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ORIGINAL ARTICLE

P.Y. Mok · B. Porter

Evolutionary optimisation of hedging points for unreliable manufacturing systems

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Abstract The production rates of manufacturing systems are notoriously difficult to control, since such systems are dynamic, uncertain and non-linear. However, the introduction of hedging-point policies for such systems has led to much progress in optimal production control. But the theoretical results so far obtained for such hedging-point policies are still far from complete, since the optimal hedging points (i.e., the optimal inventory levels) are analytically available only for simple systems and under restrictive assumptions. In this paper, an evolutionary stochastic optimisation procedure is proposed to estimate the short-run optimal hedging points for failure-prone manufacturing systems under crisp-logic control. This methodology is illustrated by examples and is validated by comparing the evolutionary results with the available analytical long-run solutions. The proposed evolutionary methodology is also shown to be capable of generating optimal hedging points for unreliable systems producing multiple products with different priorities. In addition, the relative merits of genetic algorithms, evolution strategies and adaptive evolution strategies in hedging-point optimisation are compared.

Keywords Evolutionary computation · Optimal inventory levels · Production control · Unreliable manufacturing systems

1 Introduction

The control of production in manufacturing systems involves the overall management of the production processes from the time an order is received until its disposition is completed, so as to ensure that goods are produced on time and at the lowest possible cost. The ultimate objective of production control is accordingly straightforward in theory – namely, to satisfy customer demands and, at the same time, minimise production costs. However, the actual achievement of such optimal production control in practice is difficult, since manufacturing systems typically contain many machines that simultaneously produce many product-types. This difficulty of control is increased since manufacturing systems invariably experience random disruptions such as machine breakdowns or unreliable material supplies. Furthermore, since manufacturing systems are essentially complex discrete-event systems, it has been necessary to develop suitable approximate continuous-flow models to provide a simpler conceptual framework for the determination of optimal production-control policies for such systems [1, 2].

In order to deal with these difficulties, various forms of rule-based production control – such as crisp-logic control or fuzzy-logic control, often implemented in real-time by programmable-logic controllers (PLCs) – have been proposed in recent years. The optimal crisp-logic control of production rates in manufacturing systems has, in particular, attracted much attention following the introduction by Kimemia and Gershwin [3] of so-called hedging-point control policies. These hedging-point policies require that, in order to meet customer demands in a timely fashion, manufacturing systems must produce more than the exogenous demands when machines are operative in order to compensate for the loss of production when machines are inoperative. Thus, the hedging-point values are in fact the optimal inventory levels for both work-in-progress and finished products that must be held in order to counteract the potentially adverse effects of the random disruption of the production process. Ever since the introduction of the hedging-point concept, much research effort has been devoted to obtaining optimal production-control policies for manufacturing systems. However, the implementation of hedging-point policies obviously requires the availability of optimal hedging-point values, which are difficult to obtain using traditional control theory. The chief difficulty is to solve simultaneously the relevant non-linear partial differential equations

P.Y. Mok (✉)
Institute of Textiles & Clothing,
The Hong Kong Polytechnic University,
Hong Kong
E-mail: tracy.mok@graduate.hku.hk
Tel.: +852-27664442
Fax: +852-27731432

B. Porter
Control Systems Centre,
University of Manchester Institute of Science and Technology,
Manchester, UK

governing the production rates and inventory costs [4]. Different optimisation techniques – for instance, dynamic programming and linear programming – have been used in endeavours to solve these equations analytically.

In the least difficult case of manufacturing systems containing completely reliable machines, Yun and Bai [5] obtained the required optimal control policies for such systems with two product-types using Pontryagin's minimum principle. In the more difficult case of unreliable manufacturing systems containing machines that fail and are repaired randomly, the complete analytical solutions have so far been obtained only for single-machine single-product-type systems [6, 7]. Thus, when considering unreliable manufacturing systems with multiple product-types, Srivatsan and Dallery [8] obtained a complete solution for single-machine two-product-type systems in the special case when the hedging points were assumed to be zero. Veatch and Caramanic [9] derived necessary and sufficient conditions for the existence of zero hedging points in two-product-type systems. Perkins and Srikant [10] solved this problem under a linear switching curve approximation and provided a numerical solution for a prioritised hedging-point policy. Sethi and Zhang [11] obtained optimal control policies for unreliable manufacturing systems producing multiple product-types in the special case when the production surplus and backlog have equal cost weightings for all product-types. Moreover, in all this work, the analytical or heuristic results obtained for the optimal hedging points of unreliable manufacturing systems are all restricted to long-run cases (i.e., production control of systems under constant demand for very long task times) in which the machine failure and repair rates are assumed to be exponentially distributed.

In this paper, an evolutionary stochastic optimisation procedure is developed to estimate the optimal short-run hedging points to be used in crisp-logic controllers for unreliable manufacturing systems. This evolutionary methodology is capable of dealing with manufacturing systems containing multiple unreliable machines and producing multiple product-types with any desired production priorities. However, in this paper, this general methodology is illustrated for manufacturing systems containing single unreliable machines and producing single or multiple product-types. Three evolutionary algorithms – namely, genetic algorithms, evolution strategies and adaptive evolution strategies – are used to estimate the optimal hedging points. The general methodology and the structures of the different evolutionary algorithms are described in Sect. 2. Illustrative examples are provided in Sect. 3. The evolutionary estimates of the short-run optimal hedging points are compared, in Sect. 4, with the theoretical long-run optimal results obtained by Bielecki and Kumar [7] for the case of single product-type systems and by Sethi and Zhang [11] for the case of multiple product-type systems. In Sect. 5, it is shown that the proposed evolutionary optimisation procedure can be used to obtain the optimal hedging points for multiple product-types when such products have different production priorities. Finally, the relative merits of genetic algorithms, evolution strategies, and adaptive evolution strategies in the optimisation of hedging points for the crisp-logic control of unreliable manufacturing systems are compared.

2 Evolutionary stochastic optimisation procedure

2.1 System description

It is convenient, before the introduction of the evolutionary methodology for the optimisation of hedging points, firstly to describe the manufacturing systems to be investigated in this paper. In the interests of brevity and clarity, the methodology described in this paper relates to crisp-logic control policies for single-machine multiple-product-type manufacturing systems (although it is important to note that this methodology is directly applicable to multiple-machine systems [12]). The manufacturing systems to be considered accordingly produce P product-types ($P \geq 1$), and satisfy the following assumptions:

(i) The manufacturing systems are "flexible" so that the machines can switch between different operations with negligible setup times.
(ii) The required raw materials and products are always available for loading into the systems. In other words, starvation never occurs.
(iii) The systems can be adequately represented by continuous-flow models.
(iv) The systems are failure-prone, and the constituent machines are subject to random failures or repairs with rates that are independent of utilisation.
(v) The various time constants are such that setup times $\ll$ operation times $\ll$ mean times between failures and repairs $\ll$ planning horizon (task time T).

These manufacturing systems are characterised by the following quantities:

$\boldsymbol{u}(t)$ production rate vector, $[u_1(t), u_2(t), \ldots, u_P(t)]$, where $u_i(t)$ denotes the production rate of the ith product-type at time t;
$\boldsymbol{\tau}$ processing time vector, $[\tau_1, \tau_2, \ldots, \tau_P]$, where τ_i denotes the processing time for the ith product-type;
$\boldsymbol{x}(t)$ finished-product inventory vector, $[x_1(t), x_2(t), \ldots, x_P(t)]$, where $x_i(t)$ denotes the inventory level of the ith finished product-type at time t;
$\boldsymbol{d}$ demand rate vector, $[d_1, d_2, \ldots, d_P]$, where d_i denotes the constant short-run demand rate for the ith finished product-type over a finite planning horizon T;
$\alpha(t)$ machine-condition variable denoting the machine condition at time t, which is such that $\alpha(t) = 1$ if the machine is operative and $\alpha(t) = 0$ if it is under repair;
T_f machine mean-time-to-failure;
T_r machine mean-time-for-repair.

In the continuous-flow models of such manufacturing systems, $\boldsymbol{x}(t)$ is the vector of the system state variables, while $\boldsymbol{u}(t)$ is the vector of the control variables. The system dynamics are described by the following differential equations:

$$\dot{x}_i(t) = u_i(t) - d_i \quad (i = 1, 2, \ldots, P) . \tag{1}$$

In these systems, the finished-product inventories, $x_i(t)$, can be positive (production surplus) or negative (production backlog).

The production rates, $u_i(t)$, must obviously be non-negative, so that

$$u_i(t) \geq 0 \quad (i = 1, 2, \ldots, P) . \tag{2}$$

In addition, the production rates are evidently limited by the capabilities of the machine. Each operation takes a finite amount of time, τ_i, and the machine clearly cannot be busy more than 100% of the time. Therefore, the machine production rates, $u_i(t)$, must be such that

$$\sum_{i=1}^{P} u_i(t)\tau_i \leq \alpha(t) , \tag{3}$$

where $\alpha(t)$ is the machine-condition variable. This implies that when the machine is "down" ($\alpha(t) = 0$), then obviously no production is possible; but that when the machine is "up" ($\alpha(t) = 1$), then the production of all product-types must be within the machine's total capacity. Thus, for example, in a manufacturing system producing three product-types, the production rates of the three product-types must at every instant be within the pyramid of capacity constraints shown in Fig. 1.

The demand, $\boldsymbol{d} = [d_1, d_2, \ldots, d_P]$, must be feasible, so that

$$\sum_{i=1}^{P} d_i\tau_i \leq E(\alpha) = \frac{T_f}{T_f + T_r} , \tag{4}$$

where $E(\alpha)$ is the average system capacity.

2.2 Evolutionary methodology

The crisp-logic controllers are to be designed so as to generate the control variables in accordance with the current values of the system state variables, and thus form a closed-loop control system for the manufacturing system. The fundamental objective of the production control problem is, of course, to satisfy the exogenous customer demand rates, $\boldsymbol{d}$, for the various product-types. Therefore, with reference to Eq. 1, the satisfaction of the customer can be revealed by the magnitude of the finished-product inventories, $x_i(t)$. If $x_i(t)$ is positive, then more material has been produced than is currently required, thus generating a surplus. If $x_i(t)$ is negative, then not enough material has been produced, thus generating a backlog. The existence of production surplus as safety stock helps to ensure that material is always available over the planning horizon. However, surplus inventories generate costs since expensive floor space and material-handling systems must be devoted to the storage of finished products. But it is even more costly when a system is backlogged, since backlogs represent unsatisfied customers – implying that sales and goodwill may be lost. It is therefore important to control production so as to satisfy customer demands and minimise production costs.

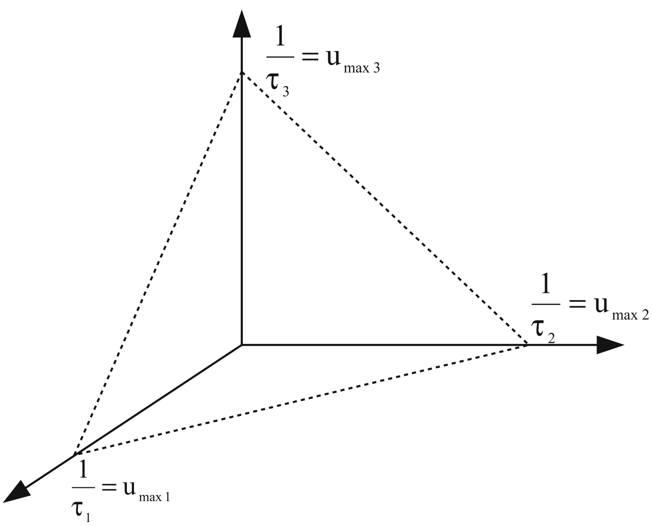

Fig. 1. Machine production capacity constraint diagram for a three-product-type manufacturing system

The relevant production cost for single-machine unreliable manufacturing systems can be defined as

$$J\left(\boldsymbol{x}^*\right) = \int_0^T \left(\sum_{i=1}^{P} g_i^+ x_i^+(t) + \sum_{i=1}^{P} g_i^- x_i^-(t) \right) dt , \tag{5}$$

where $x_i^+(t) = \max(x_i(t), 0)$ and $x_i^-(t) = \max(-x_i(t), 0)$ are the respective surplus and backlog of the ith product-type, and g_i^+ and g_i^- are the corresponding inventory cost weighting parameters. It is assumed that a hedging-point policy [3] is used to provide production control for such systems. The essential feature of such a control policy is to maintain the individual product inventory as close to its hedging point (i.e., the target inventory level), x_i^*, as possible. The hedging-point control policy can be expressed as follows [11]:

$$u_i(t) = \begin{cases} 0 , & \text{if } i \in I(x) \\ d_i , & \text{if } i \in J(x) \\ \left(1 - \sum\limits_{j \in J(x)} \tau_j d_j\right) d_i / \tau_i \sum\limits_{k \in K(x)} d_k , & \text{if } i \in K(x) \end{cases} \tag{6}$$

where $I(x) = \{i : x_i > x_i^*\}$, $J(x) = \{j : x_j = x_j^*\}$, $K(x) = \{k : x_k < x_k^*\}$ are sets of product-types with inventory levels above, at, or below the corresponding hedging points. In the hedging-point policy, clearly, no production is possible if the machine is "down". However, when the machine is "up", if the inventory level of a product-type is above its hedging point, then that product-type is not produced; if it is at its hedging point, then it is produced at a rate equal to that of its demand; and, finally, if it is below its hedging point, then it is produced at a rate that is proportional to its demand rate. The analytical determination of such optimal hedging points is very difficult. In fact, the only available analytical results for unreliable manufacturing systems are all for long-run control (i.e., for control as the task time $T \to \infty$). Furthermore, in systems producing multiple product-types, the available analytical results are based on the assumptions of equal cost weightings for surplus and backlog and of constant demand rates for all product-types [11]. However, such optimal hedging points can be estimated in the short-run by the use of evolutionary algorithms.

The crucial step in the evolutionary stochastic optimisation procedure is the calculation of the expected value of the cost function, $J(\boldsymbol{x}^*)$, defined in Eq. 5 for specified values of $\boldsymbol{d}$ and $\boldsymbol{x}(0)$. It should be noted that constant long-run demand rates are assumed in all the analytical solutions, and that the optimal hedging points obtained are, therefore, independent of initial states. However, demand rates will never in practice be constant for infinite task times; and, if the task time is short, then the initial state of the system will have a major influence on the optimal hedging-point values. In the evolutionary stochastic optimisation procedure, the calculation of the expected value of the cost function can be effected by introducing an appropriately large set, $\{\alpha^{(1)}(t), \alpha^{(2)}(t), \ldots, \alpha^{(n)}(t)\}$, of n scalar machine-condition variables. The time-domain behaviour of each such variable, $\alpha^{(j)}(t)$ $(1 \leq j \leq n)$, represents a particular history of machine failure and repair over a given task time, T, and a typical $\alpha(t)$-trajectory is shown in Fig. 2.

In each of these machine-condition variables, the times for the transitions (between unity and zero, or between zero and unity) are instances of exponentially distributed times-to-failure and times-for-repair with respective mean values T_f and T_r. However, it is important to note that the present evolutionary stochastic optimisation procedure differs from previous analytical procedures in not restricting such machine failure and repair times to be exponentially distributed. Such an assumption is made in the present investigations solely to facilitate the comparison of the evolutionarily optimised results with the available analytic results. In the case of any fixed hedging point, $\boldsymbol{x}^* = [x_1^*, x_2^*, \ldots, x_P^*]$, in the control rule given by Eq. 6, the cost function in Eq. 5 is evaluated for every machine-condition variable in the entire set $\{\alpha^{(1)}(t), \alpha^{(2)}(t), \ldots, \alpha^{(n)}(t)\}$, thus producing a set $\{J^{(1)}, J^{(2)}, \ldots, J^{(n)}\}$ of n corresponding cost functions. The expected value of the cost function over the entire set of n machine-condition variables is then given by the equation

$$E\left[J\left(\boldsymbol{x}^*\right)\right] = \frac{1}{n}\sum_{j=1}^{n} J^{(j)}\left(\boldsymbol{x}^*\right) . \tag{7}$$

In the evolutionary stochastic optimisation procedure, the individual hedging point, x_i^* $(i = 1, 2, \ldots, P)$, for each product-type is represented by a substring of binary digits. These P substrings representing the respective hedging-point values for the P product-types are then concatenated to form a complete binary string that represents the entire vector of inventory level settings, $\boldsymbol{x}^* = [x_1^*, x_2^*, \ldots, x_P^*]$, as shown in Fig. 3.

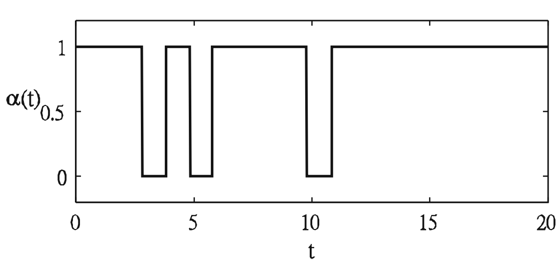

Fig. 2. Typical machine-condition variable

The Darwinian fitness, $\Phi(\boldsymbol{x}^*)$, of each such complete binary string is defined as

$$\Phi\left(\boldsymbol{x}^*\right) = \frac{\lambda}{E\left[J\left(\boldsymbol{x}^*\right)\right]} , \tag{8}$$

where $\lambda > 0$ is an appropriate constant.

2.3 Evolutionary procedure

In the evolutionary stochastic optimisation procedure, genetic algorithms (GAs) [13], non-adaptive $(\mu + \lambda)$-evolution strategies (ESs) [14], and adaptive $(\mu + \lambda)$-evolution strategies (AESs) [15] are used to estimate the optimal short-run inventory levels (hedging-point values). In each of these evolutionary algorithms, the procedure begins by randomly generating an initial population of binary strings in which each such string represents a hedging point, $\boldsymbol{x}^*$, as illustrated in Fig. 3, in the control rule given by Eq. 6.

Evolution is then caused to occur in this population of binary strings in accordance with appropriate forms of crossover, mutation and selection. In this evolutionary process, the Darwinian fitness of each binary string is evaluated by substituting into Eq. 8 the expected value, $E\left[J\left(\boldsymbol{x}^*\right)\right]$, of the cost function over the entire set of relevant machine-condition variables. This evolutionary process is allowed to continue until no significant further increase is obtained in the fitness of the fittest binary string. This fittest binary string is then decoded, and thus provides the estimate of the optimal value of the hedging point, $\boldsymbol{x}^*_{\text{opt}}$, for the specified values of $\boldsymbol{d}$ and $\boldsymbol{x}(0)$. It is evident that this evolutionary optimisation procedure can be repeated for any desired values of $\boldsymbol{d}$ and $\boldsymbol{x}(0)$ within the operational envelope of the manufacturing system.

In the case of genetic algorithms, evolution is caused to occur by subjecting successive generations of μ parental binary strings to mutation, crossover and biased roulette-wheel selection with elitism to produce μ offspring binary strings. In the case of non-adaptive $(\mu + \lambda)$-evolution strategies without recombination, evolution is caused to occur by mutating successive generations of μ parental binary strings to produce $\lambda (\geq \mu)$ offspring binary strings and by then selecting the μ fittest binary strings from the combined population of parents and offspring as the μ parental binary strings for the next generation. In the case, finally, of adaptive $(\mu + \lambda)$-evolution strategies without recombination, evolution proceeds just as for non-adaptive evolution strategies except that the mutation probability associated with every individual bi-

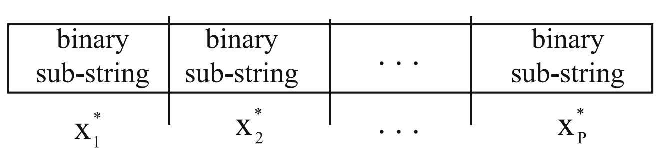

Fig. 3. Binary string representation of inventory levels

nary string is continuously adjusted on-line [15]. Figure 4 illustrates the general processes of (a) genetic algorithms, (b) evolution strategies and (c) adaptive evolution strategies.

3 Illustrative examples

3.1 Optimisation of short-run hedging points for single-product-type systems

This evolutionary stochastic optimisation procedure for determining the short-run optimal hedging points in unreliable manufacturing systems can be illustrated by considering a particular system producing a single product-type for which $\tau = 0.2$ s/piece, $T_f = 5$ s, and $T_r = 1.5$ s. In this case, it is assumed that the task time $T = 20$ s and that the weighting parameters for surplus and backlog of the single product-type are, respectively, $g^+ = 1$ and $g^- = 2$. In this example, the evolutionary stochastic optimisation procedure is used to estimate the optimal short-run inventory levels, x^*_{opt}, in the control rule given by Eq. 6 so as to minimise the expected cost function given by Eq. 5 when the demand, d, has any value in the set $\{2, 2.5, 3, 3.5\}$ pieces/s, and the initial state, $x(0)$, has any value in the set $\{-4, -2, 0, 2, 4, 6\}$ pieces. In each case, the optimal hedging point is obtained over a set of 100 machine-condition variables, $\left\{\alpha^{(1)}(t), \alpha^{(2)}(t), \ldots, \alpha^{(100)}(t)\right\}$, representing failure and repair histories with mean-time-to-failure, $T_f = 5$ s, and mean-time-for-repair, $T_r = 1.5$ s, i.e., $n = 100$ in Eq. 7.

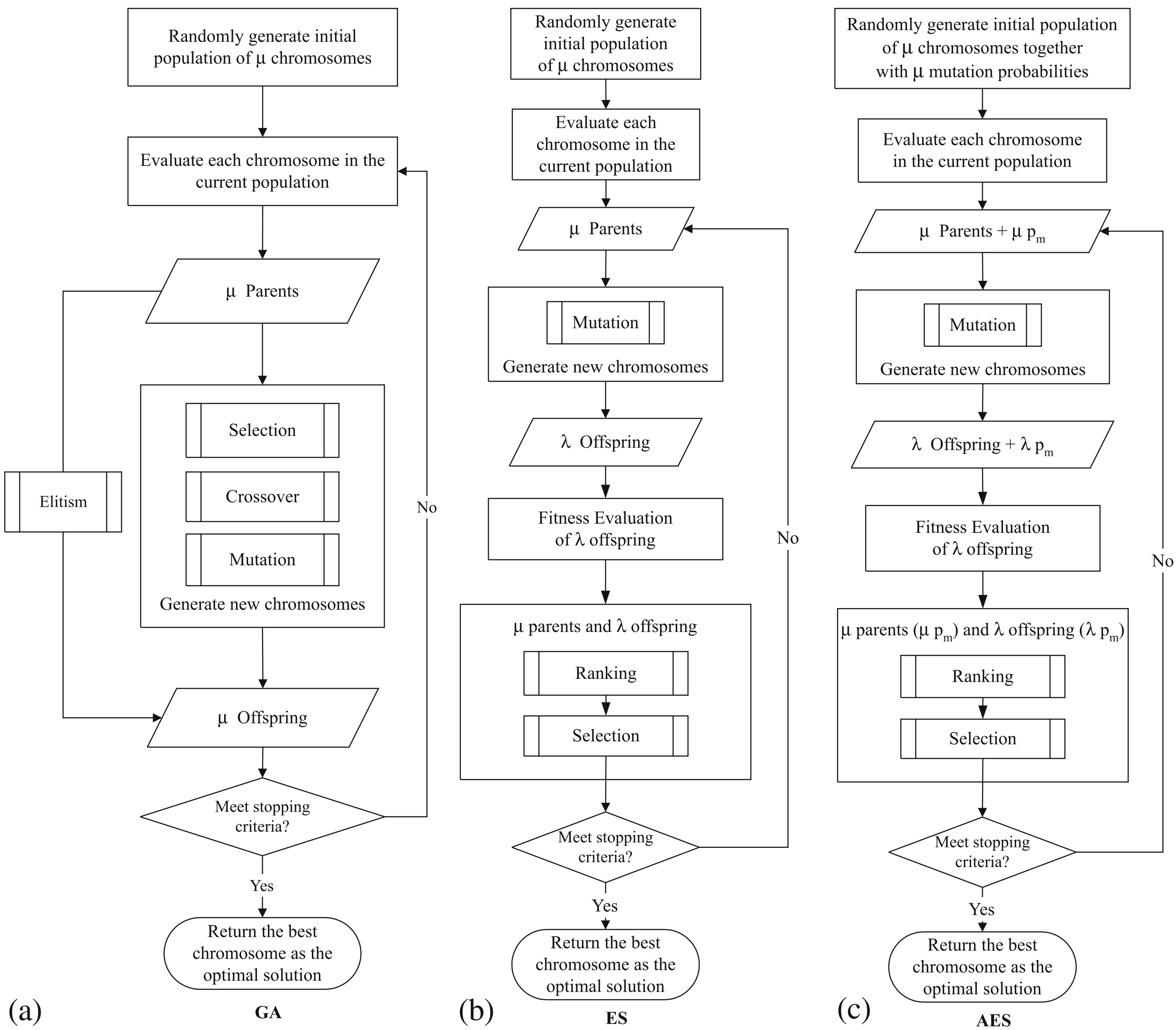

Fig. 4a–c. Flow diagram of **a** genetic algorithms, **b** $(\mu+\lambda)$-evolution strategies and **c** $(\mu+\lambda)$-adaptive evolution strategies

This procedure for determining the hedging points is used in connection with all three evolutionary algorithms. The best hedging points obtained using genetic algorithms (GAs), non-adaptive evolution strategies (ESs), and adaptive evolution strategies (AESs) in each case are summarised in Table 1. It is evident from this table that the optimal hedging points obtained from the three different evolutionary algorithms are almost identical.

3.2 Optimisation of short-run hedging points for multiple product-type systems

The evolutionary stochastic optimisation procedure for short-run hedging points can also be illustrated by considering a particular unreliable multiple product-type manufacturing system. In this system, the processing times, τ_1 and τ_2, of the two product-types are both 0.2113 s/piece, the mean-time-to-failure, T_f, and mean-time-for-repair, T_r, are 4 s and 2 s, respectively, and the task time, T, is 20 s.

In order to facilitate the comparison (see Sect. 4) between the evolutionary estimates of the optimal hedging points and the theoretical results obtained by Sethi and Zhang [11], which are the only available analytic results, it is assumed that the cost weightings of surplus and backlog for both product-types are equal, i.e., $g_1^+ = g_1^- = g_2^+ = g_2^- = 1$. In this case, a set of 100 machine-condition variables, $\{\alpha^{(1)}(t), \alpha^{(2)}(t), \ldots, \alpha^{(100)}(t)\}$, is again introduced to represent failure and repair histories with the relevant mean-time-to-failure and the mean-time-for repair. The expected value of the cost function for the corresponding hedging point is then calculated according to Eq. 7. The Darwinian fitness of the binary string representing the hedging point is finally obtained using Eq. 8.

This procedure for determining the hedging points is used in connection with all three evolutionary algorithms. It is again found that GAs, ESs, and AESs provide nearly identical results. The estimates of the optimal hedging points for $\boldsymbol{d} \in \{[1, 1], [1, 1.5], [1, 2], [1.5, 1], [1.5, 1.5]\}$ when $\boldsymbol{x}(0)$ varies from [−4, −4] to [4, 4], are summarised in Table 2.

Table 1. Evolutionary estimates of optimal short-run inventory levels for single-product-type manuacturing systems

x^*_{opt}	d											
	2			2.5			3			3.5		
$x(0)$	GA	ES	AES	GA	ES	AES	GA	ES	AES	GA	ES	AES
−4	0.22	0.29	0.22	0.95	1.25	0.95	1.76	1.76	1.76	2.52	2.52	2.59
−2	0.29	0.29	0.29	1.05	1.05	1.05	1.94	1.94	2.05	3.11	3.11	3.11
0	0.21	0.21	0.36	1.00	1.00	1.05	2.18	2.18	2.24	3.74	3.74	3.74
2	0.22	0.22	0.22	0.95	0.95	0.99	2.36	2.36	2.32	4.12	4.12	4.12
4	0.22	0.22	0.22	1.00	1.00	0.99	2.24	2.24	2.22	4.23	4.23	4.23
6	0.22	0.22	0.22	0.95	0.95	0.99	2.18	2.18	2.12	4.02	4.02	4.02

Table 2. Evolutionary estimates of optimal short-run inventory levels for two-product-type manufacturing systems

$\boldsymbol{x}^*_{opt}$	$\boldsymbol{d} = [d_1, d_2]$				
$\boldsymbol{x}(0)$	[1, 1]	[1, 1.5]	[1, 2]	[1.5, 1]	[1.5, 1.5]
[−4, −4]	[0, 0]	[0, 0.01]	[0.10, −0.01]	[0.01, 0]	[0.18, 0.21]
[−4, −2]	[0, 0]	[0.10, 0.01]	[0.16, −0.01]	[0.10, 0]	[0.25, 0.01]
[−4, 0]	[0, 0]	[0.12, 0.01]	[0.37, −0.01]	[0.18, 0]	[0.51, 0.01]
[−4, 2]	[0, 0]	[0.18, 0.10]	[0.33, 0.29]	[0.19, 0.05]	[0.51, 0.27]
[−4, 4]	[0, 0]	[0.18, 0.06]	[0.47, 0.72]	[0.14, 0]	[0.66, 0.47]
[−2, −4]	[0, 0]	[0, 0.10]	[0.12, 0.29]	[0.01, 0.10]	[0.01, 0.25]
[−2, −2]	[0, 0]	[0.11, 0.08]	[0.33, 0.35]	[0.08, 0.11]	[0.45, 0.53]
[−2, 0]	[0, 0]	[0.14, 0.01]	[0.34, 0.30]	[0.21, 0]	[0.53, 0.06]
[−2, 2]	[0, 0]	[0.16, 0.14]	[0.49, 0.87]	[0.21, 0.10]	[0.66, 0.66]
[−2, 4]	[0, 0]	[0.14, 0.06]	[0.52, 0.80]	[0.19, 0]	[0.64, 0.53]
[0, −4]	[0, 0.01]	[0, 0.18]	[0, 0.69]	[0.01, 0.10]	[0.01, 0.51]
[0, −2]	[0, 0]	[0, 0.21]	[0, 0.66]	[0.01, 0.14]	[0.06, 0.53]
[0, 0]	[0, 0]	[0.18, 0.31]	[0.47, 0.96]	[0.31, 0.18]	[0.71, 0.70]
[0, 2]	[0, 0]	[0.18, 0.23]	[0.55, 1.03]	[0.31, 0.10]	[0.75, 0.68]
[0, 4]	[0, 0]	[0.14, 0.06]	[0.51, 0.83]	[0.16, 0]	[0.70, 0.55]
[2, −4]	[0, 0]	[0.05, 0.19]	[0.23, 0.71]	[0.10, 0.18]	[0.27, 0.51]
[2, −2]	[0, 0]	[0.10, 0.21]	[0.34, 0.91]	[0.19, 0.19]	[0.66, 0.66]
[2, 0]	[0, 0]	[0.10, 0.31]	[0.41, 0.90]	[0.23, 0.18]	[0.68, 0.75]
[2, 2]	[0, 0]	[0.15, 0.23]	[0.38, 0.99]	[0.23, 0.15]	[0.73, 0.73]
[2, 4]	[0, 0]	[0.12, 0.08]	[0.45, 0.87]	[0.12, 0]	[0.70, 0.55]
[4, −4]	[0, 0]	[0, 0.14]	[0, 0.62]	[0.06, 0.18]	[0.47, 0.66]
[4, −2]	[0, 0]	[0, 0.19]	[0.01, 0.78]	[0.06, 0.14]	[0.53, 0.64]
[4, 0]	[0, 0]	[0, 0.16]	[0.03, 0.77]	[0.06, 0.14]	[0.55, 0.70]
[4, 2]	[0, 0]	[0, 0.12]	[0.04, 0.91]	[0.08, 0.12]	[0.55, 0.70]
[4, 4]	[0, 0]	[0, 0.18]	[0.01, 0.78]	[0.18, 0]	[0.60, 0.60]

4 Comparative analyses

It is interesting to compare the evolutionary estimates of the optimal short-run inventory levels with the corresponding theoretical solutions. In the case of manufacturing systems containing single machines producing single product-types, Bielecki and Kumar [7] obtained the complete optimal hedging-point values for minimising the long-run average cost. These theoretical optimal hedging points are, unfortunately, therefore only available for long-run cases when the task time, T, becomes infinite and where the results are, consequently, independent of the initial inventory level, $x(0)$. In the case of the particular system described in Sect. 3.1, the theoretical values of Bielecki and Kumar [7] for the optimal hedging points, x^*_{opt}, are 0.54, 1.74, 4.49, 14.63, and infinity when the demand rates are 2, 2.5, 3, 3.5, and 4 pieces/s, respectively.

These long-run optimal values are evidently very different from the short-run optimal values of the hedging points shown in Table 1, but it is very interesting that these short-run optimal values of the hedging points are found to approach the theoretical values of Bielecki and Kumar [7] as the task time, T, increases. This behaviour of the values for x^*_{opt} obtained by using the adaptive evolution strategy, for example, is demonstrated in Table 3, which lists the evolutionary estimates of the optimal hedging points, x^*_{opt}, for $T = 20$ s, 40 s, 70 s, 100 s, 200 s, 300 s, and 400 s, when $(d, x(0))$ are (2, 0), (2.5, 0), (3, 0), (3.5, 0) and (4, 0), respectively. If the inequality Eq. 4 is not satisfied, then the demand rate is unachievable, and the theoretical optimal hedging point is infinite for such cases. Such an infinite theoretical hedging point for an infeasible demand implies that a machine should always produce at its maximum capacity whenever it is operative. It is evident, from the case of the infeasible demand rate $d = 4$ pieces/s in Table 3, that the evolutionarily optimised hedging points increase as the task time, T, increases. This phenomenon agrees with the theoretical results.

In the case of manufacturing systems producing multiple product-types, the evolutionarily optimised results can be compared to the theoretical long-run results obtained by Sethi and Zhang [11], which are the only available analytical solutions for multiple product-type systems. Thus, with reference to the manufacturing system described in Sect. 3.2, it is expected that the theoretical long-run optimal hedging points will be quite different from the short-run optimal values shown in Table 2, for which the task time, T, is only 20 s. However, it can be anticipated that the evolutionarily optimised short-run hedging-point values will approach the theoretical values of Sethi and Zhang [11] as the task time, T, increases. This anticipation is substantiated by Table 4, which compares the long-run theoretical hedging points with the short-run optimal values with increasing task time, T, and zero initial inventory levels, $\boldsymbol{x}(0) = [0, 0]$.

Table 3. Evolutionarily optimised short-run hedging points with increasing T versus theoretical long-run hedging points (single-product-type systems)

	Task time T	d = 2	2.5	3	3.5	4
Evolutionarily	20	0.36	1.05	2.24	3.74	4.65
optimised	40	0.58	1.52	3.65	6.65	10.00
short-run	70	0.36	1.25	3.22	6.33	14.00
inventory	100	0.51	1.54	3.71	7.99	16.55
levels	200	0.49	1.67	4.06	10.01	22.69
	300	0.44	1.62	4.10	11.48	32.00
	400	0.45	1.53	4.00	11.13	39.22
Theoretical results	∞	0.54	1.74	4.49	14.64	∞

It is noted that the available theoretical results are all for unreliable manufacturing systems in which the demand rates are constant for infinite time. Such an assumption is obviously unrealistic in cases of actual production. However, the evolutionarily optimised short-run inventory levels obtained in this paper can be readily used to construct look-up tables for use in gain-scheduled adaptive controllers for manufacturing systems with variable demands [12, 16].

It is also noted that the results presented in this paper concern unreliable manufacturing systems in which the machine failure and repair times are assumed to follow exponential distributions. Such an assumption is necessary (as mentioned previously) to facilitate the comparison of the evolutionarily optimised results with theoretical results. However, the properties of the exponential distribution on which analytical methods heavily depend are often not good models of the behaviour of well-engineered workstations [17]. The evolutionary optimisation methodologies proposed in this paper are not subject to such a restriction, but are applicable to unreliable manufacturing systems in which failure and repair times follow other random distributions. In-

Table 4. Evolutionarily optimised short-run hedging points with increasing T versus theoretical long-run hedging points (two-product-type systems)

	Task T	$\boldsymbol{d} = [d_1, d_2]$ = [1, 1]	[1, 1.5]	[1, 2]	[1.5, 1]	[1.5, 1.5]
Evolutionarily	20	[0, 0]	[0.18, 0.31]	[0.47, 0.96]	[0.31, 0.18]	[0.71, 0.70]
optimised	40	[0.18, 0.19]	[0.73, 1.09]	[1.41, 2.88]	[1.09, 0.74]	[2.11, 2.16]
short-run	70	[0.3, 0.27]	[0.89, 1.33]	[1.64, 3.36]	[1.35, 0.89]	[2.54, 2.50]
inventory	100	[0.27, 0.25]	[1.01, 1.46]	[2.36, 4.76]	[1.44, 0.99]	[3.50, 3.56]
levels	200	[0.36, 0.34]	[1.21, 1.87]	[3.36, 6.76]	[1.83, 1.23]	[5.08, 5.03]
	300	[0.32, 0.32]	[1.22, 1.87]	[4.07, 8.23]	[1.83, 1.25]	[6.11, 6.16]
	400	[0.42, 0.42]	[1.51, 2.11]	[4.77, 9.58]	[2.18, 1.42]	[7.15, 7.20]
Theoretical results	∞	[0.45, 0.45]	[1.57, 2.36]	[8.95, 17.91]	[2.36, 1.57]	[13.43, 13.43]

deed, this evolutionary stochastic optimisation procedure can be readily used to deal with random disturbances caused by events such as machine setups with various statistical distribution characteristics [12, 18].

5 Product prioritisation

The investigation by Sethi and Zhang [11] is very important because it provides the only available theoretical results for multiple-product-type systems. However, in obtaining these results, it is assumed [11] that all product-types have equal cost weightings for both surplus and backlog, and that they therefore have equal production priorities. In contrast, by using the evolutionary methodology for the optimisation of hedging points for multiple product-type systems, it is easy to specify any desired priorities among different product-types by assigning appropriate cost weightings to the different product-types. The evolutionary optimisation of hedging points for different priority assignments can be conveniently illustrated by the following two cases of a two-product-type unreliable manufacturing system:

(i) the first and the second product-types have equal priority, such that $g_1^+ = g_1^- = g_2^+ = g_2^- = 1$;
(ii) the second product-type has priority over the first product-type, such that $g_1^+ = g_1^- = 1$ and $g_2^+ = g_2^- = 10$.

Table 5. Prioritised short-run hedging points for a system with $\boldsymbol{d} = [1.5, 1.5]$

$\boldsymbol{x}(0)$	When both product-types have equal priority	When the second product-type has priority over the first
$[-4, -4]$	$[0.18, 0.21]$	$[-5.62, 0.49]$
$[-2, -2]$	$[0.45, 0.53]$	$[-2.95, 0.76]$
$[0, 0]$	$[0.71, 0.70]$	$[0.45, 0.71]$

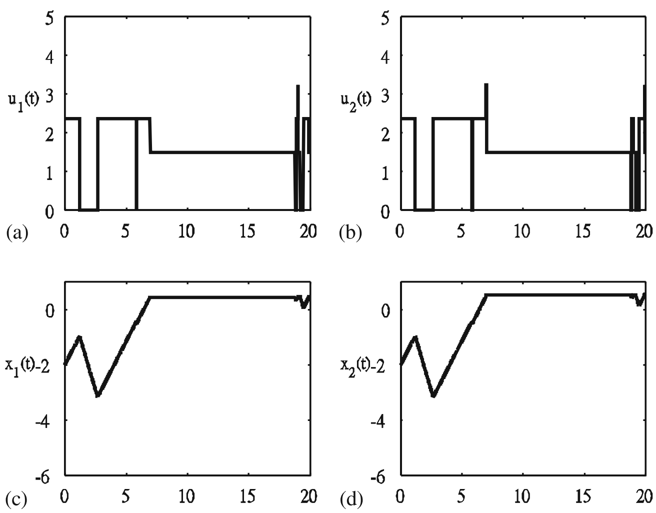

Fig. 5. Time-domain behaviour when both product-types have equal priority

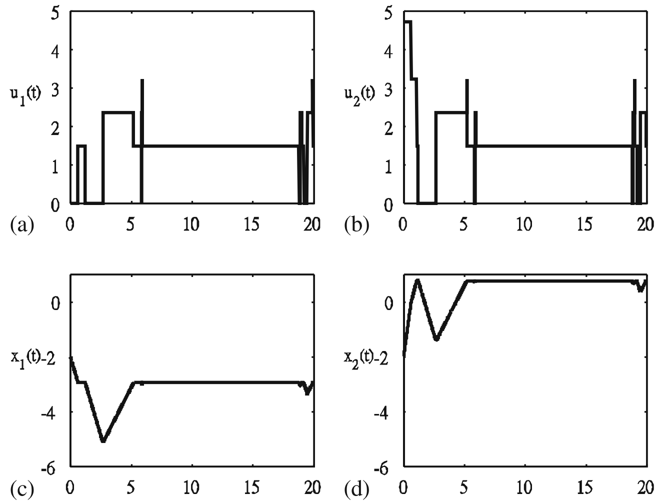

Fig. 6. Time-domain behaviour when the second product-type has priority over the first product-type

Table 5 shows the estimates of the prioritised hedging points when $\boldsymbol{d} = [1.5, 1.5]$ and $\boldsymbol{x}(0) = [-4, -4]$, $[-2, -2]$ and $[0, 0]$ obtained using the adaptive evolution strategy.

The time-domain behaviour of the manufacturing system under crisp-logic control when $\boldsymbol{d} = [1.5, 1.5]$ and $\boldsymbol{x}(0) = [-4, -4]$ for these two cases of production priority is shown in Figs. 5 and 6, respectively. In the case of equal priority, the crisp-logic controllers evidently automatically generate the maximum permissible production rates for both product-types in order to clear both backlogs simultaneously. However, when the second product-type has priority over the first product-type, it is clear from Fig. 6 that the crisp-logic controllers first generate automatically the maximum permissible production rate for the second product-type in order to clear the associated backlog and then automatically switch production to the first product-type.

6 Performance of evolutionary algorithms

Evolutionary algorithms of three types – namely, genetic algorithms, $(\mu + \lambda)$-evolution strategies and adaptive $(\mu + \lambda)$-evolution strategies, were used in the procedures for optimising hedging points. The relative effectiveness of these algorithms can be conveniently illustrated by considering the multiple-product-type manufacturing system described in Sect. 3.2 when $\boldsymbol{d} = [1, 2]$ and $\boldsymbol{x}(0) = [0, 0]$.

In this case, the performance of the genetic algorithm over 20 generations with a population size of 10 is shown in Fig. 7a for different values of the probabilities of mutation, p_m, and crossover, p_c. It is thus evident that, for this genetic algorithm, the most rapid evolution occurred with $p_m = 0.03$ and $p_c = 0.6$, and that the best solution was obtained after 14 generations.

The corresponding results for the non-adaptive $(10+10)$-evolution strategy without recombination over 20 generations

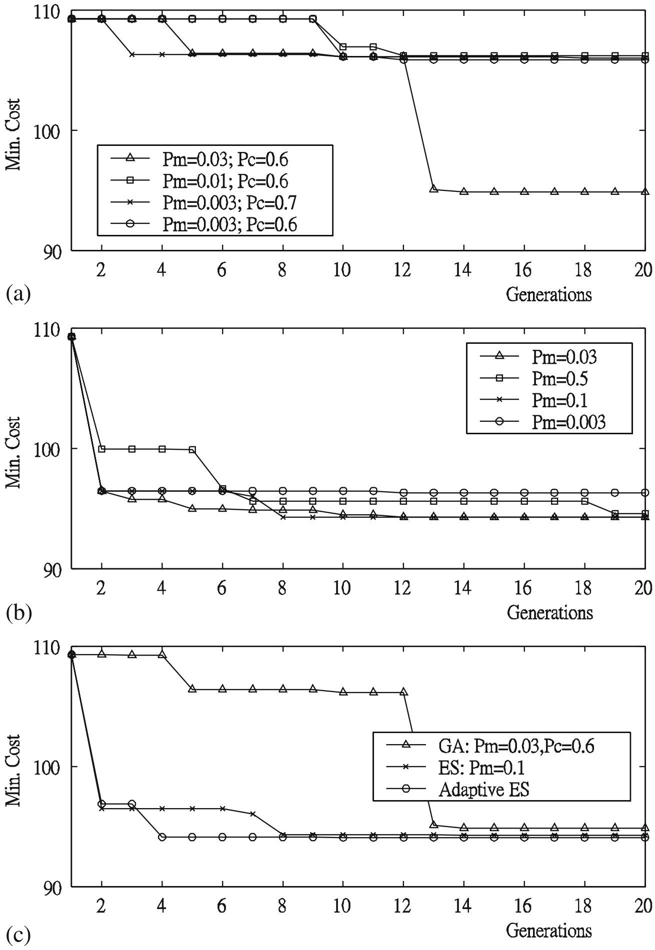

Fig. 7a–c. Evolutionary optimisation performance comparison. **a** GA minimum-of-generation cost. **b** ES minimum-of-generation cost. **c** GA, ES and AES minimum-of-generation cost

are shown in Fig. 7b for different values of the probability of mutation, p_m. It is thus evident that, for this non-adaptive evolution strategy without recombination, the most rapid evolution occurred with $p_m = 0.1$, and that the best solution was obtained after 8 generations.

The corresponding results for the adaptive $(10+10)$-evolution strategy without recombination over 20 generations are compared with these best results for the genetic algorithm and the non-adaptive $(10+10)$-evolution strategy in Fig. 7c. It is thus evident that, for this adaptive evolution strategy without recombination, the best solution was obtained after four generations. Indeed, it is clear from Fig. 7c that the adaptive $(10+10)$ evolution strategy is more effective than the other evolutionary algorithms in estimating the optimal hedging points for this unreliable manufacturing system under crisp-logic control.

In this particular case, it thus transpires that the adaptive evolution strategy is the most effective evolutionary algorithm among the three alternatives for estimating the optimal hedging points for unreliable manufacturing systems under crisp-logic control. However, it is also important to note that even though the adaptive evolution strategy without recombination may not always provide the most rapid evolution, it is the simplest evolutionary algorithm to use since it requires no *a priori* selection of either mutation probability or crossover probability.

7 Conclusion

In this paper, an evolutionary stochastic optimisation procedure has been proposed to estimate the optimal hedging points (i.e. optimal inventory levels) for unreliable manufacturing systems producing either single product-types or multiple product-types under crisp-logic control. The methodology has been illustrated by examples, and has been validated by comparing the hedging points produced by evolutionary algorithms with those obtained from the theoretical long-run solutions. It has been also shown that the evolutionary stochastic optimisation procedure can be used to obtain prioritised optimal hedging points, i.e. hedging points when the cost weightings are different among the different products. The proposed methodology is not restricted to unreliable manufacturing systems with exponentially distributed random machine failures and repairs, but is applicable to such random events with other distribution characteristics. It has also been pointed out that the evolutionarily optimised short-run inventory levels can be readily used to construct look-up tables for adaptive gain-scheduled controllers for unreliable manufacturing systems with variable demands. Finally, the relative merits of genetic algorithms, evolution strategies, and adaptive evolution strategies have been compared in the optimisation of hedging points for unreliable manufacturing systems. Indeed, it has thus been indicated that the adaptive evolution strategy without recombination is the simplest algorithm to use since it requires no *a priori* selection of mutation probability or crossover probability.

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