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ORIGINAL ARTICLE

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Concurrent process tolerance design based on minimum product manufacturing cost and quality loss

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Abstract In a concurrent design environment, a robust optimum method is presented to directly determine the process tolerances from multiple correlated critical tolerances in an assembly. With given distributions of multiple critical assembly dimensions, the Taguchi quadric quality loss function is first derived. The quality loss is then expressed as the function of pertinent process tolerances. A nonlinear optimal model is established to minimize the summation of manufacturing costs and product quality loss. An example illustrates the proposed model and the solution method.

Keywords Concurrent tolerance design · Cost-tolerance function · Process planning · Quality loss · Tolerance optimization

1 Introduction

In manufacturing practice, actual dimensions are impossible as well as unnecessary to determine exact values. Under stable fabrication conditions, the processed dimensions often vary within certain controlled ranges. Tolerances are specified to control the actual dimensions of processed features within allowable variation zones for product functional requirements and manufacturing costs [1–7].

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The contemporary practice of tolerance design has two sequential phases: product tolerance design and process tolerance design [3].

In product tolerance design, designers use their knowledge and expertise to determine the assembly critical tolerances by computation or design handbooks. These tolerances will then be allocated to component design tolerances (blueprint tolerances) in terms of component structures, assembly restrictions, and given design criteria. If a mathematical model is used, the objective function is usually to minimize manufacturing costs or to maximize weighted component tolerances. The constraints are often tolerance stack-up and economical tolerance ranges of each component part [8–11]. Swift et al. [8] presented a tolerance optimization model in assembly stacks based on capacity design. In their research, systematic analysis for estimating process capability levels at the design stage is used in conjunction with statistical methods for optimization of tolerances in assembly stacks. Ngoi and Min [9] presented a new approach for optimum tolerance allocation in assembly. Their method allows all blueprint (BP) tolerances to be determined while ensuring that all assembly requirements are satisfied. Ngoi and Ong [10] presented a complete tolerance charting in the assembly phase. Their method integrates product tolerance design and process tolerance design. The objective is to maximize the summation of weighted process tolerances. Huang and Gao [11] presented a discrete hierarchy optimal approach for allocating the optimum component tolerance based on estimated process capability. They minimize the total manufacturing cost by using a cost-tolerance function.

In process tolerance design, manufacturing engineers develop component process planning to determine manufacturing methods, machine tools, fixtures, cutting tools, cutting conditions, manufacturing routines, and process tolerances. At this stage, BP tolerances are the most important factors. If they are too tight and cannot guarantee the economic fabrication for components by using selected process planning, more precise machine tools, special fixtures, and expensive measurements should be introduced [12]. This inevitably increases the manufacturing cost of the product. The manufacturing engineers may ask for revision of BP tolerances or of the process plan. In process tolerance design, the most popular methods are also the optimal design for minimum manufacturing cost or maximum process tolerances. Huang et al. [13] presented an optimal planar tolerance design approach to allocate dimensional and orientation geometric tolerances. A special relevance graph (SRG) was used to represent the relationships between manufactured elements and their size and tolerance information. In addition, the SRG is also applied for the geometric dimensions and tolerances. A linear programming model was established to solve the problem. Huang, Gao, and Xu [14] presented a nonlinear programming model for optimal process tolerance balancing. A linear programming model to determine process dimensions and process tolerances was used in Ji [15] and Ngoi and Teck [16]. Similar methods to determine optimum process tolerances were proposed by Wei and Lee 1995 [17] and Chang et al. [18].

Though the above methods have been used successfully to distribute both component design tolerances and process tolerances in two different phases, they over-emphasize manufacturing factors and seldom consider quality aspects. Systematically, product satisfaction conflicts with manufacturing cost. In other words, a better product satisfaction requires smaller tolerances and a higher manufacturing cost. Taguchi quality loss is a useful monetary specification to evaluate the quality factors [19, 20]. Therefore the best policy is to consolidate manufacturing cost and quality loss in the same optimization objective to best balance quality satisfaction and tolerances [11, 20]. Using this method, the research work has been carried out in product design and component process planning stages, respectively. Lee and Tang [1] presented an optimization model for controlling dimensional tolerances of components with multiple functional characteristics by minimizing the sum of manufacturing cost and quality loss. Jeang [21] introduced a mathematical optimization model to integrate manufacturing cost and quality loss for tolerance charting balancing during machining process planning. Jeang also [22] discussed a set of models to determine the optimal product tolerance and to minimize combined manufacturing and related costs.

Although tolerance assignment in the product design and process planning stages is often interdependent and interactive and affects overall production costs and product satisfaction, research into these areas is often conducted separately [3]. There are some inherent shortcomings in this method. Firstly, in product tolerance design, designers are unable to allocate the real optimal BP tolerances to components because there is no manufacturing information available at this stage. Secondly, in process tolerance design, manufacturing engineers develop process planning in terms of the component information obtained from mechanical drawings, technical notes, and others such as title bars. They are less concerned with functional roles of components than with their manufacturing capabilities. This sequential tolerance design method would result in some problems in cooperation, continuity, and consistency between two separate design stages. Therefore, rework or redesign cannot be avoided.

Until recently, the concurrent tolerancing method has attracted the attention of some engineers [2–7, 24]. Zhang [2] first systematically presented mathematical methods for concurrent tolerancing and developed a general model of optimal tolerancing that supports concurrent engineering. Ngoi and Teck [3] proposed a concurrent tolerancing method for product design in which the assembly tolerance can be allocated to the component design tolerance in an early stage of product design. Huang et al. [4] proposed a special relative hierarchical hypergraph (SRHG) to represent the assembly. Through use of SRHG, assembly and process tolerance chains can be generated automatically. The method can allocate required assembly tolerances to process tolerances concurrently. Huang and Gao [5] and Chen et al. [6] proposed a concurrent method to allocate the optimal process tolerances in early product design stages. Here, a nonlinear optimization model is established to minimize the total manufacturing cost. Fang and Wu [7] proposed a mathematical model to minimize the cost of sum machining. The constraints include assembly functional requirements, machining methods, stock remove tolerances, and economically attainable accuracies. Fang et al. [24] proposed a concurrent tolerancing method to determine the optimum process tolerances with manufacturing cost and quality loss being considered simultaneously. However, only a single assembly critical tolerance is related.

So far no design method has been presented to directly allocate multiple correlated critical tolerances to their process tolerances in a concurrent design environment. Therefore, the purpose of this paper is to introduce a concurrent optimal tolerancing method to realize this goal. To implement optimal robust tolerance design from product design stage to manufacturing stage, we first derive the quality loss function of multiple correlated critical tolerances in terms of manufacturing tolerances. A nonlinear optimization model is then given to minimize the summation of total component manufacturing cost and product quality loss. Finally, the optimal processes are obtained by solving the model.

This paper is divided into the following sections. Section 2 describes concurrent dimensioning and dimensioning. In Sect. 3 we derive the quality loss of multiple correlated critical dimensions in terms of the process tolerances. In Sect. 4 we develop the optimal tolerance design model, whereas Sect. 5 examines the implementation for a specific example. The concluding remarks are given in Sect. 6.

2 Concurrent dimensioning and tolerancing

In assembling a complex product, normally several critical dimensions evaluate the functional performance requirements. These critical dimensions are controlled simultaneously within certain variation ranges for the best working performances. Let the critical dimension vector $y = \left[y_1\, y_2 \ldots y_p\right]^T$, and the deviation vector $w = \left[w_1\, w_2 \ldots w_p\right]^T$, $w_i = y_i - y_{0i}$, $i = 1, 2, \ldots, p$, where y_{0i} is the nominal/target value of y_i. In a concurrent design environment, the assembly restrictions, topological relationships, and nominal dimensions of the main component have been determined by the assembly structure design. Let

$x = [x_1\, x_2 \ldots x_n]^T$ be the vector of component design dimensions. Normally, x_j $(j = 1, 2, \ldots, n)$ is the combination of a set of pertinent process dimensions of a component. Let the process dimension vector $z_j = \left[z_{j1}\, z_{j2} \ldots z_{jm_j}\right]^T$, $(j = 1, 2, \ldots, n)$, where m_j is the number of the operations related to dimension x_j. Finally the assembly functional equations [2] are expressed:

$$y_i = f_i(x) \quad i = 1, 2, \ldots, p \tag{1}$$

In process planning, the machining equations [2] are generally expressed as:

$$x_j = g_j\left(z_j\right) \quad j = 1, 2, \ldots, n \tag{2}$$

Since there is no need or way for critical dimensions to be controlled in the exact nominal/target value, a rational variation zone should be assigned for each design dimension. From Eq. 1, the actual critical dimension deviations due to their design dimension deviations are expressed as:

$$w_i = y_i - f_i\left(\bar{x}\right) = \sum_{j=1}^{n} \left.\frac{\partial f_i(x)}{\partial x_j}\right|_{\bar{x}} \Delta x_j \tag{3}$$

where $f_i\left(\bar{x}\right)$ is the nominal value obtained by evaluating the assembly functional Eq. 1 with its nominal design dimension vector $\bar{x}$. Δx_j is the algebraic difference between x_j and $\bar{x}_j$.

In tolerance design, accumulated design tolerances must be less than or equal to their critical tolerance, so Eq. 3 needs some adjusting. For worst-case tolerance stack-up, each differential coefficient is positive, therefore, the absolute value of each differential coefficient is required. w_i and Δx_j are replaced by t_i and tx_j. Where t_i and tx_j are respectively the tolerance of critical dimension y_i and design dimension x_j. With these substitutions, Eq. 3 changes into inequality:

$$t_i \geq \sum_{j=1}^{n} \left| \left.\frac{\partial f_i(x)}{\partial x_j}\right|_{\bar{x}} \right| tx_j \tag{4}$$

Similarly, from Eq. 2 the actual design dimension deviations due to their process dimension deviations can be expressed as:

$$x_j - g_j\left(\bar{z}_j\right) = \sum_{k=1}^{m_j} \left.\frac{\partial g_j\left(z_j\right)}{\partial z_{jk}}\right|_{\bar{z}_j} \Delta z_{jk} \tag{5}$$

where $g_j\left(\bar{z}_j\right)$ is the nominal value obtained by evaluating the machining Eq. 2 with its nominal process dimension vector $\bar{z}_j$. Δz_{jk} is the algebraic difference of z_{jk} and $\bar{z}_{jk}$.

When component design tolerances are allocated to process tolerances, Eq. 5 changes into inequality:

$$tx_j \geq \sum_{k=1}^{m_j} \left| \left.\frac{\partial g_j\left(z_j\right)}{\partial z_{jk}}\right|_{\bar{z}_j} \right| t_{jk} \tag{6}$$

where t_{jk} is jkth process tolerance of design dimension z_{jk}.

Assume that all process dimensions are of normal distributions. Because design dimensions are functions of process dimensions and assembly critical dimensions are functions of design dimensions, according to statistical theory, both critical dimensions and design dimensions are of normal distributions. From Eq. 1, we get variance equations:

$$\operatorname{var}\left(w_i\right) = \sum_{j=1}^{n} \left(\left.\frac{\partial f_i(x)}{\partial x_j}\right|_{\bar{x}} \right)^2 \operatorname{var}\left(\Delta x_j\right) \quad i = 1, 2, \cdots, p \tag{7}$$

where variance $\operatorname{var}\left(\Delta x_j\right)$ is obtained from Eq. 3 and expressed as:

$$\operatorname{var}\left(\Delta x_j\right) = \sum_{k=1}^{m_j} \left(\left.\frac{\partial g_j\left(z_j\right)}{\partial z_{jk}}\right|_{\bar{z}_j} \right)^2 \operatorname{var}\left(z_{jk}\right) \quad k = 1, 2, \cdots, n \tag{8}$$

where $\operatorname{var}\left(w_i\right)$, $\operatorname{var}\left(\Delta x_j\right)$, and $\operatorname{var}\left(z_{jk}\right)$ are variances of w_i, Δx_j, and z_{jk}, respectively.

Equations 4 and 6 reveal the worst-case tolerance stack-up effect related to two stages, respectively. In Eq. 4, component design stack-up tolerance must be less than or equal to functional critical tolerances. Similarly in Eq. 6, component process stack-up tolerance must be less than or equal to design tolerances. As discussed above, interdependent tolerancing is divided into two separate stages. In initial product design, designers care more about product satisfaction than about subsequent production capabilities and costs. On the other hand, process planners are more concerned about component manufacturing capabilities than their functional roles in assembly. This conventional method can obtain only the optimum solutions within two separate stages. The best policy is to integrate the two stages into one.

In concurrent engineering, however, the two separate phases are integrated into only one stage [2, 3]. This makes it easy for design and manufacturing to collaborate. Essentially, the product designer can consider more fabrication issues when initially designing the product, while manufacturing engineers can cope with the manufacturing problems based on the component functional roles. This balances the different targets related to product satisfaction and production costs. Mathematically, by substituting the machining equation into functional equations the concurrent design equation can be obtained as:

$$t_i \geq \sum_{j=1}^{n} \sum_{k=1}^{m_j} \left| \left.\frac{\partial f_i(x)}{\partial x_j}\right|_{\bar{x}} \right| \left| \left.\frac{\partial g_j\left(z_j\right)}{\partial z_{jk}}\right|_{\bar{z}_j} \right| t_{jk} \quad i = 1, 2, \ldots, p \tag{9}$$

The above equation represents the relationships between assembly critical functional tolerances and related process tolerances in concurrent tolerance design. It essentially reveals pertinent process tolerance stack-up processes by presenting a worst-case model. This equation is particularly significant because critical functional tolerances can be directly allocated into component manufacturing tolerances by this equation when pertinent component process planning is given. Tolerance design has been extended directly from the product tolerance design stage to the process tolerance design stage. The rework and redesign existing in conventional methods have been eliminated.

3 Quality loss of multiple correlated critical dimensions

High quality and low cost are two fundamental requirements for product design and manufacturing. In an assembly, critical tolerances must be guaranteed for functional requirements. It is well known that the tighter tolerance is, the higher the cost is, and vice versa. For a selected machining operation, if process tolerance becomes smaller and smaller until it reaches a certain value, it will result in the infinite theoretical manufacturing cost. To simplify the computation, let the best product performance be the point where tolerance is zero. At that point, the theoretical manufacturing cost is infinite. For a single critical dimension case, when the critical dimension deviates from its target, the symmetric quadratic Taguchi quality loss function is [19]:

$$L(y) = k\left(y - \bar{y}\right)^2 \tag{10}$$

where y and $\bar{y}$ are respectively the actual and target values of critical dimension, and k is a positive constant coefficient

To determine the value of k, provided that when dimension y deviates from its target in value w, will cause the loss of A \$. Thus the following equation will be satisfied:

$$k = A/w^2 \tag{11}$$

where $w = y - \bar{y}$.

For a p-dimensional multivariate vector w, Le and Tang [1] presented a general formula to evaluate the total quality loss due to w:

$$L(w) = w^T K w \tag{12}$$

where K is a $p \times p$ symmetric constant matrix. $k_{ij} = k_{ji}$, for $i \neq j$, $i, j = 1, 2, \ldots, p$. If $p(p+1)/2$ set of product quality deviations and corresponding quality losses are available. The elements of K are related by:

$$\sum_{i=1}^{p}\sum_{j=1}^{p} k_{ij} w_i^{(k)} w_j^{(k)} = A_k \quad \text{where } k = 1, 2, \ldots, p(p+1)/2 \tag{13}$$

Since manufacturing dimension distribution is dependent upon the related manufacturing process random factors such as machine tools, fixtures, tool wearing, system vibration, temperature fluctuation, operators, and measurement devices, etc, each actual process dimension z_{jk} is obviously a random variable. In terms of Eqs. 2 and 1, design dimension x_j is the combination of process dimension z_{jk} and critical dimension y_i is the combination of design dimension x_j, so design dimension x_j and critical dimension y_i are also random variables. The distribution of critical dimension y_i is finally dependent upon the density distribution functions of pertinent process dimensions. The product quality loss is determined by all critical dimension distributions. For a batch of products, average quality loss rather than individual loss should be considered. When a product has only a single critical dimension y, let the density function of y be function $\psi(w)$, the average loss of a batch product could be obtained by integration:

$$E(L(w)) = \int_{-\infty}^{+\infty} \psi(w) k w^2 dw \tag{14}$$

As for the multiple critical dimensions, the expectation loss is obviously the summation of individual contributions derived from Eq. 14:

$$E(L(w)) = \sum_k \Psi\left(w^{(k)}\right)\left(w^{(k)^T} K w^{(k)}\right) \tag{15}$$

where

$$\sum_k \Psi\left(w^{(k)}\right) = 1\,. \tag{16}$$

For the design vector x, the density function is continuous within an interval. Expected quality loss function is [1]:

$$E(L(w)) = \text{Trace}\,[KV(w)]\,, \tag{17}$$

where $V(w)$ is the variance-covariance matrix of the parameter vector w expressed by:

$$V(w) = \begin{bmatrix} \text{var}\,(w_1) & \text{cov}\,(w_1, w_2) & \cdots & \text{cov}\,(w_1, w_p) \\ \text{cov}\,(w_1, w_2) & \text{var}\,(w_2) & \cdots & \vdots \\ \vdots & \cdots & \ddots & \vdots \\ \text{cov}\,(w_1, w_p) & \cdots & \cdots & \text{var}\,(w_p) \end{bmatrix}, \tag{18}$$

where variance $\text{var}\,(w_i)$ is determined by Eq. 7. The covariance between the ith and the lth critical dimensions is:

$$\text{cov}\,(w_i, w_l) = \sum_{k=1}^{n} \left.\frac{\partial f_i(x)}{\partial x_k}\right|_{\bar{x}} \left.\frac{\partial f_l(x)}{\partial x_k}\right|_{\bar{x}} \text{var}\,(\Delta x_k)\,. \tag{19}$$

For tolerance design, each dimension variance should be expressed as the function of its dimension tolerance. Under stable machining conditions, provided that process dimensions are normally distributed. Therefore when component design tolerances are expressed as process tolerances in the process planning stage, the relation between design tolerance and process variance is:

$$t_j = \frac{2}{C_j}\left[\text{var}\,\left(\Delta x_j\right)\right]^{1/2} = \frac{2}{C_j}\left[\sum_{k=1}^{m_j}\left(\left.\frac{\partial g_j\left(z_j\right)}{\partial z_{jk}}\right|_{\bar{z}}\right)^2 \text{var}\,\left(z_{jk}\right)\right]^{1/2}, \tag{20}$$

where t_j is bilateral tolerance of design dimension x_j. C_j is a constant factor depending on the probability distribution of the dimension variations concerned. $C_j = 1/3$ for normally distributed process dimensions with 99.73% probability.

When the above equation is substituted into Eqs. 7 and 19, the variance and covariance of critical dimensions can be expressed by:

$$\begin{aligned}\operatorname{var}(w_i) &= \frac{1}{4}\sum_{j=1}^{n} C_j^2 \left(\left.\frac{\partial f_i(x)}{\partial x_j}\right|_{\bar{x}} \right)^2 t_j^2 \\ &= \frac{1}{4}\sum_{j=1}^{n} C_j^2 \left(\left.\frac{\partial f_i(x)}{\partial x_j}\right|_{\bar{x}} \right)^2 \sum_{k=1}^{m_j} \left(\left.\frac{\partial g_j(z_j)}{\partial z_{jk}}\right|_{\bar{z}} \right)^2 t_{jk}^2 \end{aligned} \quad (21)$$

$$\begin{aligned}\operatorname{cov}(w_i, w_l) &= \frac{1}{4}\sum_{j=1}^{n} C_j^2 \left(\left.\frac{\partial f_i(x)}{\partial x_j}\right|_{\bar{x}} \right)\left(\left.\frac{\partial f_l(x)}{\partial x_j}\right|_{\bar{x}} \right) t_j^2 \\ &= \frac{1}{4}\sum_{j=1}^{n} C_j^2 \left(\left.\frac{\partial f_i(x)}{\partial x_j}\right|_{\bar{x}} \right)\left(\left.\frac{\partial f_l(x)}{\partial x_j}\right|_{\bar{x}} \right) \sum_{k=1}^{m_j} \left(\left.\frac{\partial g_j(z_j)}{\partial z_{jk}}\right|_{\bar{z}} \right)^2 t_{jk}^2 \end{aligned} \quad (22)$$

where t_j is an m_jth process tolerance vector, i.e., $t_j = \left[t_{j1}\, t_{j2} \ldots t_{jk} \ldots t_{jm_j}\right]^T$, and $k = 1, 2, \ldots, n$.

4 Optimal tolerance assignment

To implement robust tolerance design, the best balance should be made between product satisfaction and manufacturing cost. In a concurrent tolerancing environment, the product quality loss is expressed as the function of pertinent process tolerances. In the optimum model, the objective is to minimize the summation of product manufacturing cost and quality loss:

$$\min \sum_{j=1}^{n}\sum_{k=1}^{m_j} c_{jk}\left(t_{jk}\right) + E(L(w)) \quad (23)$$

where $c_{jk}\left(t_{jk}\right)$ is manufacturing cost of jkth process operation, and $E(L(w))$ is expected quality loss function of the product.

To determine the manufacturing cost, cost-tolerance functions can be used. With regard to cost-tolerance functions, several types of models have been presented [2, 7]. Regression techniques are often applied to the acquired discrete cost-tolerance data and determine the unknown constant coefficients for each model. The models with the highest regression precision are used as cost-tolerance functions. Based on this method, Fang et al. presented a set of cost-tolerance functions suitable for middle quantitative production in manufacturing enterprises. The one suitable for planar features is [7]:

$$c_{jk}\left(t_{jk}\right) = 50.261 \exp\left(-15.8903 t_{jk}\right) + t_{jk}/\left(0.3927 t_{jk} + 0.1176\right) \quad (24)$$

In actual manufacturing, each process dimension z_{jk} has an economical tolerance range. It can be expressed mathematically by:

$$t_{jk}^{-} \le t_{jk} \le t_{jk}^{+} \quad (25)$$

where t_{jk}^{-} and t_{jk}^{+} are respectively the lower and upper bounds of process tolerance t_{jk}.

In a concurrent tolerancing environment, the complete optimization model can be introduced as:

$$\min \sum_{j=1}^{n}\sum_{k=1}^{m_j} c_{jk}\left(t_{jk}\right) + E(L(w))$$

s.t.

$$ty_i^{-} \le \sum_{j=1}^{n} \left| \left.\frac{\partial f_i(x)}{\partial x_j}\right|_{\bar{x}} \right| \sum_{k=1}^{m_j} \left| \left.\frac{\partial g_j(z_j)}{\partial z_{jk}}\right|_{\bar{z}_j} \right| t_{jk} \le ty_i^{+}$$

$$t_{jk}^{-} \le t_{jk} \le t_{jk}^{+}$$

where ty_i^{-} and ty_i^{+} are the lower and upper bounds of assembly critical tolerance ty_i, respectively. They are given as input data in terms of product quality and manufacturing cost. The optimum ty_i is determined by solving the optimal model.

Two kinds of constraints are proposed for the optimal model. The first are concurrent design equations. These equations present the tolerance stack-up effects between assembly critical tolerances and pertinent manufacturing tolerances by worst-case or statistical model. In the concurrent design equation critical tolerance must be greater than or equal to its pertinent sum manufacturing tolerance. The second constraints are process capabilities. According to selected fabrication methods and machining tools, each processed tolerance should specify an economical variation range.

5 Illustrative example

Figure 1 shows a wheel assembly. Assume that nominal design dimensions have already been assigned based on the requirements in size, strength, structure, assembly, and maintenance,

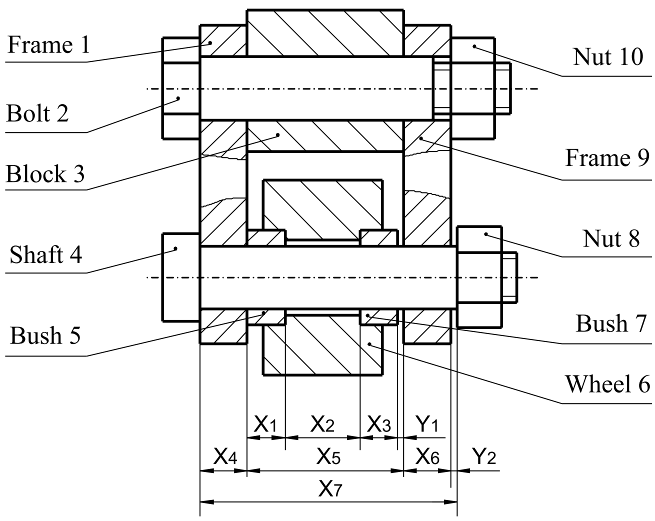

Fig. 1. Wheel assembly

etc., they are: $x_1 = 9$, $x_2 = 20$, $x_3 = 9$, $x_4 = 12$, $x_5 = 38.2$, $x_6 = 12$, $x_7 = 62.4$ (unit: mm). Two critical dimensions $y_1 = 0.2 \pm 0.080 \sim 0.140$, and $y_2 = 0.2 \pm 0.075 \sim 0.130$. y_1 is the critical axial gap between bush 7 and frame 9. y_2 is another critical axial gap between nut 8 and frame 9. It is not difficult to formulate the assembly functional equations using the method presented by Huang et al. [4].

$$y_1 = -x_1 - x_2 - x_3 + x_5$$
$$y_2 = -x_4 - x_5 - x_6 + x_7$$

According to Eq. 3, the deviation equations of critical dimensions are:

$$w_1 = y_1 - \bar{y}_1 = -\Delta x_1 - \Delta x_2 - \Delta x_3 + \Delta x_5$$
$$w_2 = y_2 - \bar{y}_2 = -\Delta x_4 - \Delta x_5 - \Delta x_6 + \Delta x_7$$

With Eq. 4, the functional tolerance inequalities by worst-case model are:

$$ty_1 \geq tx_1 + tx_2 + tx_3 + tx_5$$
$$ty_2 \geq tx_4 + tx_5 + tx_6 + tx_7$$

Provided that the manufacturing process takes place under stable conditions, each process dimension will be of normal distribution. For simplification, assume that the distribution center of each process dimension is just equal to its nominal value. Each critical dimension variance can be expressed as the function of its design tolerance:

$$\mathrm{var}\,(w_1) = \frac{1}{36}\left(tx_1^2 + tx_2^2 + tx_3^2 + tx_5^2\right)$$
$$\mathrm{var}\,(w_2) = \frac{1}{36}\left(tx_4^2 + tx_5^2 + tx_6^2 + tx_7^2\right)$$

Similarly, the covariance of the two correlated critical dimensions can be expressed as the function of the pertinent design tolerances:

$$\mathrm{cov}\,(w_1, w_2) = -\frac{1}{36} tx_5^2$$

The critical tolerance ranges of y_1 and y_2 in Fig. 1 are determined both by performance satisfaction and manufacturing cost of this assembly. To finally determine the optimum tolerance of these two critical dimensions and then allocate them to the related process dimensions, quality loss and manufacturing cost must first be determined . Provided that when critical dimension y_1 and y_2 deviate from their target (nominal) vector with values $\underline{w}^{(1)} = \left[\underline{w}_1^{(1)}, 0\right]^T = [0.160, 0]^T$, $\underline{w}^{(2)} = \left[0, \underline{w}_2^{(2)}\right]^T = [0, 0.150]^T$, or $\underline{w}^{(3)} = \left[\underline{w}_1^{(3)}, \underline{w}_2^{(3)}\right]^T = [0.140, 0.130]^T$ will result in product failure and cause a quality loss of \$ 300. The constant matrix K can thus be decided by Eq. 12:

$$k_{11} = A_1 / \left(\underline{w}_1^{(1)}\right)^2 = 300/0.16^2 = 11\,718.75$$
$$k_{22} = A_2 / \left(\underline{w}_2^{(2)}\right)^2 = 300/0.15^2 = 13\,333.33$$
$$\begin{aligned} k_{12} &= k_{21} \\ &= \left(A_3 - A_1\left(\underline{w}_1^{(3)}\right)^2 / \left(\underline{w}_1^{(1)}\right)^2 - A_1\left(\underline{w}_2^{(3)}\right)^2\right) / \left(\underline{w}_2^{(2)}\right)^2\Big) \\ &\quad / \left(2\underline{w}_1^{(3)}\underline{w}_2^{(3)}\right) \\ &= \left(300 - 300 \times 0.14^2/0.16^2 - 300 \times 0.13^2/0.15^2\right)/2 \\ &\quad \times 0.14 \times 0.13 \\ &= -4258.81 \end{aligned}$$

With this, total expected loss is:

$$\begin{aligned} E(L(w)) &= \mathrm{Trace}\,[KV(w)] \\ &= \frac{1}{36}\Big[k_{11}tx_1^2 + k_{11}tx_2^2 + k_{11}tx_3^2 + k_{22}tx_4^2 \\ &\qquad + (k_{11} - 2k_{12} + k_{22})\,tx_5^2 + k_{22}tx_6^2 + k_{22}tx_7^2\Big] \end{aligned}$$

Figure 2 shows related structure and design dimension for each machining part. For the corresponding process plan, look at the economical process tolerance bounds for each machining part in Table 1.

Using the method presented by Huang et al. [4], the machining equations are obtained from given component process plans:

$$x_1 = z_{11} - z_{12}$$
$$x_2 = z_{24} - z_{23} - z_{25}$$
$$x_3 = z_{31} - z_{32}$$
$$x_4 = z_{44}$$
$$x_5 = z_{54}$$
$$x_6 = z_{64}$$
$$x_7 = z_{74} - z_{73}$$

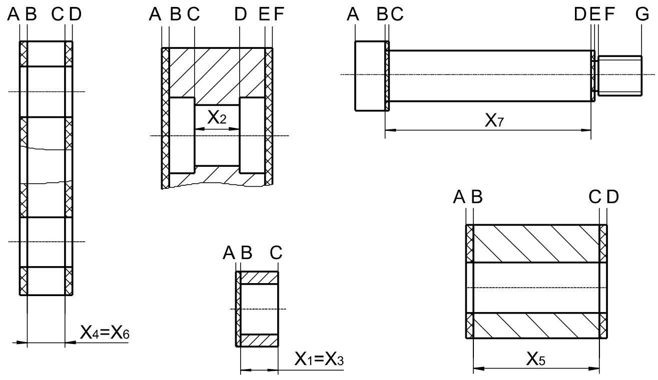

Fig. 2. Process plan of related parts

Table 1. Axial process plan for related parts

Parts	No.	Process name	Measure reference	Machined plane	Process dimension z_{kq}	Process tolerance t_{kq}	Tolerance bounds $\left[t_{kq}^{-}, t_{kq}^{+}\right]$
Bush 5	11	Parting-off	C	A	$z_{11}=11$	t_{11}	[0.027 ∼ 0.070]
and 7	12	L plane by FL	A	B	$z_{12}=2$	t_{12}	[0.010 ∼ 0.025]
Wheel 6	21	R plane by RL	A	F	$z_{21}=36$	t_{21}	[0.054 ∼ 0.140]
	22	L plane by FL	F	B	$z_{22}=34$	t_{22}	[0.054 ∼ 0.140]
	23	L pole by L	B	C	$z_{23}=6$	t_{23}	[0.012 ∼ 0.030]
	24	R plane by FL	B	E	$z_{24}=32$	t_{24}	[0.025 ∼ 0.062]
	25	R pole by L	E	D	$z_{25}=6$	t_{25}	[0.012 ∼ 0.030]
Frame 1	41	R plane by RM	A	D	$z_{41}=16$	t_{41}	[0.043 ∼ 0.110]
and 9	42	L plane by RM	D	B	$z_{42}=16$	t_{42}	[0.043 ∼ 0.110]
	43	R plane by FM	B	C	$z_{43}=14$	t_{43}	[0.043 ∼ 0.110]
	44	L plane by FM	C	B	$z_{44}=12$	t_{44}	[0.027 ∼ 0.070]
Block 3	51	R plane by RM	A	D	$z_{51}=42.2$	t_{51}	[0.062 ∼ 0.160]
	52	L plane by RM	D	B	$z_{52}=42.2$	t_{52}	[0.062 ∼ 0.160]
	53	R plane by FM	B	C	$z_{53}=40.2$	t_{53}	[0.039 ∼ 0.100]
	54	L plane by FM	C	B	$z_{54}=38.2$	t_{54}	[0.039 ∼ 0.100]
Shaft 4	71	Step by RL	G	C	$z_{71}=80.4$	t_{71}	[0.054 ∼ 0.140]
	72	Step by RL	G	E	$z_{72}=18$	t_{72}	[0.027 ∼ 0.070]
	73	Step by FL	G	D	$z_{73}=20$	t_{73}	[0.021 ∼ 0.052]
	74	Step by FL	G	B	$z_{74}=82.4$	t_{74}	[0.035 ∼ 0.087]
	75	Truncation	G	A	$z_{75}=90$	t_{75}	[0.054 ∼ 0.140]

Notes: FM stands for finish milling, RM stands for rough milling, FL stands for finish lathing, RL stands for rough lathing, L stands for lathing, R stands for right and L stands for left.

The design tolerance inequalities are:

$$tx_1 \geq t_{11}+t_{12}$$
$$tx_2 \geq t_{23}+t_{24}+t_{25}$$
$$tx_3 \geq t_{31}+t_{32}$$
$$tx_4 \geq t_{44}$$
$$tx_5 \geq t_{54}$$
$$tx_6 \geq t_{64}$$
$$tx_7 \geq t_{73}+t_{74}$$

The component design tolerance can be formulated as the function of its related process tolerances with Eq. 10:

$$tx_1^2 = t_{11}^2+t_{12}^2$$
$$tx_2^2 = t_{23}^2+t_{24}^2+t_{25}^2$$
$$tx_3^2 = t_{31}^2+t_{32}^2$$
$$tx_4^2 = t_{44}^2$$
$$tx_5^2 = t_{54}^2$$
$$tx_6^2 = t_{64}^2$$
$$tx_7^2 = t_{73}^2+t_{74}^2$$

In a concurrent tolerancing environment, when machining equations are substituted into assembly functional equations, product quality loss is finally obtained as:

$$\begin{aligned}E(L(w)) &= \frac{1}{36}\Big[k_{11}tx_1^2+k_{11}tx_2^2+k_{11}tx_3^2+k_{22}tx_4^2 \\ &\quad + \left(k_{11}-2k_{12}+k_{22}\right)tx_5^2+k_{22}tx_6^2+k_{22}tx_7^2\Big] \\ &= 325.52\left(t_{11}^2+t_{12}^2+t_{23}^2+t_{24}^2+t_{25}^2+t_{31}^2+t_{32}^2\right) \\ &\quad +370.37t_{44}^2+33\,569.7t_{54}^2+370.37t_{64}^2 \\ &\quad +370.37\left(t_{73}^2+t_{74}^2\right)\end{aligned}$$

In this example, we only consider the manufacturing cost of process dimensions that are involved in assembly functional equations. The reason is that the other process dimensions can use the most economical tolerances, and manufacturing costs of these operations are minimal. Furthermore, these process dimensions don't contribute to quality loss. The manufacturing cost of these considered operations is:

$$\begin{aligned}C_M &= \sum_{j=1}^{n}\sum_{k=1}^{m_j} c_{jk}\left(t_{jk}\right) \\ &= c_{11}+c_{12}+c_{23}+c_{24}+c_{25}+c_{31}+c_{32}+c_{44}+c_{54} \\ &\quad +c_{64}+c_{73}+c_{74}\end{aligned}$$

The summation of C_M and $E(L(w))$ is:

$$\begin{aligned}C &= C_M+E(L(w)) \\ &= c_{11}+c_{12}+c_{23}+c_{24}+c_{25}+c_{31}+c_{32}+c_{44}+c_{54}+c_{64} \\ &\quad +c_{73}+c_{74}+325.52\left(t_{11}^2+t_{12}^2+t_{23}^2+t_{24}^2+t_{25}^2+t_{31}^2+t_{32}^2\right) \\ &\quad +370.37t_{44}^2+33\,569.7t_{54}^2+370.37t_{64}^2+370.37\left(t_{73}^2+t_{74}^2\right)\end{aligned}$$

Table 2. The comparison results of the two methods (μm)

Method	t_{11}	t_{12}	t_{23}	t_{24}	t_{25}	t_{31}	t_{32}	t_{44}	t_{54}	t_{64}	t_{73}	t_{74}	total
C_M+C_L	43	25	30	25	30	43	25	43	25	43	21	35	388
C_M	43	25	30	49	30	43	25	43	35	43	52	87	505

Finally, the entire optimization problem is formulated as:

$$\min\Big\{c_{11}+c_{12}+c_{23}+c_{24}+c_{25}+c_{31}+c_{32}+c_{44}+c_{54}+c_{64} + c_{73}+c_{74}+325.52\left(t_{11}^2+t_{12}^2+t_{23}^2+t_{24}^2+t_{25}^2+t_{31}^2+t_{32}^2\right) + 370.37\left(t_{44}^2+t_{64}^2+t_{73}^2+t_{74}^2\right)+33\,569.7t_{54}^2\Big\}$$

where

$$c_{jk}=c_{jk}\left(t_{jk}\right) = 5.0261\exp\left(-15.8903t_{jk}\right)+t_{jk}/\left(0.3927t_{jk}+0.1176\right)$$

Subjected to:

The concurrent tolerance stack-up constraints by worst-case model:

$$0.160=t_1^- \le t_{11}+t_{12}+t_{23}+t_{24}+t_{25}+t_{31}+t_{32}+t_{54} \le t_1^+ = 0.280$$

$$0.150=t_2^- \le t_{44}+t_{54}+t_{64}+t_{73}+t_{74}\le t_2^+=0.260$$

where $t_1^-=0.160$, $t_1^+=0.280$ is the lower and upper tolerance bound of critical dimension y_1, $t_2^-=0.150$, $t_2^+=0.260$ is the lower and upper tolerance bound of critical dimension y_2, respectively.

The economical process tolerance ranges for each process operation are as follows:

$$0.018=t_{11}^-\le t_{11}\le t_{11}^+=0.043$$
$$0.010=t_{12}^-\le t_{12}\le t_{12}^+=0.025$$
$$0.012=t_{23}^-\le t_{23}\le t_{23}^+=0.030$$
$$0.025=t_{24}^-\le t_{24}\le t_{24}^+=0.062$$
$$0.012=t_{25}^-\le t_{25}\le t_{25}^+=0.030$$
$$0.018=t_{31}^-\le t_{31}\le t_{31}^+=0.043$$
$$0.010=t_{32}^-\le t_{32}\le t_{32}^+=0.025$$
$$0.018=t_{44}^-\le t_{44}\le t_{44}^+=0.043$$
$$0.025=t_{54}^-\le t_{54}\le t_{54}^+=0.062$$
$$0.018=t_{64}^-\le t_{64}\le t_{64}^+=0.043$$
$$0.021=t_{73}^-\le t_{73}\le t_{73}^+=0.052$$
$$0.035=t_{74}^-\le t_{74}\le t_{74}^+=0.087$$

The proposed optimization model is solved by the nonlinear optimal method. In order to test the validity of the proposed approach, a similar optimal model is also introduced. This model removes the quality loss from the objective function. The constraints are the same for these two different models. The optimization results of the two models are given in Table 2 for comparison. Obtained process tolerance $t_{11}, t_{12}, t_{23}, t_{25}, t_{31}, t_{32}, t_{44}$, and t_{64} are the same for both approaches. However, t_{24}, t_{54}, t_{73}, and t_{74} are different. For the proposed method, these tolerances are of smaller values to maintain less quality loss.

6 Concluding remarks

This paper has presented a robust optimization method in a concurrent tolerancing environment. This method can determine multiple correlated critical tolerances and directly allocate them to process tolerances by using component process plans.

In a concurrent environment, the product tolerance design and process tolerance design can be integrated into one stage. Tolerance design has been extended directly from the product design to the manufacturing stage. The necessity of redesign and rework between product tolerance design and process tolerance design has been eliminated, increasing the design efficiency. In a conventional tolerance design, the optimal model is established for two separate stages, and the optimum solutions are for different stages but not for the entire product design process.

Though Lee and Tang [1] in their research introduced a method to implement tolerance design for products with correlated characteristics, they only dealt with tolerancing problems within the product design stage. The basic method they used has now been extended profoundly to the concurrent environment to determine multiple correlated critical product tolerances, and then allocate them directly to pertinent process tolerances.

The purpose of this paper is to propose a robust optimum tolerance design method in a concurrent environment to balance the conflict design targets between manufacturing tolerances and product satisfaction. The design targets are quantified in monetary ways in the optimization objective function. The focus is on establishment of quality loss of product with multiple correlated critical tolerances in a concurrent tolerance design environment. The paper presents an approach to provide the product quality loss function, which is finally expressed as the function of process tolerances.

A wheel assembly example presented by Huang and Gao [5] has also been applied. The simulation results show the validity of the proposed method. If cost-tolerance function and related information of product quality loss are available, the rational tolerances can be obtained in actual design and production.

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