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## Determination of optimum parameters for multi-performance characteristics in drilling by using grey relational analysis

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**Abstract** The theory of grey systems is a new technique for performing prediction, relational analysis and decision making in many areas. In this paper, the use of grey relational analysis for optimising the drilling process parameters for the work piece surface roughness and the burr height is introduced. Various drilling parameters, such as feed rate, cutting speed, drill and point angles of drill were considered. An orthogonal array was used for the experimental design. Optimal machining parameters were determined by the grey relational grade obtained from the grey relational analysis for multi-performance characteristics (the surface roughness and the burr height). Experimental results have shown that the surface roughness and the burr height in the drilling process can be improved effectively through the new approach.

**Keywords** Burr height · Drilling · Grey relational analysis · Optimization · Surface roughness

### 1 Introduction

With regard to the quality characteristics of drilled parts, some of the problems encountered include hole surface roughness, burr height and tool wear. Among these three characteristics, surface roughness and burr height play the most important roles in the performance of a drilled part. Cutting speed, feed rate, drill and work piece material, drill point angle and coolant conditions are the drilling parameters which highly affect the performance measures. In order to improve machining efficiency, reduce the machining cost, and improve the quality of machined parts, it is necessary to select the most appropriate machining conditions. The setting of drilling parameters relies strongly on the experience of operators. It is difficult to utilize the highest performance

of a machine because there are too many adjustable machining parameters.

In order to minimize these machining problems, there is need to develop scientific methods to select cutting conditions for damage-free drilling of materials. Most of published works focused on optimisation of parameters for machining of metals. Taylor [1] introduced the concept of optimum speed for metal cutting operations. Since then, many approaches have been proposed for optimizing machining parameters for better economic performance. Bhattacharyaa [2] used the Lagrangian function method in searching for optimum cutting parameters. Ermer [3] used the geometric programming method. Davim and Antonio [4] conducted drilling tests with the aim of developing optimal drilling conditions using a genetic algorithm approach. Enemuoh et al. [5] presented a new comprehensive approach to select cutting parameters for damage-free drilling in carbon fibre-reinforced epoxy composite material. Tosun [6] investigated the effect of machining conditions on surface roughness and microstructure in drilled holes.

Hoshi et al. [7] introduced the circular edge, which has an S-shaped cutting edge formed by connecting two circular cutting edges at the centre point of the drill. This high-speed steel drill, coated with TiN, was used in drilling 0.45%C containing plain steel. Lin [8] investigated the tool life, surface roughness and burr formation in high-speed drilling of stainless steel using TiN-coated carbide drill.

In a system that is complex and multi-variate, the relationship between various factors such as those described above is unclear. Such systems often are called as “grey” implying poor, incomplete, and uncertain information. Their analysis by classical statistical procedures may not be acceptable or reliable without large data sets that satisfy certain mathematical criteria. The grey theory, on the other hand, makes use of relatively small data sets and does not demand strict compliance to certain statistical laws, such as simple or linear relationships among the observables [9].

The purpose of the present work is to introduce the use of grey relational analysis in selecting optimum drilling conditions on multi-performance characteristics, namely, the hole surface

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roughness and the burr height. To the best knowledge of the author of this work, there is no published work evaluating the optimisation and the effect of cutting parameters on the multi-performance characteristics in drilling process by using grey relational analysis. The setting of drilling parameters was accomplished using the Taguchi experimental design method. In addition, the most effective factor and the order of importance of the controllable factors to the multi- performance characteristics in the drilling process were determined. Thus, by properly adjusting the control factors, we can improve work efficiency and produce quality parts.

## 2 Experimental works

The experimental studies were performed on a Tezsan M35ES drilling machine tool (manufactured in Gebze, Turkey). Three different tool materials were utilized: an HSS drill, a TiN-coated HSS drill, and a carbide drill. The tests were performed dry at different cutting speeds, feed rates and drill point angles. The summary of experimental conditions is listed in Table 1. A 5 mm drill diameter was used in the experiments. The thickness of the TiN coating is 2–3  $\mu\text{m}$ . AISI 4140 steel (DIN 42CrMo4) with 10 mm thickness was used as work piece material.

The experimental results after drilling were evaluated in terms of the following measured machining performance: (1) surface roughness ( $R_a$ ); (2) burr height ( $h$ ). The surface roughness of drilled holes was measured using a Mitutoyo Surfes SJ-201 instrument. The instrument was set to a cut-off length of 0.8 mm and the number of samplings of 5 (i.e. traverse length of 4 mm). Direction of surface roughness measurements was perpendicular to the hole circumference. The surface roughness values given in this study are the mathematical average of three measurements taken from the same hole surface. A Scherr&Tumico (USA) measuring microscope with an accuracy of 0.002 mm was used for the burr height measurements. The burr height values are

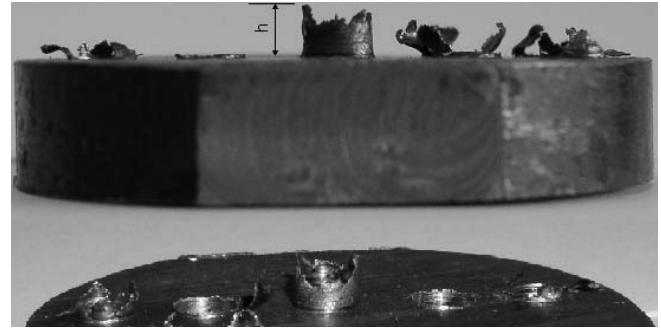


Fig. 1. Front and top view of the burrs produced during the experiments

the mathematical average of two measurements taken from the different specimens drilled in the same experiment conditions. Fig. 1 shows burrs formed in the holes during the experiments.

Normally, the full-factorial design would require  $3^4 = 81$  experimental runs. However, the effort and experimental cost for such a design could be prohibitive and unrealistic. In this study, the Taguchi method - a powerful tool for parameter design of performance characteristics- was used. According to the Taguchi quality design concept [10, 11], a  $L_9$  orthogonal arrays with nine rows (corresponding to the number of experiments) was used for the experiments (Table 2). Four drilling parameters were chosen as control factors and each parameter was designed to have three levels, denoted 1, 2 and 3 (Table 1). The experimental results are summarized in Table 2.

## 3 Grey relational analysis

In grey relational analysis, black represents having no information and white represents having all information. A grey system has a level of information between black and white. In other words, in a grey system, some information is known

Table 1. The experimental conditions

Levels of drilling parameter	Drilling parameters			
	Drill type	Cutting speed (rpm)	Feed rate (mm/rev)	Point angle ( $^\circ$ )
1	HSS	450	0.1	90
2	TiN	1120	0.2	118
3	Carbide	1800	0.3	130

Table 2. Experimental design using  $L_9$  orthogonal array

Exp. no	A Drill type	B Cutting speed	C Feed rate	D Point angle	Surface roughness $R_a$ ( $\mu\text{m}$ )	Burr height $h$ (mm)
1	1	1	1	1	3.41	2.528
2	1	2	2	2	3.54	2.998
3	1	3	3	3	2.51	4.028
4	2	1	2	3	3.77	0.506
5	2	2	3	1	3.20	1.392
6	2	3	1	2	1.56	1.306
7	3	1	3	2	3.10	0.360
8	3	2	1	3	2.73	0.228
9	3	3	2	1	1.81	0.510

and some information is unknown. In a white system, the relationships among factors in the system are certain; in a grey system, the relationships among factors in the system are uncertain [12, 13].

Grey relational analysis is an impacting measurement method in grey system theory that analyzes uncertain relations between one main factor and all the other factors in a given system. In the case when experiments are ambiguous or when the experimental method cannot be carried out exactly, grey analysis helps to compensate for the shortcomings in statistical regression [14]. Grey relational analysis is actually a measurement of the absolute value of the data difference between sequences, and it could be used to measure the approximate correlation between sequences [15].

### 3.1 Data pre-processing

Data pre-processing is normally required since the range and unit in one data sequence may differ from the others. Data pre-processing is also necessary when the sequence scatter range is too large, or when the directions of the target in the sequences are different. Data pre-processing is a means of transferring the original sequence to a comparable sequence. Depending on the characteristics of a data sequence, there are various methodologies of data pre-processing available for the grey relational analysis [15].

If the target value of original sequence is infinite, then it has a characteristic of the “higher is better”. The original sequence can be normalized as follows:

$$x_i^*(k) = \frac{x_i^o(k) - \min x_i^o(k)}{\max x_i^o(k) - \min x_i^o(k)} \tag{1}$$

When the “lower is better” is a characteristic of the original sequence, then the original sequence should be normalized as follows:

$$x_i^*(k) = \frac{\max x_i^o(k) - x_i^o(k)}{\max x_i^o(k) - \min x_i^o(k)} \tag{2}$$

However, if there is a definite target value (desired value) to be achieved, the original sequence will be normalized in form:

$$x_i^*(k) = 1 - \frac{|x_i^o(k) - x^o|}{\max x_i^o(k) - x^o} \tag{3}$$

Or, the original sequence can be simply normalized by the most basic methodology, i.e. let the values of original sequence be divided by the first value of the sequence:

$$x_i^*(k) = \frac{x_i^o(k)}{x_i^o(1)} \tag{4}$$

where  $i = 1, \dots, m$ ;  $k = 1, \dots, n$ .  $m$  is the number of experimental data items and  $n$  is the number of parameters.  $x_i^o(k)$  denotes the original sequence,  $x_i^*(k)$  the sequence after the data pre-processing,  $\max x_i^o(k)$  the largest value of  $x_i^o(k)$ ,  $\min x_i^o(k)$  the smallest value of  $x_i^o(k)$  and  $x^o$  is the desired value.

### 3.2 Grey relational coefficient and grey relational grade

In grey relational analysis, the measure of the relevancy between two systems or two sequences is defined as the grey relational grade. When only one sequence,  $x_0(k)$ , is available as the reference sequence, and all other sequences serve as comparison sequences, it is called a local grey relation measurement. After data pre-processing is carried out, the grey relation coefficient  $\xi_i(k)$  for the  $k$ th performance characteristics in the  $i$ th experiment can be expressed as [14–16]:

$$\xi_i(k) = \frac{\Delta_{\min} + \zeta \Delta_{\max}}{\Delta_{oi}(k) + \zeta \Delta_{\max}} \tag{5}$$

where,  $\Delta_{oi}$  is the deviation sequence of the reference sequence and the comparability sequence.

$$\begin{aligned} \Delta_{oi} &= \|x_0^*(k) - x_i^*(k)\| \\ \Delta_{\min} &= \min_{j \in i} \min_{\forall k} \|x_0^*(k) - x_j^*(k)\| \\ \Delta_{\max} &= \max_{j \in i} \max_{\forall k} \|x_0^*(k) - x_j^*(k)\| \end{aligned}$$

$x_0^*(k)$  denotes the reference sequence and  $x_i^*(k)$  denotes the comparability sequence.  $\zeta$  is distinguishing or identification coefficient:  $\zeta \in [0, 1]$  (the value may be adjusted based on the actual system requirements). A value of  $\zeta$  is the smaller and the distinguished ability is the larger.  $\zeta = 0.5$  is generally used.

After the grey relational coefficient is derived, it is usual to take the average value of the grey relational coefficients as the grey relational grade [14, 16]. The grey relational grade is defined as follows:

$$\gamma_i = \frac{1}{n} \sum_{k=1}^n \xi_i(k) \tag{6}$$

However, in a real engineering system, the importance of various factors to the system varies. In the real condition of unequal weight being carried by the various factors, the grey relational grade in Eq. 6 was extended and defined as [14, 16]:

$$\gamma_i = \frac{1}{n} \sum_{k=1}^n w_k \xi_i(k) \quad \sum_{k=1}^n w_k = 1 \tag{7}$$

where  $w_k$  denotes the normalized weight of factor  $k$ . Given the same weight, Eqs. 6 and 7 are equal.

The grey relational grade  $\gamma_i$  represents the level of correlation between the reference sequence and the comparability sequence. If the two sequences are identical, then the value of grey relational grade is equal to 1. The grey relational grade also indicates the degree of influence that the comparability sequence could exert over the reference sequence. Therefore, if a particular comparability sequence is more important than the other comparability sequences to the reference sequence, then the grey relational grade for that comparability sequence and reference sequence will be higher than other grey relational grades [15].

### 4 Analysis and discussion of experimental results

In the present study, the work piece surface roughness and burr height values in different drilling parameters are listed in Table 2. In the drilling process, lower surface roughness and burr height are indications of better performance. For data pre-processing in the grey relational analysis process, both surface roughness and burr height are taken as the “lower is better” (LB). Let the results of nine experiments be the comparability sequences  $x_i^O(k)$ ,  $i = 1-9, k = 1$ . All the sequences after data pre-processing using Eq. 2 are listed in Table 3 and denoted as  $x_0^*(k)$  and  $x_i^*(k)$  for reference sequence and comparability sequence, respectively.

The deviation sequences  $\Delta_{0i}$  can be calculated as follows:

$$\Delta_{01}(1) = |x_0^*(1) - x_1^*(1)| = |1.00 - 0.1629| = 0.8371,$$

$$\Delta_{01}(2) = |x_0^*(2) - x_1^*(2)| = |1.00 - 0.3947| = 0.6053$$

so  $\Delta_{01} = (0.8371, 0.6053)$ .

The same calculation method was performed for  $i = 1-9$  and the results of all  $\Delta_{0i}$  for  $i = 1-9$  are given in Table 4. Using Table 4,  $\Delta_{max}$  and  $\Delta_{min}$  can be found as follows:

$$\Delta_{max} = \Delta_{04}(1) = \Delta_{03}(2) = 1.00,$$

$$\Delta_{min} = \Delta_{06}(1) = \Delta_{08}(2) = 0.00$$

The distinguishing coefficient  $\zeta$  can be substituted into Eq. 5 to produce the grey relational coefficient. If all the process parameters are of equal weighting, then  $\zeta$  is 0.5. The grey relational

**Table 4.** The deviation sequences

Deviation sequences	$\Delta_{0i}(1)$	$\Delta_{0i}(2)$
Exp. no. 1	0.8371	0.6053
Exp. no. 2	0.8959	0.7289
Exp. no. 3	0.4299	1.0000
Exp. no. 4	1.0000	0.0732
Exp. no. 5	0.7421	0.3063
Exp. no. 6	0	0.2837
Exp. no. 7	0.6968	0.0347
Exp. no. 8	0.5294	0
Exp. no. 9	0.1131	0.0742

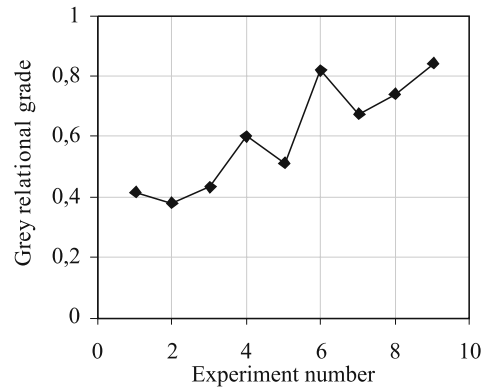
coefficients and grade values for each experiment of the  $L_9$  orthogonal array were calculated by applying Eqs. 5 and 7 (Table 5).

According to the performed experiment design, it is clearly observed from Table 5 and Fig. 2 that the drilling parameters’ setting of experiment no. 9 has the highest grey relational grade. Therefore, experiment no. 9 is the optimal machining parameters’ setting for minimum surface roughness and minimum burr height simultaneously (i.e. the best multi-performance characteristics) among the nine experiments.

In addition to the determination of optimum drilling parameters for surface roughness and burr height, the response table for the Taguchi method was used to calculate the average grey relational grade for each level of the drilling parameters. The procedure is: i) group the grey relational grades by factor level for each column in the orthogonal array, ii) take the average of

**Table 3.** The sequences after data pre-processing

Reference/Comparability sequence	Ra	h
Reference sequence	1.0000	1.000
Comparability sequence		
Exp. no. 1	0.1629	0.3947
Exp. no. 2	0.1041	0.2711
Exp. no. 3	0.5701	0
Exp. no. 4	0	0.9268
Exp. no. 5	0.2579	0.6937
Exp. no. 6	1.0000	0.7163
Exp. no. 7	0.3032	0.9653
Exp. no. 8	0.4706	1.0000
Exp. no. 9	0.8869	0.9258



**Fig. 2.** Grey relation grades for the minimum  $R_a$  and h

**Table 5.** The calculated grey relational coefficient and grey relational grade for nine comparability sequences

Exp. no	A	B	C	D	Grey relational coefficient $R_a$ ( $\mu\text{m}$ )	Grey relational coefficient h (mm)	Grey relational grade	Orders
1	1	1	1	1	3.41	2.528	0.4132	8
2	1	2	2	2	3.54	2.998	0.3825	9
3	1	3	3	3	2.51	4.028	0.4355	7
4	2	1	2	3	3.77	0.506	0.6028	5
5	2	2	3	1	3.20	1.392	0.5113	6
6	2	3	1	2	1.56	1.306	0.8190	2
7	3	1	3	2	3.10	0.360	0.6764	4
8	3	2	1	3	2.73	0.228	0.7429	3
9	3	3	2	1	1.81	0.510	0.8431	1

them. For example, the grey relational grade for factor A at level 1 can be calculated as follow:

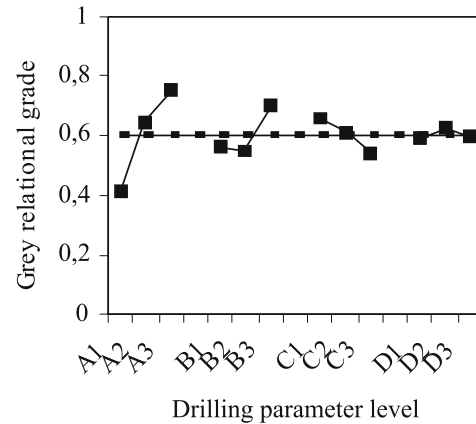
$$\gamma_{A1} = \frac{1}{3}(0.4132 + 0.3825 + 0.4355) = 0.4104$$

The grey relational grade values for each level of the drilling parameters were calculated using the same method. The grey relational grade values are shown in Table 6. Since the grey relational grade represents the level of correlation between the reference sequence and the comparability sequence, the greater value of the grey relational grade means that the comparability sequence has a stronger correlation to the reference sequence [15]. In other words, regardless of category of the performance characteristics, a greater grey relational grade value corresponds to better performance [17]. Therefore, the optimal level of the machining parameters is the level with the greatest grey relational grade value. An asterisk (\*) indicates that the level value results in a better drilling performance. Based on the grey relational grade values given in Table 6, the optimal machining performance for both the surface roughness and the burr height was obtained for carbide drill (level 3), 1800 rpm cutting speed (level 3), 0.1 mm/rev feed rate (level 1) and 118° drill point angle (level 2) combination. The optimal drilling parameter levels can be shortly given as A3, B3, C1 and D2. Fig. 3 shows the effect of drilling parameters on the multi-performance characteristics (the surface roughness and the burr height) and the response graph of each level of the drilling parameters for the performance. The greater values in Fig. 3 give the low burr height and good surface finish quality.

As listed in Table 6, the difference between the maximum and the minimum value of the grey relational grade of the drilling parameters is as follow: 0.3437 for drill type, 0.1537 for cutting speed, 0.1173 for feed rate and 0.0367 for point angle. The most effective factor affecting performance characteristics is determined by comparing these values. This comparison will give the level of significance of the controllable factors over the multi-performance characteristics. The most effective controllable factor was the maximum of these values. Here, the maximum value among 0.3437, 0.1537, 0.1173 and 0.0367 is 0.3437. The value indicates that the drill type has the strongest effect on the multi-performance characteristics among the other drilling parameters. On the other hand, the significance of role that every controllable factor plays over the multi-performance characteristics can be obtained by examining these values. The order of importance of the controllable factors to the multi-performance characteristics in the drilling process, in sequence can be listed as: factor A (drill type), B (cutting speed), C (feed rate) and D (point angle) (i.e.

**Table 6.** The response table for grey relational grade

Drilling parameters	Average grey relational grade by factor level			
	Level 1	Level 2	Level 3	Max-Min
A	0.4104	0.6443	0.7541*	0.3437
B	0.5641	0.5455	0.6992*	0.1537
C	0.6583*	0.6094	0.5410	0.1173
D	0.5892	0.6259*	0.5937	0.0367



**Fig. 3.** Effect of drilling parameter levels on the multi-performance

0.3437 > 0.1537 > 0.1173 > 0.0367). Factor A (drill type) was the most effective factor to the performance. This indicates that the drilling performance was strongly affected by the drill type.

## 5 Conclusion

In this paper, the optimal drilling parameters were determined for the multi-performance characteristics (surface roughness and burr height) in the drilling process by using the grey relational analysis.

The grey relational analysis, based on the Taguchi method's response table, was proposed as a way of studying the optimisation of drilling process factors. The surface roughness and the burr height were selected to be the quality targets. From the response table of the average grey relational grade, the largest value of grey relational grade for the drilling parameters was found. These values are the recommended levels of controllable drilling parameters for the multi-performance characteristics. It was found that the drill type has the strongest effect among the other drilling parameters used on the multi-performance characteristics. In other words, the most influential factor is drill. The order of importance of the controllable factors to the multi-performance characteristics is drill type, cutting speed, feed rate and drill point angle. Experimental results have shown clearly that the surface roughness and the burr height in the drilling process can be improved effectively through the proposed approach.

This study indicated that grey relational analysis approach can be applied successfully to other operations in which performance is determined by many parameters at multiple quality requests.

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