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ORIGINAL ARTICLE

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Robust pin layout design for sheet-panel locating

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Abstract Sheet panels, represented as freeform surfaces in a CAD system, are widely used in manufacturing processes. Locating blocks and pins, collectively known as locators, are the most common fixtures for the joining and assembly of sheet panels. In this paper, a robust design approach is utilized to optimize the pin layout so that the sheet panel variations, expressed as translational and orientational variations at certain key product/process characteristic points (KPCs), are minimized. The advantage of this approach is that the locating variations at any given point on the panel can be obtained quantitatively, and hence, the optimal pin design can be selected from among an infinite set of feasible designs. Based on the analyses, some useful pin design guidelines will also be proposed.

Keywords Fixture optimization · Pin locating · Robust design · Sheet panel · Variation analysis

1 Introduction

Fixture research has been accelerated in the past decade due to the increasing need for advanced manufacturing. Asada and By [1] conducted kinematics analysis for automatically reconfigurable fixtures. In their research, the conditions for deterministic locating and total restraining were derived. Mani and Wilson [2] proposed an approach to decompose the 3D fixture design problem to 2D by considering workpiece cross sections. Chou et al. [3] formulated mathematical theories for automatic configuration of machining fixtures for prismatic workpieces. Sayeed and DeMeter [4] developed fixture design analysis software that considered kinematics restraint, total restraint, and tool path clearance requirements. In the above research, kinematics was the dominant tool.

In many manufacturing processes, deformations of workpieces and/or fixtures are significant effects, and these were studied by Shawki and Abdel-Aal [5, 6] through experiments. In 1991, Menassa and DeVries [7], based on the effort of Lee and Haynes [8] to utilize the finite element method in fixture design, solved the fixture optimization problem. Their method selected the fixture locations that could minimize workpiece deflection, using the finite element modeling technique. The design variables were three locators on a primary datum, as required by the "3-2-1" principle. In the sheet panel fixture research field, Ceglarek and Shi [9] used a pattern recognition method to perform fixture failure diagnosis for autobody assembly. Cai et al. [10], extending the work by Rearick et al. [11], proposed the "*N*-2-1" locating principle for sheet panel fixturing, where "*N*" represents the locating blocks, and "2-1" represents the two locating pins. Built on the "*N*-2-1" principle, an optimal fixture design method was proposed to achieve minimum workpiece deformation.

The locating accuracy of a workpiece and its interactions with the fixturing system is an important topic. Weill et al. [12] analyzed workpiece locating variations, where the "function comparison method," an optimization method for discrete variables, was employed to choose the best locating set-up from randomly selected locating candidates. Cai et al. [13] developed a variational method for robust fixture configuration design to minimize workpiece locating variations due to source variations such as workpiece/fixture surface form variations and locator set-up variations. An optimal locating configuration could be found from among an infinite number of feasible locating schemes that can ensure deterministic locating and total fixturing conditions. Source codes were developed to simulate 3D robust fixture design.

This paper utilizes the robust fixture design technique presented by Cai et al. [13] to optimize the locating pin design for sheet panels. By using the robust design technique, the locating

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quality can be evaluated quantitatively, and the best pin layout with minimum workpiece variations can be obtained. Some previously unknown locating guidelines will be established, and analytical arguments for some well-known design rules (such as those described in Reid [14]) will be presented.

2 Algorithms for robust locator design

This section summarizes the key algorithms for robust fixture design proposed by Cai et al. [13], followed by the 2D formula for robust locating pin design in sheet panel applications.

1. Robust design refers to the minimization of workpiece variations due to source variations such as workpiece/locator form and/or locator setup variations in a deterministic or unique locating environment.
2. A 2D workpiece requires three locators to be deterministically located (Fig. 1). The translational and orientational variations at any key product/process characteristic point (KPC) can be expressed as a vector:

$$\delta\mathbf{q_0} \equiv \left[\delta x_0 \;\; \delta y_0 \;\; \delta\phi_0\right]^T = -\mathbf{J}^{-1}\mathbf{\Phi_R}\delta\mathbf{R} \tag{1}$$

where the Jacobian is expressed as

$$\mathbf{J} = \begin{bmatrix} -n_{1x} & -n_{1y} & n_{1y}x_1 - n_{1x}y_1 \\ -n_{2x} & -n_{2y} & n_{2y}x_2 - n_{2x}y_2 \\ -n_{3x} & -n_{3y} & n_{3y}x_3 - n_{3x}y_3 \end{bmatrix} \tag{2}$$

and

$$\mathbf{\Phi_R} = \begin{bmatrix} n_{1x} & n_{1y} & 0 & 0 & 0 & 0 \\ 0 & 0 & n_{2x} & n_{2y} & 0 & 0 \\ 0 & 0 & 0 & 0 & n_{3x} & n_{3y} \end{bmatrix} \tag{3}$$

Here, $\delta\mathbf{R} \equiv \left[\delta x_1 \;\; \delta y_1 \;\; \delta x_2 \;\; \delta y_2 \;\; \delta x_3 \;\; \delta y_3\right]^T$ represents the source variation vector at locating points, $(x_i,\ y_i)$, $i = 1, 2, 3$. The $\mathbf{n_i} \equiv \left[n_{ix} \; n_{iy}\right]^T$, $i = 1, 2, 3$ denotes the surface normal vector at locating points. Note that it is assumed that only one KPC is studied at a time, and that the KPC also serves as the origin of the coordinate system, based on which all of the parameters in Eqs. 2 and 3 are defined. Also note that the Jacobian $\mathbf{J}$ cannot be singular, which is guaranteed if the workpiece is deterministically located. In this way, the workpiece resultant variation vector $\delta\mathbf{q_0}$ is linked to the source variation vector $\delta\mathbf{R}$. The purpose of the robust design is then to choose locator positions to reduce the $\delta\mathbf{q_0}$ as much as possible, given the quantity of $\delta\mathbf{R}$.

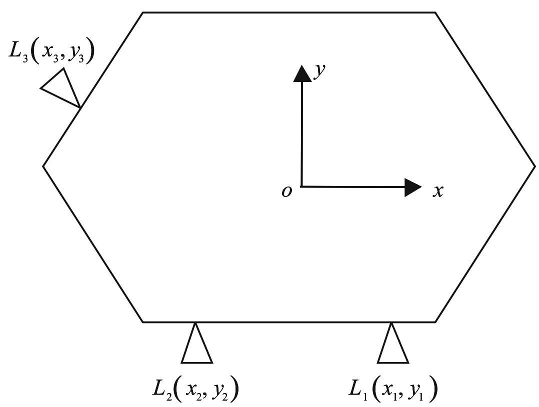


Fig. 1. A deterministically located 2D workpiece

3. A software program has been developed for the robust design of a locating system. The program solves the following optimization problem, stated here in 2D format:
Minimize

$$F(\mathbf{X}) = \frac{1}{\sqrt{2N_{\mathrm{KPC}}}}\sqrt{\sum_{i=1}^{N_{\mathrm{KPC}}}\left[\sigma^2(\delta x_0)_i + \sigma^2(\delta y_0)_i\right]} \tag{4a}$$

Subject to

$$\mathbf{G}_1(\mathbf{X}) = 0 \tag{4b}$$
$$\mathbf{G}_2(\mathbf{X}) \geq 0$$

where $\mathbf{X}$ is called the design variable vector for optimization, which, in this case, represents the locator coordinates, i.e., $\mathbf{X} \equiv (x_1, y_1, x_2, y_2, x_3, y_3)^T$. $F(\mathbf{X})$ is called the objective function for optimization, and N_{KPC} is the number of KPCs. The term $\sigma^2(\delta x_0)_i + \sigma^2(\delta y_0)_i$ represents variations at the ith KPC in the x and y directions. Hence, the objective function in Eq. 4a is the pooled standard deviation of resultant errors at all KPCs. The $\mathbf{G}_1(\mathbf{X})$ represents equality constraints for design variables, i.e., locators must always satisfy workpiece surface equations. The $\mathbf{G}_2(\mathbf{X})$ represents inequality constraints of the design variables, i.e., workpiece surface regions where locators can move.

3 Robustness analyses for pin locating

In contrast to a rigid workpiece, whose locating is usually achieved through six independent blocks, a sheet panel is located through a combination of blocks and pins. It is the common practice that N block locators and two locating pins are used to achieve "N-2-1" locating, as shown in Fig. 2a. The pins function together with a hole and a slot. The pin associated with a hole is called a 4-way pin, which restrains the workpiece in two orthogonal directions, and the other pin associated with the slot is a 2-way pin, which imposes one directional constraint only. Since pins are used to restrain rigid body motion in the in-plane direction, their arrangement does not affect the deformation. They can, however, affect the locating accuracy of the workpiece in in-plane directions due to source variations (such as clearances between the pin and hole/slot). From Fig. 2, we can see that a sheet panel locating system using four block locators and two locating pins (Fig. 2a) is equivalent to Fig. 2b using "4-2-1" locating blocks. Therefore, in the following subsections, we can use the robust design approach presented in Sect. 2 to perform the variation analyses and determine the best layout for

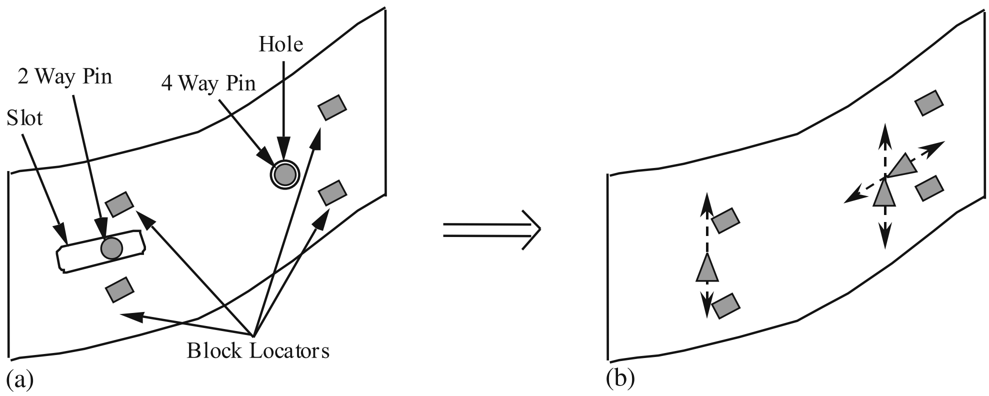


Fig. 2. The "4-2-1" locating system for a sheet panel workpiece

the locating pins. In this section, the locating accuracies of the 1-pin/hole + 1-pin/slot system and 3-pins/slot pair system are studied in detail.

3.1 Influence of slot orientation on locating variations

The first issue to study is the influence of the slot orientation in the 1-pin/hole + 1-pin/slot system, as shown in Fig. 3, where $\theta \in [-90°, 90°]$ represents the slot orientation. The purpose of the analysis is to identify the θ that gives minimum workpiece locating variations. We can choose to study one or more components of the resultant locating variations, depending on applications. For example, if we want primarily to study the rotational behavior of the workpiece as shown in Fig. 3, then the component $\delta\phi_0$ in Eq. 1 is most critical. If we choose to evaluate the overall locating performance, then all of the translational and/or rotational variations should be included.

An equivalent representation of Fig. 3 is shown in Fig. 4 by using three locating blocks, L_1, L_2, and L_3, where L_2 and L_3 share the same coordinates. Following the robust design approach described in Sect. 2, the centerline of the slot can be represented as $ax + by + c = 0$, and the Jacobian is

$$\mathbf{J} = \begin{bmatrix} -a & -b & bx_1 - ay_1 \\ 0 & -1 & x_2 \\ -1 & 0 & -y_1 \end{bmatrix} \tag{5}$$

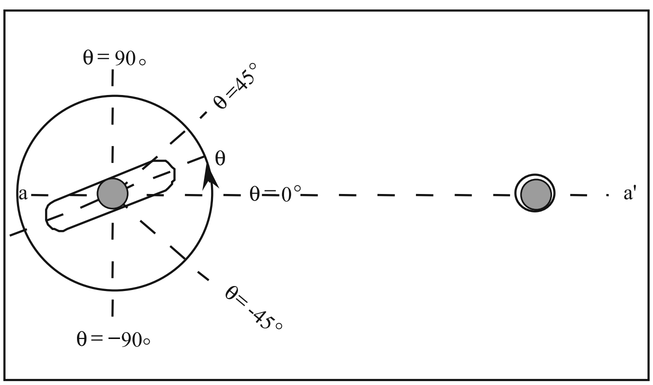


Fig. 3. Different slot orientations

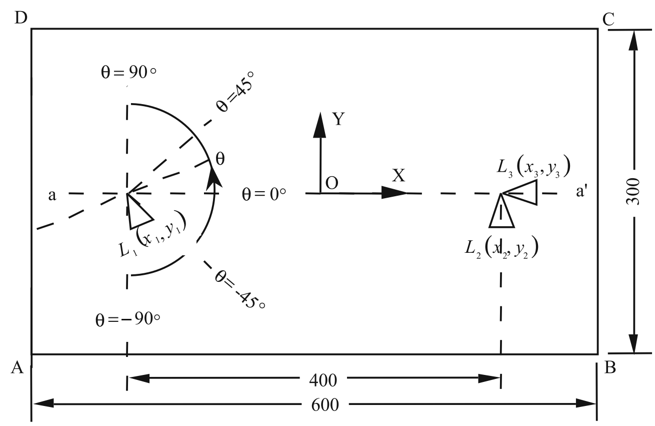


Fig. 4. The schematics of slot orientations for a workpiece-locating system

whose determinant is

$$|\mathbf{J}| = b\,(x_2 - x_1) \tag{6}$$

Eq. 6 indicates that necessary conditions

$$\begin{cases} x_1 \neq x_2 \\ b \neq 0 \Leftrightarrow \theta \neq \pm 90° \end{cases} \tag{7}$$

must be satisfied in order to meet the deterministic locating condition and perform robust design.

Define $k \equiv -\frac{a}{b} = \tan\theta$. Then, the workpiece locating variations at point can be calculated as

$$\delta\mathbf{q_0} = \begin{bmatrix} \delta x_0 \\ \delta y_0 \\ \delta\phi_0 \end{bmatrix} = \begin{bmatrix} \delta x_2 + \frac{\delta y_1 - \delta y_2}{x_1 - x_2} y_1 - k \frac{\delta x_1 - \delta x_2}{x_1 - x_2} y_1 \\ \frac{x_1 \delta y_2 - x_2 \delta y_1}{x_1 - x_2} + k \frac{\delta x_1 - \delta x_2}{x_1 - x_2} x_2 \\ \frac{\delta y_2 - \delta y_1}{x_1 - x_2} + k \frac{\delta x_1 - \delta x_2}{x_1 - x_2} \end{bmatrix} \tag{8a}$$

and its standard deviations are

$$\sigma\,(\delta\mathbf{q_0}) = \begin{bmatrix} \sigma\,(\delta x_0) \\ \sigma\,(\delta y_0) \\ \sigma\,(\delta\phi_0) \end{bmatrix} = \frac{\sigma_0}{x_2 - x_1} \begin{bmatrix} \sqrt{(x_2 - x_1)^2 + 2\left(1 + k^2\right) y_1^2} \\ \sqrt{x_1^2 + \left(1 + 2k^2\right) x_2^2} \\ \sqrt{2\left(1 + k^2\right)} \end{bmatrix} \tag{8b}$$

Here, as is the practice throughout this paper, it is assumed that $(\delta x_1, \delta y_1)$ and $(\delta x_2, \delta y_2)$ follow independent, normal distributions, i.e., $N(0, \sigma_0^2)$.

The workpiece locating deviations due to different slot orientations are plotted in Fig. 5, where in Fig. 5a, the center point O in Fig. 4 has been chosen as the KPC, and in Fig. 5b, all of the points – O, A, B, C, and D –are selected as KPCs. Note that the rotational deviation is close to zero because it is of arc unit as compared to the length unit for translational deviations. As can be seen from the figures, the minimum workpiece locating deviation is achieved when the slot angle $\theta = 0$, indicating that the slot should lie on the straight line of the two locating pins to reduce the workpiece locating variations. The results also show that the average workpiece locating variations are generally magnified several times, implying that small source variations can lead to significant resultant variations.

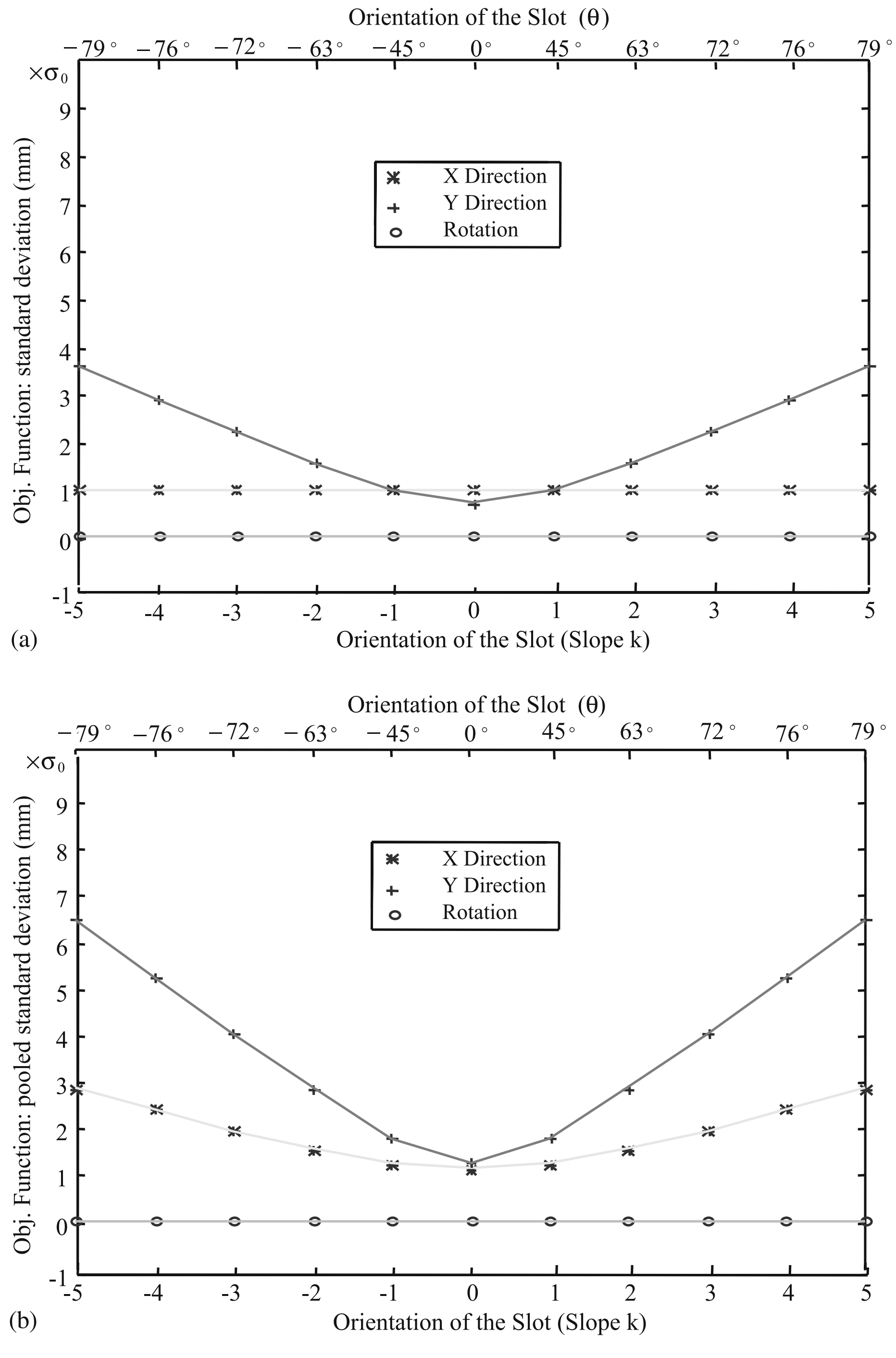


Fig. 5. Standard deviations for different slot orientations

3.2 Influence of the pin distance

Another important issue to study is the optimal selection of the distance between two locating pins. The locating variations of the sheet panel can be obtained by setting $k=0$ in Eq. 8b. The results (the average locating variations for four KPCs, i.e., A, B, C, and D) are plotted in Fig. 6, where the locating variations decrease monotonically as the pin distance increases. In conclusion, the robust design suggests that locating pins should be placed as far away as possible.

3.3 Design of the 3-pins/slot pairs locating system

As an alternative to the 1-pin/hole + 1-pin/slot scheme discussed so far, another locating strategy is to use three separate pin/slot pairs, as shown in Fig. 7a. In this 3-pin/slot pair scenario, the first two slots are typically aligned, i.e., they share a common centerline. The third slot has the freedom to "float" anywhere on the part, depending on process requirements and constraints. In this subsection, the robust design issues with the 3-pin/slot pair system are investigated.

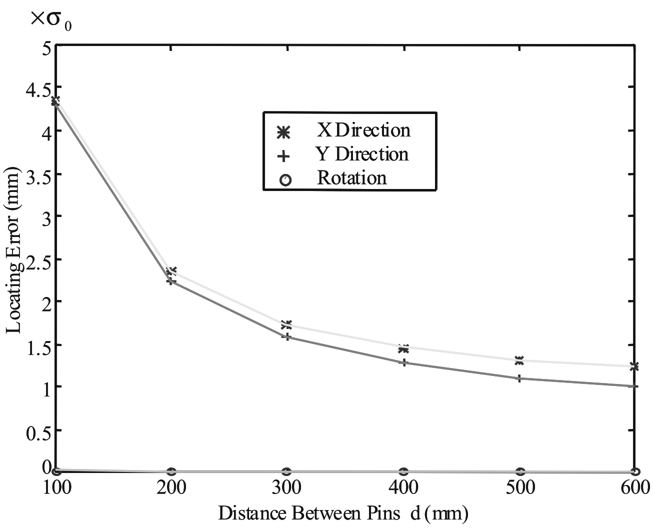


Fig. 6. The inversely proportional relationship between locating variation and pin distance

Define $\xi \equiv \frac{b_3}{a_3}$ for $L3$, which lies on the line $ax+by+c=0$ as shown in Fig. 7b. Then, the workpiece locating variations at an arbitrarily selected point O (also serving as the origin of the coordinate system XOY) can be calculated as

$$\delta \mathbf{q_0} = \begin{bmatrix} \delta x_0 \\ \delta y_0 \\ \delta \phi_0 \end{bmatrix} = \begin{bmatrix} \delta x_3 + \xi \delta y_3 + \frac{[y_3+(x_2-x_3)\kappa]\delta y_1 - [y_3-(x_3-x_1)\xi]\delta y_2}{x_1-x_2} \\ \frac{x_1\delta y_2 - x_2 \delta y_1}{x_1-x_2} \\ \frac{\delta y_2 - \delta y_1}{x_1 - x_2} \end{bmatrix} \tag{9a}$$

and the variances are

$$\sigma^2(\delta \mathbf{q_0}) \equiv \begin{bmatrix} \sigma_x^2 \\ \sigma_y^2 \\ \sigma_\phi^2 \end{bmatrix} = \frac{\sigma_0^2}{(x_2-x_1)^2} \tag{9b}$$

$$\times \begin{bmatrix} \left(1+\xi^2\right)(x_2-x_1)^2 \\ \qquad + [y_3 + (x_2-x_3)\xi]^2 + [y_3 - (x_3-x_1)\xi]^2 \\ x_1^2 + x_2^2 \\ 2 \end{bmatrix}$$

Equation 9b indicates that σ_y^2 and σ_ϕ^2 for any KPCs are determined by the x coordinates of L_1 and L_2 only, while being independent of the locator L_3. The σ_x^2 component, however, depends on all three locators, including the orientation of L_3. Eq. 9b only shows the variances at one particular point of interest. In practice, it is the overall (or average) variance that matters. Therefore, a better formula for variance output is to use the average variance at several KPCs. With this in mind, we have the average variance at points A, B, C, D given by

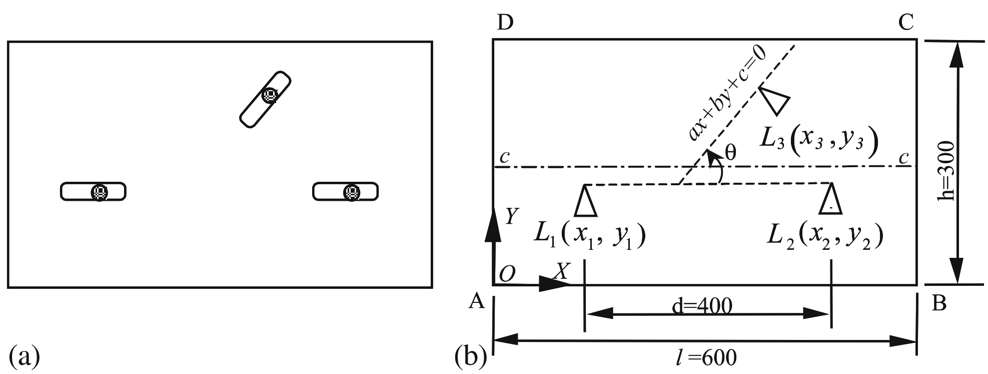
$$\overline{\sigma^2(\delta \mathbf{q_0})} \equiv \begin{bmatrix} \overline{\sigma_x^2} \\ \overline{\sigma_y^2} \\ \overline{\sigma_\phi^2} \end{bmatrix} \tag{9c}$$

$$= \frac{\sigma_0^2}{d_{12}^2} \begin{bmatrix} \left(d_{12}^2 + d_{13}^2 + d_{32}^2\right)\xi^2 + (2y_3 - h)(d_{32} - d_{13})\xi \\ \qquad + \left(d_{12}^2 + 2y_3^2 - 2y_3 h + h^2\right) \\ x_1^2 + x_2^2 \\ 2 \end{bmatrix}$$

where it has been defined that

$$\begin{cases} d_{12} \equiv x_2 - x_1 \\ d_{13} \equiv x_3 - x_1 \\ d_{32} \equiv x_2 - x_3 \end{cases} \tag{9d}$$

Fig. 7a,b. Sheet panel locating system using 3-pin/slot pair **a** The 3-pin/slot locating scheme **b** The equivalent block locating scheme



Therefore, when

$$\xi = \frac{(2y_3 - h)(d_{13} - d_{32})}{2\left(d_{12}^2 + d_{13}^2 + d_{32}^2\right)} \tag{9e}$$

$\overline{\sigma_x^2}$ reaches its minimum of

$$\left[\overline{\sigma_x^2}\right]_{\min} = \frac{\sigma_0^2}{d_{12}^2}\left\{\left(d_{12}^2 + 2y_3^2 - 2y_3 h + h^2\right) + \frac{[(2y_3 - h)(d_{13} - d_{32})]^2}{4\left(d_{12}^2 + d_{13}^2 + d_{32}^2\right)}\right\} \tag{9f}$$

To illustrate the characteristics of Eq. 9e and Eq. 9f, ξ (unitless) and $\left[\overline{\sigma_x^2}\right]_{\min}$ (unit: $\times\sigma_0^2\text{mm}^2$) are plotted in Fig. 8, assuming $l = 600$ mm, $h = 300$ mm, $x_1 = 50$ mm, $x_2 = 450$ mm for the workpiece shown in Fig. 7. The $\left[\overline{\sigma_x^2}\right]_{\min}$ is a function of ξ, with $\xi \in [-0.2156, 0.2156]$, or equivalently, $\theta \in [77.83°, 102.17°]$, and $\left[\overline{\sigma_x^2}\right]_{\min} \in [1.2812, 1.5623] \times \sigma_0^2\text{mm}^2$. Therefore, in general, the third slot should not be perpendicular to the other two slots to achieve the minimum variation.

As a special case, if we choose to let $\xi = 0$, as would be the case in the common practice in production of making the third slot L_3 perpendicular to the first two slots L_1 and L_2, Eq. 9c reduces to

$$\overline{\sigma^2(\delta q_0)} \equiv \begin{bmatrix} \overline{\sigma_x^2} \\ \overline{\sigma_y^2} \\ \overline{\sigma_\phi^2} \end{bmatrix} = \frac{\sigma_0^2}{(x_1 - x_2)^2}\begin{bmatrix} (x_1 - x_2)^2 + 2y_3^2 - 2y_3 h + h^2 \\ x_1^2 + x_2^2 \\ 2 \end{bmatrix} \tag{9g}$$

The $\overline{\sigma_x^2}$ and $\overline{\sigma_y^2}$ components in Eq. 9g are plotted in Fig. 9, with $\overline{\sigma_x^2}$ ranging from $1.28\sigma_0^2$ (minimum, when $y_3 = h/2 = 150$ mm), and $1.56\sigma_0^2$ (maximum when $y_3 = 0$ or $y_3 = h = 300$ mm). Note that in Fig. 9, $\overline{\sigma_y^2}$ is constant and coincidentally equals $\overline{\sigma_x^2}$ when $y_3 = h/2 = 150$ mm.

Based on the analyses in this section, with the assumptions that the first two slots are aligned, and that the four vertices of the rectangular workpiece are the KPCs, we can make the following conclusions regarding the 3-pin/slot pair locating design:

1. The locating accuracy is a complex function of the locations of the three slots, as well as the orientation of the third slot (Eq. 9c). To ensure the best locating accuracy, the third slot should be oriented according to Eq. 9e, varying with x_1, x_2, x_3, and y_3.
2. When the third slot is designed perpendicular to the first two slots, it should be placed at the workpiece centerline (*cc* line in Fig. 7b) to achieve a minimum in locating variations (Eq. 9g and Fig. 9).
3. The sensitivity of the locating variations to the position and orientation of the third slot (Fig. 8 & Fig. 9) in the 3-pin/slot design is much less significant than that for the second slot orientation (Fig. 5) and distance between the first and the second pins (Fig. 6).

Prior to concluding this section, we would like to compare the locating quality between the traditional 1-pin/hole + 1-pin/slot design and the 3-pin/slot pair design. The latter design has the flexibility to have the third slot placed on the workpiece centerline (*cc* line in Fig. 7b) to achieve the minimum in locating variations regardless of the positions of the first two slots. The minimum variations achieved under this condition equal those for the 1-pin/hole + 1-pin/slot locating when both the hole and slot are placed onto the *cc* centerline. If, due to either product or process constraints, the 1-hole + 1-slot cannot be placed at the *cc* centerline, the locating variations increase accordingly, as shown in Fig. 9, albeit not significantly. In automotive manufacturing, the efficiency and cost savings from piecing one hole and one slot on the workpiece instead of piecing three slots can generally justify the dominant practice of using 1-pin/hole + 1-pin/slot locating instead of the 3-pin/slot design.

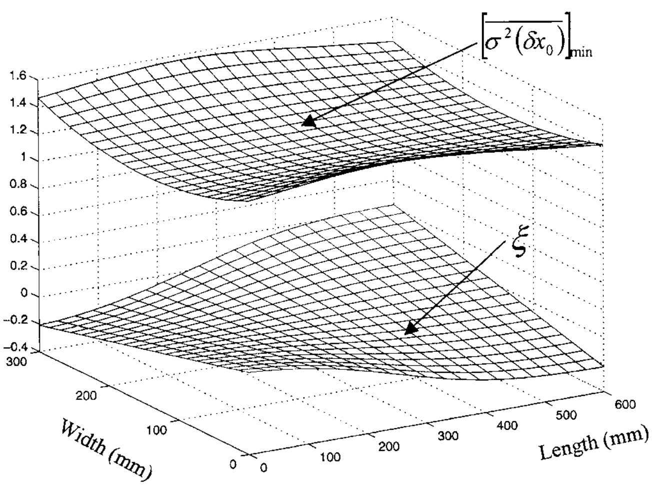


Fig. 8. ξ and $\left[\overline{\sigma_x^2}\right]_{\min}$ as functions of all three locators

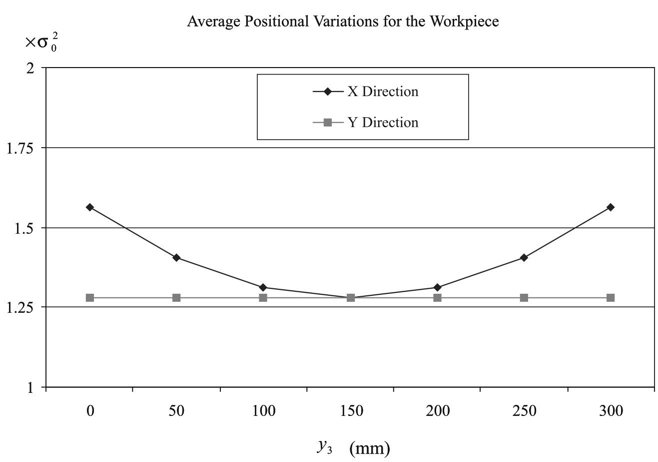


Fig. 9. Variation analysis considering the third pin/slot position

4 A case study: analysis of rear compartment locating (with an auxiliary pin)

We investigate locating (datum) strategies for an automobile rear compartment panel involving an auxiliary pin. The existing datum strategy is shown in Fig. 10, where four up/down (U/D) blocks, one cross car (C/C) and fore/aft (F/A) pin, one C/C pin, plus one auxiliary C/C pin, are used. In the following four subcases ((a) through (d)), the in-plane (i.e., the C/C and F/A) variations will be calculated for different datum strategies for comparison. Variation sources come from pin/hole and pin/slot clearances, which are assumed to follow independent normal distributions, i.e., $N\left(0, \sigma^2\right)$, with $6\sigma_0 = 0.5$ mm. Typical dimensions are used, i.e., $PP_1 = P_2P_3 = 1200$ mm, $\angle P_1P_2P = 30°$, and $P_1P_2 = P_3P = 2078.46$ mm.

Subcase (a) in Fig. 11a shows the datum strategy when F/A + C/C at P_1 and C/C at P_2 are in effect (without the auxiliary C/C locating at P_3). Subcase (b) in Fig. 11b shows the strategy when F/A + C/C at P_1 and the auxiliary C/C at P_3 are in effect (without the C/C locating at P_2). Subcase (c) in Fig. 11c is derived from subcase (b) by aligning the centerline with line P_1P_3. The variations at point P_3 were calculated, and the results are tabulated in Table 1, where σ_x and σ_y are standard deviations in the x and y directions, and $\sigma \equiv \sqrt{\sigma_x^2 + \sigma_y^2}$.

Subcase (d) includes F/A + C/C at P_1, and C/C at P_2, with the consideration of the auxiliary pin at P_3. The primary purpose of the auxiliary pin is to help reduce possible large variations. Assuming the auxiliary pin/slot also has a 6σ clearance of 0.5 mm (a typical value), it will not help reduce variation at all if the 6σ value at point P_3 (under the 1-pin/hole + 1-pin/slot locating system as in subcase (a)) is less than 0.5 mm. If, on the other hand, the 6σ variation at point P_3 is greater than 0.5 mm, then the auxiliary pin will function to keep the variation at P_3 to the 0.5 mm level. Therefore, the purpose of this subcase is to find out the maximum variation direction[1] at P_3 so that the auxiliary slot can be oriented to most effectively restrain the variation in that direction. Two steps are necessary to achieve this purpose: (i) calculate the variation components at P_3 (expressed in a local $x'y'$ coordinate system) under the locating scheme in subcase (a); (ii) determine the maximum variation direction and arrange the auxiliary slot accordingly. The following will derive the explicit relationship between the slot angle α and the variation in the local y' direction. Note that only the variation in the y' direction is considered because the pin is a two-way pin (associated with a slot rather than a hole).

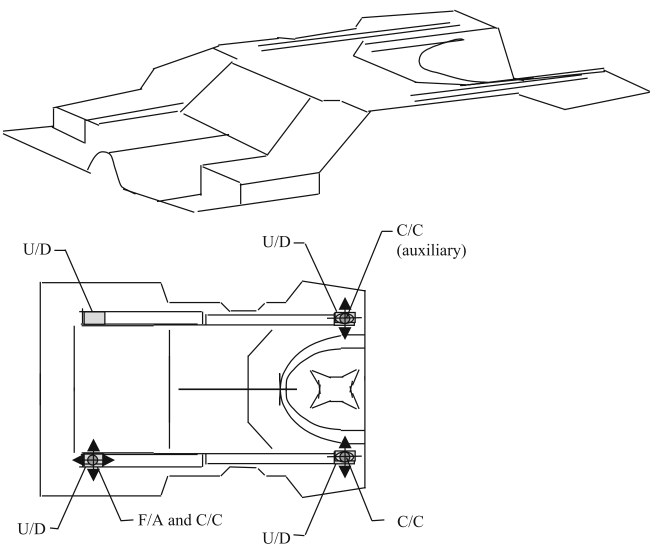


Fig. 10. Datum strategy for an automobile rear compartment assembly

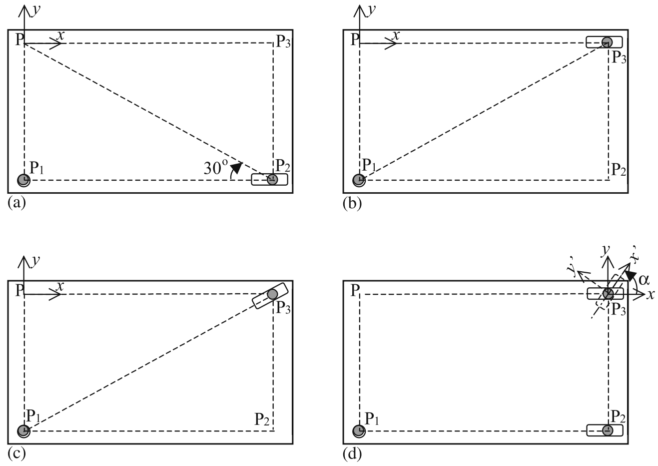


Fig. 11. Schematics for an automotive rear compartment locating

Based on Eq. 8a, the variation components in the global x and y directions at point P_3 are

$$\delta\mathbf{q_0} = \begin{bmatrix} \delta x_0 \\ \delta y_0 \\ \delta\phi_0 \end{bmatrix} = \begin{bmatrix} \delta x_2 + \frac{\delta y_1 - \delta y_2}{x_1 - x_2} y_1 - k\frac{\delta x_1 - \delta x_2}{x_1 - x_2} y_1 \\ \frac{x_1\delta y_2 - x_2\delta y_1}{x_1 - x_2} + k\frac{\delta x_1 - \delta x_2}{x_1 - x_2} x_2 \\ \frac{\delta y_2 - \delta y_1}{x_1 - x_2} + k\frac{\delta x_1 - \delta x_2}{x_1 - x_2} \end{bmatrix} \tag{10a}$$

[1] The variation at P_3, σ, is insensitive to orientation, though its two orthogonal components, σ_x and σ_y are functions of orientation. The equation $\sigma^2 = \sigma_x^2 + \sigma_y^2$ always holds, as is true for Eq. 10b.

Table 1. Variations at point P for subcases (a), (b), and (c)

Subcases	(a) $6\sigma_x$	$6\sigma_y$	6σ	(b) $6\sigma_x$	$6\sigma_y$	6σ	(c) $6\sigma_x$	$6\sigma_y$	6σ
Variations	0.646	0.500	0.817	0.612	0.408	0.736	0.587	0.395	0.707
% of reduction	Benchmark			5.3%	18.4%	9.9%	9.3%	21.0%	13.5%

When expressed in x' and y' directions, the variation components are

$$\delta \mathbf{q}_0' = \begin{bmatrix} \delta x_0' \\ \delta y_0' \end{bmatrix} = \begin{bmatrix} \delta x_0 \cos\alpha + \delta y_0 \sin\alpha \\ -\delta x_0 \sin\alpha + \delta y_0 \cos\alpha \end{bmatrix} \tag{10b}$$

Since only the y' component is of interest, we have

$$\delta y_0' = -\sin\alpha \delta x_2 - \frac{x_2 \cos\alpha + y_1 \sin\alpha}{x_1 - x_2} \delta y_1 + \frac{x_1 \cos\alpha + y_1 \sin\alpha}{x_1 - x_2} \delta y_2 \tag{10c}$$

Its standard deviation is

$$\begin{aligned} \sigma_{y'} &\equiv \sigma\left(\delta y_0'\right) \\ &= \sigma_0 \sqrt{(-\sin\alpha)^2 + \left(\tfrac{x_2 \cos\alpha + y_1 \sin\alpha}{x_1 - x_2}\right)^2 + \left(\tfrac{x_1 \cos\alpha + y_1 \sin\alpha}{x_1 - x_2}\right)^2} \\ &= \sigma_0 \sqrt{r \sin(2\alpha + \beta) + \frac{c_1 + c_2}{2}} \end{aligned} \tag{10d}$$

where

$$c_1 \equiv \frac{(x_1 - x_2)^2 + 2y_1^2}{(x_1 - x_2)^2}$$

$$c_2 \equiv \frac{x_1^2 + x_2^2}{(x_1 - x_2)^2}$$

$$c_3 \equiv \frac{(x_1 + x_2)\, y_1}{(x_1 - x_2)^2}$$

$$r \equiv \sqrt{c_3^2 + \left(\frac{c_2 - c_1}{2}\right)^2}$$

and β is determined by $\sin\beta = (c_2 - c_1)/2r$ and $\cos\beta = c_3/r$.

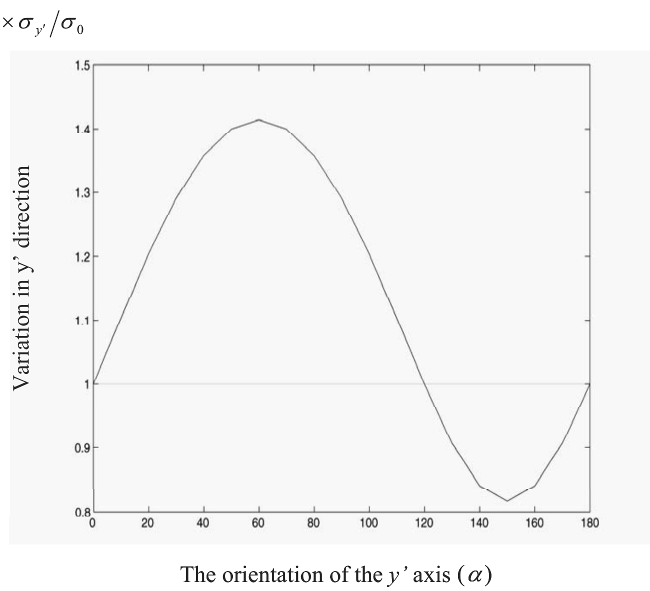


Fig. 12. The variations at point P_3 in the y' direction

Eq. 10d indicates that the y' direction variation, σ_y', at Point P_3 is orientation-dependent. Therefore, to maximize the restraining function of the auxiliary pin locating system, the slot orientation should be chosen such that the maximum σ_y' occurs, i.e.,

$$\alpha = \left(90° - \beta\right)/2 \tag{10e}$$

and the corresponding maximum variation is:

$$\left[\sigma_y'\right]_{\max} = \sigma_0 \sqrt{r + \frac{c_1 + c_2}{2}} \tag{10f}$$

Figure 12 shows the functional relationship between slot angle α and the variation σ_y' when the actual dimensions are used. As can be seen from the figure, the auxiliary pin does not function at all when the slot angle is 0°, or within the [120°, 180°] range, and, the pin functions most efficiently at 60°. Since the slot angle for the auxiliary pin in Fig. 10 is zero, the pin does not help reduce variations as one might think.

5 Summary

This paper presents the applications of a robust design strategy in locating pin layout design for sheet panels. Several important locating criteria are obtained:

1. For the 1-pin/hole + 1-pin/slot locating system, the slot centerline should be aligned with the pin-connecting line to achieve best locating accuracy. As the slot angle becomes larger (such as over 30 degrees), the locating variation will increase from more than 10% to infinity (Sect. 3.1). Therefore, a slot angle should be designed as small as possible, and any angle greater than 30 degrees should be used very cautiously.
2. The distance between the two pins plays a significant role in locating accuracy. The two pins should always be placed as far apart as possible (Sect. 3.2).
3. The 1-pin/hole + 1-pin/slot strategy for sheet panel in-plane locating is generally only a near-optimum choice, rather than the best choice, unless the pin/hole/slot are placed along the longest centerline[2] of a workpiece to achieve the most the optimal locating (Sect. 3.3).
4. When selecting an auxiliary pin, one needs to study its effectiveness. In certain circumstances, the auxiliary pin may not function as intended. Thus, further analysis is recommended to pursue the most effective slot angle for the auxiliary pin (the rear compartment case, subcase (d)). The procedure is to calculate the variations at any candidate points on the workpiece, and then choose the point with maximum $\sigma_{y'}$ as the auxiliary locating point.

As a result, the 1-pin/hole + 1-pin/slot locating guideline should be stated as such: pins should be placed on the longest centerline of the workpiece, as apart as possible, and with zero slot angle. Though one can rely on the above principle to adequately

[2] This statement holds only if the workpiece is symmetric and has a centerline. If not, detailed robust analyses should be carried out, preferably using the simulation codes developed.

design locating pins, quantitative variations can always be obtained through the robust analysis as demonstrated in this paper, or equivalently, through simulation codes.

To end this paper, it is worth mentioning that the 1-pin/hole + 1-pin/slot locating is not the sole practice in industry, though definitely a dominant one. Other alternative locating practices, such 2-pin/slot + 1-block, 1-pin/slot + 2-blocks, etc., do exist [15]. One can always, however, convert different strategies to the generic format of "3-2-1" and perform the robust analysis for the "2-1" locators in the in-plane direction accordingly.

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