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ORIGINAL ARTICLE

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Integrating multivariate engineering process control and multivariate statistical process control

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Abstract Multivariate engineering process control (MEPC) and multivariate statistical process control (MSPC) are two strategies for quality improvement that have developed independently. MEPC aims to minimize variability by adjusting process variables to keep the process output on target. On the other hand, MSPC aims to reduce variability by monitoring and eliminating assignable causes of variation. In this paper, the use of MEPC alone is compared to using the MEPC coupled with MSPC. We use simulations to evaluate the average run lengths (ARL) and the averages of the performance measure. The simulation results show that the use of both MEPC and MSPC can always outperform the use of either alone. To detect small sustained shifts of the mean vector, combing MEPC with a multivariate generally weighted moving average (MGWMA) chart (MEPC/MGWMA) is more sensitive than the MEPC/multivariate exponentially weighted moving average (MEWMA) chart and MEPC/Hotelling's χ^2 chart. An example of the application, based on the proposed method, is also given.

Keywords Automatic process control · Control chart · EWMA · GWMA · Multivariate engineering process control · Multivariate statistical process control

1 Introduction

Statistical process control (SPC) uses measurements to monitor a process and looks for major changes. Most SPC techniques assume that the process data can be described in terms of statistically independent observations that fluctuate around a constant mean. On the other hand, engineering process control (EPC) and automatic process control (APC) make regular changes to adjustable process variables to keep the output quality characteristic on target. Box and Kramer [1] mention that the origin of statistical process monitoring was in the parts industry, whereas EPC had its origins in the process industry. The concept of integrating EPC and SPC techniques uses EPC to reduce the effect of predictable quality variations, and uses SPC to monitor the process for detection of assignable causes. MacGregor [2] and Box and Kramer [1] have presented overview descriptions of this integration concept.

Montgomery et al. [3] (including the minimum mean squared error (MMSE) control rule [4]) and Keats et al. [5] (including the proportional integral derivative (PID) control rule [4]) showed that proper use of both EPC and SPC can always outperform the use of either alone. Sachs et al. [6] developed a run-to-run controller (RTR) which combines EPC/SPC to automate the response to shifts and drifts, and has proven to be a successful application. More recent discussions on the EPC/SPC integration can be seen in Tsung and Shi [7], Tsung [8], and Pan and del Castillo [9]. However, these studies are restricted to single input and single output (SISO) models. In practice, many manufacturing processes, such as the silicon epitaxy process and the chemical-mechanical polishing process, are multiple input and multiple output (MIMO). The integration of MEPC and MSPC has a practical necessity. Although Raich and Cinar [10] have applied principal components and discriminant analysis to quantitatively describe and interpret disturbances in the multivariate process, integrating MEPC and MSPC still has received very little attention in the literature.

The purpose of this paper is to demonstrate the potential effectiveness of integrating MEPC and MSPC in a reasonably general situation. We use the multivariate EWMA controller proposed by Tseng et al. [11] as a feedback controller of the MIMO process model, and apply some multivariate control charts to detect the assignable causes. The remainder of this paper is organized as follows: in Sect. 2 the MIMO process model and the multivariate EWMA controller are introduced. Sect. 3 describes some MSPC charts, such as Hotelling's χ^2 control chart,

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the MEWMA control chart, and the MGWMA control chart. In Sect. 4 we compare the use of MEPC alone to using the MEPC coupled with MSPC. An example of the application, based on the proposed method, is given. In Sect. 5 the numerical simulation is used to evaluate the average run lengths (ARL) before the sustained shift of the mean vector is detected and the average Euclidean distance of the deviations from the target vector. Finally, we offer our conclusions in Sect. 6.

2 The MIMO EWMA controller

Focusing on the process control problem, Ingolfsson and Sachs [12] considered a first-order model for the process and discuss the process stability conditions of a single exponentially weighted moving average (EWMA) controller. Bulter and Stefani [13] proposed a double EWMA controller to eliminate the deterministic drift within the process. Recently, in the MIMO case, Tseng et al. [11] proposed a multivariate EWMA controller for a linear MIMO model. Del Castillo and Rajagopal [14] proposed an MIMO double EWMA feedback controller for drifting processes. Tseng et al. [11] described a linear MIMO system with m inputs and p outputs as follows:

$$\mathbf{y}_i = \boldsymbol{\alpha} + \boldsymbol{\beta}\mathbf{c}_{i-1} + \boldsymbol{\varepsilon}_i. \tag{1}$$

In the above equation, $\mathbf{y}_i$ is a $(p \times 1)$ vector containing the quality characteristics (outputs), $\boldsymbol{\alpha}$ is a $(p \times 1)$ vector containing the offset parameter of each output, $\boldsymbol{\beta}$ is a $(p \times m)$ process gain matrix, $\mathbf{c}_{i-1}$ is an $(m \times 1)$ vector giving the levels of the input recipes (or controllable factors), and $\boldsymbol{\varepsilon}_i$ is a $(p \times 1)$ vector denoting the process disturbance. It is assumed that the dynamics come from $\boldsymbol{\varepsilon}_i$.

The intercepts, $\boldsymbol{\alpha}$, will be estimated on-line and updated after each run. Let $\hat{\boldsymbol{\alpha}}_0$ denote the estimate of $\boldsymbol{\alpha}$ at the beginning with $i = 0$. For simplicity, assume that an estimate, $\mathbf{B}$, of the process gain, $\boldsymbol{\beta}$, will be obtained off-line using methods such as linear regression and design of experiments techniques. Then, the predicted model is as follows:

$$\hat{\mathbf{y}}_i = \hat{\boldsymbol{\alpha}}_0 + \mathbf{B}\,\mathbf{c}_{i-1}.$$

When the feedback control scheme is not implemented, the input recipe $\mathbf{c}_0$ will be:

$$\mathbf{c}_0 = \mathbf{B}^{-1}(\boldsymbol{\tau} - \hat{\boldsymbol{\alpha}}_0), \tag{2}$$

where $\boldsymbol{\tau}$ is the target vector. Therefore, the expected initial bias $\boldsymbol{\gamma}_0$, a $(p \times 1)$ vector, will be $\boldsymbol{\gamma}_0 = \boldsymbol{\alpha} + \mathbf{B}\mathbf{c}_0 - \boldsymbol{\tau}$.

Similar to the single EWMA controller proposed by Ingolfsson and Sachs [12], the multivariate EWMA controller can be proposed as follows:

$$\begin{aligned}\hat{\boldsymbol{\alpha}}_i &= \omega(\mathbf{y}_i - \mathbf{B}\mathbf{c}_{i-1}) + (1-\omega)\hat{\boldsymbol{\alpha}}_{i-1} \\ &= \hat{\boldsymbol{\alpha}}_{i-1} + \omega(\mathbf{y}_i - \boldsymbol{\tau}),\end{aligned} \tag{3}$$

where ω is a discount factor $(0 < \omega < 1)$. In Tseng et al. [11], the stability conditions of the multivariate EWMA controller and the feasible region of ω are derived, and the determination of an optimal discount factor ω is within a finite number of production runs, such that the trace of the total MSE is minimized. When the ith run is completed, $\hat{\boldsymbol{\alpha}}_i$ will be updated. Then, the new input vector $\mathbf{c}_i$ can be written as follows:

$$\mathbf{c}_i = (\mathbf{I} - \mathbf{B}'(\mathbf{B}\mathbf{B}')^{-1}\mathbf{B})\mathbf{c}_{i-1} + \mathbf{B}'(\mathbf{B}\mathbf{B}')^{-1}(\boldsymbol{\tau} - \hat{\boldsymbol{\alpha}}_i). \tag{4}$$

Let $\boldsymbol{\varepsilon}_0 = \mathbf{0}$ and $\boldsymbol{\tau} = \mathbf{0}$; then, the off-target amount at run i can be expressed as:

$$\begin{aligned}\mathbf{y}_i - \boldsymbol{\tau} &= \mathbf{y}_i \\ &= (1-\omega)^{i-1}\boldsymbol{\gamma}_0 + \sum_{t=0}^{i-1}(1-\omega)^t(\boldsymbol{\varepsilon}_{i-t} - \boldsymbol{\varepsilon}_{i-t-1}).\end{aligned} \tag{5}$$

When $\boldsymbol{\varepsilon}_i$ in Eq. 1 is a white noise series with mean vector $\boldsymbol{\mu}$ and a common covariance matrix $\boldsymbol{\Sigma}$, the expected value of $\mathbf{y}_i$ (from Eq. 5) will be:

$$E(\mathbf{y}_i - \boldsymbol{\tau}) = E(\mathbf{y}_i) =$$
$$(1-\omega)^{i-1}\boldsymbol{\gamma}_0 + E(\boldsymbol{\varepsilon}_i) - \omega E\left(\sum_{t=0}^{i-2}(1-\omega)^t\,\boldsymbol{\varepsilon}_{i-t-1}\right), \tag{6}$$

the covariance matrix of $\mathbf{y}_i$ will be:

$$\begin{aligned}\boldsymbol{\Sigma}_{\mathbf{y}_i} &= \boldsymbol{\Sigma} + \frac{\omega}{2-\omega}\left(1-(1-\omega)^{2(i-1)}\right)\boldsymbol{\Sigma} \\ &= \left(1 + \frac{\omega}{2-\omega}\left(1-(1-\omega)^{2(i-1)}\right)\right)\boldsymbol{\Sigma}.\end{aligned} \tag{7}$$

When $0 < \omega \leq 1$, the process is asymptotically stable. That is, when $i \to \infty$, Eqs. 6 and 7 can be reduced to:

$$\lim_{i\to\infty} E(\mathbf{y}_i) = E(\boldsymbol{\varepsilon}_i) = \boldsymbol{\mu}, \tag{8}$$

and

$$\lim_{i\to\infty} \boldsymbol{\Sigma}_{\mathbf{y}_i} \cong (2/(2-\omega))\boldsymbol{\Sigma} < \infty. \tag{9}$$

If $\boldsymbol{\mu} = \mathbf{0}$, from Eq. 8, $\lim_{i\to\infty} E(\mathbf{y}_i) = \mathbf{0}$.

When $\boldsymbol{\varepsilon}_i$ in Eq. 1 is a multivariate IMA (1,1) time series, that is

$$\boldsymbol{\varepsilon}_i - \boldsymbol{\varepsilon}_{i-1} = \mathbf{a}_i - \boldsymbol{\Theta}\mathbf{a}_{i-1},$$

where $\{a_i\}_{i=1}^{\infty}$ is a white noise series with a common covariance matrix $\boldsymbol{\Sigma}$, the covariance matrix of $\mathbf{y}_i$ will be as follows:

$$\boldsymbol{\Sigma}_{\mathbf{y}_i} = \boldsymbol{\Sigma} + \frac{1-(1-\omega)^{2(i-1)}}{1-(1-\omega)^2}[(1-\omega)\,\mathbf{I} - \boldsymbol{\Theta}]\,\boldsymbol{\Sigma}\,[(1-\omega)\,\mathbf{I} - \boldsymbol{\Theta}]'.$$

If the multivariate EWMA controller defined in Eq. 4 is asymptotically stable, from Eq. 5, $\lim_{i\to\infty} E(\mathbf{y}_i) = \boldsymbol{\tau}$ and $\lim_{i\to\infty} \boldsymbol{\Sigma}_{\mathbf{y}_i} < \infty$. For simplicity, we consider the case that $\boldsymbol{\varepsilon}_i$ is a white noise series only. For more detail refer to Tseng et al. [11].

The control action of Eq. 3 assumes that there are no assignable causes present. The only source of disturbance is the white noise series $\boldsymbol{\varepsilon}_i$ in Eq. 1. We now investigate how this system operates when additional assignable causes occur. Assume that this MIMO process model is appropriate and that the statistical monitoring scheme will only signal external changes (i.e., assignable causes). Applying MSPC to monitor the output deviation from target can result in rapid detection of assignable causes; we assume the assignable cause takes the form of a sustained shift in the process mean vector. If the assignable causes are eliminated, then the output deviation will be reduced. Using MEPC alone in the MIMO process will be compared to using MEPC coupled with MSPC. In this paper, three different MSPC charts for monitoring the output deviation from the target are used: Hotelling's χ^2 control chart, the MEWMA control chart, and the MGWMA control chart.

3 Some MSPC charts

3.1 Hotelling's χ^2 control chart

Hotelling's χ^2 control chart is a direct analog of the univariate Shewhart $\bar{X}$ control chart for monitoring the mean vector of the process. In this paper, we denote the set of positive integers as I^+ (i.e., $I^+ = \{1, 2, 3, \ldots\}$). According to the MIMO system defined in Sect. 2, let the white noise series $\boldsymbol{\varepsilon}_i$, $i \in I^+$, in Eq. 1 be independent multivariate normal random vectors with mean vectors $\boldsymbol{\mu}_i$ and a common covariance matrix $\boldsymbol{\Sigma}$ (i.e., $\boldsymbol{\varepsilon}_i \sim N(\boldsymbol{\mu}_i, \boldsymbol{\Sigma})$, $\boldsymbol{\Sigma}$ is non-singular). By measuring deviation from the target vector ($\boldsymbol{\tau} = \mathbf{0}$), $\mathbf{y}_i$, $i \in I^+$, we denote the known covariance matrix of $\mathbf{y}_i$ as $\boldsymbol{\Sigma}_{\mathbf{y}_i}$ (Eq. 7). Then, Hotelling's χ^2 control chart gives an out-of-control signal as soon as the statistic T_i^2, as

$$\begin{aligned} T_i^2 &= \mathbf{y}'_i \boldsymbol{\Sigma}_{\mathbf{y}_i}^{-1} \mathbf{y}_i \\ &= \mathbf{y}'_i \left(\left(1 + \frac{\omega}{2-\omega} \left(1 - (1-\omega)^{2(i-1)} \right) \right) \boldsymbol{\Sigma} \right)^{-1} \mathbf{y}_i > h_1 \end{aligned} \tag{10}$$

at time i, where the upper control limit (UCL) h_1 (> 0) is chosen to achieve a specified in-control ARL (ARL_0). Details concerning Hotelling's χ^2 control chart can be found in Hotelling [15] and Montgomery [16].

3.2 The MEWMA control chart

Because Hotelling's χ^2 control chart is based on only the most recent observation, it is not sensitive to small shifts in the mean vector. Lowry et al. [17] proposed an EWMA-based multivariate control procedure (MEWMA) for monitoring the process mean vector. If $\boldsymbol{\varepsilon}_i$ in Eq. 1 is a white noise series with mean vector $\mathbf{0}$ and a common covariance matrix $\boldsymbol{\Sigma}$, and there is no a priori reason to weight past observations differently for the p quality characteristics being monitored, the equation for the MEWMA control chart is as follows:

$$\mathbf{Z}_i = r\mathbf{y}_i + (1-r)\mathbf{Z}_{i-1}, \tag{11}$$

where $\mathbf{Z}_i$ is a $(p \times 1)$ vector , $i \in I^+$, $\mathbf{Z}_0 = \mathbf{0}$, $0 < r \leq 1$. From Eqs. 7 and 11, the covariance matrix of $\mathbf{Z}_i$ is:

$$\begin{aligned} \boldsymbol{\Sigma}_{\mathbf{Z}_i} &= [r(1-(1-r)^{2i})/(2-r)]\boldsymbol{\Sigma}_{\mathbf{y}_i} \\ &= \left[r(1-(1-r)^{2i})/(2-r) \right] \\ &\quad \times \left[1 + \frac{\omega}{2-\omega} \left(1 - (1-\omega)^{2(i-1)} \right) \right] \boldsymbol{\Sigma}. \end{aligned} \tag{12}$$

The MEWMA control chart gives an out-of-control signal as soon as the statistic T_i^2,

$$T_i^2 = \mathbf{Z}'_i \boldsymbol{\Sigma}_{\mathbf{Z}_i}^{-1} \mathbf{Z}_i > h_2, \tag{13}$$

where the UCL h_2 (> 0) is chosen to achieve a specified ARL_0.

3.3 The MGWMA control chart

Sheu and Lin [18] proposed a generally weighted moving average (GWMA) control chart which is a generalization of the EWMA control chart. Due to the added adjustment parameter α, the GWMA control chart has been shown to perform much better than Shewhart's chart and the EWMA chart in monitoring small shifts of the process mean under the univariate case. In the multivariate case, we propose the multivariate GWMA (MGWMA) control chart which is a natural extension of the GWMA control chart. If $\boldsymbol{\varepsilon}_i$ in Eq. 1 is a white noise series with mean vector $\mathbf{0}$ and a common covariance matrix $\boldsymbol{\Sigma}$, and there is no a priori reason to weight past observations differently for the p quality characteristics being monitored, then the equation for the MGWMA control chart is as follows:

$$\mathbf{g}_i = \sum_{t=1}^{i} \left[q^{(i-t)^\alpha} - q^{(i-t+1)^\alpha} \right] \mathbf{y}_t, \qquad i \in I^+, \tag{14}$$

where $\mathbf{g}_i$ is a $(p \times 1)$ vector ($\mathbf{g}_0 = \mathbf{0}$), the design parameter q is constant ($0 \leq q < 1$), and the adjustment parameter α is determined by the practitioner. From Eqs. 7 and 14, the covariance matrix of $\mathbf{g}_i$ is:

$$\begin{aligned} \boldsymbol{\Sigma}_{\mathbf{g}_i} &= \text{var} \left\{ \sum_{t=1}^{i} \left[q^{(i-t)^\alpha} - q^{(i-t+1)^\alpha} \right] \mathbf{y}_t \right\} \\ &= (q^{0^\alpha} - q^{1^\alpha}) \boldsymbol{\Sigma}_{\mathbf{y}_i} (q^{0^\alpha} - q^{1^\alpha}) + (q^{1^\alpha} - q^{2^\alpha}) \boldsymbol{\Sigma}_{\mathbf{y}_i} (q^{1^\alpha} - q^{2^\alpha}) \\ &\quad + \cdots + (q^{(i-1)^\alpha} - q^{i^\alpha}) \boldsymbol{\Sigma}_{\mathbf{y}_i} (q^{(i-1)^\alpha} - q^{i^\alpha}) \\ &= Q_i \boldsymbol{\Sigma}_{\mathbf{y}_i} \\ &= Q_i \left[\left(1 + \frac{\omega}{2-\omega} \left(1 - (1-\omega)^{2(i-1)} \right) \right) \boldsymbol{\Sigma} \right], \end{aligned} \tag{15}$$

where $Q_i = (q^{0^\alpha} - q^{1^\alpha})^2 + (q^{1^\alpha} - q^{2^\alpha})^2 + \cdots + (q^{(i-1)^\alpha} - q^{i^\alpha})^2$. The MGWMA control chart gives an out-of-control signal as soon as

$$\begin{aligned} T_i^2 &= \mathbf{g}'_i [Q_i \boldsymbol{\Sigma}_{\mathbf{y}_i}]^{-1} \mathbf{g}_i \\ &= \mathbf{g}'_i \left[Q_i \left(1 + \frac{\omega}{2-\omega} \left(1 - (1-\omega)^{2(i-1)} \right) \right) \boldsymbol{\Sigma} \right]^{-1} \mathbf{g}_i > h_3, \end{aligned} \tag{16}$$

where the UCL h_3 (> 0) is chosen to achieve a specified ARL_0.

Here, the in-control mean vector of $\mathbf{y}_i$ is $\boldsymbol{\mu} = \boldsymbol{\mu}_0 = \mathbf{0}$, and the out-of-control mean vector is $\boldsymbol{\mu} = \boldsymbol{\mu}_1$. Lowry et al. [17] have shown that the ARL performance of the MEWMA control chart depends only on the mean vector $\boldsymbol{\mu}_1$ and covariance matrix $\boldsymbol{\Sigma}_{\mathbf{y}_i}$ through the value of the non-centrality parameter λ, where:

$$\lambda = \left[(\boldsymbol{\mu}_1 - \boldsymbol{\mu}_0)' \boldsymbol{\Sigma}_{\mathbf{y}_i}^{-1} (\boldsymbol{\mu}_1 - \boldsymbol{\mu}_0)\right]^{1/2} = \left[\boldsymbol{\mu}'_1 \boldsymbol{\Sigma}_{\mathbf{y}_i}^{-1} \boldsymbol{\mu}_1\right]^{1/2}. \qquad (17)$$

In the MGWMA case, if we let

$$\begin{aligned} \mathbf{g}_i^* &= \mathbf{M}\mathbf{g}_t \\ &= \sum_{t=1}^{i} [q^{(i-t)^\alpha} - q^{(i-t+1)^\alpha}]\mathbf{M}\mathbf{y}_t, \end{aligned}$$

where $i \in I^+$, $\mathbf{M}$ is a $p \times p$ full-rank matrix. It follows that:

$$T_i^{*^2} = \mathbf{g}_i^{*'} \boldsymbol{\Sigma}_{\mathbf{g}_i^*}^{-1} \mathbf{g}_i^* = \mathbf{g}_i' \boldsymbol{\Sigma}_{\mathbf{g}_i}^{-1} \mathbf{g}_i = T_i^2, \qquad i \in I^+.$$

It means that the values of the MGWMA statistic in Eq. 16 are invariant to any full-rank transformation of the data. Therefore, the ARL performance of the MGWMA procedure also depends only on $\boldsymbol{\mu}_1$ and $\boldsymbol{\Sigma}_{\mathbf{y}_i}$ through the value of the non-centrality parameter λ in Eq. 17.

When $\alpha = 1$ and $q = 1 - r$, $\mathbf{g}_i$ (from Eq. 14) will be as follows:

$$\begin{aligned} \mathbf{g}_i &= \sum_{t=1}^{i} [(1-r)^{(i-t)} - (1-r)^{(i-t+1)}]\mathbf{y}_t \\ &= r\mathbf{y}_i + (1-r)\mathbf{g}_{i-1}, \end{aligned}$$

which is similar to Eq. 11 of the MEWMA control chart. Then, $\boldsymbol{\Sigma}_{\mathbf{g}_i}$ and T_i^2 (from Eqs. 15 and 16) will be

$$\begin{aligned} \boldsymbol{\Sigma}_{\mathbf{g}_i} &= [(q^0 - q^1)^2 + (q^1 - q^2)^2 + \cdots + (q^{(i-1)} - q^i)^2]\boldsymbol{\Sigma}_{\mathbf{y}_i} \\ &= [r(1-(1-r)^{2i})/(2-r)]\boldsymbol{\Sigma}_{\mathbf{y}_i} \end{aligned}$$

and

$$T_i^2 = \mathbf{g}'_i [[r(1-(1-r)^{2i})/(2-r)]\boldsymbol{\Sigma}_{\mathbf{y}_i}]^{-1} \mathbf{g}_i,$$

which is similar to Eqs. Eq. 12 and Eq. 13. That is, the MEWMA control chart is a special case in the MGWMA control chart when $\alpha = 1$.

When the process is under control, ARL_0 should be sufficiently large to avoid false alarms; however, when the process is out of control, the ARL (named ARL_1) should be sufficiently small to rapidly detect shifts. The design parameters of the MGWMA control chart are the value of q, α, and h to achieve a specified ARL_0. Simulation [19] is used to estimate the ARL of the MGWMA control chart. With various design parameter $q \in \{0.7, 0.8, 0.9\}$, different adjustment parameters $\alpha \in \{0.7, 0.8, 0.9, 1.0\}$, and in-control ($\lambda = 0$) ARL_0 is maintained at approximately 200 by changing the width of the control limits (h_3). Each simulation runs 20 000 iterations and each iteration ends when $T_i^2 > h_3$. The ARL performance for several MGWMA control schemes is shown in Table 1 and Table 2 with $p = 2$ and $p = 4$, respectively. When $\alpha = 1.0$, the MGWMA control chart reduces to the MEWMA control chart. Based on Table 1 and Table 2, the adjustment parameter α of the MGWMA control chart is more sensitive to small shifts in

Table 1. ARLs of MGWMA control charts ($p = 2$)

	$q = 0.7$				$q = 0.8$				$q = 0.9$			
	$\alpha = 0.7$	0.8	0.9	1.0	0.7	0.8	0.9	1.0	0.7	0.8	0.9	1.0
λ	$h_3 = 10.29$	10.23	10.20	10.13	9.96	9.83	9.75	9.71	9.25	9.03	8.85	8.78
0.00	200.0	200.0	200.0	200.0	200.0	200.0	200.0	200.0	200.0	200.0	200.0	200.0
0.25	**99.2**	**104.9**	**109.7**	110.9	**74.8**	**79.8**	**88.7**	95.5	**63.7**	**64.0**	**69.3**	73.9
0.50	**37.3**	**39.3**	**41.6**	43.6	**28.7**	**30.0**	**31.5**	33.6	**23.6**	**23.7**	**23.9**	25.0
0.75	**17.7**	**17.8**	**18.8**	19.4	**14.7**	**14.5**	**14.7**	15.3	12.7	**12.2**	**12.2**	12.5
1.00	**10.7**	**10.3**	**10.3**	10.7	9.3	**9.0**	**8.8**	9.1	8.0	7.7	7.7	7.7
1.25	6.9	6.8	**6.7**	6.8	6.3	**6.1**	**5.9**	6.2	5.6	5.4	5.4	5.2
1.50	5.0	4.9	4.8	4.8	4.6	4.5	4.5	4.4	4.2	4.1	4.0	4.0
2.00	3.1	3.0	3.0	2.9	2.9	2.8	2.8	2.8	2.7	2.6	2.5	2.5

Table 2. ARLs of MGWMA control charts ($p = 4$)

	$q = 0.7$				$q = 0.8$				$q = 0.9$			
	$\alpha = 0.7$	0.8	0.9	1.0	0.7	0.8	0.9	1.0	0.7	0.8	0.9	1.0
λ	$h_3 = 14.58$	14.50	14.43	14.36	14.28	14.12	14.05	13.93	13.48	13.23	13.08	12.93
0.00	200.0	200.0	200.0	200.0	200.0	200.0	200.0	200.0	200.0	200.0	200.0	200.0
0.25	**83.4**	**86.4**	**90.3**	94.0	**64.8**	**67.9**	**74.2**	77.9	**48.6**	**49.6**	**53.2**	56.1
0.50	**26.0**	**26.4**	**28.2**	29.9	**20.9**	**20.6**	**21.3**	22.6	16.9	**16.6**	**16.8**	16.9
0.75	**11.8**	**11.9**	**11.9**	12.3	10.3	**10.0**	**9.9**	10.1	8.8	8.7	**8.5**	8.6
1.00	7.0	6.7	6.7	6.6	6.3	6.0	**5.9**	6.0	5.5	5.4	**5.3**	5.3
1.25	4.6	4.4	4.4	4.4	4.3	4.2	4.1	4.1	3.9	3.8	3.7	3.7
1.50	3.3	3.3	3.2	3.2	3.2	3.1	3.1	3.0	2.9	2.8	2.8	2.8
2.00	2.1	2.0	2.0	2.0	2.0	2.0	2.0	1.9	1.9	1.9	1.8	1.8

the process mean vector than to that of the MEWMA control chart.

4 An example

We first show a simple example ($m = p = 2$), and then give the results of a more comprehensive simulation study. Let the number of production runs $n = 100$. Assume the white noise series $\boldsymbol{\varepsilon}_i$ in Eq. 1 follows the bivariate normal distribution. The mean vector of $\boldsymbol{\varepsilon}_i$ is on target at $[0\ 0]'$ for the first 20 observations. At time $i = 21$, a disturbance consisting of a sustained shift of magnitude (0.875, 0) units is introduced into the process. That is, $\boldsymbol{\mu}_0 = [0\ 0]'$, $\boldsymbol{\mu}_1 = [0.875\ 0]'$. Let $\omega = 0.1$, $\hat{\boldsymbol{\alpha}}_0 = [1\ 1]'$,

$$\mathbf{B} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \boldsymbol{\Sigma} = \begin{bmatrix} 1.0 & 0.5 \\ 0.5 & 1.0 \end{bmatrix}, \quad \boldsymbol{\gamma}_0 = [0.2\ \ 0.2]'.$$

From Eqs. 2, 9, and 17, we get $\mathbf{c}_0 = \mathbf{B}^{-1}(\boldsymbol{\tau} - \hat{\boldsymbol{\alpha}}_0) = [-1\ -1]'$,

$$\boldsymbol{\Sigma}_{\mathbf{y}_i} \cong \frac{2}{2-\omega}\boldsymbol{\Sigma} = \frac{2}{1.96}\begin{bmatrix} 1.0 & 0.5 \\ 0.5 & 1.0 \end{bmatrix},$$

and $\lambda = \left(\boldsymbol{\mu}'_1 \boldsymbol{\Sigma}_{\mathbf{y}_i}^{-1} \boldsymbol{\mu}_1\right)^{1/2} = 1.0$.

After Johnson and Wichern [20], if we want to simulate a random vector $\boldsymbol{\varepsilon}$ from a multivariate normal distribution with mean $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$ (i.e., $\boldsymbol{\varepsilon} \sim N(\boldsymbol{\mu}, \boldsymbol{\Sigma})$), let κ_i be the ith eigenvalue of the covariance matrix $\boldsymbol{\Sigma}$, $\mathbf{e}_i$ be the ith normalized eigenvector, and $\mathbf{N}$ be a $p \times 1$ vector of independent standard normal deviates (i.e., $\mathbf{N} \sim N(\mathbf{0}, \mathbf{I})$). To generate one random multivariate normal vector $\boldsymbol{\varepsilon}$ from a population with mean $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$, begin by generating p random standard normal deviates as the elements of vector $\mathbf{N}$. Let

$$\mathbf{N} = \mathbf{A}(\boldsymbol{\varepsilon} - \boldsymbol{\mu}),$$

where

$$\mathbf{A} = \left[\frac{1}{\sqrt{\kappa_1}}\mathbf{e}'_1, \frac{1}{\sqrt{\kappa_2}}\mathbf{e}'_2, \cdots, \frac{1}{\sqrt{\kappa_p}}\mathbf{e}'_p\right]'.$$

Then we can get

$$\boldsymbol{\varepsilon} = \mathbf{A}^{-1}\mathbf{N} + \boldsymbol{\mu}. \tag{18}$$

According to Eq. 18 and the data mentioned above, the first 20 random vectors of $\boldsymbol{\varepsilon}_i$ are computed as follows:

$$\begin{aligned} \boldsymbol{\varepsilon}_i &= \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{bmatrix} = \mathbf{A}^{-1}\mathbf{N} + \boldsymbol{\mu}_0 \\ &= \frac{1}{\sqrt{2}}\begin{bmatrix} \sqrt{0.5} & \sqrt{1.5} \\ -\sqrt{0.5} & \sqrt{1.5} \end{bmatrix}\begin{bmatrix} N_1 \\ N_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \\ i &= 1, 2, \ldots, 20. \end{aligned}$$

The last 80 random vectors of $\boldsymbol{\varepsilon}_i$ are computed as follows:

$$\begin{aligned} \boldsymbol{\varepsilon}_i &= \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{bmatrix} = \mathbf{A}^{-1}\mathbf{N} + \boldsymbol{\mu}_1 \\ &= \frac{1}{\sqrt{2}}\begin{bmatrix} \sqrt{0.5} & \sqrt{1.5} \\ -\sqrt{0.5} & \sqrt{1.5} \end{bmatrix}\begin{bmatrix} N_1 \\ N_2 \end{bmatrix} + \begin{bmatrix} 0.875 \\ 0 \end{bmatrix}, \\ i &= 21, 22, \ldots, 100. \end{aligned}$$

Figure 1 shows the output for 100 observations of the process under the MIMO process model given by Eq. 1 with the MEWMA controller given by Eqs. 3 and 4. At time $i = 21$, a disturbance consisting of a sustained shift of magnitude (0.875, 0) units is introduced into the process. Figure 2 shows the resulting control actions from Eq. 4. Without using MSPC to detect the shift, the MEWMA controller (c_1) compensates for this sustained shift to a large degree.

Figures 3 to 5 show the process output assuming that in addition to the MEPC rule, a Hotelling's χ^2 chart, an MEWMA chart, and an MGWMA chart are applied to the output deviation from target, respectively. The values of T_i^2 that correspond to Hotelling's χ^2 chart are obtained using Eq. 10. The values of T_i^2 that correspond to the MEWMA chart are obtained using Eq. 13 with the design parameter $r = 0.20$. The values of

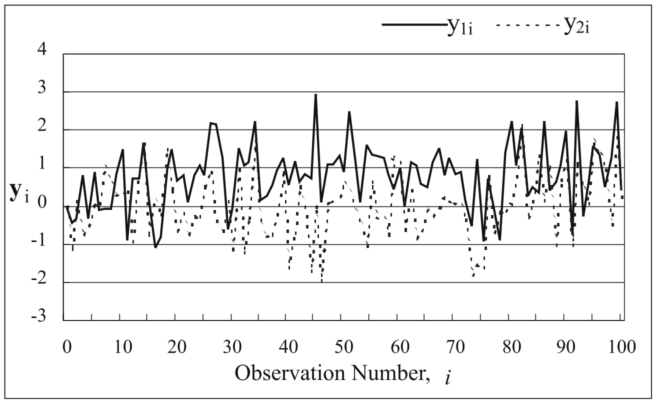

Fig. 1. Output deviations from the target using MEPC alone. A sustained shift of (0.875, 0) units occurs at $i = 21$ and PM = 1.296

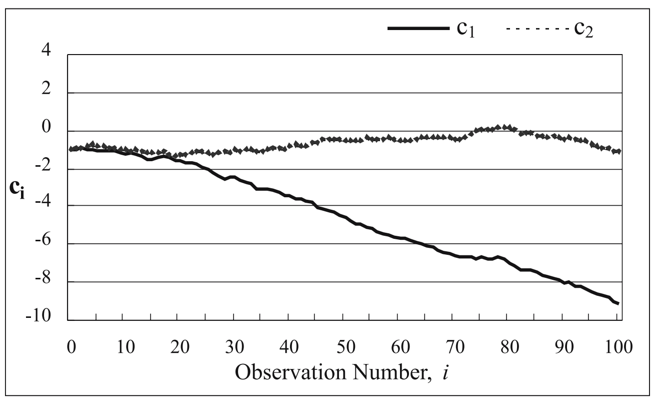

Fig. 2. Control actions for the process in Fig. 1 (using MEPC alone)

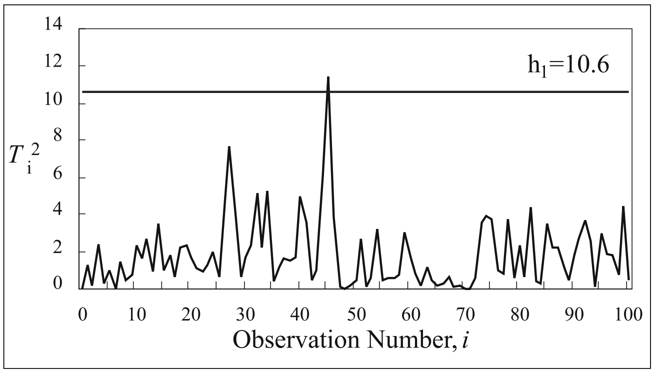

Fig. 3. Output deviations from the target using MEPC and a Hotelling's χ^2 chart. A sustained shift of (0.875, 0) units occurs at $i = 21$ and PM $= 1.109$

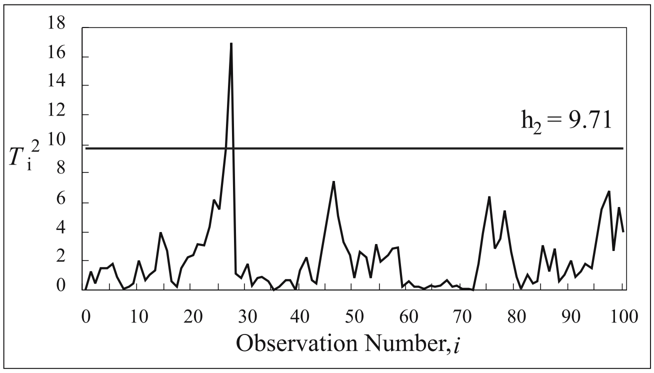

Fig. 4. An MEWMA chart applied to the output deviations from the target with $r = 0.2$. A sustained shift of (0.875, 0) units occurs at $i = 21$ and PM $= 1.054$

T_i^2 that correspond to the MGWMA chart are obtained using Eq. 16 with the design parameter $q = 0.8$ (equivalent to $r = 0.2$ in MEWMA) and the adjustment parameter $\alpha = 0.9$. The control limits $h_1 = 10.6$, $h_2 = 9.71$ (with $q = 0.8$, $\alpha = 1.0$), and $h_3 = 9.75$ (with $q = 0.8$, $\alpha = 0.9$) are obtained from Lowry et

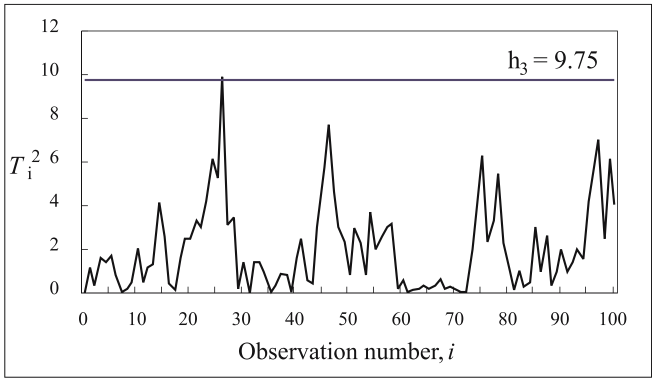

Fig. 5. An MGWMA chart applied to the output deviations from the target with $q = 0.8$ and $\alpha = 0.9$. A sustained shift of (0.875, 0) units occurs at $i = 21$ and PM $= 1.046$

al. [17] and Table 1 to provide ARL_0's of 200. The control limits (h_1, h_2, and h_3) are shown on the control charts. Assume that the shift is eliminated as soon as it is detected. The Hotelling's χ^2 chart (in Fig. 3) signals out-of-control after the 45th observation, whereas the MEWMA chart (in Fig. 4) signals after the 27th observation, and the MGWMA chart (in Fig. 5) signals after the 26th observation. Under the assigned parameters described above, it takes only 11.7 samples and 11.6 samples in average for the MEPC/MEWMA chart and the MEPC/MGWMA chart after the shift to detect an out-of-control signal, while 59.6 samples are needed for the MEPC/Hotelling's χ^2 chart (see Table 3).

The performance measure we used is the average Euclidean distance of the deviation from the target **0**. That is,

$$PM = \frac{1}{n} \sum_{j=1}^{n} \left[\sum_{i=1}^{p} y_{ij}^2 \right]^{1/2}, \tag{19}$$

where, in this case, $n = 100$, $p = 2$. The performance measures (PM) in Figs. 1, 3, 4, and 5 are PM = 1.296, 1.109, 1.054, and 1.046, respectively. Because the sustained shift is detected most

Table 3. ARL_1s and averages of PM (in parentheses) for MEPC/MSPC charts ($ARL_0 = 200$, $m = p = 2$, $n = 500$)

λ	Prior to Shift	(1) MEPC alone	(2) MEPC/ Hotelling's χ^2, $h_1 = 10.6$	(3) MEPC/MGWMA (4) MEPC/MEWMA $q = 0.7$ (3) $\alpha = 0.7$ $h_3 = 10.29$	(4) $\alpha = 1.0$ $h_2 = 10.1$	$q = 0.8$ (3) $\alpha = 0.8$ $h_3 = 9.83$	(4) $\alpha = 1.0$ $h_2 = 9.71$	$q = 0.9$ (3) $\alpha = 0.8$ $h_3 = 9.03$	(4) $\alpha = 1.0$ $h_2 = 8.78$
0.25			168.3	**121.1**	129.4	**103.50**	113.8	**80.9**	89.9
	(1.253)	(1.264)	(1.262)	**(1.258)**	(1.259)	**(1.256)**	(1.258)	**(1.254)**	(1.256)
0.50			137.8	**48.0**	56.1	**37.4**	42.4	**30.5**	31.0
	(1.253)	(1.293)	(1.293)	**(1.263)**	(1.267)	**(1.260)**	(1.262)	**(1.258)**	(1.258)
0.75			96.8	**22.1**	24.4	**18.4**	19.3	17.2	16.5
	(1.253)	(1.342)	(1.317)	**(1.263)**	(1.265)	**(1.260)**	(1.262)	(1.259)	(1.260)
1.00			59.6	**13.2**	13.3	**11.6**	11.7	11.8	11.2
	(1.253)	(1.407)	(1.321)	**(1.263)**	(1.264)	**(1.261)**	(1.262)	(1.261)	(1.261)
2.00			9.2	4.7	4.6	4.8	4.8	5.4	5.4
	(1.253)	(1.782)	(1.283)	(1.264)	(1.264)	(1.264)	(1.264)	(1.267)	(1.267)
5.00			2.1	**2.1**	2.3	**2.3**	2.5	**2.6**	2.9
	(1.253)	(3.230)	(1.266)	**(1.266)**	(1.270)	**(1.269)**	(1.273)	**(1.274)**	(1.278)

quickly by the MGWMA chart and is eliminated as soon as possible, the PM value of MEPC/MGWMA is smallest. Thus, among the four control schemes, MEPC/MGWMA is the best, and MEPC alone is the worst. Figures 6 to 8 show the resulting

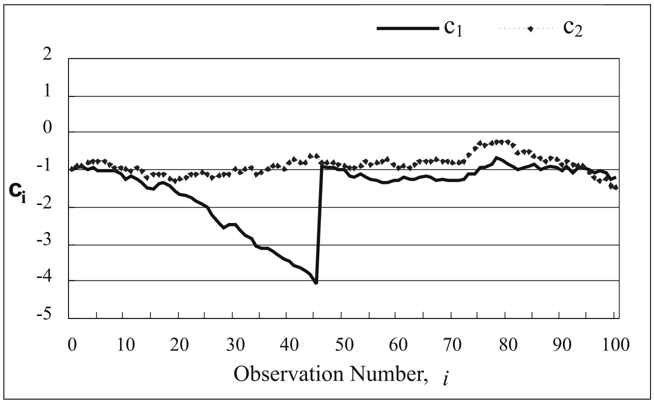

Fig. 6. Control actions for the process in Fig. 3 (MEPC/Hotelling's χ^2 chart). A sustained shift of (0.875, 0) units occurs at $i = 21$ and the assignable cause is eliminated after $i = 45$

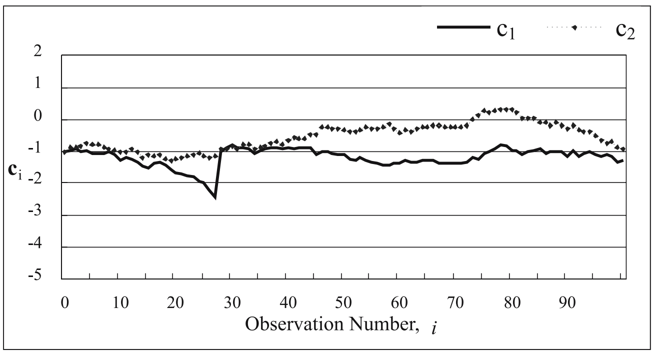

Fig. 7. Control actions for the process in Fig. 4 (MEPC/MEWMA chart). A sustained shift of (0.875, 0) units occurs at $i = 21$ and the assignable cause is eliminated after $i = 27$

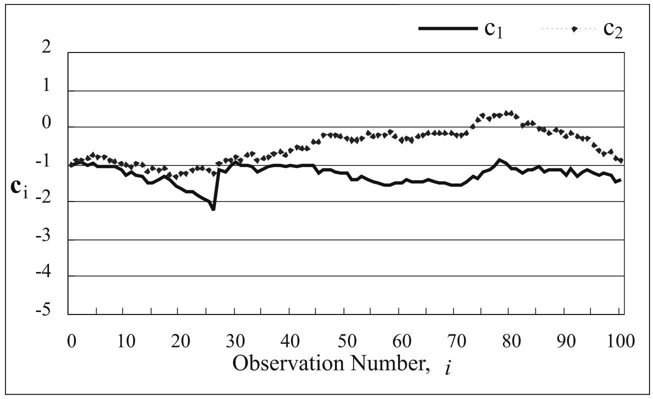

Fig. 8. Control actions for the process in Fig. 5 (MEPC/MGWMA chart). A sustained shift of (0.875, 0) units occurs at $i = 21$ and the assignable cause is eliminated after $i = 26$

control actions (c_1, c_2) from Eq. 4. As soon as the shift is detected and is eliminated, the magnitude of the control actions (c_1, c_2), especially the c_1 value (i.e., the solid line in Figs. 6 to 8), will be reduced immediately.

5 Simulation results

The simulation study is performed to further investigate the performance of this integrated MEPC rule and MSPC rule. Assume the assignable cause is a sustained shift. Several different MSPC control charts for the output deviation from the target are investigated. An in-control ($\lambda = 0$) ARL (ARL_0) is maintained at approximately 200 by changing the width of the control limits (h). The shift magnitudes investigated are $\lambda = 0.25$, 0.5, 0.75, 1, 2, and 5. When $\alpha = 1.0$, the MGWMA chart is reduced to the MEWMA chart. The assignable cause occurs at $i = 251$ and is eliminated as soon as it is detected by the MSPC chart. The out-of-control ARL_1s and the averages of performance measures are used for comparison. The ARL comparisons based on the non-

Table 4. ARL_1s and averages of PM (in parentheses) for MEPC/MSPC charts ($ARL_0 = 200$, $m = p = 4$, $n = 500$)

λ	Prior to Shift	(1) MEPC alone	(2) MEPC/ Hotelling's χ^2, $h_1 = 14.9$	(3) MEPC/MGWMA (4) MEPC/MEWMA $q = 0.7$ (3) $\alpha = 0.7$ $h_3 = 14.58$	$q = 0.7$ (4) $\alpha = 1.0$ $h_2 = 14.36$	$q = 0.8$ (3) $\alpha = 0.8$ $h_3 = 14.12$	$q = 0.8$ (4) $\alpha = 1.0$ $h_2 = 13.93$	$q = 0.9$ (3) $\alpha = 0.8$ $h_3 = 13.23$	$q = 0.9$ (4) $\alpha = 1.0$ $h_2 = 12.93$
0.25			177.5	**145.1**	151.2	**128.5**	137.5	**101.7**	113.0
	(1.879)	(1.894)	(1.883)	**(1.880)**	(1.881)	**(1.880)**	(1.880)	**(1.878)**	(1.880)
0.50			160.3	**69.0**	81.6	**52.0**	61.1	**38.3**	41.3
	(1.879)	(1.917)	(1.911)	**(1.888)**	(1.892)	**(1.884)**	(1.887)	**(1.881)**	(1.882)
0.75			128.9	**30.4**	35.3	**23.9**	26.1	20.7	20.3
	(1.879)	(1.954)	(1.942)	**(1.888)**	(1.891)	**(1.884)**	(1.886)	(1.883)	(1.883)
1.00			91.7	**17.0**	18.1	**14.5**	14.7	14.1	13.3
	(1.879)	(2.005)	(1.958)	**(1.887)**	(1.889)	**(1.885)**	(1.885)	(1.884)	(1.884)
2.00			15.0	5.5	5.3	5.5	5.4	6.1	6.0
	(1.879)	(2.314)	(1.920)	(1.887)	(1.887)	(1.887)	(1.887)	(1.889)	(1.889)
5.00			2.2	**2.2**	2.5	2.5	2.7	2.8	3.0
	(1.879)	(3.650)	(1.888)	**(1.890)**	(1.893)	(1.893)	(1.896)	(1.898)	(1.901)

centrality parameter (λ) assume that a shift to $\boldsymbol{\mu} = \boldsymbol{\mu}_1$ will be detected as quickly as a shift to $\boldsymbol{\mu} = \boldsymbol{\mu}_2$, if $\lambda = \left(\boldsymbol{\mu}'_1 \boldsymbol{\Sigma}^{-1} \boldsymbol{\mu}_1\right)^{1/2} = \left(\boldsymbol{\mu}'_2 \boldsymbol{\Sigma}^{-1} \boldsymbol{\mu}_2\right)^{1/2}$. PM results are calculated across 500 periods ($n = 500$) per simulation run; the MEPC rule continues for all 500 periods. The random vector $\boldsymbol{\varepsilon}_i$ in Eq. 1 is assumed to be an independent multivariate normal distribution with mean $\boldsymbol{\mu}_i$ and a common covariance matrix $\boldsymbol{\Sigma}$ (i.e., $\boldsymbol{\varepsilon}_i \sim N(\boldsymbol{\mu}_i, \boldsymbol{\Sigma})$). We consider the numbers of inputs (m) and outputs (p) are ($m = p = 2$) and ($m = p = 4$) only. Let the discount factor be $\omega = 0.1$. Each simulation runs 20 000 iterations. All programs are written in the SAS programming language (the SAS system for Windows, Release 8.02).

Table 3 (with $m = p = 2$) and Table 4 (with $m = p = 4$) show the simulation results of ARL_1s and the averages of PM (in parentheses). The second columns of Tables 3 and 4 give the performance measures prior to the introduction of the shift (i.e., for periods 1-250). The next column gives the performance measures for period 251-500 for either the MEPC alone or some combination of MEPC and an MSPC chart. Based on the boldface numbers in Tables 3 and 4, a variance of the adjustment parameter α ($\alpha < 1$) indicates that the MGWMA chart is more sensitive to small shifts than the MEWMA chart ($\alpha = 1$) with the same q value, and the Hotelling's χ^2 chart. When q is smaller, the properties become even more obvious. For instance, when ($q = 0.8$, $\alpha = 0.8$) or ($q = 0.7$, $\alpha = 0.7$), and $\lambda \leq 1$, the ARL_1s and the averages of PM are smaller than those of the MEWMA chart and the Hotelling's χ^2 chart.

6 Conclusion

Most of MEPC schemes are designed to react to process disturbances and do not make any effort to remove the assignable causes. The MSPC chart can be used to monitor, identify, and subsequently eliminate the assignable causes. In this paper, we have demonstrated the potential effectiveness of integrating MEPC and MSPC in a reasonably general situation. Although we consider the case that the disturbance is only a white noise series, when the disturbance is a multivariate, IMA (1, 1) time series will get the similar results.

Combining MEPC and MSPC charts always results in the reduction of overall variability if the process has external assignable causes that lead to sustained shifts. Especially in detecting small shifts of the mean vector (due to the added adjustment parameter), the combined MEPC/MGWMA chart is more sensitive than a MEPC/MEWMA chart and MEPC/Hotelling's χ^2 chart. We conclude that proper use of both MEPC and MSPC can always outperform the use of either one alone.

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