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A simulated investigation on the machining instability and dynamic surface generation

Abstract In this paper, the authors propose the generic concept of machining instability based on the analysis of all kinds of machining instable behaviours and their features. The investigation covers all aspects of the machining process, including the machine tool structural response, cutting process variables, tooling geometry and workpiece material property in a full dynamic scenario. The paper presents a novel approach for coping with the sophisticated machining instability and enabling better understanding of its affect on the surface generation through a combination of the numerical method with the characteristic equations and using block diagrams/functions to represent implicit equations and non-linear factors. It therefore avoids the lengthy algebraic manipulations in deriving the outcome and the solution scheme is thus simple, robust and intuitive. Several machining case studies and their simulation results demonstrate the proposed approach is feasible for shop floor CNC machining optimisation in particular. The results also indicate the proposed approach is useful to monitor the machining instability and surface topography and to be potentially applied in adaptive control of the instability in real time.

Keywords Machining instability · Simulation · Surface generation · Surface topography

Nomenclature

b	Width of cut (mm)
F_f	Feed force (N)
F_r	Normal force (N)
F_t	Main cutting force (N)

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$F_x^{(m)}, F_y^{(m)}, F_z^{(m)}$	Laplace transforms of the cutting forces in the X, Y and Z directions, respectively
$G_{ab}(w)$	The relative response of the structure in ath direction due to a force acting in the bth direction when the other two force components are zero
G	Transfer matrix
h	Uncut chip thickness (mm)
$\mathbf{H}$	Height vector of the intersection points of the sequence tool path
i	The oblique angle (degrees)
$\mathbf{r}$	The projection of the intersection points of the sequence tool path on the XZ plane
R	Tool nose radius (mm)
t_i	Index for the tool tip position in the tangential direction
t_j	Index for the tool tip position along the feed direction
α_n	Normal rake angle of the cutting tool (degrees)
β_n	Normal friction angle (degrees)
Φ_n	Normal shear angle (degrees)
η	Chip flow angle (degrees)
τ_s	Shearing yield stress (MPa)

1 Introduction

Chatter vibration as a primarily involved machining instability has been studied for several decades. The basic mechanism of chatter is the "regeneration of waviness", i.e. the vibrating tool leaves a wavy surface, which in turn excites the tool at the same frequency during the next tooth passage. Since the cutting force is proportional to the chip thickness, the structural modes in the overall machining system are self-excited and lead to unstable vibrations [1]. Since the 1960s, many researchers including Tobias and Tlusty have developed the basic theory of machining chatter [2, 3]. Based on their work, many useful guidelines were formulated for both machine tools designers and manufacturing engineers.

Hanna and Tobias carried out a study of face-milling and turning processes in the 1970s [1], in which both structural

non-linearities and cutting force non-linearities were taken into account. Quadratic and cubic non-linearities were included in a differential equation with constant coefficients and the stability of the zero solution was studied. A two-term harmonic balance scheme was used to determine the amplitude of the periodic motion following the instability. Tlusty and Ismail investigated the basic non-linearity in machining chatter. Both turning and milling are considered. The mode coupling self-excitation was demonstrated [3]. Altintas et al. have undertaken a lot of research on milling processes with stability analysis [4, 5]. They looked at the processes with many kinds of cutters and dropped the feed term from the expression for the instantaneous chip thickness prior to the stability analysis. To determine the onset of instability, they examined the stability of the zero solution of a linear homogeneous differential equation with constant coefficients. This equation was obtained after time averaging the periodic terms associated with the cutting force. They obtained an analytical expression for roots of the characteristic equation in terms of the system transfer function. This is useful for predicting the onset of instability, but not for investigating the post-instability motion. Davies et al. have carried out the investigations on chatter in turning processes [6, 7]. In the studies, they provided experimental, numerical and analytical evidence suggesting that the onset of segmented chip formation was the result of a hopf bifurcation in material flow. They introduced a concept of the local plastic deformation zone that accounted for indentation of the material near the tool tip. They also simulated and measured the chatter in diamond turning of aluminum [8]. A non-linear chip area model was formulated as a function of the change in depth of cut between consecutive passes. The cutting model was deduced based on the chip area model and further extended to include an impact disturbance to the cutting process. The simulations of the resulting tool displacement showed close agreement with experimental measurements.

So far, many aspects of self-excited machine tool vibrations or chatters have been discussed. In practice, however, many problems of poor work surface finish are due to forced vibrations and the methods of reducing forced vibrations should be known. Forced vibrations are usually caused by an out-of-balance force associated with a component integral with, or external to, the machine tool. Whereas a self-excited vibration is spontaneous and increases rapidly from a low vibratory amplitude to a large one, the forced vibration results in an oscillation of constant amplitude. The exploration on the chatter vibration enables the better understanding of machining instability in practice. However, there are many sources of non-linearities in the machining process, which are difficult to be precisely defined by the regeneration theory alone.

In the work presented, the authors not only have researched the dynamic instability based on linear regeneration chatter theory but also attempted to cover all of the variables in the machining system. In this paper the turning process is taken as an example as it is the most typical machining process where a stationary tool normally removes chips from the circumference of a rotating workpiece. Milling processes are similar from the viewpoint of chatter analysis, where the cutting tool rotates and the workpiece remains stationary. Based on the corresponding cutting force, a novel modelling approach is proposed to model the dynamic turning and milling process. The approach is in the light of transfer functions and block diagrams which are used to represent the non-linear factors or functions in particular. The approach and models developed can not only be used to predict the onset of instability, but also to illustrate the post-instability motion in time domain and to simulate the generation of the surface topography and the effects of various process variables on the surface generation.

2 Conception of machining instability

From the machining point of view, with the designed machining conditions, a desired surface finish will be produced under the stable machining process. However, as a complicated dynamic system, various mechanisms inherent in the machining process may lead the innately stable machining system to work at a dynamic instable status which invariably results in an unsatisfactory workpiece surface quality [9]. The machining instability coined here is a new generalised concept, which includes all phenomena making the machining process depart from what it should be. For instance, a variety of disturbances affect the machining system such as self-excited vibration [10], thermo-mechanical oscillations in material flow [11], feed drive hysteresis [12], etc., but the most important one is self-excited vibrations resulting from the dynamic instability of the overall machine-tool/machining-process system [5]. However, sometimes the machining process is carried out with a relative vibration between the workpiece and the cutting tool especially in heavy cutting and rough machining in order to obtain high material removal rates. The relative vibration is not necessarily a sign of the machining instability for the designed machining conditions and prescribed surface finish. In another extreme case, ultra-precision machining or micro/nano metric machining, the relative vibration between the workpiece and the cutting tool is too small to be measured, but the machining is sensitive to environmental disturbances. The surface generated may be unsatisfactory because of the disturbance, even though the machining system itself operates in the stable state. Therefore, the machining instability is related to the level of the surface quality required and the designed machining conditions.

Depending on the above conception, the authors summarise all kinds of machining instability and their features as listed in Table 1 [13, 14]. The instability is classified as the chatter vibration, which includes regenerative, frictional, mode-coupling and thermo-mechanical chatter, and the random or free vibration usually includes any shock or impulsive loading on the machine tool. A typical random vibration is the tool vibration when the tool strikes a hard spot during the cutting process. The tool will bounce or vibrate relative to the workpiece, which is the beginning of the phenomenon of the self-excited vibration. The initial vibration instigated by the hard spot is heavily influenced by the dynamic characteristics of the machine tool structure which must be included in any rational chatter analysis. Being different from

Table 1. The classification of machining instability

| | Machining instability | | | | | |
	Chatter vibrations				Random and free vibrations	
	Regenerative (dominate)	Frictional	Mode coupling	Thermomechanical	Tool dependent	Workpiece dependent	Environment dependent
Location	Between cutting edge and workpiece	Tool flank –workpiece; chip-to rake face	In cutting and thrust force directions	Tool-chip plastic zone	Tool flank –workpiece; chip-rake face	Cutting zone	Whole cutting process
Causes	Overlapping cut	Rubbing on the flank face and the rake face	Friction on the rake and clearance faces; chip-thickness variation, shear angle oscillation etc.	Temperature and strain-rate affects chip formation	Tool wear and breakage; BUE, etc.	Material soften and harden; hard grain and other kinds of flaws	Environmental disturbances
Features	Self-excited vibration; left a wavy surface on workpiece	Self-excited vibration; vibration amplitude depends on the system damping	Mode coupling vibration; simultaneous vibration in two directions	Velocity-dependent vibration	Random and chaotic; depends on cutting conditions	Random and chaotic; depends on material property and its heat treatment	Random and chaotic; depends on work environment
Suppression method	Select proper depth of cut and spindle speed according to regenerative stability chart	Select proper clearance and rake angles	Change the tool path; select proper cutting variables	Select proper cutting speed	Select high quality tool materials and proper cutting parameters	Select proper cutting tool and cutting parameters	If needed, isolate the machine tool
Modeling	Linear differential equation	Nonlinear differential equation	A set of linear differential equations	A set of partial and ordinary differential equations	Nonlinear factors in cutting process	Nonlinear factors in cutting process	Nonlinear factors in cutting process

the work carried out by others, this paper is not only focused on the chatter vibration, but also on the random and free vibration. Using the approach presented, the non-linearities in the machining process will be modelled and analysed and thus the random or free vibration can be quantitatively determined.

3 Modelling

3.1 Cutting force model of the oblique turning process

The basic cutting force model is as follows [15] for the typical oblique turning process as shown in Fig. 1.

$$F_t = bh \bullet \left[\frac{\tau_s}{\sin\phi_n} \frac{\cos(\beta_n - \alpha_n) + \tan i \tan\eta \sin\beta_n}{\sqrt{\cos^2(\phi_n + \beta_n - \alpha_n) + \tan^2\eta \sin^2\beta_n}} \right] \quad (1)$$

$$F_f = bh \bullet \left[\frac{\tau_s}{\sin\phi_n \cos i} \frac{\sin(\beta_n - \alpha_n)}{\sqrt{\cos^2(\phi_n + \beta_n - \alpha_n) + \tan^2\eta \sin^2\beta_n}} \right] \quad (2)$$

$$F_r = bh \bullet \left[\frac{\tau_s}{\sin\phi_n} \frac{\cos(\beta_n - \alpha_n)\tan i - \tan\eta \sin\beta_n}{\sqrt{\cos^2(\phi_n + \beta_n - \alpha_n) + \tan^2\eta \sin^2\beta_n}} \right], \quad (3)$$

where F_t, F_f and F_r are the main cutting force, feed force and normal force respectively. h is the uncut chip thickness, η is the chip flow angle and b is the width of cut. τ_s is the shearing yield stress, β_n is the normal friction angle and Φ_n is the normal shear angle. α_n is the tool normal rake angle and i is the oblique angle.

3.2 Model analysis of the machine tool structure

Although the machine tool/cutting system is a very complex system, for its dynamics analysis it is sufficient to focus on the structural dynamics only at the cutting zone. The cutting force applied on the tool is decomposed into three components along their perpendicular directions as illustrated in Fig. 1. If $F_x^{(m)}$, $F_y^{(m)}$, $F_z^{(m)}$ are Laplace transforms of these forces, and $x^{(m)}$,
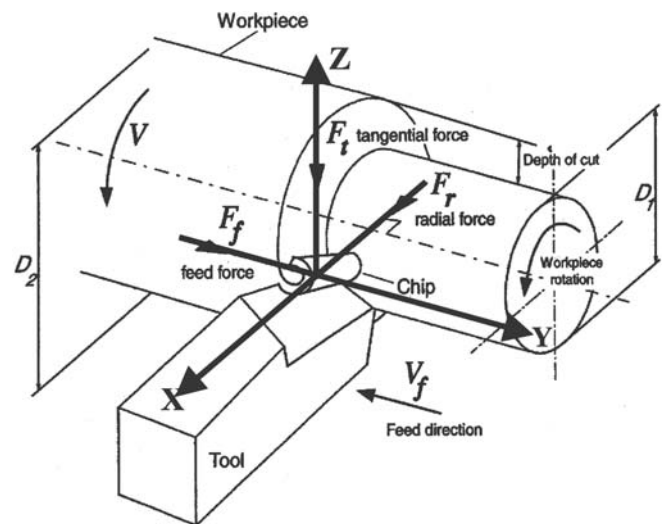
Fig. 1. Oblique turning process illustration

$y^{(m)}$, $z^{(m)}$ are Laplace transforms of the relative displacement components between the tool and the workpiece, then the dynamics at the cutting zone can be expressed by the following matrix equation:

$$\begin{bmatrix} x^{(m)} \\ y^{(m)} \\ z^{(m)} \end{bmatrix} = \begin{bmatrix} G_{xx} & G_{xy} & G_{xz} \\ G_{yx} & G_{yy} & G_{yz} \\ G_{zx} & G_{zy} & G_{zz} \end{bmatrix} \begin{bmatrix} F_x^{(m)} \\ F_y^{(m)} \\ F_z^{(m)} \end{bmatrix} \quad (4)$$

where G is the transfer matrix. This matrix consists of nine transfer function elements. $G_{ab}(\omega)$ provides the relative response of the structure in the ath direction due to a force acting in the bth direction when the other two force components are zero. For a linear system, the matrix **G** is symmetric ($G_{ab} = G_{ba}$). The assumption of linearity is generally accepted, although it is known that the static stiffness of the machine tool structure and guideways behave with non-linear characteristics. On the other hand,

the non-linearities can be expressed by a specially designed single block in the modelling and simulation. The nine transfer functions in Eq. 1 can be determined by structural dynamics test. The reference [15] provides quite useful information on the feasibility and experimental practices of determining the transfer functions.

3.3 Major aspects within the machining system

There are four categories of elements that particularly affect the machining process. They are the machine tool, operational factors, cutting tool and workpiece material [16]. There are also many non-linear factors in the machining process. These non-linear factors were normally ignored in the past because of the difficulty and limitation of the modelling. Figure 2 illustrates the major aspects associated with the four categories of elements as well as the non-linearity.
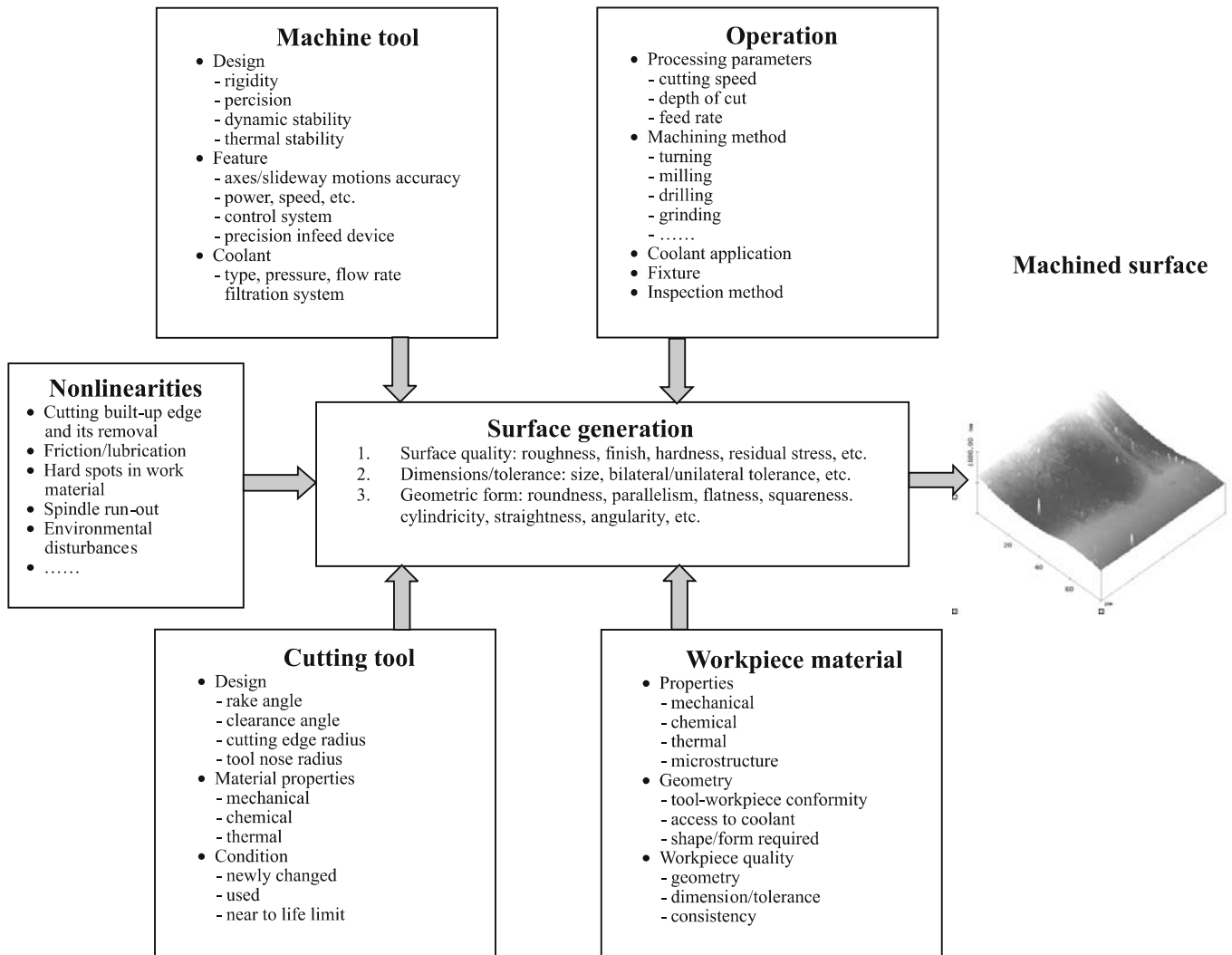

Machine tool
- Design
  - rigidity
  - percision
  - dynamic stability
  - thermal stability
- Feature
  - axes/slideway motions accuracy
  - power, speed, etc.
  - control system
  - precision infeed device
- Coolant
  - type, pressure, flow rate filtration system

Operation
- Processing parameters
  - cutting speed
  - depth of cut
  - feed rate
- Machining method
  - turning
  - milling
  - drilling
  - grinding
  - ……
- Coolant application
- Fixture
- Inspection method

Machined surface

Nonlinearities
- Cutting built-up edge and its removal
- Friction/lubrication
- Hard spots in work material
- Spindle run-out
- Environmental disturbances
- ……

Surface generation
1. Surface quality: roughness, finish, hardness, residual stress, etc.
2. Dimensions/tolerance: size, bilateral/unilateral tolerance, etc.
3. Geometric form: roundness, parallelism, flatness, squareness. cylindricity, straightness, angularity, etc.

Cutting tool
- Design
  - rake angle
  - clearance angle
  - cutting edge radius
  - tool nose radius
- Material properties
  - mechanical
  - chemical
  - thermal
- Condition
  - newly changed
  - used
  - near to life limit

Workpiece material
- Properties
  - mechanical
  - chemical
  - thermal
  - microstructure
- Geometry
  - tool-workpiece conformity
  - access to coolant
  - shape/form required
- Workpiece quality
  - geometry
  - dimension/tolerance
  - consistency

Fig. 2. Major aspects associated with the machining process

3.4 Modelling of the overall machining system

It is extremely difficult to model the whole machining system by using an exact mathematical model, since there are not only linear factors but also non-linear factors in the process. Therefore, a block diagram model is proposed as shown in Fig. 3, to model the complicated machining process including the non-linearity of the process. The model was implemented with MAT-LAB SIMULINK programming. The non-linearity is built into the model by using different kinds of functions as specified. For instance, the sinusoidal function is used to denote the spindle imbalance which imposes an additional force acting on the machining system, the assumption is consistence with the error analysis on ultra-precision diamond turning machines by Precitech Inc. [17]. The amplitude of the sinusoidal function equals to the spindle specification provided by the machine tool manufacturer, where its frequency is that of the spindle revolution. The intervally repeated ramp function denoted for the variation of the tool rake angle duo to the generation and removal of the built-up edge during the machining process, and the frequency and amplitude of the ramp function can be acquired from the measured cutting forces; a series of impulse values randomly generated at regular intervals to emulate the workpiece hard spots and their effects on the work material's characteristics, i.e. the changing of shear stress during the machining process. Furthermore, those non-linear factors can be switched on or off very easily and their effects on the surface generation can thus be interactively visualised and quantitatively investigated on the basis of an in-dividual isolated event as specified. The proposed model can be used to predict the surface generated and to optimise the cutting conditions in the light of machining instability avoidance.

3.5 Modelling of the surface generation

Having set up the cutting conditions and the system transfer function and defined the non-linear factors, the dynamic displacement between the cutting tool and the workpiece can be achieved by running the cutting system model. Taking the turning process as a case study, the modelling outcome will be the real tool path. After getting the real tool path on the workpiece, the intersection points of the sequence tool path are calculated by the following equation:

$$\begin{cases} r_{ti,tj} = \dfrac{2R(y_{ti,tj}-y_{ti+1,tj})-(x_{ti+1,tj}^2-x_{ti,tj}^2)}{2(x_{ti,tj}-x_{ti+1,tj})} \\ H_{ti,tj} = y_{ti+1,tj} + \dfrac{(r_{ti,tj}-x_{ti+1,tj})^2}{2R} \end{cases} \quad (5)$$

Where $\mathbf{r}$ and $\mathbf{H}$ are the projection on plane XZ and height vector of the intersection points of the sequence tool path, respectively. t_i is the index for the tool tip position in the tangential force direction. t_j is the index for the tool tip position along the feed direction. R is the tool nose radius. Trimming the line above the intersection points, the machined surface will be generated. Using the similar method, a milled surface can also be modelled and digitally generated.
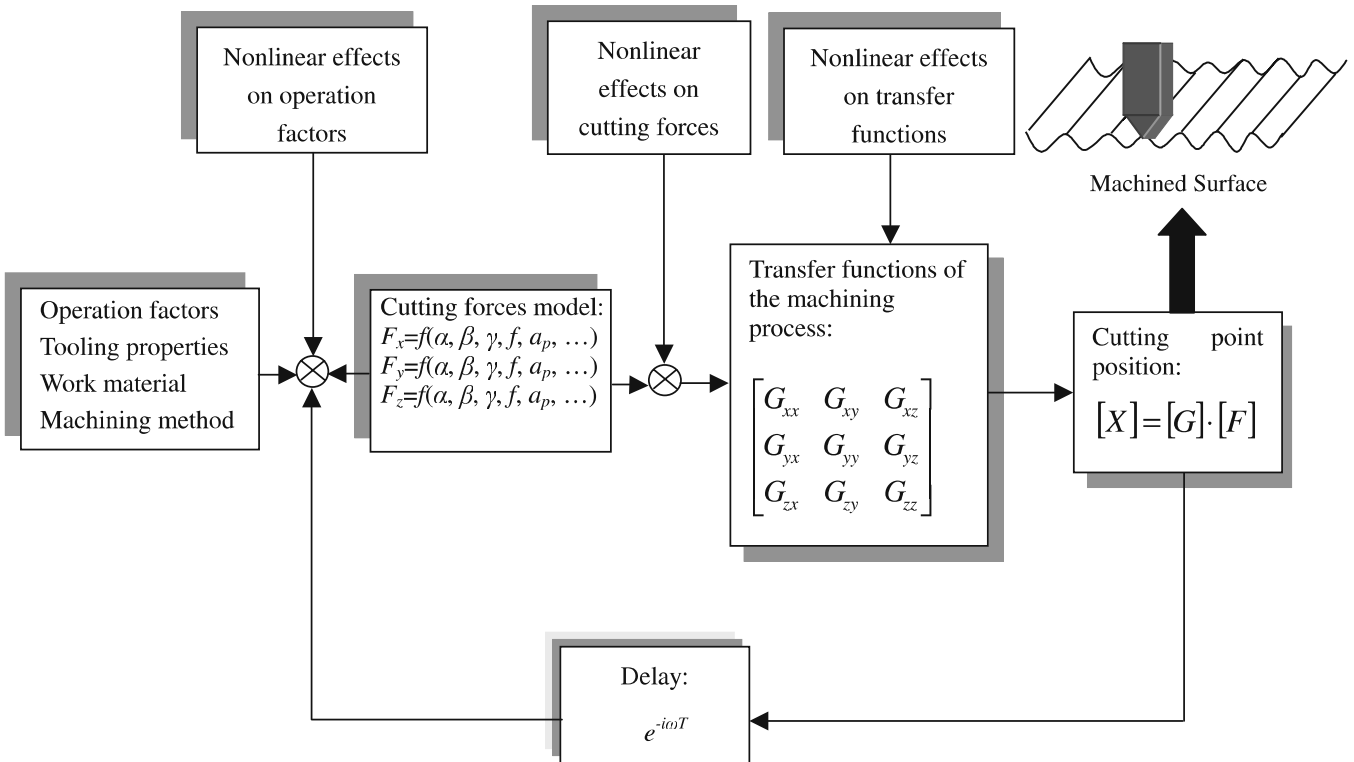

Fig. 3. A comprehensive model of the machining process

4 Simulation and experimental validations

The whole machining dynamics model is implemented in a MATLAB simulink environment. It includes the cutting force module and machining system response module. In the turning process, the frequency of the ramp function for simulating the effects of BUE is about 5.25 Hz. It is assume that the increasing of the shear stress is ten percent of the initial shear stress due to the existence of hard spots in the workpiece materials. A delay function is used to represent the regenerative vibration effects on the variation of the depth of cut and the feed rate, its frequency is that of the spindle revolution.

Machining trials are carried out on a lathe to validate the model and simulation. The experimental configuration is shown in Fig. 4. The dynamic cutting forces are measured by a Kistler dynamometer, 9257BA, on which the carbide tool insert is mounted. The machined surfaces are measured by the Zygo Newview 5000 optical microscope. The aluminium alloy and steel sample components are turned in the experiments. The machining conditions are listed in Table 2.

Figure 5 shows the simulated and measured cutting forces in the radial direction when the cutting is undertaken at the conditions: cutting speed = 1400 rpm, feed rate = 0.0397 ∼ 0.3175 mm/rev and depth of cut = 0.01 mm. It can be seen that the simulated cutting forces agree well with the measured results (about 38% lower than the measured results). The assumptions about the non-linear factors are reasonable. It also shows the tendency that the radial cutting force increases with the increment of the feed rate. When the feed rate 0.0397 mm/rev is applied, the simulated machined surface and the measured surfaces are shown in Fig. 6. The direction of the lay is evident in both the simulated surface and the experimentally generated sur-
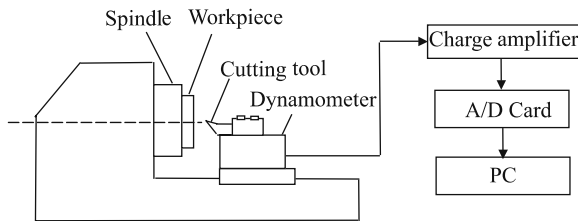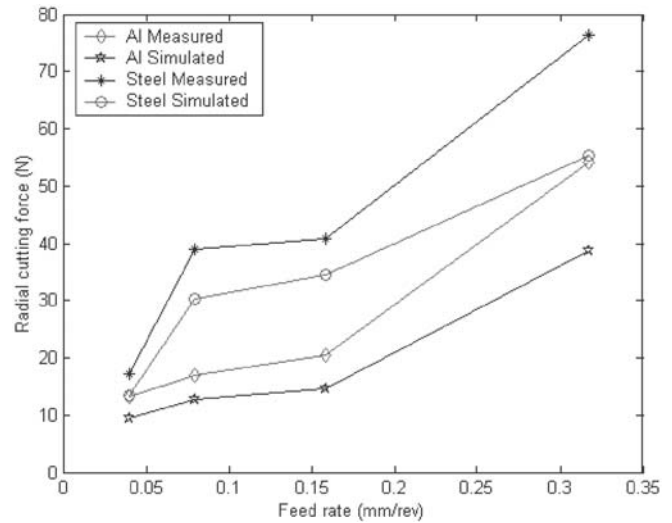

Fig. 5. The variation of the radial cutting forces with different feed rate

face. The root-mean square deviation S_q of the simulated surface is 0.071 μm, which is close to the experimental result 0.14 μm. The difference between the simulation result and experimental result may be caused by the estimated static machining structural parameters, but the simulation results are still in the reasonable deviation scale.

Two machining process cases are presented below. The turning process simulation is carried out with the spindle speed = 1400 rpm, depth of cut = 1.0 mm and Feed rate = 0.1 mm/rev. Figures 7a and 7b show two surfaces generated with and without taking account of non-linear factors respectively. From the simulation results above, it can be easily found that these non-linear factors have a great affect on the surface topography and texture produced.

5 Conclusions

In this paper, a novel modelling approach and the simulation system are presented with the case studies on turning and milling processes. The preliminary research findings include:

1. The modelling and simulation approach proposed is based on combining a numerical computing method, cutting mechanics, block diagrams and non-linear functions to simulate the complexity of the machining system as a whole. It therefore avoids the lengthy algebraic manipulations in deriving the outcome and thus improves the simulation accuracy and comprehensiveness.
2. A set of models are developed, which represents the dynamic characteristics of the machining system and also includes major non-linear factors within the machining process.
3. The modelling and simulation developed can be used to predict the onset of the machining instability, but also to observe the post-instability motion in the time domain.
4. The approach contributes to the comprehensive understanding of the machining system. The models and simulation will
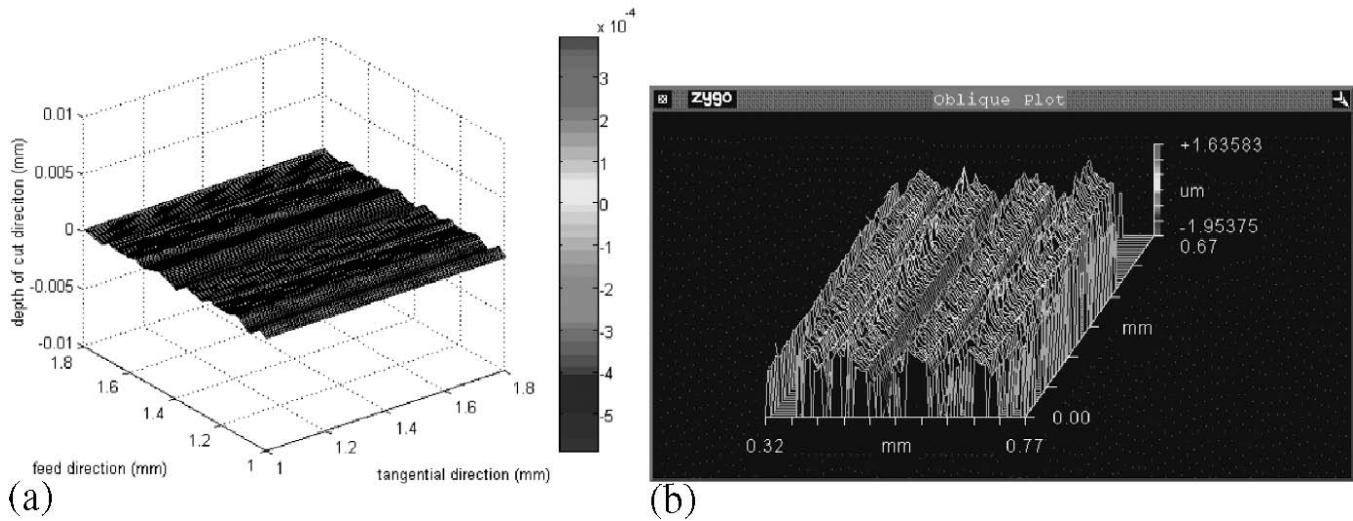

Fig. 4. Experimental configuration of the cutting trials

Table 2. Machining trial conditions

Workpiece	Material	Al alloy/low carbon steel
	Diameter	ϕ 50 mm
Cutting tool	Material	Carbide insert
	Nose radius	0.4, 0.8, 1.2 mm
	Initial side rake angle	0°, ± 5°
	Side clearance angle	0°, 5°, 7°
	Back rake angle	10°
	Back clearance angle	6°
Operation conditions	Spindle speed	490 rpm – 1400 rpm
	Feed rate	0.0397 mm/rev – 0.3175 mm/rev
	Depth of cut	0.01 mm, 0.1 mm, 0.5 mm

Fig. 6. Comparison between the simulated surface and machined surface
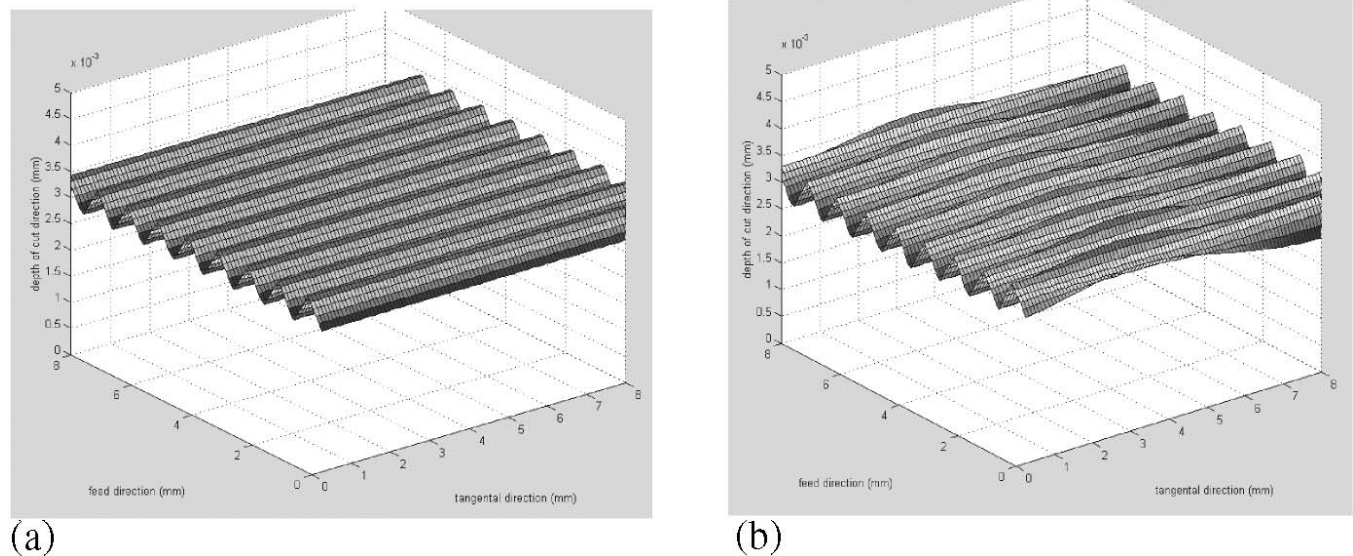

Fig. 7. Machined surfaces generated by the simulation

assist the machining operators to select optimal machining parameters.

5. The modelling and simulation will potentially lay down a foundation for researching on-line monitoring and control of the machining instability and its control algorithms.

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