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ORIGINAL ARTICLE

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Process capability analysis of non-normal process data using the Burr XII distribution

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Abstract This paper proposes a novel modification of Clements's method using the Burr XII distribution to improve the accuracy of estimates of indices associated with one-sided specification limits for non-normal process data. This work proposes a novel Burr-based method, and compares it with Clements's method by simulation. Finally, an example application to semiconductor manufacturing is presented.

Keywords Burr-based method · Burr XII distribution · Clements's method · Non-normal distributions · Process capability index

1 Introduction

In recent decades, process capability analysis has been widely applied in the field of quality control to monitor the performance of industrial processes. The purpose of a process capability analysis is to estimate, monitor, and reduce the variability of industrial processes [1, 2]. Additionally, process capability analysis provides a common standard of product quality for suppliers and customers. The most popular way to assess process capability is to use histograms and process capability indices (PCIs). The most extensively used PCIs in semiconductor manufacturing are defined as follows:

$$C_p = \frac{USL - LSL}{6\sigma} \tag{1}$$

$$C_{pk} = \min(C_{pu}, C_{pl}) \tag{2}$$

where

$$C_{pu} = \frac{USL - \mu}{3\sigma} \tag{3}$$

$$C_{pl} = \frac{\mu - LSL}{3\sigma} \tag{4}$$

USL is the upper specification limit, LSL is the lower specification limit, μ is the process mean, and σ is the process standard deviation.

C_p and C_{pk} are used in cases of bilateral specifications; C_{pu} and C_{pl} are used in cases of unilateral specifications.

Such PCIs have been generally defined based on three basic assumptions. The first assumption is that the system determining which data are collected is under control. The second assumption is that the collected process data are independent and identically distributed. The third assumption is that the collected process data are normally distributed; that is, the process must be a normal process. In practice, industrial production involves many non-normal processes, especially in the semiconductor industry, so the use of PCIs based on an assumption of a normality assumption may yield misleading results.

Several generalizations and modifications of classical PCIs have been proposed to try to solve this problem and handle non-normal processes. The simplest way to treat non-normal data is to transform the data using mathematical functions into normally distributed data. Johnson [3] built a system of distributions based on the moment method, called the Johnson transformation system. Box and Cox [4] presented a useful family of power transformations. Somerville and Montgomery [5] used a square-root transformation to transform a skewed distribution into a normal one; however, their approach was based on data transformations, is difficult to implement using a standard method, and tends to be computationally intensive. Another conceptually simple way to treat non-normal data is to use non-normal percentiles to modify classical PCIs. Clements [6] proposed the method of non-normal percentiles to calculate C_p and C_{pk} indices for a distribution of any shape, using the Pearson family of curves. The main advantage is that no complicated distribution fitting is required. These modified indices are easily understood by non-statisticians

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and are the most commonly used in industry. Gruska et al. [7], and Kotz and Lovelace [8] constructed tables of standardized tails of Pearson curves as functions of kurtosis and skewness for skewness ranging from −2 to 2 and kurtosis ranging from −1.4 to 12.2. Based on skewness and kurtosis, standardized percentiles can be easily obtained and then actual percentiles can be estimated.

Pearn and Kotz [9] also applied Clements's method to construct the second-generation index C_{pm} and the third-generation C_{pmk} for non-normal data. These percentile-based indices, C_p, C_{pk}, C_{pm} and C_{pmk}, for non-normal data, are defined as follows:

$$C_{p(q)} = \frac{USL - LSL}{x_{99.865} - x_{0.135}} \tag{5}$$

$$C_{pu(q)} = \frac{USL - x_{50}}{x_{99.865} - x_{50}} \tag{6}$$

$$C_{pl(q)} = \frac{USL - x_{50}}{x_{99.865} - x_{50}} \tag{7}$$

$$C_{pk(q)} = \min\left(C_{pu(q)}, C_{pl(q)}\right) \tag{8}$$

$$C_{pm(q)} = \frac{\min(USL - T, T - LSL)}{3 * \sqrt{\left(\frac{x_{99.865} - x_{0.135}}{6}\right)^2 + (x_{50} - T)^2}} \tag{9}$$

$$C_{pmk(q)} = \min\left(\frac{USL - x_{50}}{3 * \sqrt{\left(\frac{x_{99.865} - x_{50}}{3}\right)^2 + (x_{50} - T)^2}}, \frac{x_{50} - LSL}{3 * \sqrt{\left(\frac{x_{50} - x_{0.135}}{3}\right)^2 + (x_{50} - T)^2}}\right) \tag{10}$$

where $x_p = p * 100$th percentile value of non-normal data, and T is the target value of the non-normal process.

Although Clements's method is the most commonly applied in industry today, Wu et al. [10] indicated that the Clements method cannot accurately measure the nominal values, especially when the underlying data distribution is skewed.

The goal of this paper is to develop a new method by modifying Clements's method using the Burr XII distribution to improve the accuracy of the estimation of indices of non-normal processes; hence, this paper proposes a novel method, which is compares with Clements's method by simulation. Finally, the method is applied to an example from semiconductor manufacturing, and conclusions are drawn and recommendations are made.

2 The Burr XII distribution

Clements's method is modified using the Burr system of distributions to provide better estimates of PCIs. Burr [11] proposed the cumulative distribution function of the Burr XII distribution:

$$\begin{aligned} F(x) &= 1 - (1 + x^c)^{-k}, \qquad \text{for } x \geqq 0 \\ &= 0, \ \text{for } x < 0 \end{aligned} \tag{11}$$

where $c > 0$, $k > 0$. The corresponding probability density function is

$$f(x) = kcx^{c-1}\left(1 + x^c\right)^{-(k+1)} \tag{12}$$

The reciprocal transformation is

$$\begin{aligned} G(y) &= \left(1 + y^{-c}\right)^{-k} \qquad \text{for } y \geqq 0 \\ &= 0, \qquad \text{for } y < 0 \end{aligned} \tag{13}$$

Burr [11] and Burr [12] tabulated the expected values, standard deviations, skewness coefficient and kurtosis coefficient of the Burr XII distribution for several combinations of c and k. These tables enable users to make a standardized transformation between a Burr variate (say, U) and another random variate (X). For a set of data, after the sample skewness and kurtosis coefficients have been estimated, the mean and standard deviation of the corresponding Burr distribution may be obtained using the tables. For instance, suppose a set of data is collected and the following sample statistics are calculated: the sample mean ($\bar{x}$) is 45.68; the sample standard deviation (s_x) is 3.51; the sample skewness coefficient (a_3) is 0.51, and sample kurtosis coefficient (a_4) is 3.87. From Table I in Burr [12], this set of data may be approximately described as a Burr XII distribution with $c = 4.2$, $k = 2.8$, $\mu = .7527$, and $\sigma = .2386$. Let μ and σ be the mean and standard deviation, respectively, of a Burr random variate. Then, the standardized transformation between a Burr variate (U) and the random variate (X) may be expressed as

$$\left(X - \bar{x}\right)/s_x = (U - \mu)/\sigma$$

$$\text{so } (X - 45.68)/3.51 = (U - 0.7527)/0.2386$$

$$X = 34.607 + 14.711U$$

Zimmer and Burr [13] found that a wide range of the skewness and kurtosis coefficients of various probability density functions can be covered by different combinations of c and k. Such probability density functions include most known functions, including the normal, Gamma, Beta, Weibull, logistic, log-logistic, log-normal, extreme value type I distribution, and other functions. For instance, the normal density function may be approximated as a Burr XII distribution with $c = 4.85437$ and $k = 6.22665$ and the Gamma distribution with shape parameter 16 can be approximated as a Burr XII distribution with $c = 3$ and $k = 6$, and the log-logistic distribution is a special case of the Burr XII distribution. Rodriguez [14] demonstrated that the Weibull distribution is a limiting distribution of the Burr XII distribution. Hence, the two-parameter Burr XII distribution can be used to describe the data in the real world. The Burr XII distribution has been applied in areas of quality control, reliability analysis, and failure time modeling. Zimmer and Burr [13] developed a method for sampling variables from non-normal populations using the Burr XII distribution. Burr [15] used his distribution to investigate the effect of non-normality on the constants of the X-bar and R control chart. Castagliola [16] derived

Burr's approach to compute the proportion of nonconforming items and then transformed this proportion into $\hat{C}_{p(q)}$. Chou and Cheng [17] extended Yourstone and Zimmer's model [18] to determine the control limits of the R control chart under non-normality. Chou et al. [19] applied the Burr XII distribution to generate an economic-statistical design of the X-bar chart for non- normally distributed data. For more information on the Burr XII distribution, please refer Wang et al. [20], Zimmer et al. [21], and Ali Mousa and Jaheen [22].

3 Process capability analysis using Burr XII distribution

Although Clements's method is the most widely used in industry today, it has one major drawback as described above. Clements's method can be modified by replacing the Pearson family of probability curves with a Burr XII distribution to improve the accuracy of the estimates of the indices for non-normal process data. Two reasons justify the use of the Burr XII distribution. The first is that the two-parameter Burr XII distribution can be used to describe data that arise in the real world, and especially those concerning non-normal processes, as described before. The second reason is that the direct use of a fitted cumulative function instead of a probability density function (like members of the Pearson family of functions) may avoid the need for a numerical or formal integration.

The procedure of the process capability analysis using the Burr XII distribution is presented as follows:

Step 1. Estimate the mean ($\bar{x}$), standard deviation (s), skewness (a_3), and kurtosis (a_4) from the process data.

Step 2. Select the parameter (c) and (k) based on estimates of skewness and kurtosis, using the Burr XII distribution table (see the appendix).

Step 3. With reference to parameters (c) and (k) obtained in step (2), use the table of standardized tails of the Burr XII distribution to determine $Z^*_{0.00135}$, $Z^*_{0.50}$, and $Z^*_{0.99865}$, where $Z^*_p =$ adjusted standardized normal variate of the process data (see the table in the appendix).

Step 4. Estimate percentiles $X_{0.00135}$, $X_{0.50}$, and $X_{0.99865}$

$$X_{0.00135} = \bar{x} + Z^*_{0.00135} \times s$$
$$X_{0.50} = \bar{x} + Z^*_{0.50} \times s$$
$$X_{0.99865} = \bar{x} + Z^*_{0.99865} \times s$$

Step 5. Estimate process capability indices, using Eqs. 5–10

4 Simulation study

The novel method is compared with Clements's method. A representative PCI for non-normal data should be compatible with that computed under normality, given the same fraction of non-conforming parts. Therefore, It is agreeable to use C_{pu}, a unilateral specification capability index, as the comparison criterion in the simulation study. In the case of normal distribution, the targeted C_{pu} value can be determined using

$$\text{Fraction of non-conforming parts} = \Phi(-3C_{pu}) \qquad (14)$$

For example, $C_{pu} = 1$, 1.5, and 2 imply non-conforming fractions of 1350 ppm, 3.4 ppm, and .001 ppm, respectively, where $1\,ppm = 1 \times 10^{-6}$ (or non-conforming parts per million). In the simulation, targeted values of $C_{pu(q)} = 1$, 1.5, and 2 were used, and corresponding USL values of underlying distributions with the same fraction of non-conforming parts are obtained. These USL values are then used to estimate the $C_{pu(q)}$ index of the simulated data using the two methods. Finally, these estimated $C_{pu(q)}$ values are compared with the target $C_{pu(q)}$ value.

A better method involves a sample mean of the estimated $C_{pu(q)}$ with a smaller deviation from the target value (greater accuracy) and with a smaller sample standard deviation (greater precision).

Beta, Gamma, and Weibull distributions are considered in the investigation of the effect of non-normal data on the PCIs in the simulation. They have parameters that can represent slight, moderate, and severe departures from normality. These distributions are known to involve significantly different tail behaviors, which may strongly influence the process capability.

A series of simulations were implemented with sample sizes of $n = 50$, 100, and 500, and with $C_{pu(q)} = 1$, 1.5, and 2 using Beta, Gamma, and Weibull distributions. Results calculated using standard PCI expressions on the assumption of normality are also included for reference. Each run was replicated 30 times to yield the average of $30\hat{C}_{pu(q)}$ values, $\bar{C}_{pu(q)}$.

Briefly, the simulation procedure is as follows:

1. Choose an underlined distribution (such as a Gamma (shape = 4, scale = 0.5))
2. Choose a targeted $C_{pu(q)}$ value and determine the corresponding USL based on the underlined distribution (such as $C_{pu(q)} = 1$, USL $= Q_{.99865} = 54.35$ for the Gamma distribution (shape = 4, scale = 0.5))
3. Use a random number generator to generate n = 50, 100, or 500 data points, following the underlined distribution, and use an $\bar{X} - R$ control chart to ensure that the simulated data are under control.
4. Calculate the sample mean, standard deviation, skewness and kurtosis.
5. Based on the sample skewness and kurtosis, determine the standardized percentiles of Pearson's curves and the standardized percentiles of Burr's distribution.
6. Estimate separate percentiles of Pearson's curves and the Burr XII distribution.
7. Estimate separately the $\hat{C}_{pu(q)}$ values of Pearson's curves and the Burr XII distribution.
8. Repeat steps 3 to 7 until they have been performed 30 times
9. Separately calculate the mean $\bar{C}_{pu(q)}$ and standard deviation of 30 $\hat{C}_{pu(q)}$ values for Pearson's curves and the Burr XII distribution.

Table 1. Non-normal distributions used in the simulation study

Distribution	Skewness
Beta (shape1 = 4.4, shape2 = 13.3)	0.506
Gamma (shape = 4, scale = 0.5)	1
Weibull (shape = 1.2, scale = 1)	1.521
Gamma (shape = 1, scale = 1)	2

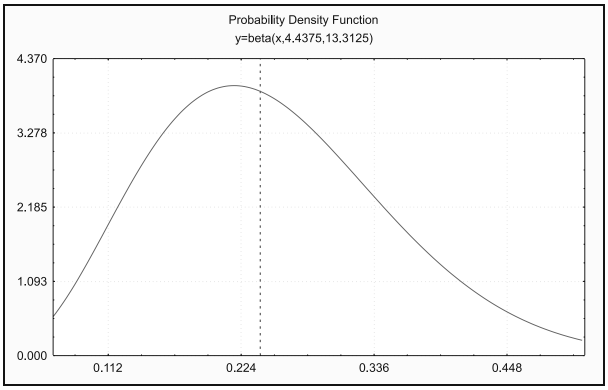

Fig. 1. PDF of a Beta (shape1 = 4.4, shape2 = 13.3) distribution (Note: the dashed line points to the location of the median)

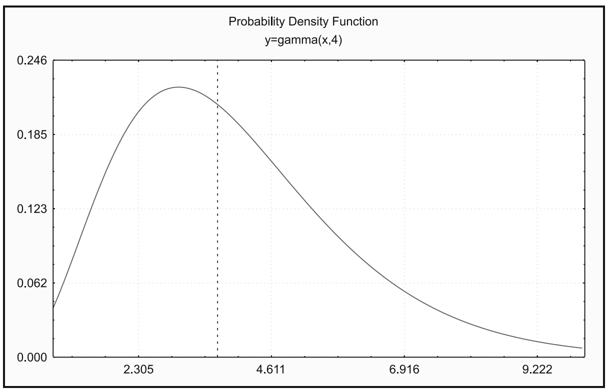

Fig. 2. PDF of a Gamma (shape = 4, scale = 1) distribution (Note: the dashed line points to the location of the median)

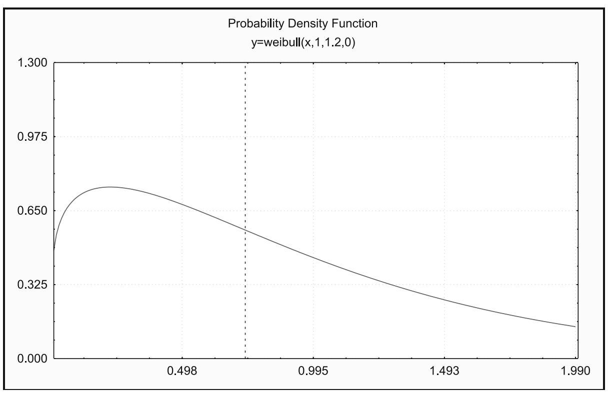

Fig. 3. PDF of a Weibull (shape = 1.2, scale = 1) distribution (Note: the dashed line points to the location of the median)

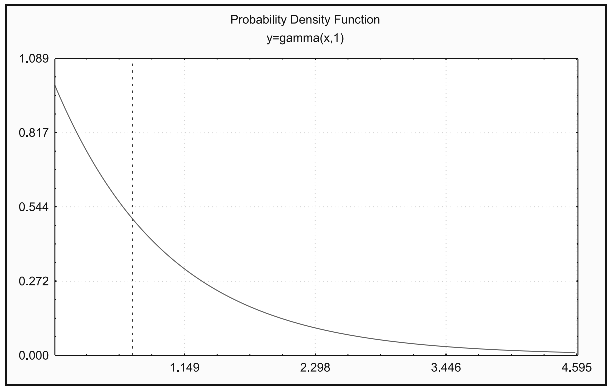

Fig. 4. PDF of a Gamma (shape = 1, scale = 1) distribution (Note: the dashed line points to the location of the median)

The results of the simulation are as follows:

Table 1 presents the non-normal distributions used in the simulation. Figures 1 to 4 present the PDFs of these non-normal distributions. Clearly, the skewnesses vary considerably among these figures.

Table 2 presents $C_{pu(q)}$ values and USLs of the non-normal distributions used in the simulation. Tables 3 to 6 summarize the simulation results obtained for non-normal distributions using the three methods. Table 3 refers to the Beta (4.4,13.3) distribution; Table 4 refers to the Gamma (4,0.5) distribution; Table 5 refers to the Weibull (1.2, 1) distribution, and Table 6 refers to the Gamma (1, 1) distribution.

To save space, Figs. 5 to 10 present only some of the simulation results obtained by applying the three methods to non-normal distributions. Figure 5 presents box plots of $\hat{C}_{pu(q)}$ obtained by applying the three methods to the Beta (4.4, 13.3) distribution with target $C_{pu(q)} = 2$. Figure 6 presents corresponding plots for the Gamma (4, 0.5) distribution; Fig. 7 presents corresponding plots for the Weibull (1.2,1) distribution and Fig. 8 presents corresponding plots for the Gamma (1,1) dis-

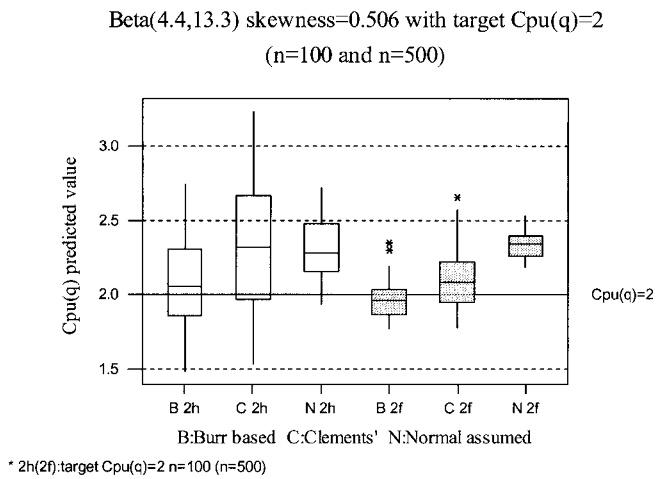

Fig. 5. Box plots of $\hat{C}_{pu(q)}$ for three methods for Beta (4.4, 13.3) with target $C_{pu}(q) = 2$

Table 2. $C_{pu(q)}$ values and USLs for non-normal distributions used in the simulation study

$C_{pu(q)} = (USL - x_{.50}) \,/\, (x_{.99865} - x_{.50})$
$USL = C^*_{pu(q)}\,(x_{.99865} - x_{.50}) + x_{.50}$

Beta (shape1 = 4.4, shape2 = 13.3)	$X_{.99865}$	$X_{.50}$	$C_{pu(q)}$	1	1.5	2
	0.5954	0.2405	USL	0.5954	0.7729	0.9504
Gamma (shape = 4, scale = 0.5)	$X_{.99865}$	$X_{.50}$	$C_{pu(q)}$	1	1.5	2
	6.3405	1.836	USL	6.3405	8.5927	10.845
Weibull (shape = 1.2, scale = 1)	$X_{.99865}$	$X_{.50}$	$C_{pu(q)}$	1	1.5	2
	4.8236	0.7368	USL	4.8236	6.867	8.9104
Gamma (shape = 1, scale = 1)	$X_{.99865}$	$X_{.50}$	$C_{pu(q)}$	1	1.5	2
	6.6078	0.6931	USL	6.6078	9.5651	12.522

Table 3. The mean $\bar{C}_{pu(q)}$ and standard deviation of 30 $\hat{C}_{pu(q)}$ values of Burr-based, Clements's, and normal-assumed methods of each C_{pu} value for the Beta simulation

Beta (shape1 = 4.4375, shape2 = 13.3125) skewness = 0.506												
$C_{pu(q)}$	1.00	Bur	Clm	Nor	1.50	Bur	Clm	Nor	2.00	Bur	Clm	Nor
mean	$n = 50$	0.99	1.07	1.14	$n = 50$	1.49	1.60	1.72	$n = 50$	1.98	2.14	2.31
std		0.17	0.22	0.14		0.25	0.33	0.19		0.32	0.43	0.25
mean	$n = 100$	1.04	1.16	1.14	$n = 100$	1.57	1.74	1.73	$n = 100$	2.09	2.32	2.32
std		0.15	0.21	0.11		0.23	0.32	0.16		0.31	0.43	0.20
mean	$n = 500$	0.98	1.05	1.15	$n = 500$	1.48	1.58	1.75	$n = 500$	1.97	2.10	2.34
std		0.07	0.10	0.05		0.10	0.15	0.08		0.14	0.20	0.10

Table 4. The mean $\bar{C}_{pu(q)}$ and standard deviation of 30 $\hat{C}_{pu(q)}$ values of Burr-based, Clements's, and normal-assumed methods for the Gamma (4,0.5) simulation

Gamma (shape = 4, scale = 0.5) skewness = 1												
$C_{pu(q)}$	1.00	Bur	Clm	Nor	1.50	Bur	Clm	Nor	2.00	Bur	Clm	Nor
mean	$n = 50$	1.22	1.30	1.52	$n = 50$	1.83	1.94	2.31	$n = 50$	2.44	2.59	3.09
std		0.20	0.25	0.20		0.31	0.38	0.28		0.42	0.51	0.37
mean	$n = 100$	1.18	1.23	1.51	$n = 100$	1.77	1.85	2.29	$n = 100$	2.36	2.46	3.07
std		0.23	0.28	0.19		0.34	0.42	0.28		0.46	0.56	0.37
mean	$n = 500$	1.01	1.01	1.46	$n = 500$	1.52	1.52	2.21	$n = 500$	2.03	2.03	2.96
std		0.06	0.07	0.06		0.10	0.10	0.08		0.13	0.13	0.11

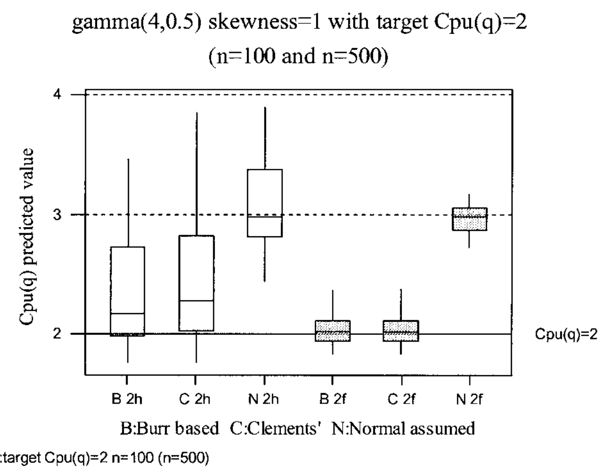

Fig. 6. Box plots of $\hat{C}_{pu(q)}$ for three methods for Gamma (4,0.5) with target $C_{pu}(q) = 2$

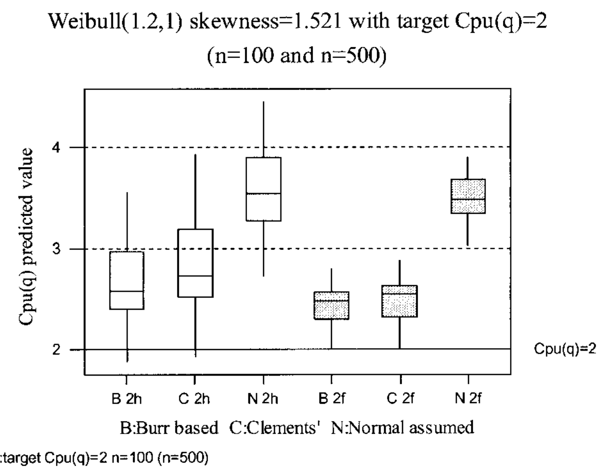

Fig. 7. Box plots of $\hat{C}_{pu(q)}$ for three methods for Weibull (1.2,1) with target $C_{pu}(q) = 2$

Table 5. The mean $\bar{C}_{pu(q)}$ and standard deviation of 30 $\hat{C}_{pu(q)}$ values of Burr-based, Clements's, and normal-assumed methods for the Weibull simulation

Weibull (shape = 1.2, scale = 1) skewness = 1.521												
$C_{pu(q)}$	1.00	Bur	Clm	Nor	1.50	Bur	Clm	Nor	2.00	Bur	Clm	Nor
mean	$n = 50$	1.33	1.40	1.71	$n = 50$	1.98	2.10	2.61	$n = 50$	2.63	2.79	3.51
std		0.39	0.47	0.38		0.59	0.69	0.55		0.78	0.92	0.72
mean	$n = 100$	1.34	1.42	1.72	$n = 100$	2.00	2.13	2.63	$n = 100$	2.66	2.83	3.53
std		0.21	0.25	0.23		0.32	0.38	0.34		0.42	0.51	0.45
mean	$n = 500$	1.22	1.24	1.70	$n = 500$	1.81	1.86	2.59	$n = 500$	2.41	2.48	3.48
std		0.10	0.12	0.11		0.15	0.18	0.17		0.21	0.24	0.22

Table 6. The mean $\bar{C}_{pu(q)}$ and standard deviation of 30 $\hat{C}_{pu(q)}$ values of Burr-based, Clements's, and normal-assumed methods for the Gamma (1,1) simulation

Gamma (shape = 1, scale = 1) skewness = 2												
$C_{pu(q)}$	1.00	Bur	Clm	Nor	1.50	Bur	Clm	Nor	2.00	Bur	Clm	Nor
mean	$n = 50$	1.59	1.67	2.11	$n = 50$	2.38	2.51	3.22	$n = 50$	3.16	3.35	4.33
std		0.41	0.49	0.41		0.63	0.74	0.60		0.84	0.98	0.79
mean	$n = 100$	1.43	1.48	1.98	$n = 100$	2.13	2.23	3.03	$n = 100$	2.84	2.97	4.07
std		0.20	0.24	0.24		0.30	0.35	0.35		0.40	0.47	0.46
mean	$n = 500$	1.39	1.43	2.02	$n = 500$	2.08	2.14	3.08	$n = 500$	2.76	2.85	4.15
std		0.12	0.13	0.16		0.18	0.19	0.24		0.24	0.26	0.31

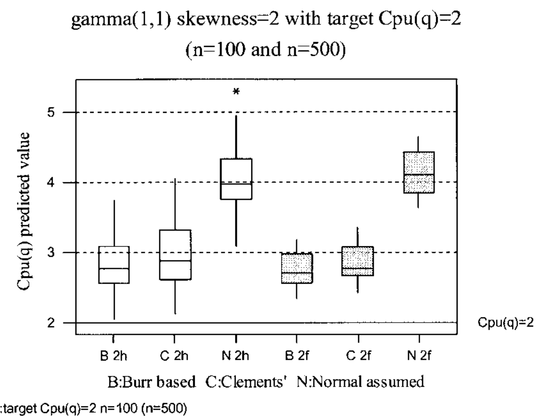

Fig. 8. Box plots of $\hat{C}_{pu(q)}$ for three methods for Gamma (1,1) with target $C_{pu}(q) = 2$

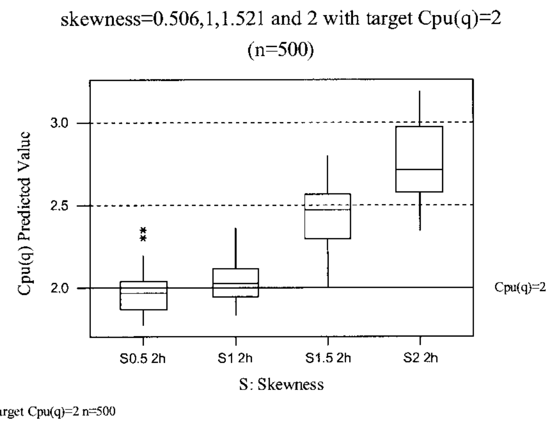

Fig. 10. Box plots of $\hat{C}_{pu(q)}$ for the Burr-based method with target $C_{pu}(q) = 2, n = 500$

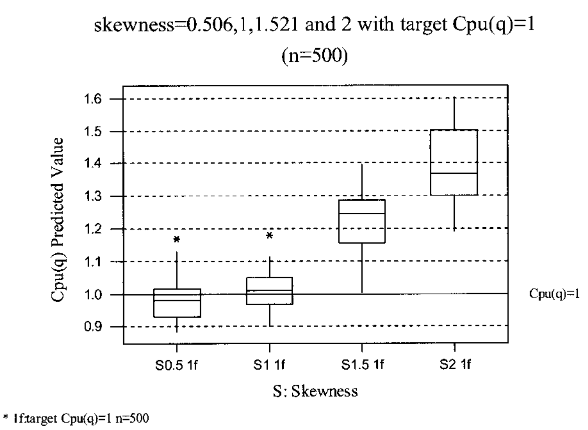

Fig. 9. Box plots of $\hat{C}_{pu(q)}$ for the Burr-based method with target $C_{pu}(q) = 1, n = 500$

tribution. Figures 9 and 10 present box plots of $\hat{C}_{pu(q)}$ for the Burr-based method with $n = 500$ and target $C_{pu(q)} = 1$ and 2, respectively.

5 Discussions

The simulation study yielded the following results:

1. The Burr-based method is the best of the three methods.
   The Burr-based method is the one for which the sample mean of the estimated $C_{pu(q)}$ deviates least from the target value (so it is the most accurate) and for which the sample standard deviation of the estimated $C_{pu(q)}$ varies least (so it is the most precise). The Burr-based method is slightly better than Clements's method. The method that assumes normal-

ity is the worst of the three methods because of its inaccurate estimates in these non-normal distributions.
2. A larger sample size n yields better estimates.
3. A larger target value of $C_{pu(q)}$ corresponds to slightly worse estimates
4. The Burr-based method is effective for slightly or moderately non-normal distributions, but it yields overestimates in cases of highly skewed distributions (skewness $\geqq 1.5$).

6 Example of the application of the proposed method in the semiconductor industry

This section presents the results obtained by applying the described method for estimating non-normal PCIs to some experimental data. Data were collected during a normal photolithographic process using a stepper at a semiconductor manufacturer in Taiwan [23]. The data are measurements of the deviation of exposure in the x-direction and the y-direction. The data can be treated as absolute deviations of exposure in the x and y directions with $\text{USL} = 0.05$. Figures 11 and 12 show histograms of the data obtained after abnormal data were screened out using the $\bar{X} - R$ control chart to ensure that the data are under control. The histograms appear to show that the underlying distributions of the two sets of data were not normal. Moreover, the data do not follow any specific distribution, according to goodness-of-fit testing, the results of which are presented in Table 7. The results show that the process capability of index $\hat{C}_{pu(q)} < 1.25$, so that the photolithographic process should be improved. The estimates made using the Burr-based method, which are lower than those made using Clements's method, can help engineers be more attentive to process improvement.

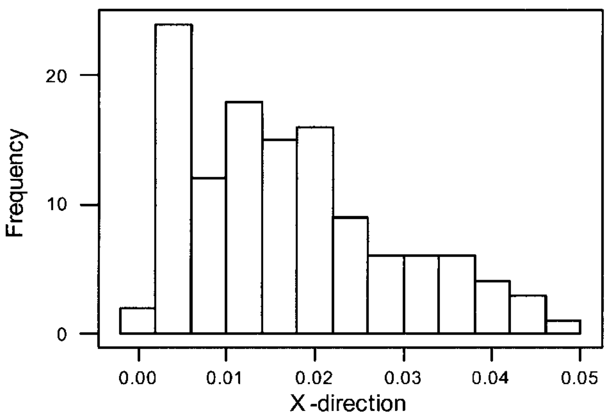

Fig. 11. Histogram of absolute deviations of exposure in the x direction with $USL = 0.05$

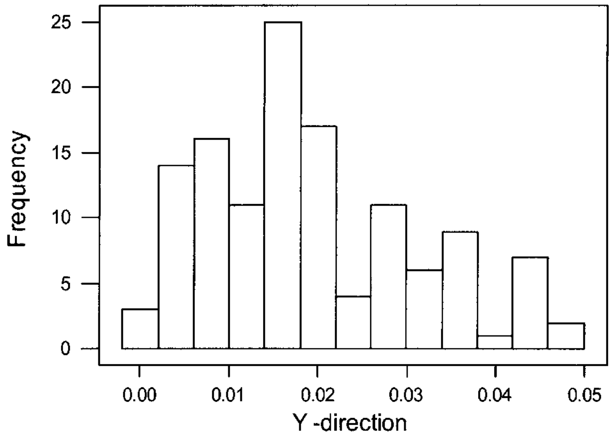

Fig. 12. Histogram of absolute deviations of exposure in the y direction with $USL = 0.05$

7 Conclusions

This article proposes a novel modification of Clements's method using the Burr XII distribution to improve the accuracy of estimates of indices associated with non-normal process data. This work proposes the novel Burr-based method, and compares it with Clements's method by simulation. Finally, an example application to semiconductor manufacturing is presented.

The following conclusions are drawn and recommendations are made:

1. This article proposes a Burr-based method for estimating the $C_{pu(q)}$ capability index, and demonstrates that it is slightly better than Clements's method. The Burr table is also made easier to use.
2. The Burr-based method works well under distributions that depart slightly or moderately from normality.
3. Clements's method and the novel Burr-based method overestimate the $C_{pu(q)}$ in cases of highly skewed distributions (skewness $\geqq 1.5$).
4. The estimates made using the Burr-based method, which are lower than those made using Clements's method, are good indicators to help engineers be more attentive to and focus on process improvement.

Table 7. The results of process analysis of the data of absolute deviations of exposure in the x- and y-directions with $USL = 0.05$

Data	Skewness	Kurtosis	Method	$\hat{C}_{pu(q)}$	K-S test for goodness-of-fit
x-direction	0.7	2.6	Burr-based	0.884	Not significant at $\alpha = 0.05$
			Clements's	0.996	Not significant at $\alpha = 0.05$
y-direction	0.6	2.5	Burr-based	0.823	Not significant at $\alpha = 0.05$
			Clements's	0.931	Not significant at $\alpha = 0.05$

The Burr XII distribution is appropriate for describing data that originate in the real world. It is computed easily because the cumulative density function of Burr XII is an algebraic form. This article strongly recommends the further investigation and application of the Burr-based method in process capability analysis and quality control.

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Appendix

Tables of the standardized percentiles of Pearson's curves (Clements's) and Burr's distribution.

Note: 1. given combinations of a_3 and a_4, values of c, k, μ, and σ can be determined for the Burr XII distribution

2. a_3 and a_4 are standardized skewness and kurtosis, respectively. ($a_3 = 0$, $a_4 = 3$) is for normal distribution.

3. $a_3 = ((n-2)/\sqrt{n(n-1)}) * \text{skew}$, $a_4 = (n-2)(n-3)/((n+1)(n-1)) * \text{kurt} + 3 * (n-1)/(n+1)$ where

$$\text{skew} = \frac{n}{(n-1)(n-2)} \cdot \sum_j \left[\frac{x_j - \bar{x}}{s}\right]^3$$

$$\text{kurt} = \left\{\frac{n(n+1)}{(n-1)(n-2)(n-3)} \cdot \sum_j \left[\frac{x_j - \bar{x}}{s}\right]^4\right\} - \frac{3(n-1)^2}{(n-2)(n-3)}$$

$\bar{x}$ and s are sample mean and sample standard deviation, respectively.

4. To save space, we present only the values for $a_3 = 0$, 0.5, 1, 1.5, and 2. For all of the details, please contact the authors or please refer to Kotz and Lovelace [8] and Burr [12]

Skewness	Kurtosis	Clements's			Burr						
a_3	a_4	$ZP_{.00135}$	$ZP_{.5}$	$ZP_{.99865}$	$BZP_{.00135}$	$BZP_{.5}$	$BZP_{.99865}$	c	k	μ	σ
0	2	−1.966	0.000	1.966	−1.843	0.022	2.396	−18.148445	0.062932	0.538666	0.290634
0	2.2	−2.210	0.000	2.210	−1.959	0.037	2.697	−13.840637	0.093482	0.574597	0.290269
0	2.4	−2.442	0.000	2.442	−2.076	0.047	2.911	−12.134081	0.120321	0.608776	0.287975
0	2.6	−2.663	0.000	2.663	−2.197	0.053	3.078	−11.251863	0.146295	0.641872	0.283920
0	2.8	−2.839	0.000	2.839	−2.735	0.008	2.914	3.938938	19.864823	0.427479	0.124319
0	3	−3.000	0.000	3.000	−2.884	0.010	3.081	4.873717	6.157568	0.644717	0.161990
0	3.2	−3.140	0.000	3.140	−3.020	0.011	3.221	6.065153	3.745010	0.767265	0.164426
0	3.4	−3.261	0.000	3.261	−3.148	0.011	3.340	7.695948	2.700685	0.850740	0.151944
0	3.6	−3.366	0.000	3.366	−3.269	0.011	3.442	10.182078	2.089559	0.911057	0.129983
0	3.8	−3.458	0.000	3.458	−3.388	0.009	3.529	14.723762	1.664480	0.955196	0.099905
0	4	−3.539	0.000	3.539	−3.509	0.015	3.609	27.068908	1.325754	0.985829	0.059999
0	4.2	−3.611	0.000	3.611	−3.642	0.001	3.659	−195.260000	0.959315	0.999691	0.009430

Skewness	Kurtosis	Clements's			Burr						
a_3	a_4	$ZP_{.00135}$	$ZP_{.5}$	$ZP_{.99865}$	$BZP_{.00135}$	$BZP_{.5}$	$BZP_{.99865}$	c	k	μ	σ
0.5	2	−1.232	−0.212	2.131	−1.225	−0.213	2.258	−46.009112	0.012681	0.368920	0.301069
0.5	2.2	−1.377	−0.169	2.422	−1.292	−0.173	2.829	−14.222488	0.046148	0.401469	0.310714
0.5	2.4	−1.539	−0.142	2.708	−1.357	−0.141	3.121	−11.000293	0.066322	0.431431	0.317882
0.5	2.6	−1.711	−0.122	2.968	−1.421	−0.115	3.325	−9.617790	0.083673	0.459691	0.323217
0.5	2.8	−1.887	−0.108	3.194	−1.487	−0.093	3.483	−8.837537	0.099951	0.486834	0.327087
0.5	3	−2.059	−0.097	3.387	−1.554	−0.076	3.612	−8.339490	0.115918	0.513287	0.329712
0.5	3.2	−2.220	−0.089	3.550	−2.085	−0.082	3.595	2.537779	12.52349	0.335292	0.147712
0.5	3.4	−2.368	−0.082	3.688	−2.202	−0.074	3.731	2.943293	6.113841	0.501498	0.201715
0.5	3.6	−2.502	−0.077	3.805	−2.313	−0.067	3.843	3.395555	4.142901	0.620824	0.227764
0.5	3.8	−2.622	−0.072	3.905	−2.419	−0.061	3.939	3.917524	3.155912	0.713383	0.237814
0.5	4	−2.727	−0.068	3.991	−2.523	−0.057	4.023	4.548291	2.540631	0.788396	0.236949
0.5	4.2	−2.817	−0.065	4.066	−2.627	−0.053	4.099	5.360672	2.100986	0.851246	0.227386
0.5	4.4	−2.893	−0.062	4.131	−2.736	−0.050	4.169	6.513761	1.751518	0.905414	0.209324
0.5	4.6	−2.957	−0.060	4.189	−2.856	−0.048	4.237	8.473843	1.440495	0.953420	0.180041
0.5	4.8	−3.011	−0.057	4.239	−3.018	−0.049	4.301	14.457708	1.084855	0.998320	0.122136
0.5	5	−3.055	−0.055	4.285	−3.034	−0.042	4.365	−13.177200	0.8078	0.983214	0.146900
1.0	2.8	−0.919	−0.413	2.608	−0.914	−0.418	2.739	−37.592244	0.009476	0.263067	0.287757
1.0	3.0	−1.000	−0.347	2.914	−0.952	−0.381	3.264	−13.068026	0.030050	0.285705	0.300232
1.0	3.2	−1.086	−0.299	3.206	−0.987	−0.350	3.535	−10.102815	0.042342	0.306405	0.310543
1.0	3.4	−1.178	−0.263	3.468	−1.020	−0.321	3.722	−8.794494	0.052544	0.325718	0.319287
1.0	3.6	−1.277	−0.235	3.693	−1.053	−0.296	3.864	−8.035677	0.061736	0.344004	0.326831
1.0	3.8	−1.381	−0.213	3.883	−1.084	−0.273	3.978	−7.534128	0.070362	0.361514	0.333420
1.0	4.0	−1.491	−0.196	4.043	−1.116	−0.252	4.072	−7.176003	0.078658	0.378436	0.339226
1.0	4.2	−1.602	−0.181	4.177	−1.522	−0.180	4.191	1.600580	75.150186	0.060742	0.039208
1.0	4.4	−1.713	−0.169	4.290	−1.598	−0.168	4.300	1.781200	13.522563	0.213171	0.129853
1.0	4.6	−1.821	−0.159	4.386	−1.671	−0.157	4.390	1.964535	7.780951	0.328516	0.189332
1.0	4.8	−1.925	−0.151	4.468	−1.740	−0.148	4.465	2.152385	5.59623	0.424414	0.231862
1.0	5.0	−2.023	−0.143	4.539	−1.808	−0.140	4.528	2.347094	4.428629	0.506046	0.262382
1.0	5.2	−2.116	−0.137	4.600	−1.873	−0.133	4.583	2.551826	3.689945	0.576758	0.283956
1.0	5.4	−2.202	−0.131	4.653	−1.936	−0.126	4.631	2.770828	3.170981	0.639034	0.298637
1.0	5.6	−2.283	−0.126	4.700	−2.000	−0.120	4.675	3.009958	2.778372	0.694794	0.307793
1.0	5.8	−2.358	−0.122	4.741	−2.075	−0.127	4.803	3.227685	2.463769	0.745571	0.312291
1.0	6.0	−2.427	−0.118	4.777	−2.129	−0.110	4.753	3.587147	2.199033	0.792649	0.312572
1.0	6.2	−2.491	−0.114	4.810	−2.197	−0.105	4.791	3.960662	1.965622	0.837209	0.308651
1.0	6.4	−2.549	−0.111	4.839	−2.272	−0.101	4.829	4.442055	1.748775	0.880562	0.299899
1.0	6.6	−2.602	−0.108	4.865	−2.358	−0.097	4.871	5.139853	1.531357	0.924740	0.284223
1.0	6.8	−2.651	−0.105	4.888	−2.478	−0.093	4.923	6.548242	1.265997	0.976403	0.252039
1.5	4.2	−0.721	−0.520	3.172	−0.719	−0.526	3.322	−28.492765	0.008116	0.188273	0.262918
1.5	4.4	−0.768	−0.474	3.463	−0.738	−0.511	3.749	−12.725454	0.019623	0.202338	0.274262
1.5	4.6	−0.815	−0.432	3.736	−0.758	−0.495	3.985	−9.947745	0.026856	0.215249	0.284055
1.5	4.8	−0.863	−0.396	3.979	−0.777	−0.479	4.149	−8.662713	0.032764	0.227275	0.292677
1.5	5	−0.913	−0.365	4.189	−0.794	−0.464	4.273	−7.899136	0.037968	0.238597	0.300383
1.5	5.2	−0.964	−0.338	4.369	−0.811	−0.450	4.371	−7.385558	0.042725	0.249344	0.307348
1.5	5.4	−1.018	−0.315	4.521	−0.828	−0.435	4.452	−7.013213	0.047172	0.259615	0.313703
1.5	5.6	−1.073	−0.295	4.649	−0.843	−0.422	4.520	−6.729262	0.051392	0.269485	0.319544
1.5	5.8	−1.131	−0.278	4.758	−0.859	−0.408	4.579	−6.504675	0.055441	0.279014	0.324948
1.5	6	−1.191	−0.263	4.850	−0.874	−0.396	4.629	−6.322081	0.059357	0.288250	0.329972
1.5	6.2	−1.253	−0.250	4.929	−1.215	−0.252	4.925	1.241925	74.22419	0.029369	0.024055
1.5	6.4	−1.317	−0.239	4.996	−1.257	−0.242	4.988	1.332837	19.55014	0.102225	0.08071
1.5	6.6	−1.381	−0.228	5.055	−1.298	−0.232	5.041	1.422317	11.68978	0.170200	0.129784
1.5	6.8	−1.446	−0.219	5.106	−1.338	−0.223	5.085	1.510611	8.52946	0.233508	0.172259
1.5	7	−1.510	−0.211	5.150	−1.376	−0.215	5.123	1.598009	6.815789	0.291987	0.208693
1.5	7.2	−1.574	−0.204	5.189	−1.413	−0.208	5.155	1.684839	5.734788	0.345803	0.239765
1.5	7.4	−1.636	−0.197	5.223	−1.449	−0.202	5.182	1.771479	4.986255	0.395319	0.266187
1.5	7.6	−1.697	−0.191	5.253	−1.483	−0.196	5.207	1.858343	4.433791	0.440973	0.288617
1.5	7.8	−1.756	−0.186	5.280	−1.517	−0.190	5.228	1.945891	4.006426	0.483218	0.307632
1.5	8	−1.813	−0.181	5.303	−1.551	−0.185	5.247	2.034632	3.663584	0.522485	0.323722
1.5	8.2	−1.867	−0.176	5.325	−1.583	−0.180	5.265	2.125136	3.38037	0.559172	0.337296
1.5	8.4	−1.920	−0.172	5.344	−1.615	−0.176	5.280	2.218055	3.140639	0.593641	0.348687
1.5	8.6	−1.970	−0.168	5.361	−1.647	−0.172	5.295	2.314143	2.933435	0.626215	0.358163
1.5	8.8	−2.019	−0.164	5.376	−1.679	−0.167	5.309	2.414297	2.751024	0.657189	0.365939
1.5	9	−2.065	−0.161	5.390	−1.712	−0.164	5.323	2.519614	2.587749	0.686833	0.372178
2.0	6.2	−0.590	−0.525	3.813	−0.589	−0.529	3.979	−21.416286	0.007433	0.137895	0.236343
2.0	6.4	−0.618	−0.513	4.072	−0.596	−0.529	4.291	−12.465414	0.013546	0.146252	0.245201
2.0	6.6	−0.646	−0.495	4.311	−0.609	−0.528	4.482	−10.020598	0.017762	0.154009	0.253089
2.0	6.8	−0.674	−0.475	4.524	−0.620	−0.526	4.619	−8.791087	0.021232	0.161277	0.260206
2.0	7.0	−0.702	−0.455	4.712	−0.630	−0.523	4.724	−8.031145	0.024275	0.168139	0.266695

Skewness	Kurtosis	Clements's			Burr						
a_3	a_4	$ZP_{.00135}$	$ZP_{.5}$	$ZP_{.99865}$	$BZP_{.00135}$	$BZP_{.5}$	$BZP_{.99865}$	c	k	μ	σ
2.0	7.2	−0.730	−0.435	4.873	−0.641	−0.520	4.808	−7.507666	0.027031	0.174654	0.272662
2.0	7.4	−0.758	−0.416	5.012	−0.650	−0.516	4.877	−7.121838	0.029578	0.180870	0.278185
2.0	7.6	−0.786	−0.399	5.131	−0.659	−0.512	4.935	−6.823937	0.031965	0.186825	0.283327
2.0	7.8	−0.815	−0.382	5.233	−0.668	−0.508	4.984	−6.585976	0.034223	0.192548	0.288139
2.0	8.0	−0.844	−0.367	5.320	−0.677	−0.503	5.027	−6.390891	0.036377	0.198066	0.292660
2.0	8.2	−0.874	−0.353	5.395	−0.685	−0.499	5.064	−6.227638	0.038442	0.203400	0.296924
2.0	8.4	−0.904	−0.340	5.460	−0.693	−0.494	5.098	−6.088733	0.040431	0.208567	0.300959
2.0	8.6	−0.935	−0.328	5.516	−0.701	−0.489	5.127	−5.968907	0.042356	0.213582	0.304789
2.0	8.8	−0.966	−0.317	5.565	−0.708	−0.484	5.154	−5.864337	0.044223	0.218459	0.308432