ORIGINAL ARTICLE

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A study of abrasive waterjet cutting of alumina ceramics with controlled nozzle oscillation

Abstract This paper presents and discusses an experimental investigation of abrasive waterjet (AWJ) cutting of alumina ceramics with controlled nozzle oscillation. Particular attention is paid to the effect of small oscillation angles on the various cutting performance measures. It is found that nozzle oscillation at small angles can equally improve the major cutting performance measures, if the cutting parameters are correctly selected. However, under high water pressures, high nozzle traverse speeds and large oscillation frequencies, nozzle oscillation may cause a decrease in some major cutting performance measures, such as surface finish. Plausible trends of cutting performance with respect to the process parameters are further considered. Finally, a predictive mathematical model for the depth of cut is developed and verified.

Keywords Abrasive waterjet cutting · Cutting performance · Modelling · Nozzle oscillation

Nomenclature

D	average particle diameter (mm)
d_j	nozzle diameter (mm)
E	modulus of elasticity for work material (GPa)
F	oscillation frequency (Hz)
h	depth of cut (mm)
H	standoff distance (mm)
K, a, b, c	constants
K_e	efficiency factor
m	average mass of a particle (g)
$\dot{m}$	abrasive mass flow rate ($\mathrm{g\,s^{-1}}$)
P	water pressure (MPa)
R	average volume of material removed by a particle ($\mathrm{mm^3}$)
T_R	kerf taper
u	nozzle traverse speed ($\mathrm{mm\,s^{-1}}$)
v	particle velocity ($\mathrm{mm\,s^{-1}}$)
w	average kerf width (mm)
w_t	top kerf width (mm)
w_m	minimum kerf width (mm)
α	particle attack angle (degrees)
θ	oscillation angle (degrees)
ρ_w	water density ($\mathrm{g\,mm^{-3}}$)

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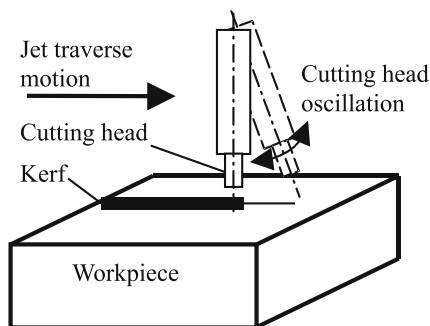Sydney, NSW 2052, Australia

1 Introduction

Abrasive waterjet (AWJ) machining is a powerful tool in cutting various materials, particularly difficult-to-cut materials. However, the cutting capacity of this technology in terms of the depth of cut and kerf quality is the major obstructions that limit its application. In the last decade, a great deal of research effort has sought to develop new techniques to enhance the cutting performance and cutting capacity of AWJ technology. Some newly developed techniques include cutting by the forward angling of the jet in the cutting plane, multipass cutting and controlled nozzle oscillation [1−3]. Among these, controlled nozzle oscillation or cutting head oscillation has been found to be the most effective way in improving the cutting performance without additional costs to the machining process. With this cutting technique, a pendulum-like, forward and backward motion of the nozzle in the cutting plane at predetermined frequency and angular amplitude is superimposed to the normal nozzle traverse motion, as shown in Fig. 1. The idea of controlled nozzle oscillation was introduced by Veltrup [4] and was then developed as the most effective cutting technique by many other researchers [5, 6].

When using the nozzle oscillation technique, oscillation angle and oscillation frequency are two of the major cutting parameters. It has been found that the nozzle oscillation cutting technique can significantly improve some major cutting performance measures such as the depth of cut and kerf surface

Fig. 1. Schematic of controlled nozzle oscillation

performance measures and under what conditions nozzle oscillation can have adverse effects on the cutting performance. Regression analysis and analysis of variance (ANOVA) are performed to comparatively study the data acquired under the normal way of cutting (cutting without nozzle oscillation at a 90° jet impact angle) and nozzle oscillation cutting, to assess the importance of each variable to the cutting performance, and to identify the best combinations of cutting conditions for the optimum cutting performance. Finally, a mathematical model for the depth of cut or depth of jet penetration is developed using a dimensional analysis technique, and verified with the experimental data.

roughness. It has been reported [1, 7] that the depth of the upper smooth zone in nozzle oscillation cutting can be increased by more than 30% compared to that without oscillation, and similarly, kerf surface finish (as measured by the centre line average R_a) can be improved by as much as 30% [6].

It appears that all the reported studies primarily discuss the use of large nozzle oscillation angles (or angular amplitudes) of ten degrees or more. Nozzle oscillation in the cutting plane (in a tangential direction to the curved profile in contouring) with such large oscillation angles results in theoretical geometric errors on the component profile in contouring and, therefore, is not preferred in practice. As a result, it is necessary to investigate if nozzle oscillation at small angles can be employed to enhance the cutting performance. In addition, it has been noticed in our early experiments that if the oscillation parameters were not correctly selected for a given combination of the other process variables, nozzle oscillation could have an adverse effect on the cutting performance. However, there has been no adequate study to thoroughly understand this phenomenon, and to examine how the cutting parameters affect cutting performance in nozzle oscillation cutting with small oscillation angles. Thus, there is a need to suggest strategies for selecting the correct or optimum oscillation parameters. Furthermore, there has been no reported study on the development of predictive mathematical models for the cutting performance in AWJ machining with this novel cutting technique. Such models are essential for the optimum selection of operating parameters in process planning.

In this study, an experimental investigation is first undertaken to examine the effect of cutting parameters on the major cutting performance measures in AWJ cutting of an 87% alumina ceramic with nozzle oscillation at small angles. Particular attention is paid to how the oscillation parameters affect the major cutting

2 Experimental procedure

2.1 Work material and AWJ cutting machine

In this experiment, the specimens used were 87% alumina ceramic plates with a thickness of 12.7 mm, to represent brittle materials. The abrasive waterjet cutting system employed was the Flow International Waterjet Cutter driven by a "Model 20X" dual intensifier pumping system, with an operating pressure of up to 380 MPa. The motion of the nozzle is numerically-controlled by a computer and a five-axis robot positioning system. The other basic components in this system consisted of an "M-263" abrasive delivery system, a "Paser II" abrasive jet cutting head, an "ASI CNC controller", a water catcher tank, and a remote terminal to program the machine.

2.2 Experimental design

In AWJ cutting, a large number of variables have an impact on the cutting performance [1, 8]. To simplify the analysis, four major variables in normal AWJ cutting, as identified in earlier studies [1, 3, 9], and two oscillation Variables, were chosen for investigation. These six variables include water pressure, nozzle traverse speed, abrasive mass flow rate, standoff distance between nozzle and workpiece surface, nozzle oscillation angle, and oscillation frequency. Their levels and corresponding values are shown in Table 1. The other parameters that were kept constant during the tests included the nominal jet impact angle (90°), orifice diameter (0.33 mm), mixing tube or nozzle diameter (1.02 mm), and abrasive material (80 mesh garnet sand).

A Taguchi experimental design array [10] was used to construct the cutting tests. Three groups of tests were used in the experimental design. The first group was the four-level, six-factor

Table 1. Experimental design

Process variables	Level 1	Level 2	Level 3	Level 4
Abrasive mass flow rate $\dot{m}$ (gs^{-1})	6.8	9.1	11.3	13.6
Standoff distance H (mm)	2	3	4	5
Water pressure P (MPa)	275	310	345	380
Nozzle traverse speed u (mms^{-1})	0.67	1.00	1.33	1.67
Oscillation angle θ (degrees)	2	4	6	8
Oscillation frequency F (Hz)	2	6	10	14

design scheme in Taguchi orthogonal arrays, with all of the six variables selected to study the influence of oscillation cutting on the cutting performance. This required 64 experimental runs (L_{64}). For comparatively studying the difference of cutting performance between oscillation cutting and normal cutting, another four-level, four-factor design scheme (L_{16}) was used, including four cutting variables. These were water pressure, nozzle traverse speed, standoff distance and abrasive mass flow rate. This resulted in 16 more runs. Furthermore, in order to facilitate the analysis using the as-measured experimental data, a third group was designed using some typical cutting conditions. This group of design included 30 tests. Thus, a total of 110 runs were undertaken in this four-level, six-factor experiment.

2.3 Data acquisition

Measurements with the assistance of metrological instruments were conducted for the major cutting performance measures, including the depth of cut, top kerf width, bottom kerf width (for through cuts only), minimum kerf width (at which kerf taper is evaluated), kerf taper, and surface roughness. Of these quantities, kerf taper was calculated using the top kerf width, the minimum kerf width, and the depth where the minimum kerf width was measured. The other five quantities were directly measured from each cut. At least three measures for each quantity on each cut were made and the average was taken as the final reading.

The top kerf width, bottom kerf width, minimum kerf width and depth of cut were measured from the end view of the kerf profile by using a "SigmaScope 500" profile projector prior to separating the specimens. Kerf surface roughness was measured after the specimens were separated to expose the kerf walls. Surface roughness, as assessed by the centre-line average R_a, was taken using a "Surtronic 3+" stylus surface profile meter. A sample length of 12.5 mm was chosen with a cut-off length of 2.5 mm for all specimens. R_a values were measured at three different locations along the cut walls:. 1 mm, 3 mm, and 5 mm from the top kerf edge, although only the surface roughness at 1 mm from the top kerf edge will be analysed here. The analysis for the major cutting performance measures is presented below.

3 Results and discussion

3.1 Surface roughness

It was noted from the measured surface roughness data that in the majority of the cases where nozzle oscillation was used, the surface is rougher than when cutting without nozzle oscillation under the corresponding cutting conditions. The average R_a value for the 94 cuts with nozzle oscillation is 6.43 μm, while that for the 16 cuts under the normal way of cutting is 5.23 μm. In general, it was found that when high water pressures and high nozzle traverse speeds were used, oscillation cutting increased surface roughness irrespective of the oscillation frequencies or angles used. The increased R_a values in oscillation cutting may

be attributed to the jet instability and system vibration caused by cutting head oscillation.

It is interesting to note that under some conditions, nozzle oscillation can improve the surface finish compared to the normal way of cutting, under the corresponding conditions. It appears that the proper combinations of cutting parameters are crucial to surface roughness. The study has found that when water pressure is high, low nozzle traverse speeds with small oscillation angles and medium to high oscillation frequencies can improve surface finish. Furthermore, if the nozzle traverse speed is high, water pressure is selected to be at a small- to medium-level, and small oscillation angles and high oscillation frequencies are used, better surface finish can be achieved. Figure 2 shows two sample surfaces produced under normal and oscillation cutting. The improvement in surface finish by using nozzle oscillation is apparent if the cutting parameters are correctly selected. When the proper combinations are used, the improvement in surface finish by oscillation cutting, as compared to the normal way of cutting, is more pronounced at high nozzle traverse speeds than at low speeds. Similar to the normal way of AWJ cutting, oscillation cutting at low traverse speed was found to reduce surface roughness.

The results of ANOVA suggest that among the six independent variables tested, water pressure and oscillation frequency have the most influence on surface roughness, while abrasive mass flow rate and oscillation angle have the least influence. The small effect of oscillation angle may be because of the small range and increment used in the study. The ANOVA of the oscillation cutting data has found that the optimum cutting conditions are: oscillation frequency at 2 Hz; oscillation angle at 2°; nozzle traverse speed at 1 mm s^{-1}; water pressure at 275 MPa; standoff distance at 5 mm; and abrasive mass flow rate at 6.8 g s^{-1}. Using this set of optimum cutting parameters can produce a surface roughness R_a of 2.05 μm.

A similar analysis has found that the optimum conditions for cutting without nozzle oscillation is a nozzle traverse speed at 1 mms^{-1}, a water pressure of 275 MPa, a standoff distance at 2 mm, and an abrasive mass flow rate of 11.3 g s^{-1}. The resulting surface roughness R_a is 2.29 μm. This analysis demonstrates that cutting with nozzle oscillation under an optimized combination of cutting parameters can improve surface finish and use less abrasive particles than cutting without nozzle oscillation. However, if the combination of cutting conditions is incorrectly
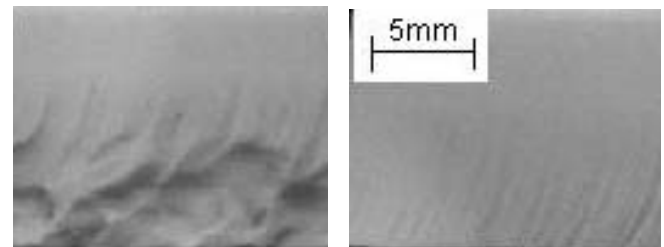

Fig. 2. Samples of surfaces produced under normal cutting (left) and oscillation cutting (right): $\dot{m} = 9.1$ g s^{-1}, $H = 4$ mm, $P = 380$ MPa, $u = 0.67$ mm s^{-1}; for oscillation cutting: $\theta = 4°$, $F = 14$ Hz

selected, cutting with nozzle oscillation can increase the surface roughness. This finding is somewhat different from the previous investigations [5–7], in which the use of nozzle oscillation technique is shown to constantly improve the cutting performance.

The effect of oscillation frequency on surface roughness is shown in Fig. 3. It can be seen from Fig. 3a that surface roughness increases with oscillation frequency monotonically and linearly at a standoff distance of 2 mm. However, standoff distance appears to affect the slope of the linear relationship between oscillation frequency and surface roughness, as shown in Fig. 3b. Here, an increase in the standoff distance results in a decrease in the slope. A standoff distance of 5 mm is associated with a negative slope in the linear relationship (i.e. surface roughness decreases slightly with the oscillation frequency). This phenomenon can be attributed to the fact that at high standoff distances, the waterjet scanning scope is widened under the same oscillation angle. This leads to an increased overlapping cutting action on the cutting front, and hence reduces the surface roughness.

Figure 4 shows the effect of oscillation angle on surface roughness. Here, it can be seen that initially, surface roughness increases slightly with an increase in oscillation angle, and reaches a maximum turning point. As the oscillation angle further increases, surface roughness starts to decrease. This may be a result of the scanning action of the jet on the cutting front. There appears to be an optimum scanning scope corresponding to a set of cutting parameters, similar to the above discussion about the effect of standoff distance. As the oscillation angle increases in the lower region, the jet scanning action cannot effectively cut off the "peaks" left on the cut surface, thus causing jet turbulence or instability and system vibration that increase the surface roughness. Larger oscillation angles increase the overlap cutting action and the number of scanning actions on a given part of surface, so that the scanning action is dominant and thus reduces the surface roughness.

3.2 Depth of cut

The depth of cut for all the tests with nozzle oscillation shows an average increase of 27.7% with respect to the cutting without nozzle oscillation under the corresponding cutting conditions. In some extreme cases, nozzle oscillation cutting increased the depth of cut by as much as 82%. This benefit is obvious in Fig. 5. Nevertheless, there are still cases where the depth of cut in oscillation cutting is less than that of the corresponding instance
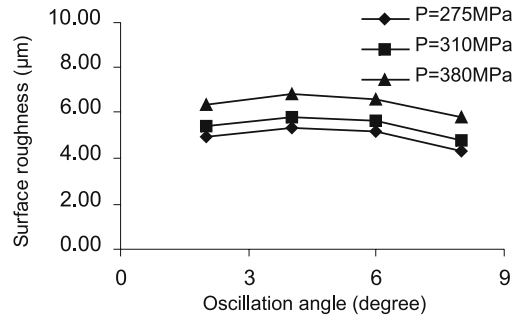

Fig. 4. Effect of oscillation angle on surface roughness ($F = 6\,\mathrm{Hz}$, $u = 1\,\mathrm{mm\,s^{-1}}$, $H = 2\,\mathrm{mm}$, $\dot{m} = 9.1\,\mathrm{gs^{-1}}$)
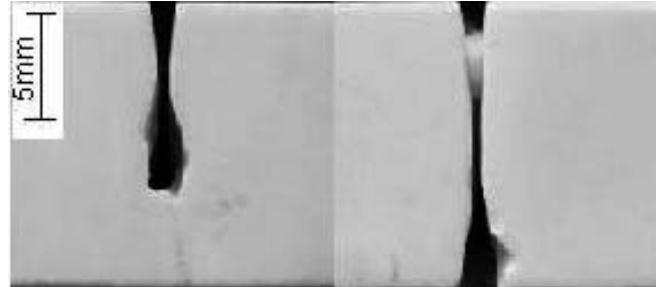

Fig. 5. Samples of kerf profiles produced under normal cutting (left) and oscillation cutting (right): $\dot{m} = 9.1\,\mathrm{gs^{-1}}$, $H = 2\,\mathrm{mm}$, $P = 310\,\mathrm{MPa}$, $u = 1.33\,\mathrm{mm\,s^{-1}}$; for oscillation cutting: $\theta = 8°$, $F = 10\,\mathrm{Hz}$

of normal cutting. This may be different from the typical expectation that oscillation techniques can constantly increase this cutting quantity. The analysis shows that whether or not nozzle oscillation can increase the depth of cut is dependent on both the oscillation angle and the oscillation frequency. Dependent on the other operating parameters, there appears to be a critical or threshold value of the product of oscillation frequency and oscillation angle above which nozzle oscillation can increase the depth of cut, as comparing to normal cutting.

This critical value is related to the other process parameters and can be determined from the analysis of experimental data. For instance, when the water pressure is 380 MPa, nozzle traverse speed is 1.33 mm s^{-1}, standoff distance is 3 mm, and abrasive mass flow rate is 11.3 gs^{-1}, the critical value is found to be approximately 8 deg · s^{-1}.

An analysis of the experimental data has found that the most dominant cutting parameters for depth of cut are oscillation

Fig. 3a,b. Effect of frequency on surface roughness: **a** $\theta = 4°$, $u = 1\,\mathrm{mm\,s^{-1}}$, $H = 2\,\mathrm{mm}$, $\dot{m} = 9.1\,\mathrm{gs^{-1}}$; **b** $\theta = 4°$, $P = 345\,\mathrm{MPa}$, $u = 1\,\mathrm{mm\,s^{-1}}$, $\dot{m} = 9.1\,\mathrm{gs^{-1}}$

(a)

(b)
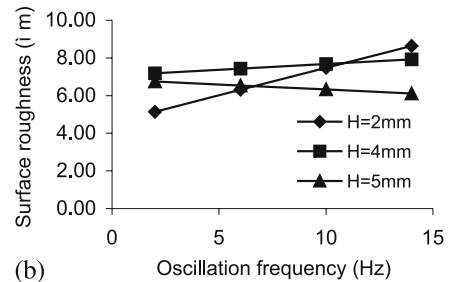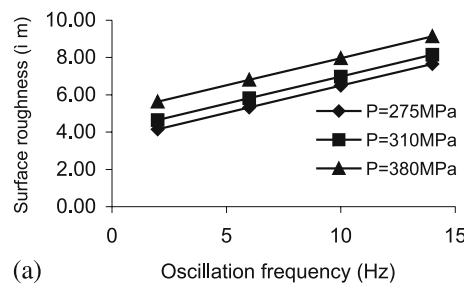
frequency, nozzle traverse speed, and water pressure. Standoff distance and oscillation angle have little effect. The result of ANOVA suggests that when an oscillation frequency of 14 Hz, an oscillation angle of 6°, a water pressure of 380 MPa, a traverse speed of 0.67 mm s^{-1}, a standoff distance of 3 mm, and an abrasive flow rate of 11.3 gs^{-1} are used, oscillation cutting can produce the maximum depth of cut of 16.3 mm. However, with the normal way of cutting, the optimum combination of the cutting parameters was found to be a water pressure 345 MPa, a nozzle traverse speed of 0.67 mm s^{-1}, a standoff distance of 2 mm, and an abrasive mass flow rate of 11.3 gs^{-1}. This yields a maximum depth of cut of 13.3 mm. Thus, statistically, nozzle oscillation cutting can increase the depth of cut by 23% with respect to the normal cutting technique under the respective optimum combinations of cutting parameters, and under the same cutting rate and abrasive mass flow rate.

The relationship between oscillation frequency and the depth of cut is plotted in Fig. 6. Figure 6a shows that the depth of cut increases approximately linearly with the oscillation frequency. It is believed that high oscillation frequency increases the number of repeated scanning actions and reduces the particle interference, which in turn increases the overall abrasive cutting capacity and increases the depth of cut. As discussed earlier, under some conditions, an increase in oscillation frequency may result in a slight decrease in the depth of cut, as shown in Fig. 6b. This figure also shows that the slope of the linear relationship between oscillation frequency and the depth of cut is affected by the nozzle traverse speed. A higher nozzle traverse speed is associated with a more rapid increase of the depth of cut, as the oscillation frequency increases.

When a low traverse speed of 0.67 mm s^{-1} is used, an increase in oscillation frequency also results in a decrease in the depth of cut. This may be explained that at low traverse speed, the jet scanning action cannot take effect and may cause increased particle interference and a reduction in the jet cutting capability.

Figure 7 shows the effect of oscillation angle on the depth of cut from the experimental data. This effect is also dependent on the oscillation frequency. At relatively large oscillation frequencies, such as 10 Hz and 14 Hz, an increase in the oscillation angle is associated with a steady decrease of the depth of cut, while the decreasing rate slightly increases with the oscillation angle. This may be due to the fact that at high oscillation frequencies, an increase in oscillation angles increases the jet instability and particle interference. This decreases the jet cutting capability and hence decreased this cutting performance measure.

By contrast, at low oscillation frequencies such as 2 Hz, as shown in Fig. 7a, an increase in oscillation angle results in a slight increase in the depth of cut, while the increasing rate decreases with the oscillation angle. As the oscillation angle further increased to beyond 6°, the depth of cut exhibits a decreasing trend. A maximum turning point for the depth of cut occurs at oscillation angles of about 4° to 6°, when small oscillation frequencies are used.

For the other oscillation frequencies in the medium range of the tested conditions, the depth of cut appears to be independent of the oscillation angles, with only a very slight decrease as this cutting variable increases, as shown in Fig. 7b at an oscillation frequency of 6 Hz. To this end, large oscillation frequencies with small oscillation angles are preferred to increase the depth of cut. Figure 7b also shows that standoff distance has an effect on the depth of cut. While an increase in the standoff distance may reduce the particle energy on the target material at the particle attack point, it may take an effect together with the oscillation angle and frequency. With a larger standoff distance, the jet scanning scope on the cutting front is increased, which may increase or decrease the depth of cut depending on the other parameters used, in a similar way to the oscillation angle.

Fig. 6a,b. Effect of oscillation frequency on the depth of cut: **a** $\theta = 4°$, $u = 1$ mm s^{-1}, $H = 2$ mm, $\dot{m} = 9.1$ gs^{-1}; **b** $\theta = 4°$, $P = 310$ MPa, $H = 2$ mm, $\dot{m} = 9.1$ gs^{-1}

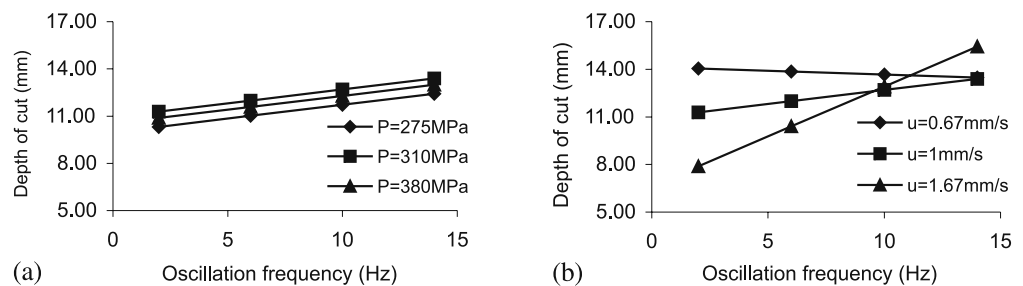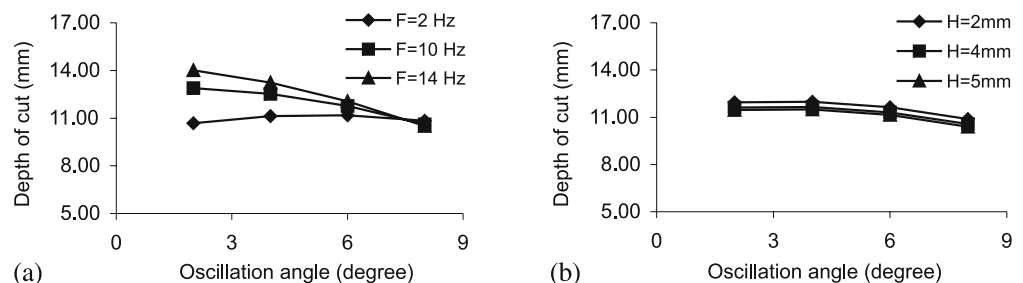**Fig. 7a,b.** Effect of oscillation angle on the depth of cut: **a** $P = 310$ MPa, $u = 1$ mm s^{-1}, $H = 3$ mm, cot $m = 9.1$ gs^{-1}; **b** $F = 6$ Hz, $P = 310$ MPa, $u = 1$ mm s^{-1}, $\dot{m} = 9.1$ gs^{-1}

3.3 Kerf taper

The value of kerf taper (T_R) for each cut was obtained from the other measured quantities, such as the top kerf width (W_t), the minimum kerf width along the kerf profile (W_m), and the depth (t) at which the minimum kerf width was measured, based on the following equation:

$$T_R = \frac{(W_t - W_m)}{2t} \tag{1}$$

The mean values of kerf taper for all the oscillation cutting and normal cutting tests were found to be 0.094 and 0.118, respectively. Based on these average kerf taper values, oscillation cutting reduced taper angle by 18.1%, compared to normal cutting under similar conditions. ANOVA analysis of the experimental data suggested that oscillation frequency accounted for most of the kerf taper formation, while nozzle traverse speed, water pressure, and standoff distance assumed the second most important role for kerf taper formation in oscillation cutting. By contrast, abrasive mass flow rate and oscillation angle had the least effect on kerf taper. The optimum combination for the smallest kerf taper has been found to be an oscillation frequency of 14 Hz, an oscillation angle of 8°, a nozzle traverse speed of 0.67 mm s^{-1}, a water pressure of 380 MPa, a standoff distance of 2 mm, and an abrasive mass flow rate of 11.3 gs^{-1}. This optimum combination can produce a kerf taper of 0.028. Similarly, the analysis of the experimental data has found that, statistically, the minimum kerf taper for the normal way of AWJ cutting is 0.061 under the optimum combination of cutting parameters (where. water pressure is at 380 MPa, nozzle traverse speed is at 0.67 mm s^{-1}, standoff distance is at 2 mm, and abrasive mass flow rate is at 11.3 gs^{-1}). Thus, under the optimum parameter combinations, cutting with nozzle oscillation can reduce the kerf taper by 54% compared to that attained with normal cutting under the same cutting rate and the same usage of abrasives.

It has been found that oscillation frequency and oscillation angle have a similar effect on kerf taper. Since oscillation frequency has little effect on kerf width but a significant effect on the jet penetration depth, an increase in oscillation frequency is associated with a reduced kerf taper, as calculated by Eq. 1. This is shown in Fig. 8. This trend is consistent for cutting under various conditions. The effect of oscillation frequency on the kerf taper becomes interesting when traverse speed is varied, as shown in Fig. 8b. Plotting kerf taper with respect to oscillation frequency, slope becomes steeper as the traverse speed increases. A similar trend is observed in the effect of oscillation angle on the kerf taper.

4 Mathematical model for depth of cut

4.1 Modelling conditions and assumptions

To model the depth of cut in AWJ cutting, one must consider a host of variables which complicate the modelling process. In addition, a number of phenomena, such as the particle interference and fragmentation, exist in abrasive jet processing. At this stage of development, there is no sufficient knowledge of these phenomena [1, 11]. Therefore, to consider all these variables and phenomena is either impossible or results in many unknown parameters in the final equation, thus making the model unrealistic for practical use. In this study, a dimensional analysis technique is used, where necessary, to establish the mathematical relationship between this cutting performance measure and the process variables, while regression analysis of the experimental data is undertaken to determine the constants in the model.

With dimensional analysis, all variables which appear to be a problem can be assembled into a smaller number of independent dimensionless products of the same dimensions. The relationships between the individual variables can be determined by algebraic expressions relating each π_i [12, 13]. To simplify the model development process, some assumptions need to be made:

- Abrasive particles are distributed uniformly over the jet cross-sectional area;
- Abrasive particles have the same velocity as the surrounding water in the jet; the jet velocity variation along the jet stream is ignored; and
- Jet side spreading is ignored; thus, kerf width (w) is approximately equal to jet diameter (d_j).

The steps involved in developing the model include:

1. Deriving the expression for the overall material removal rate in terms of the material removed by individual abrasive particles and the number of particles in the jet for the time span;
2. Formulating the relationship between the material removed by an individual particle and the other influencing variables; and
3. Estimating the constants in the equation by the multiple regression analysis.

Fig. 8a,b. Effect of frequency on taper: **a** $\theta = 4°$, $u = 1$ mm s^{-1}, $H = 3$ mm, $\dot{m} = 9.1$ gs^{-1} **b** $\theta = 4°$, $P = 310$ MPa, $H = 2$ mm, $\dot{m} = 9.1$ gs^{-1}

(a)

(b)
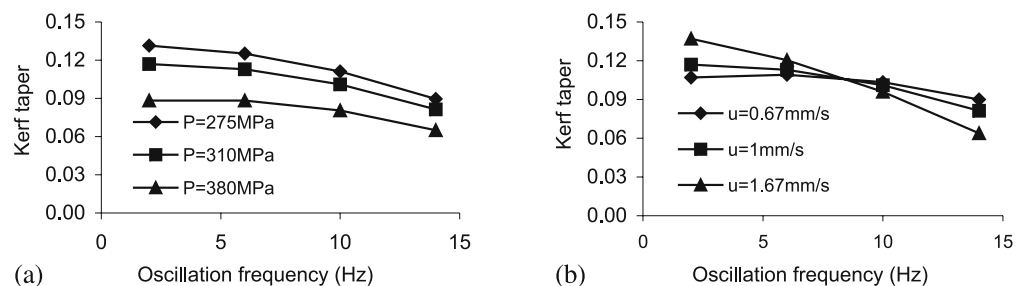
The underlying principle in constructing the predictive model for depth of cut is that the overall material removal rate is equal to the accumulated volume of material removed by individual abrasive particles in a given time span [14]. Assuming that the depth of cut is h, the overall material removal rate can be expressed as uhw, where u is nozzle traverse speed and w is the average kerf width. If the abrasive mass flow rate is $\dot{m}$ and the average mass of an individual particle is m, then the number of abrasive particles in the jet per unit time is $\dot{m}/m$. The total accumulated volume of material removed by the abrasive particles can be represented as $R\dot{m}/m$, where R is the average material removed by an individual particle contributing to the material removal process. In abrasive waterjet cutting, not all particles in the jet will impinge the material or have sufficient energy to cut the target material. Some particles may collide with other particles and are therefore not involved in the cutting action. To consider this phenomenon, an efficiency factor (K_e) may be introduced so that the following volumetric relationship can be drawn to relate the overall material removal rate to the accumulated volume of material removed by individual particles:

$$uhw = K_e \frac{\dot{m}}{m} R \tag{2}$$

By ignoring jet side spreading, it may be assumed that the average kerf width is equal to the jet diameter, which in turn is approximated by the nozzle diameter (i.e. $w = d_j$). Hence,

$$uhd_j = K_e \frac{\dot{m}}{m} R \tag{3}$$

4.2 Material removal by individual particles

To determine the mathematical expression for the average volume of material removed by an individual particle, a dimensional analysis technique was used. It was noted that there are a number of variables that affect the material removed by a particle (R). The most dominant of these are the modulus of elasticity for the target material (E), particle velocity (v), the angle between the material surface and particle flow direction at the point of attack (particle attack angle – α), and the average mass of an individual particle (m). Mathematically, the relationship between the material removed by a particle and the above variables can be written in the form of:

$$R = f(E, v, m, \alpha) \tag{4}$$

This set of variables depends on three fundamental dimensions: length L, mass M, and time T. Since α is already a dimensionless variable, two independent dimensionless products can be formed from Eq. 4:

$$\pi_1 = \frac{RE}{mv^2} \tag{5}$$

$$\pi_2 = \alpha \tag{6}$$

It then follows from Eq. 5 and Eq. 6 that the functional relation between these two dimensionless products is:

$$\pi_1 = f(\pi_2) \tag{7}$$

or

$$\frac{RE}{mv^2} = f(\alpha) \tag{8}$$

A non-dimensional quantity is proportional to the product of other dimensionless products raised to a rational power [15]. Because of its simplicity and wide use, the power law formulation is applied to Eq. 8, and the complete dimensional equation is given as follows:

$$\frac{RE}{mv^2} = A\alpha^{a_1} \tag{9}$$

Eq. 9 can be rewritten as:

$$R = \frac{Amv^2}{E}\alpha^{a_1} \tag{10}$$

where A and a_1 are two constants introduced.

It has been reported that the particle attack angle (α) is a variable along the cutting front [1]. To determine the attack angle for each particle is a difficult task. Therefore, an average attack angle for all particles impinging the target material is used in this study. To derive the relationship between the average particle attack angle and its influencing factors, dimensional analysis is again used. According to the reported investigations for AWJ cutting without the use of nozzle oscillation [16, 17], the modulus of elasticity for target material (E), nozzle traverse speed (u), and average particle diameter (D) have a significant effect on the particle attack angle. When the nozzle oscillation cutting technique is used, the oscillation parameters have been found to be additional variables affecting the depth of cut, as in the foregoing analysis. In addition, the standoff distance between the nozzle and work surface has changed its role in affecting the cutting process in nozzle oscillation cutting. Specifically, the jet scanning or oscillating scope in the cutting front is not only related to the oscillation angle, but also to the standoff distance. A study has found the underlying theme of why nozzle oscillation cutting can increase the depth of cut [1]; it shows that the successive traces of particles on the cut surface with nozzle oscillation are steeper than those without nozzle oscillation. The changed particle traces in fact affect the particle attack angle. Consequently, three additional parameters – oscillation angle (θ), oscillation frequency (F) and standoff distance (H) – are considered in determining the particle attack angle in nozzle oscillation cutting.

Thus, the particle attack angle can be expressed as a function of the six variables:

$$\alpha = \Phi(\theta, F, H, u, D, E) \tag{11}$$

All these seven parameters in Eq. 11 are quantities with regards to three fundamental dimensions: length (L), mass (M), and time (T). Since α and θ are already dimensionless variables, four independent dimensionless groups can be formed from the dimensional analysis:

$$\pi_1 = \alpha \tag{12}$$

$$\pi_2 = \theta \tag{13}$$

$$\pi_3 = \frac{FH}{u} \tag{14}$$

$$\pi_4 = \frac{D}{H} \tag{15}$$

Those groups are related by the function of:

$$\pi_1 = \Phi(\pi_2, \pi_3, \pi_4) \tag{16}$$

Thus,

$$\alpha = \Phi\left(\theta, \frac{FH}{u}, \frac{D}{H}\right) \tag{17}$$

Again, based on the power law formulation, α can be expressed, using those four groups, as:

$$\alpha = B\theta^{a_2}\left(\frac{FH}{u}\right)^{b_2}\left(\frac{D}{H}\right)^{c_2} \tag{18}$$

where B, a_2, b_2, and c_2 are all constants.

Substituting Eq. 18 into Eq. 10 and generalizing the constants and exponents by new constants C, a, b, and c gives:

$$R = \frac{Cmv^2}{e}\theta^a\left(\frac{FH}{u}\right)^b\left(\frac{D}{H}\right)^c \tag{19}$$

4.3 Depth of cut model

Having established the equation for the average volume of material removed by an individual particle, the depth of cut can be achieved by substituting this equation (Eq. 19) into the overall material removal rate (Eq. 3). After making the necessary transformations, the depth of cut is given by:

$$h = \frac{K_1\dot{m}v^2}{Ed_ju}\theta^a\left(\frac{FH}{u}\right)^b\left(\frac{D}{H}\right)^c \tag{20}$$

where $K_1 = CK_e$ and is a dimensionless constant.

Assuming that the water is incompressible and the friction loss in the supply system is negligible, the water velocity can be calculated by Bernoulli's Equation:

$$v_w = \sqrt{\frac{2P}{\rho_w}} \tag{21}$$

where P is water pressure and ρ_w is water density.

Similar to the previous studies [2], for the cutting conditions used in this study, it has been found, that the particle velocity can be expressed as:

$$v = K_2 v_w \tag{22}$$

Thus, by introducing Eq. 22 into Eq. 20, the final form of the depth of cut equation can be given by:

$$h = \frac{K\dot{m}P}{E\rho_w d_j u}\theta^a\left(\frac{FH}{u}\right)^b\left(\frac{D}{H}\right)^c \tag{23}$$

where $K = 2K_1K_2$, is an empirical constant to be determined by experiment.

4.4 Model verification

A mathematical model for the depth of cut has been established. This model is in its general form, and the constants in the model need to be determined from experimental data before it can be of any use. For this purpose, a regression analysis was performed on the experimental data obtained under the conditions given in Sect. 2 of the paper. The constants K, a, b, and c in Eq. 23 have been determined at a 95% confidence interval. Substituting these constants into the depth of cut model results in:

$$h = \frac{1.025\dot{m}P}{E\rho_w d_j u}\theta^{0.043}\left(\frac{FH}{u}\right)^{0.054}\left(\frac{D}{H}\right)^{0.044} \tag{24}$$

Re-arranging the equation gives:

$$h = 1.025\frac{\dot{m}P\theta^{0.043}\,F^{0.054}\,D^{0.044}\,H^{0.01}}{E\rho_w d_j u^{1.054}} \tag{25}$$

where the symbols and theirs units are specified in the Nomenclature.

Equation 25 is valid for nozzle oscillation cutting of an 87% alumina ceramic under the ranges of cutting parameters given in Sect. 2 of the paper. An examination of the equation reveals that the form of the model is generally feasible and consistent with the experimental trends of the depth of cut, with respect to the major process variables. Specifically, it represents a realistic model of the effects of water pressure, nozzle traverse speed, and abrasive mass flow rate, as discussed earlier in this paper. For instance, the model suggest that an increase in the water pressure and abrasive mass flow rate and a decrease in nozzle traverse speed both result in an increase in the depth of cut. It has been reported in early investigations [1] that larger particles carry more energy and have the potential to removal more material in a cutting action. This variable has a positive exponent in the numerator of the equation and is therefore considered to be correctly incorporated in the model. Similarly, the model correctly reflects the trends of the depth of cut with respect to the oscillation parameters discussed earlier in the paper. Overall, both the oscillation frequency and oscillation angle have a positive effect on the depth of cut. It is interesting to note that standoff distance has a positive exponent in the numerator. An analysis of the experimental data has found that in a considerable number of cases, an increase in standoff distance initially results in a slight increase in the depth of cut. As the standoff distance further increases, some tests show a slightly decreasing trend for the depth of cut. Thus, it is not surprising that standoff distance has a positive effect on the depth of cut, as represented by the model. Consequently, the basic form of the model is considered as correct.

In order to check the adequacy of the model, a qualitative assessment has been made by comparing the predicted depths of cut with the corresponding experimental data. Some typical and representative samples of the comparisons are given in Fig. 9. It can be seen from the figures that the model's predictions are in agreement with the experimental data. A quantitative comparison between the predicted and experimental depths of cut

Fig. 9a,b. Comparisons between model predictions and experimental data: **a** $\theta = 4°$, $P = 310$ MPa, $u = 1.33$ mm s^{-1}, $H = 3$ mm, $\dot{m} = 11.3$ gs^{-1} **b** $\theta = 2°$, $P = 275$ MPa, $u = 0.67$ mm s^{-1}, $H = 2$ mm, $\dot{m} = 6.8$ gs^{-1}
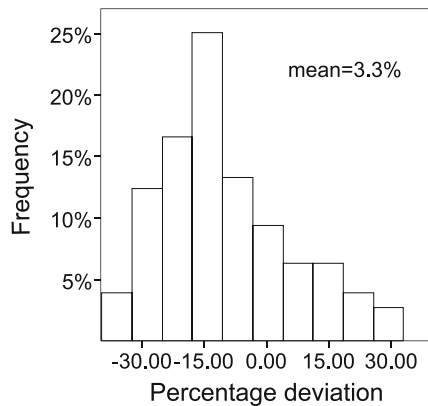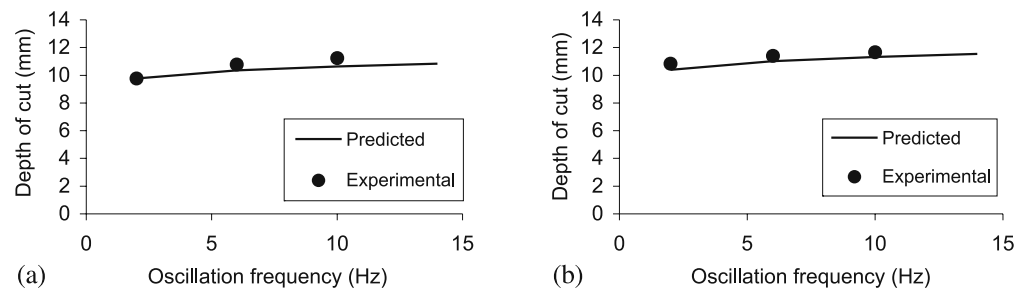

Fig. 10. Histogram for percentage deviation between predicted and experimental depth of cut

has also been carried out. It compares the percentage deviation of the model's predicted value to the corresponding experimental result, as shown in the histogram in Fig. 10. The comparison shows that the model's prediction yields an average percentage deviation of 3.3%, with a standard deviation of 20.1%. The large standard deviation is attributed to the scatter of the experimental data which were used to determine the constants in the model. Thus, it can be deduced that the developed model can be used for adequate predictions of the depth of cut in process planning for the ranges of the parameters used in this study.

5 Conclusions

An experimental investigation has been carried out in AWJ cutting of an 87% alumina ceramic with controlled nozzle oscillation. The investigation focuses on major cutting performance measures, such as surface roughness, depth of cut, and kerf taper, when nozzle oscillation cutting at small oscillation angles. As with oscillation cutting at large oscillation angles, it has been shown that oscillation at small angles can have an equally significant impact to the cutting performance. It has been found that if the cutting parameters are not selected properly, nozzle oscillation cutting can reduce some major cutting performance measures. Plausible trends of the major cutting performance measures with respect to the various cutting variables have been analysed, and the benefits of using the nozzle oscil-

lation cutting technique demonstrated. Nozzle oscillation cutting at small oscillation angles can increase the depth of cut by as much as 82%. When the optimum cutting parameters are used for both nozzle oscillation and normal cutting, the former can statistically increase the depth of cut by 23%, and reduce kerf taper by 54%. Similarly, under the optimum cutting conditions, nozzle oscillation can significantly reduce the surface roughness.

A predictive mathematical model for the depth of cut in AWJ cutting with controlled nozzle oscillation has been developed by using a dimensional analysis technique. A numerical analysis has verified the model and demonstrated the adequacy of the model's prediction. This model provides an essential basis for the development of optimization strategies for the effective use of the AWJ cutting technology when the nozzle oscillation technique is used.

References

1. Wang J (2003) Abrasive waterjet machining of engineering materials. Trans Tech Publications, Uetikon-Zuerich, Switzerland
2. Wang J (1999) Abrasive waterjet machining of polymer matrix composites: cutting performance, erosive analysis and predictive models. Int J Adv Manuf Technol 15:757–768
3. Wang J, Kuriyagawa T, Huang CZ (2003) An experimental study to enhance the cutting performance in abrasive waterjet machining. Mach Sci Technol 7(2):191–207
4. Veltrup EM (1976) Application of oscillating nozzles for cutting and cleaning. In: Proceedings of the Third International Symposium on Jet Cutting Technology, Chicago, IL, pp C1-1–C1-13
5. Siores E, Wong WCK, Chen L, Wager JG (1996) Enhancing abrasive waterjet cutting of ceramics by head oscillation techniques. Ann CIRP 45(1):327–330
6. Lemma E, Chen L., Siores E, Wang J (2002) Optimising the AWJ cutting process of ductile materials using nozzle oscillation technique. Int J Mach Tools Manuf 42(7):781–789
7. Chen L, Siores E, Wang WCK (1998) Optimising abrasive waterjet cutting of ceramic materials. J Mater Process Technol 74(1–3): 251–254
8. Hashish M (1984) A modelling study of metal cutting with abrasive waterjets. J Eng Mater Technol Trans ASME 106(1):88–100
9. Wang J (1999) A study on abrasive waterjet cutting of metallic coated sheet steels. Int J Mach Tools Manuf 39:855–870
10. Roy R K (1990) A primer on the Taguchi method. Society of Manufacturing Engineers, Dearborn, MI

11. Wang J, Guo DM (2002) A predictive depth of penetration model for abrasive waterjet cutting of polymer matrix composites. J Mater Proc Technol 121(2–3):390–394

12. Isaacson E, Isaacson M (1975) Dimensional methods in engineering and physics: reference sets and the possibilities of their extension. Edward Arnold, London

13. Svobodny T (1998) Mathematical modelling for industry and engineering. Prentice-Hall, Upper Saddle River, New Jersey

14. Zeng J, Wu S, Kim TJ (1994) Development of a parameter prediction model for abrasive waterjet turning. In: Proceedings of 12th International Conference on Jet Cutting Technology, Rouen, France, Mechanical Engineering Publication Limited, London, pp 601–617

15. Thurairajasingam E, Shayan E, Masood E (2002) Modelling of a continuous food pressing process by dimensional analysis. Comput Ind Eng 42(2–4):343–351

16. Zeng J, Kim TJ (1992) Development of an abrasive waterjet kerf model for brittle materials. In: Proceedings of 11th International Conference on Jet Cutting Technology, Bedford, UK, pp 483–501

17. Hashish M (1989) Model for abrasive-waterjet (AWJ) machining. J Eng Mater Technol Trans ASME 111(2):154–162