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ORIGINAL ARTICLE

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A single-vendor multiple-buyer inventory model with a multiple-shipment policy

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Abstract This paper presents a new methodology to obtain the joint economic lot size in the case where multiple buyers are demanding one type of item from a single vendor. The shipment policy is found and a new model is proposed to minimise the joint total relevant cost (JTRC) for both vendor and buyer(s). Considering the two-buyer and the more-than-two-buyer cases, an analytical solution and numerical solution are obtained. A significant savings in joint total relevant cost is achieved when the total demand rate is close to the production rate. A sensitivity analysis is also conducted to test the robustness of the new model.

Keywords Distribution system · Inventory · Joint economic lot size · Multiple shipment policy

1 Introduction

The emergence of supply-chain management has created a new trend in establishing a partnership between two or more related parties. Through this partnership, an organisational chain is formed. This allows information critical to satisfying customer demands to be better shared. In order to utilise the information and create a win-win situation among the parties, the implemented tools have to be able to take into account the whole chain instead of one specific party only. Therefore, due to this need, the concept of joint economic lot size (JELS) is introduced to refine the well-known classical economic order quantity (EOQ) method. The basic idea of the concept is to take into account the collaboration of two parties to determine a more profitable economic lot size and thus move away from the adversarial bargaining process.

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JELS was developed and introduced by Banerjee [1]. He assumed that the vendor produces on a lot-by-lot basis in response to orders from a single purchaser. The demand is deterministic, and the vendor is the sole supplier. The notion of this concept is that the vendor (supplier) and purchaser are value chain partners in manufacturing and delivering a high quality product to the purchaser's customers.

From the outcome of this model, Banerjee concludes that joint determination of the economic lot size for both parties can reduce their total cost substantially, moving toward a win-win situation. This outcome has been supported by several authors, such as Hall [2], Li et al. [3] and Miller and Kelle [4].

Goyal [5] examined a joint total relevant cost model for a single-vendor–single-buyer production inventory system. He assumed that the vendor's lot size is an integer multiple of the purchaser's order size in order to achieve a better joint total relevant cost as compared to Banerjee's JELS model. Relaxing Goyal's assumption, Lu [6] proposed a model that the vendor can actually supply the purchaser in a number of equal small lots even before completing the entire lot. It is obvious that both models will increase the transportation cost substantially. However, this situation is still acceptable, since the reduction acquired in the holding cost would be sufficient to compensate the increase of the transportation cost. Implementing a similar equal-size multiple-shipment concept, Hahm and Yano [7] and Aderohunmu et al. [8] developed a model to minimise the total relevant inventory cost for both vendor and buyer, which includes transportation costs. Banerjee and Kim [9], Khan and Sarker [10], and Khan and Sarker [11] proposed a model to optimise the activities of both raw material purchasing and production lot sizing simultaneously by taking all of the operating parameters into consideration.

The previous studies consider a situation where the demand for an item comes from a single buyer. Little attention has been given to the problem of having a single-vendor multi-buyer situation, with no attention given to the case where all of the buyers are demanding a single type of item. This is a general case that an inventory manager could face, especially when forward postponement or standardisation is integrated into the chain [12].

The joint total relevant cost (JTRC) function for a single-vendor multi-buyer situation based on an equal-size multiple-shipment policy is developed. The production and ordering policies are structured to give minimum JTRC. An analytical solution was obtained for the unconstrained multiple-buyer problem giving the shipment number, production and ordering cycle to minimise JTRC. Incorporating constraints, the analytical solution for the two-buyer case is obtained, and a direct numerical solution is applied for more than two buyers.

2 New joint economic lot size model

It is assumed that the vendor is the sole supplier for all buyers, and the buyers demand only one specific item. To satisfy the orders, the vendor will deliver a number of equal-size shipments to each buyer. However, the shipment size might differ from each buyer in accordance with his or her demand rate and related cost parameters.

The production is organised in such a way that the first shipment for each buyer is done in a sequence. Following this sequence, the first delivery starts from the first buyer followed by the second, the third and so on. The duration from one delivery to the next is fixed for each buyer. In this study, it is assumed that the order cycle time for each buyer and the production cycle time for the vendor are equal. Throughout this paper, we will refer to the order cycle time as the time between each order and the production cycle time as the time between each production setup. The overall system inventory level is illustrated in Figs. 1 and 2.

Analysing Fig. 1, it is clear that the sum of the area described by the dashed lines is less than the area described by the solid lines. On the other hand, in Fig. 2, the area described by the dashed lines for each buyer is bigger than the sum of the area described by the solid line. This confirms the earlier discussion on the equal-size multiple-shipment policy. The savings incurred in the buyers' holding cost, which is represented in Fig. 2, outstrips the additional costs on transportation and the vendor's holding cost.

In addition to the assumptions made in the classical EOQ and EPQ (Economic Production Quantity) models the following assumptions have been made for this preliminary study.

1. The production rate is greater than the sum of the demand rate ($P > \sum_{i=1}^{y} D_i$).
2. The production lot size is equal to the sum of the order amount from all of the buyers, which only consists of a single type of item.
3. The production lot size for each buyer is delivered in a number of equal-size shipments, where the number and the size of the shipments might be different for each buyer.

All of the symbols used in this study are defined in Table 1.

In order to formulate an equation that can represent the vendor's total average inventory, the methodology used by Joglekar [13] is implemented, and this is shown in Fig. 3.

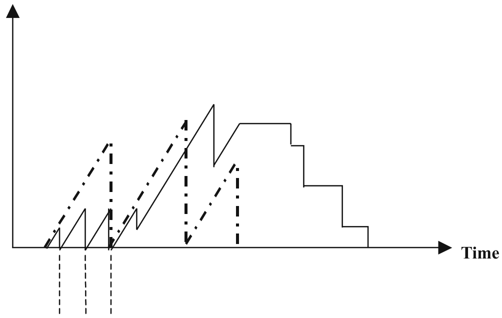

Fig. 1. Vendor's inventory level against time. The solid lines and the dashed lines represent the inventory level with and without multiple shipments, respectively

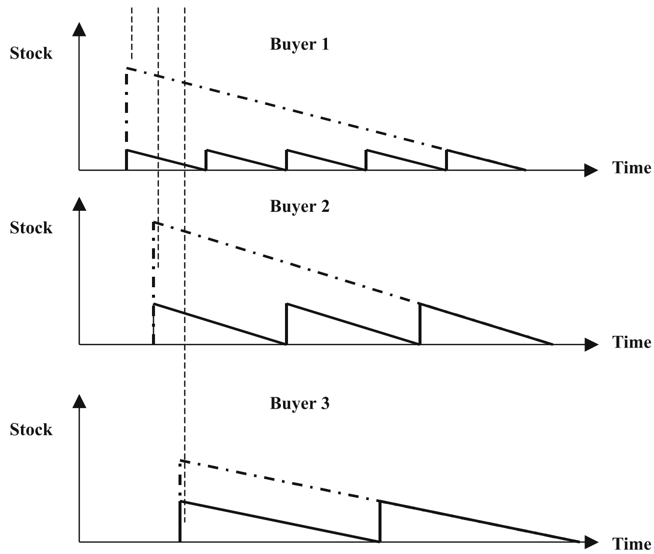

Fig. 2. Buyer's inventory level against time. The solid lines and the dashed lines represent the inventory level with and without multiple shipments, respectively

Table 1. The definitions of the symbols

D	Demand rate per unit per unit time
P	Production rate per unit per unit time
A	Ordering cost per order
S	Setup cost per lot
$JTRC$	Joint total relevant cost
$JELS$	Joint economic lot size
C	Ordering or production cycle time
y	Number of buyers
n	Number of shipments
H_V	Vendor's holding cost per unit per unit time
H_b	Buyer's holding cost per unit per unit time
A_T	Transportation cost per shipment
k	Ratio between production rate and demand rate
λ	Lagrange multiplier factor
i	A subscript used to represent different buyers

The step-like rectangles are the accumulation of the products delivered to buyers. Subtracting the overall area (KLMN) of these rectangles, the equation for the vendor's average inventory

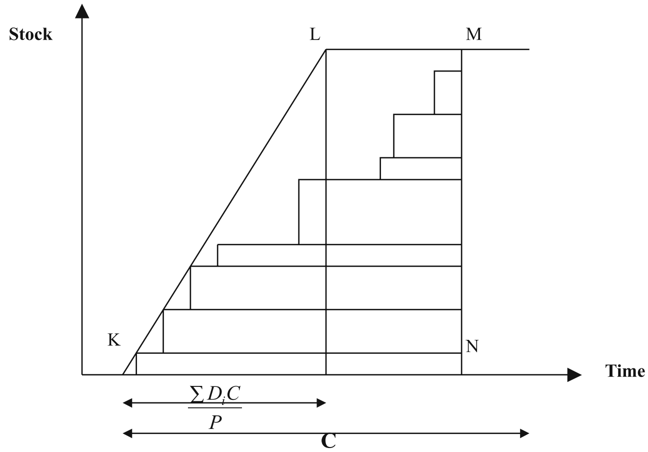

Fig. 3. Vendor's average inventory against time with a production cycle time of C.

can be defined in a simple form, as shown below. The step-by-step derivation of this equation is given in the appendix.

Vendor's average inventory

$$= \frac{C}{2P}\left(\left(\sum_{i=1}^{y} D_i\right)^2 + 2\sum_{i=1}^{y} D_i R_B - R_C\right) \tag{1}$$

where

$$R_B = \frac{P}{n_1}\left(\frac{D_1}{P} + n_1 - 1\right) - \sum_{i=1}^{y} D_i$$

$$R_C = \sum_{i=1}^{y} D_i \times \left(\frac{2n_i\left(P\left(n_1-1\right) + D_1 - n_1\sum_{j=1}^{i}\frac{D_j}{n_j}\right) - Pn_1\left(n_i-1\right)}{n_1 n_i}\right)$$

Thus, the joint total relevant cost for the entire system is

$$\begin{aligned} JTRC =& \frac{1}{C}\left(S + \sum_{i=1}^{y}\left(A_i + n_i A_{Ti}\right)\right) \\ &+ \frac{C}{2}\left(\frac{H_V}{P}\left(\left(\sum_{i=1}^{y} D_i\right)^2 + 2\sum_{i=1}^{y} D_i R_B - R_C\right)\right. \\ &\left. + \sum_{i=1}^{y} H_{bi}\frac{D_i}{n_i}\right) \end{aligned} \tag{2}$$

subject to

$$n_1, n_2, n_3, \ldots, n_y \geq 1$$

$$\frac{C}{n_i} \geq \frac{C}{P}\sum_{i=1}^{y}\frac{D_i}{n_i}$$

The first term from the cost function above is the setup cost for the vendor (S/C) and the sum of ordering and transportation costs for the buyers

$$\left(\frac{1}{C}\left(\sum_{i=1}^{y}\left(A_i + n_i A_{Ti}\right)\right)\right).$$

The second term of the function is the vendor's holding cost

$$\left(\frac{C}{2}\frac{H_V}{P}\left(\left(\sum_{i=1}^{y} D_i\right)^2 + 2\sum_{i=1}^{y} D_i R_B - R_C\right)\right)$$

and the buyers total holding cost

$$\left(\frac{C}{2}\sum_{i=1}^{y} H_{bi}\frac{D_i}{n_i}\right).$$

The cost function Eq. 2 is subjected to two constraints. The first constraint is that there must be at least one shipment for each buyer. The second assumption is that the principle assumption of the new method – that the first delivery to each buyer has to be done in sequence – holds. Therefore, restricted with this constraint, the second shipment to any buyer is done after all buyers in the system have received their first lot. Note that when substituting $n_i = 1$, the cost function Eq. 2 is equal to the cost function without multiple shipments.

Through an algebraic manipulation, JTRC can be expressed as

$$\begin{aligned} JTRC =& \frac{1}{C}\left(S + \sum_{i=1}^{y} A_i + \sum_{i=1}^{y} n_i A_{Ti}\right) \\ &+ \frac{C}{2}\left\{\frac{H_v}{P}\sum_{i=1}^{y} D_i\left(P - \sum_{i=1}^{y} D_i\right)\right. \\ &\left. + \sum_{i=1}^{y}\left[\frac{D_i}{n_i}\left(\frac{2H_v}{P}\sum_{j=i}^{y} D_j + H_{bi} - H_v\right)\right]\right\} \end{aligned} \tag{3}$$

Relaxing the second constraint, the optimal values of C and n_i can be derived as

$$C = \left[\frac{2P\left(S + \sum_{i=1}^{y} A_i\right)}{H_v\sum_{i=1}^{y} D_i\left(P - \sum_{i=1}^{y} D_i\right)}\right]^{1/2} \tag{4}$$

$$n_i = C\left[\frac{D_i\left(\frac{2H_v}{P}\sum_{j=i}^{y} D_j + H_{bi} - H_v\right)}{2A_{Ti}}\right]^{1/2} \tag{5}$$

where $i = 1, 2, 3, \ldots, y$.

Since it is impossible to have a fractional number of shipments, the value of n_i has to be rounded to the nearest integer. Or, using the equation below,

$$n_i\left(n_i+1\right) \geq \frac{PD_i\left(S + \sum_{i=1}^{y} A_i\right)\left(\frac{2H_v}{P}\sum_{j=i}^{y} D_j + H_{bi} + H_v\right)}{A_{Ti}H_v\sum_{i=1}^{y} D_i\left(P - \sum_{i=1}^{y} D_i\right)} \geq n_i\left(n_i-1\right)$$

From Eq. 5, it shows that the value of n_i is dependent on the decision of the order of the buyers, i.e. which buyer is positioned as the first buyer, second, third and so on. In other words, if the hierarchy of the buyers is restructured, the value of "n_i" will also change due to the change of its denominator – specifically, the value of $\sum_{j=i}^{y} D_j$. From empirical findings, in order to obtain the best result of the cost function Eq. 3, alternative ordering must be considered. This can be approximately achieved by following the algorithm given in the next section.

The value of n_i can be derived from Eq. 5 if only none of the resulted value is restricted with the constraints. However, when this is not true, Eq. 5 is not applicable. Therefore, incorporating the constraints, two different solutions are given to deal with the two- and more-than-two-buyer case.

For the two-buyer case, an analytical solution can be found by using Lagrange multipliers. Given:

$$\begin{aligned}\text{Min JTRC} =& \frac{1}{C}\left(S+\sum_{i=1}^{y} A_i + n_1 A_{T1} + n_2 A_{T2}\right)\\ &+\frac{C}{2}\left[\frac{H_v}{P}\sum_{i=1}^{y} D_i\,(P-D_1)+\frac{D_1}{n_1}a+\frac{D_2}{n_2}b\right]\\ &+\lambda\left(\frac{\mathrm{P}}{n_1}-\sum_{1}^{2}\frac{D_i}{n_i}\right)\end{aligned}$$

when

$$n_1=\left[\frac{C^2}{2A_{T1}}\left[(P-D_1)\,a+D_1 b\right]-\frac{n_2^2 A_{T2}}{D_2 A_{T1}}\,(P-D_1)\right]^{1/2} \tag{6}$$

$$n_2=C\left[\frac{D_2\left((P-D_1)\,a+D_1 b\right)}{2\,(P-D_1)\left((P-D_1)\,\frac{A_{T1}}{D_2}+A_{T2}\right)}\right]^{1/2} \tag{7}$$

where

$$a=\frac{2H_v}{P}D_2+H_{b2}-H_v$$

and

$$b=\frac{2H_v}{P}\sum_{i=1}^{2}Di+H_{b1}-H_v.$$

Similar with the unconstrained condition C can be calculated using equation Eq. 4, since the constraints are not restricting the value of C. Furthermore, the values of n_1 and n_2 that resulted from Eq. 6 and Eq. 7 have to be rounded up to the nearest integer in order to make it applicable.

For the more than two buyer case the complexity of an analytical solution becomes such that a direct numerical solution is more efficient. The example of the solution is given in the numerical illustration section, example three. To achieve the solution the following algorithm can be applied, which is also applicable for any number of buyers case.

The algorithm for the optimisation problem is as follows:

Step 1 Calculate C using Eq. 4.

Step 2 With a given number of buyers, a position is allocated for each of them.
The first buyer will be the one that has a maximum value of n1, the second buyer will be the one that has a maximum value of n2 and so on. n is derived using Eq. 5.

Step 3 Test the validity of each n_i ($i = 1, 2, 3, \ldots, y$) with respect to the constraints.

Step 4 If all n_i are valid then let n_i be the optimal number of shipment for the associated buyer. Where the shipment size can be determined using:

$$q_i=\frac{D_i C}{n_i} \tag{8}$$

Step 5 If one or more n_i ($i = 1, 2, 3, \ldots, y$) are invalid, then new n_i ($i = 1, 2, 3, \ldots, y$) are derived from a full search through a numerical procedure. Or if only deal with two buyers, Eq. 4, Eq. 6, and Eq. 7 can be applied to determine the solution. The shipment size can be calculated using Eq. 8.

3 Effects of the shipment constraint on the hierarchy of buyers

In the event where one or more shipment numbers do not satisfy the second constraint, the hierarchy of the buyers should not be changed because the constraint does not restrict the hierarchy. It is applied to the cost function in order to achieve shipment numbers or shipment cycles that could ensure one delivery to each buyer before the second delivery for the first buyer. This would allow the vendor to set a production rate to ensure enough items be delivered on schedule. Any shipment number that does not satisfy the constraint may cause the associated buyer to incur a time delay before the order may be fulfilled. During this delay, backorders or lost sales would happen when the demand is incurred at a constant rate. This is an undesirable situation when dealing with a deterministic system. Therefore, following the given algorithm, the shipment quantities have to be recalculated for all buyers considering the constraint.

The constraint does not restrict the hierarchy, any changes from the initial hierarchy would lead to a higher joint total relevant cost (JTRC). Following the initial hierarchy, the resultant shipment numbers would tend to have the first and the last shipment within the production cycle delivered to the first buyer. For this case, it is optimal for the vendor's holding period during the production cycle to be the shortest. Shifting the position of the buyers – ignoring whether the shipment number(s) is (are) restricted by the constraint or not – would alter the shipment number for each buyer accordingly. Consequently, the vendor's holding period would increase, as the sequence of the shipment numbers does not follow $n_1 \geq n_2 \geq \ldots \geq n_y$. This would imply that the last shipment is scheduled for any buyer other than the first one. In this situation, the production cycle would be the sum of the shipment cycles for the buyer with the last shipment plus the production time needed to produce the first shipment quantity for the buyer and for any other buyers prior to this one. As a result, the vendor's holding cost is increased.

Buyers' holding and transportation costs would also be increased due to the restructuring of the hierarchy and/or recalculating of the shipment numbers, n_i. However, in the latter method of recalculating n_i, the extra costs are incurred due solely to the constraint. To ensure the validity of the constraint, the first buyer's shipment number is reduced to the largest integer that satisfies the constraint. For the rest of the buyers, their shipment numbers would still be the same or increased accordingly. Thus, the first buyer has to gain an extra holding cost due to the reduction of their shipment number. While the others, whose shipment numbers are increased, have to accept an increase on their transportation cost. However, all of these extra costs are mitigated by savings incurred on the transportation cost for the first buyer and the holding costs for the other buyers. On the other hand, for the former method, of restructuring the hierarchy, it would incur the same costs as above in addition to the cost of not having the shipment numbers as $n_1 \geq n_2 \geq \ldots \geq n_y$. Thus, to ensure minimum JTRC, it is necessary to find the n_i iteratively such that the hierarchy is maintained and the constraint is met.

4 Numerical illustration

In this section, the advantage of the new model is illustrated in comparison with the cost function without multiple shipments, the so-called base-cost function. The parameters used are tabulated in Table 2. Three different examples are given, where each example has a different number of buyers.

In example 1, only one buyer is considered: buyer A. The results are contained in Table 3. Dealing with one buyer, where the demand rate is less than the production rate, the second constraint given in Eq. 2 is certainly satisfied $\left(\frac{P}{n} > \frac{D}{n}\right)$. Furthermore, the results also confirm that the new model will be equal to the previous one-vendor one-buyer model when dealing with only one buyer. This implies that the new model is also applicable in the case of dealing with a single buyer. The percentage savings in the last column results from comparing the new model with the base-cost function.

In example 2, in addition to buyer A, buyer B is included. Since two buyers are considered, a position has to be allocated for each buyer in order to achieve the optimal solution. After the solution algorithm was applied, the results given in Table 4 were obtained.

If the demand rate for buyer B is increased to 25 000 unit/unit time as shown in Table 4, the value of n_1 and n_2 become invalid. Thus, both values have to be recalculated by using Eq. 6 and Eq. 7, whereas C is still the same because it is unrestricted by the constraint.

Finally, in example 3, all buyers (A, B and C) are considered. Similar to example 2, the problem was solved by the solution algorithm, as shown below. The results are summarised in Table 5.

As illustrated in examples 1, 2 and 3, the new model has a higher cycle time than the base-cost function. This means that each buyer will order a larger amount and without multiple shipments, it is obvious that their holding cost is increased. However, with the new model, the buyer's holding costs are significantly reduced and still give an advantage to the vendor of a cut to their setup cost with a longer production cycle time. By comparison, the new model has a considerably lower joint total cost than the base-cost function with a highest cost difference of \$13272.75 or a reduction of 33.95% in example 3.

Table 2. Input parameters

Buyer	A_{Ti} (\$)	H_{bi} (\$)	D_i (unit/unit time)	A_i (\$)
A	30	8	10 000	100
B	30	8	13 000	100
C	20	8	17 000	80
P (unit/unit time)	S (\$)	H_v (\$)		
55 000	200	4		

Table 3. The solution for example 1 by only considering buyer A

	C (unit time)	JTRC (\$)	Savings (%)
New method	0.14	6240.36	17.78
Base-cost function	0.09	7589.47	
One-vendor one-buyer	0.14	6240.36	17.78

Table 4. The solution for example 2 with two different demand rates for buyer B, which are 13 000 unit/unit time and 25 000 unit/unit time, respectively, for unconstrained and constrained scenarios.

	Unconstrained new model	Base-cost function	Constrained new model	Base-cost function
C [Unit time]	0.12	0.07	0.13	0.05
n_1 (unit)	5	1.00	9	1.00
n_2 (unit)	4	1.00	3	1.00
JTRC (\$)	10820.67	13684.98	13875.33	17495.97
Savings	20.93%		20.69%	

Table 5. The solution for example 3 with buyer C, B and A as buyer 1, 2 and 3, respectively

	New model	Base-cost function
C (unit time)	0.13	0.06
n_1 (unit)	8	1.00
n_2 (unit)	6	1.00
n_3 (unit)	4	1.00
JTRC (\$)	12570.06	20096.06
Saving	30.99%	

5 Sensitivity analysis

A sensitivity analysis was performed to test the robustness of the new model in facing uncertainties or changes in the given parameters, P, D, A_{Ti}, H_{bi} , A_i, S, and $H_{v,}$ on the incurred joint total cost savings. An analysis of the numerical examples in the previous section shows that having more buyers increases the incurred savings. This implies that the closer the total demand rate to the production rate, the greater the savings that can be obtained by implementing the new model. In other words, by gradually declining the ratio of the production rate and the total demand rate (k) the obtained percentage of cost savings is increased. In opposite, by inclining the value of k, the saving is decreasing. However it does not mean that the savings diminish to zero as k becomes significantly high, as indicated in Fig. 4.

Table 6 illustrates the effects on the joint total cost savings with a 50% increase and a decrease of the given parameters. If the parameters are decreased by 50%, the effect on the cost improvement has an opposite result than when they are increased by 50%. The explanation for such an impact (50% increase) on the joint total cost savings is given below.

1. Transportation cost (A_{Ti}): the number of deliveries is reduced due to the increase of the cost of multiple shipments, which reduces the benefit of having equal small-size shipments.
2. Buyer holding cost (H_{bi}): when the buyers' holding costs increase, the value of the inventory that is shifted to the vendor is increased. This will give more savings from shifting the inventory, as the vendor's holding cost is the same.
3. Ordering cost (A_i): as the ordering cost increases, the buyers tend to order in a large amount, and as a result, the number of shipments will also increase. Consistent with the increase of the number of shipments, buyers' inventory will decrease, and consequently, greater savings are obtained.
4. Vendor's setup cost (S): increasing the setup cost has the same consequences as an increase of A_i, where increasing the setup cost will increase the size of the joint economic lot size and thus force the system to have more frequent deliveries.
5. Vendor's holding cost (H_v): the impact of higher H_v contradicts the impact of a higher H_{bi}. By increasing the holding cost for the vendor, the value of the inventory that has been shifted increases, which reduces the incurred savings.

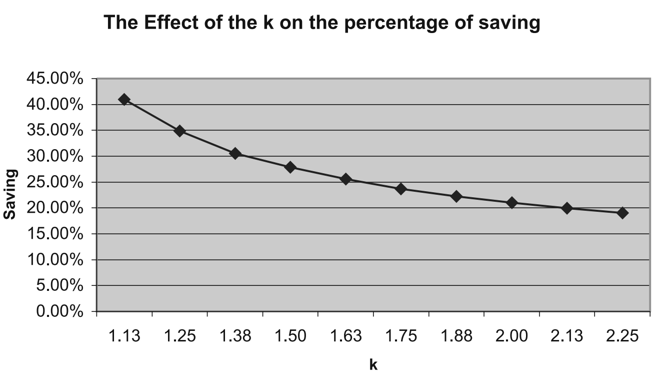

Fig. 4. Sensitivity of percentage of saving to the value of k, where k is the ratio of production rate and total demand rate

Table 6. The effect of parameter changes on the joint total cost savings

Parameter change	Obtained joint total cost savings (initial value = 33.94%) A_{Ti}	H_{bi}	A_i	S	H_v
+50%	29.67%	38.27%	38.65%	36.15%	29.18%
−50%	41.74%	25.78%	29.55%	30.99%	41.11%

6 Conclusion

The multiple-shipment policy for the joint economic lot size concept is proven to be beneficial as it reduces the joint total relevant cost. Further, it is shown that a multiple-shipment policy is more beneficial than a single-shipment policy considered by Banerjee [1]. The incurred savings are increased as the total demand rate approaches the production rate. This means that as long as the first assumption is still satisfied, the better the production rate is utilised, and the greater the savings will be. Moreover, since the increase or decrease of the input parameters by 50% only result in a change in savings of no more than 10%, this implies that the new model is robust to any uncertainties of the input parameters.

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Appendix

The area of the triangle

$$= \frac{\left(\sum_{i=1}^{y} D_i C\right)}{2} \cdot \frac{\left(\sum_{i=1}^{y} D_i C\right)}{P}$$
$$= \frac{\left(\sum_{i=1}^{y} D_i C\right)^2}{2P} \tag{9}$$

The area of the rectangle

$$= \sum_{i=1}^{y} D_i C \left(\left(\frac{C}{n_1}\right)(n_1 - 1) + \frac{C D_1}{n_1 P} - \frac{\sum_{i=1}^{y} D_i C}{P} \right)$$
$$= \frac{\sum_{i=1}^{y} D_i C^2}{P} \left(\frac{P}{n_1} \left(\frac{D_1}{P} + n_1 - 1 \right) - \sum_{i=1}^{y} D_i \right) \tag{10}$$

The area of the rectangles for each buyer
For buyer 1, the area is given by

$$\frac{C}{n_1} D_1 \left(\frac{C}{n_1}(n_1 - 1) + \frac{C}{n_1}(n_1 - 2) + \frac{C}{n_1}(n_1 - 3) + \ldots + \frac{C}{n_1} \right)$$
$$= \frac{C^2}{n_1^2} D_1 \left(\frac{n_1 (n_1 - 1)}{2} \right)$$

For buyer 2, the area is given by

$$\frac{C}{n_2} D_2 \left(\frac{C}{n_2}(n_2 - 1) + \frac{C}{n_2}(n_2 - 2) + \frac{C}{n_2}(n_2 - 3) + \ldots + \frac{C}{n_2} \right)$$
$$+ n_2 \left(\frac{C}{n_2} D_2 \left(\frac{C}{n_1}(n_1 - 1) - \frac{C}{n_2}(n_2 - 1) - \frac{D_2 C}{n_2 P} \right) \right)$$

For buyer 3, the area is given by

$$\frac{C}{n_3} D_3 \left(\frac{C}{n_3}(n_3 - 1) + \frac{C}{n_3}(n_3 - 2) + \frac{C}{n_3}(n_3 - 3) + \ldots + \frac{C}{n_3} \right)$$
$$+ n_3 \left(\frac{C}{n_3} D_3 \left(\frac{C}{n_1}(n_1 - 1) - \frac{C}{n_3}(n_3 - 1) - \frac{D_2 C}{n_2 P} - \frac{D_3 C}{n_3 P} \right) \right)$$

For buyer i, the area is given by

$$= \frac{C}{n_i} D_i \left(\frac{C}{n_i}(n_i - 1) + \frac{C}{n_i}(n_i - 2) + \frac{C}{n_i}(n_i - 3) + \ldots + \frac{C}{n_i} \right)$$
$$+ n_i \left(\frac{C}{n_i} D_i \left(\frac{C}{n_1}(n_1 - 1) - \frac{C}{n_i}(n_i - 1) + \frac{D_1 C}{n_1 P} - \sum_{j=1}^{i} \frac{D_j C}{n_j P} \right) \right)$$
$$= D_i C^2 \left(\frac{(n_i - 1)}{2n_i} + \frac{(n_1 - 1)}{n_1} - \frac{(n_i - 1)}{n_i} + \frac{D_1}{n_1 P} - \sum_{j=1}^{i} \frac{D_j}{n_j P} \right)$$
$$= \frac{D_i C^2}{2P} \left(\frac{P n_1 (n_i - 1) + 2P n_i (n_1 - 1) - 2P n_1 (n_i - 1) + 2 n_i D_1 - 2 n_1 n_i \sum_{j=1}^{i} \frac{D_j}{n_j}}{n_1 n_i} \right)$$
$$= \frac{D_i C^2}{2P} \left(\frac{2 n_i \left(P(n_1 - 1) + D_1 - n_1 \sum_{j=1}^{i} \frac{D_j}{n_j} \right) - P n_1 (n_i - 1)}{n_1 n_i} \right)$$

The sum of the rectangles for all buyers

$$= \frac{C^2}{2P} \left(\sum_{i=1}^{y} D_i \times \frac{2 n_i \left(P(n_1 - 1) + D_1 - n_1 \sum_{j=1}^{i} \frac{D_j}{n_j} \right) - P n_1 (n_i - 1)}{n_1 n_i} \right) \tag{11}$$

Time-weighted average inventory for the vendor

$$= (4) + (5) - (6)$$
$$= \frac{\left(\sum_{i=1}^{y} D_i C\right)^2}{2P} + \frac{\sum_{i=1}^{y} D_i C^2}{P} \left(\frac{P}{n_1} \left(\frac{D_1}{P} + n_1 - 1 \right) - \sum_{i=1}^{y} D_i \right)$$
$$- \frac{C^2}{2P} \left(\sum_{i=1}^{y} D_i \left(\frac{2 n_i \left(P(n_1 - 1) + D_1 - n_1 \sum_{j=1}^{i} \frac{D_j}{n_j} \right) - P n_1 (n_i - 1)}{n_1 n_i} \right) \right) \tag{12}$$

Thus, the vendor's average inventory is given by

$$(4)/C$$
$$= \left(\frac{\left(\sum_{i=1}^{y} D_i\right)^2 C^2}{2P} + \frac{\sum_{i=1}^{y} D_i C^2}{P} R_B - \frac{C^2}{2P} R_C \right) \frac{1}{C}$$
$$= \frac{C}{2P} \left(\left(\sum_{i=1}^{y} D_i \right)^2 + 2 \sum_{i=1}^{y} D_i R_B - R_C \right) \tag{1}$$

Buyer i's average inventory is given by

$$\frac{C D_i}{2 n_i} \tag{13}$$

Total setup cost for vendor and order and transportation costs for all buyers:

$$= \frac{S + \sum_{i=1}^{y} (A_i + n_i A_{Ti})}{C}$$

Thus, the joint total relevant cost is given by

$$JTRC = \frac{1}{C} \left(S + \sum_{i=1}^{y} (A_i + n_i A_{Ti}) \right)$$
$$+ \frac{C}{2} \left(\frac{H_V}{P} \left(\left(\sum_{i=1}^{y} D_i \right)^2 + 2 \sum_{i=1}^{y} D_i R_B - R_C \right) + \sum_{i=1}^{y} H_{bi} \frac{D_i}{n_i} \right) \tag{2}$$

$$C=\left(\frac{2\left(S+\sum_{i=1}^{y}\left(A_i+n_iA_{Ti}\right)\right)}{\left(\frac{H_V}{P}\left(\left(\sum_{i=1}^{y}D_i\right)^2+2\sum_{i=1}^{y}D_iR_B-R_C\right)+\sum_{i=1}^{y}H_{bi}\frac{D_i}{n_i}\right)}\right)^{1/2} \tag{4}$$

$$JTRC=\left(2\left(S+\sum_{i=1}^{y}\left(A_i+n_iA_{Ti}\right)\right)\right.$$
$$\left.\times\left(\frac{H_V}{P}\left(\left(\sum_{i=1}^{y}D_i\right)^2+2\sum_{i=1}^{y}D_iR_B-R_C\right)+\sum_{i=1}^{y}H_{bi}\frac{D_i}{n_i}\right)\right)^{1/2} \tag{14}$$