0000	−0.5236	13.2330	−7.6401	4.1517	13.5209	−7.8121	4.2533	13.6528	−7.9534	3.7894
27	6.0000	−0.5236	13.9830	−8.0731	4.4937	14.3405	−8.2795	4.6084	14.4642	−8.4179	4.1442
28	7.0000	−0.5236	14.7330	−8.5061	4.8358	15.1501	−8.7469	4.9635	15.2754	−8.8827	4.4989
29	8.0000	−0.5236	15.4830	−8.9391	5.1778	15.9596	−9.2143	5.3186	16.0865	−9.3478	4.8538
30	9.0000	−0.5236	16.2330	−9.3721	5.5198	16.7692	−9.6817	5.6737	16.8974	−9.8131	5.2086

With Gaussian noise: max. repeatability error=5 μm							
Sl.no	Gaussian random number	Gaussian distributed repeatability error along coordinate directions, mm			Probe center point with repeatability error, mm		
		X	Y	Z	Xm	Ym	Zm
1	0.0602	−0.0001	0.0000	0.0003	9.6778	5.4871	−0.1489
2	0.1447	−0.0003	0.0000	0.0007	10.4842	5.9605	0.2055
3	0.3809	−0.0007	−0.0001	0.0018	11.2908	6.4329	0.5609
4	−0.2095	0.0004	0.0001	−0.0010	12.0992	6.9049	0.9126
5	−0.0477	0.0001	0.0000	−0.0002	12.9065	7.3762	1.2679
6	−0.2084	0.0004	0.0001	−0.0010	13.7146	7.8470	1.6218
7	0.0223	0.0000	0.0000	0.0001	14.5222	8.3174	1.9776
8	0.1927	−0.0003	−0.0001	0.0009	15.3300	8.7875	2.3332
9	−0.1640	0.0003	0.0001	−0.0008	16.1390	9.2575	2.6863
10	−0.0626	0.0001	0.0000	−0.0003	16.9472	9.7270	3.0417
11	0.0602	−0.0001	0.0001	0.0003	11.1247	−0.0869	0.9344
12	0.1447	−0.0003	0.0001	0.0007	12.0598	−0.0802	1.2889
13	0.3809	−0.0007	0.0003	0.0018	12.9945	−0.0743	1.6442
14	−0.2095	0.0004	−0.0001	−0.0010	13.9307	−0.0698	1.9959
15	−0.0477	0.0001	0.0000	−0.0002	14.8654	−0.0653	2.3512

Table 4 (continued)

With Gaussian noise: max. repeatability error=5 μm							
Sl.no	Gaussian random number	Gaussian distributed repeatability error along coordinate directions, mm			Probe center point with repeatability error, mm		
		X	Y	Z	Xm	Ym	Zm
16	−0.2084	0.0004	−0.0001	−0.0010	15.8007	−0.0616	2.7051
17	0.0223	0.0000	0.0000	0.0001	16.7353	−0.0580	3.0609
18	0.1927	−0.0003	0.0001	0.0009	17.6699	−0.0548	3.4165
19	−0.1640	0.0003	−0.0001	−0.0008	18.6055	−0.0523	3.7697
20	−0.0626	0.0001	0.0000	−0.0003	19.5402	−0.0497	4.1250
21	0.0602	−0.0001	0.0001	0.0003	9.5908	−5.6376	2.0178
22	0.1447	−0.0002	0.0002	0.0007	10.4040	−6.0994	2.3722
23	0.3809	−0.0004	0.0006	0.0018	11.2165	−6.5616	2.7276
24	−0.2095	0.0002	−0.0003	−0.0010	12.0294	−7.0258	3.0793
25	−0.0477	0.0001	−0.0001	−0.0002	12.8412	−7.4893	3.4346
26	−0.2084	0.0003	−0.0003	−0.0010	13.6530	−7.9537	3.7885
27	0.0223	0.0000	0.0000	0.0001	14.4642	−8.4179	4.1443
28	0.1927	−0.0002	0.0003	0.0009	15.2752	−8.8825	4.4998
29	−0.1640	0.0002	−0.0002	−0.0008	16.0867	−9.3480	4.8530
30	−0.0626	0.0001	−0.0001	−0.0003	16.8975	−9.8132	5.2083

(r_b=10.95 mm; b=1.394443 mm; L=12.566 mm; α=20.0°)

where p is the lead per radian, and u and θ are surface parameters. For right side flank, the values of α and b are made negative. The surface normal is obtained by the Eq. 4.

$$N = \frac{\partial p_w}{\partial u} \times \frac{\partial p_w}{\partial \theta} N = \begin{bmatrix} N_x \\ N_y \\ N_z \end{bmatrix} = \begin{bmatrix} -p\cos(\alpha)\sin(\theta) - (r_b + u\ \cos(\alpha))\cos(\theta)\sin(\alpha) \\ p\cos(\alpha)\cos(\theta) - (r_b + u\ \cos(\alpha))\sin(\theta)\sin(\alpha) \\ (r_b + u\ \cos(\alpha))\cos(\alpha) \end{bmatrix} \tag{4}$$

4.2 Normal deviation

Normal deviation is computed as the distance of the point from the foot of the normal, which lies on the ideal surface. Suppose $p_c(x_c, y_c, z_c)$ is a point on the real surface, then the normal line passing through this point is given by Eq. 5.

$$\frac{x_w - x_c}{N_x} = \frac{y_w - y_c}{N_y} = \frac{z_w - z_c}{N_z} \tag{5}$$

Surface parameter u can be obtained as a function of θ by substituting Eqs. 3 and 4 into the first part of Eq. 5.

$$u = \frac{p\{x_c \cos(\theta) + y_c \sin(\theta)\}}{p\cos(\alpha) + \sin(\alpha)\{x_c \sin(\theta) - y_c \cos(\theta)\}} - \frac{r_b}{\cos(\alpha)} \tag{6}$$

Equation 7 is obtained as a function of θ by substituting Eqs. 3, 4 and 6 into the second part of Eq. 5.

$$\begin{aligned} &(r_b + u\cos(\alpha))\{(r_b + u\cos(\alpha))\cos(\theta) + x_c\} \\ &+ \{b + u\sin(\alpha) - z_c - p\theta\}\{(r_b + u\cos(\alpha)) \\ &\tan(\alpha)\cos(\theta) + p\sin(\theta)\} = 0 \end{aligned} \tag{7}$$

Since Eq. 7 is non-linear, a numerical method is to be used to solve the equation. In this case, the Newton-Raphson method is used and a convergence condition of 10^{-6} is taken. An initial value of θ for the numerical method is obtained using Eq. 8.

$$\theta_{initial} = \frac{1}{p}\left(b + \left(\sqrt{(x_c^2 + y_c^2)} - r_b\right)\tan\alpha - z_c\right) \tag{8}$$

The value of u can be obtained by substituting the value of θ in Eq. 6. Substituting the values of u and θ in Eq. 3 the foot of the normal $p_w(x_w, y_w, z_w)$ is obtained. Then the normal deviation is calculated as the distance between p_c and p_w using Eq. 9.

$$nd = \sqrt{(x_w - x_c)^2 + (y_w - y_c)^2 + (z_w - z_c)^2} \tag{9}$$

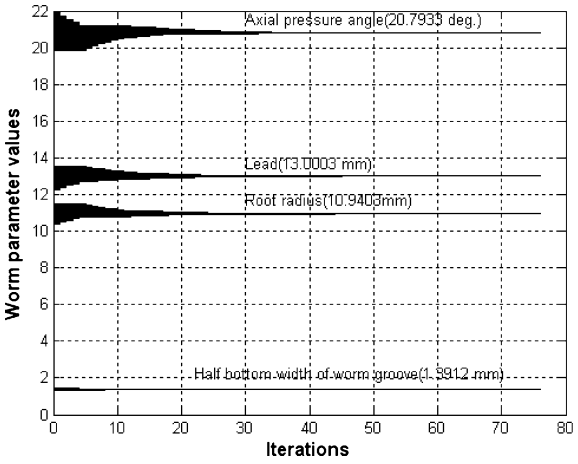


Fig. 5 Convergence pattern in OA based algorithm

4.3 Validation of the algorithm

Validation is done with input data points with known errors. Hence, the measured data points are generated from the theoretical surface equation of the worm with known errors introduced in some critical parameters. The design details of the worm under study are given in Table 3. Equation 10 gives the probe center point coordinates $p_m(x_m, y_m, z_m)$ for left side flank as shown in Fig. 4.

$$p_m = \begin{bmatrix} x_m \\ y_m \\ z_m \end{bmatrix} = \begin{bmatrix} (r_b + u\ \cos(\alpha))\cos(\theta) - r_p n_x \\ (r_b + u\ \cos(\alpha))\sin(\theta) - r_p n_y \\ b + u\ \sin(\alpha) - p\theta - r_p n_z \end{bmatrix} \quad (10)$$

where

$$n_x = \frac{N_x}{\|N_x\|};\ n_y = \frac{N_y}{\|N_y\|};\ n_z = \frac{N_z}{\|N_z\|}$$

In this case study, the four worm design parameters discussed above are considered and an L_8 array has been selected. Systematic errors of +0.434 mm in lead and +0.8° in axial pressure angle have been introduced, as the other two parameters are not critical. Probe center coordinates are generated using Eq. 10 with a probe radius of 0.5 mm. Parameter u is varied between 0 mm and 9 mm in steps of 1 mm and three values are taken for parameter θ ($-\pi/6$, 0, $\pi/6$) to obtain 30 generated points on left side flank of right hand worm. In order to make measurement data more realistic, a maximum repeatability error of 5 μm has been included in a Gaussian random manner. The gaussian random

Table 5 Comparison of results

OA based algorithm						
Conditions	With systematic error only		With noise only		With systematic error and noise	
Initial values	r_b=10.95 mm b=1.39 mm L=12.9 mm α=20.9°		r_b=10.95 mm b=1.39 mm L=12.7 mm α=20.2°		r_b=10.95 mm b=1.39 mm L=12.9 mm α=20.9°	
Parameters	Predicted	Difference	Predicted	Difference	Predicted	Difference
r_b, mm	10.9459	−0.0041	10.9475	−0.0025	10.9403	−0.0097
b, mm	1.3924	−0.0020	1.3932	−0.0012	1.3912	−0.0032
L, mm	12.9996	−0.0004	12.5663	+0.0003	13.0003	+0.0003
α, degrees	20.8071	+0.0071	20.0059	+0.0059	20.7933	−0.0067
Minimum value of objective function (10^{-5})	0.5		3.4		2.2	
Number of iterations	73		72		75	
Number of objective function evaluations	584		576		600	
Genetic algorithm						
r_b, mm	10.8888	−0.0612	10.8786	−0.0714	10.8722	−0.0778
b, mm	1.3744	−0.0200	1.3682	−0.0262	1.3623	−0.0321
L, mm	13.0031	+0.0031	12.5811	+0.0151	13.0081	+0.0081
α, degrees	20.7649	−0.0351	20.0015	+0.0015	20.8371	+0.0371
Minimum value of objective function (10^{-5})	9.3		5.5		17.8	
Number of generations	48		77		106	
Number of objective function evaluations	1440		2310		3180	

(r_b=10.95 mm; b=1.394443 mm; L=12.566 mm; α=20.0°) (S_r=0.05, L_m=0.1) (Nc=30, Ng=4, Nd=14, $Pcross$=0.8, $Pmute$=0.02, Parameter range ±5% from initial value)

Table 6 Deviation of measurement data from best-fit surface

Sl.no	Systematic error only* mm				Systematic error with noise** mm			
	Xm	Ym	Zm	$nd\text{-}r_p$	Xm	Ym	Zm	$nd\text{-}r_p$
1	9.6779	5.4871	−0.1492	−0.00039	9.6778	5.4871	−0.1489	0.00009
2	10.4845	5.9605	0.2049	−0.00031	10.4842	5.9605	0.2055	−0.00042
3	11.2915	6.4330	0.5591	−0.00013	11.2908	6.4329	0.5609	−0.00174
4	12.0988	6.9049	0.9136	−0.00008	12.0992	6.9049	0.9126	0.00109
5	12.9064	7.3761	1.2681	0.00009	12.9065	7.3762	1.2679	0.00017
6	13.7142	7.8470	1.6228	0.00017	13.7146	7.8470	1.6218	0.00085
7	14.5222	8.3174	1.9775	0.00031	14.5222	8.3174	1.9776	−0.00041
8	15.3304	8.7875	2.3323	0.00043	15.3300	8.7875	2.3332	−0.00142
9	16.1387	9.2574	2.6871	0.00059	16.1390	9.2575	2.6863	0.00033
10	16.9471	9.7270	3.0420	0.00067	16.9472	9.7270	3.0417	−0.00036
11	11.1248	−0.0870	0.9342	−0.00050	11.1247	−0.0869	0.9344	0.00012
12	12.0601	−0.0803	1.2882	−0.00031	12.0598	−0.0802	1.2889	−0.00046
13	12.9952	−0.0746	1.6425	−0.00023	12.9945	−0.0743	1.6442	−0.00171
14	13.9303	−0.0696	1.9969	−0.00008	13.9307	−0.0698	1.9959	0.00116
15	14.8653	−0.0653	2.3515	−0.00001	14.8654	−0.0653	2.3512	0.00021
16	15.8003	−0.0614	2.7061	0.00015	15.8007	−0.0616	2.7051	0.00091
17	16.7353	−0.0580	3.0608	0.00031	16.7353	−0.0580	3.0609	−0.00036
18	17.6703	−0.0549	3.4156	0.00043	17.6699	−0.0548	3.4165	−0.00137
19	18.6052	−0.0522	3.7704	0.00058	18.6055	−0.0523	3.7697	0.00029
20	19.5401	−0.0497	4.1253	0.00066	19.5402	−0.0497	4.1250	−0.00032
21	9.5909	−5.6377	2.0175	−0.00050	9.5908	−5.6376	2.0178	0.00007
22	10.4042	−6.0996	2.3715	−0.00031	10.4040	−6.0994	2.3722	−0.00039
23	11.2169	−6.5622	2.7258	−0.00023	11.2165	−6.5616	2.7276	−0.00173
24	12.0292	−7.0254	3.0802	−0.00009	12.0294	−7.0258	3.0793	0.00111
25	12.8411	−7.4892	3.4348	0.00000	12.8412	−7.4893	3.4346	0.00019
26	13.6528	−7.9534	3.7894	0.00018	13.6530	−7.9537	3.7885	0.00087
27	14.4642	−8.4179	4.1442	0.00022	14.4642	−8.4179	4.1443	−0.00039
28	15.2754	−8.8827	4.4989	0.00041	15.2752	−8.8825	4.4998	−0.00129
29	16.0865	−9.3478	4.8538	0.00049	16.0867	−9.3480	4.8530	0.00033
30	16.8974	−9.8131	5.2086	0.00067	16.8975	−9.8132	5.2083	−0.00024

*(r_b=10.9459 mm; b=1.3924 mm; L=12.9996 mm; α=20.8071°)
**(r_b=10.9403 mm; b=1.3912 mm; L=13.0003 mm; α=20.7933°)

numbers are generated between −0.5 and 0.5 using Boxmuller transformation. The repeatability error of a measured point is obtained by multiplying the Gaussian random number generated for that coordinate with maximum repeatability error. This error is then multiplied with the unit normal vectors along the x, y, z directions and added to the x_m, y_m, z_m coordinate values. Table 4 shows the ideal surface coordinates for left side flank and the sequence of generation of probe center point with systematic error in parameters with a probe of 0.5 mm radius. Gaussian repeatability error is also incorporated as measurement noise and the corresponding probe center values are also included in the table. A termination criterion is taken as ten iterations over which there is no change in objective function value. With the generated probe center points and initial values, the OA based algorithm predicts the worm parameters. Figure 5 shows the pattern of range convergence of each parameter.

The same problem has been attempted through a single objective multi-parameter genetic algorithm (GA) with roulette wheel selection, single point cross over and unary mutation operator. The results of both the algorithms are compared in Table 5 for different conditions. It is found from the results that the OA based iterative algorithm proposed in this paper involves fewer objective function evaluations compared to genetic algorithm. The results are repeatable with OA based algorithm as no random parameter is involved; however, with GA, results are not repeatable as it operates with random parameters.

With the parameters obtained from the OA based algorithm, best-fit worm surface is generated. The deviation of measured point from best-fit surface is obtained from Eq. 9 and subtracting the probe radius as given in Table 6. It is seen that the maximum deviation is 0.67 μm with systematic error introduced, and around 1.74 μm with noise added along with the systematic error. This shows that the best-fit surface is in conformity with the measurement data.

5 Conclusion

Measurement of worm using CMM to predict the deviation of worm parameters using an OA based iterative optimization algorithm is explained in this paper. The algorithm proposed makes use of the effect of individual parameters on the objective function using design of experiments and ANOM. The validation of this algorithm is also carried out and the algorithm is able to predict the parameters within reasonable accuracy. When compared with genetic algorithm, the proposed algorithm is faster, due to fewer evaluations of objective function and is highly repeatable as there are no random operations involved. Computerized prediction of deviation of worm parameters helps to enhance the geometric quality by design and setting corrections. The algorithm proposed in this paper can be used in many engineering problems where the objective function is to be evaluated by large time-consuming programs.

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