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ORIGINAL ARTICLE

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Feedrate optimization for variant milling process based on cutting force prediction

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Abstract Machining process modeling, simulation and optimization is one of the kernel technologies for virtual manufacturing (VM). Optimization based on physical simulation (in contrast to geometrical simulation) will bring better control of a machining process, especially to a variant cutting process – a cutting process so complex that cutting parameters, such as cutting depth and width, change with cutter positions. In this paper, feedrate optimization based on cutting force prediction for milling process is studied. It is assumed that cutting path segments are divided into micro-segments according to a given computing step. Heuristic methods are developed for feedrate optimization. Various practical constraints of a milling system are considered. Feedrates at several segments or micro-segments are determined together but not individually to make milling force satisfy constraints and approach an optimization objective. After optimization, an optimized cutting location data file is outputted. Some computation examples are given to show the optimization effectiveness.

Keywords Cutting force · Feedrate optimization · Milling process

1 Introduction

Although the area of cutting process is 'age-old', there remain many unsolved or unexplainable problems, and the development of manufacturing technology raises many new problems. To solve these problems, cutting process modeling, simulation and optimization is emphasized. It is regarded as one of the kernel technologies of virtual manufacturing (VM) [9]. In 1995, a 'machining operation modeling' work group was established in CIRP to stimulate the development of models which can be used to quantitatively predict the performance of metal machining operations [19]. In recent years, several state-of-the-art reviews have been published [6, 14, 19].

In this paper, cutting processes are classified into two types according to whether a cutter experiences the same process in each cutting cycle of a cut (a cutting cycle generally refers to a rotation of a cutter or a workpiece). If a cutter experiences the same process in each cutting cycle, the cutting process is called constant cutting process (CCP), and variant cutting process (VCP) otherwise. Today, VCP accounts for a significant percentage of production, especially in mould and die production. Unfortunately, much less work has been done on VCP than CCP. This paper focuses on the optimization of milling VCP based on physical simulation (cutting force prediction).

A CAM software system generates cutter location data (CLD) or an NC programme without consideration of the physical behaviours of a real machining process. Thus the generated CLD/NC for a VCP may produce a part which fails to meet quality requirements or cause damage such as cutter breakage. Moreover, most CAM systems only allow one to set machining parameters (spindle speed, feedrate, etc.) once for an operation (for example, to mill a cavity). To obtain a stable process and avoid cutter damage, accuracy violation, excessive deformation, vibration or failure of fixing, the selected parameters are often so conservative that efficiency is very low during a large part of the process. It is always difficult to modify the CLD/NC manually either because it requires complicated calculation or the CLD/NC is fairly long. That is why we need automatic/automated optimization. To optimize a VCP, we should first know how it works physically. Thus a good solution would be to take physical simulation results as the input for optimization.

2 Literature review

Although the research on machining parameter optimization with a consideration of part geometry and machining process physical behaviour dates back to the 1970s, for example [13] or earlier, it grew in the 1980s [3, 20] and boomed in the 1990s.

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There are several kinds of machining operations, and each machining operation has several parameters. However, this paper focuses on feedrate optimization for milling operation. Thus most references given in the following discussion have that same focus. Ideally, optimization of machining operation should consider cutting path (machining/feedrate direction is especially important) and parameters at the same time, especially for sculptured surfaces. Some work has been done in this direction, such as [8, 11, 12]. Lim and Menq [11, 12] proposed a cutting path adaptive feedrate strategy by which machining time was reduced by cutting along low-force-low-error machining directions and by maximizing feedrates. Feng and Su [8] studied an integrated approach to the concurrent optimization of tool path and feedrate for the finishing machining of 3D plane surfaces using ball-end milling. Cutting path generation is a very complicated and specialized area, and industry prefers the mature commercial CAM system for cutting path generation; thus most researchers study machining parameter optimization problems with existing cutting path data. Furthermore, combined optimization of cutting path and feedrate could mainly be done locally because a combination exploration problem is more serious when concurrently optimizing cutting directions and machining parameters at multiple positions of a sculptured surface. It is the authors' contention that combined and separated optimization of cutting directions and machining parameters both have their advantages and disadvantages.

Previous work on machining parameter optimization can be classified into three categories: experiment-based methods, mathematical optimization methods and analytical methods. Experiment-based methods use real cutting experiment data to establish a relationship between optimization objectives and machining parameters and then use the established relationship to solve a special application problem. Multi-regression is a traditional method used to established an explicit relationship. Recently, artificial neural networks (ANN) [4] have been widely used in an attempt to establish an implicit relationship. Experiment-based methods are applicable to CCP. However, they would fail to solve VCP optimization problems.

Mathematical optimization methods [1, 2, 5, 15, 17, 18] explicitly define objective functions and constraints by a group of mathematical expressions. Thus a machining process optimization problem is transformed into a mathematical optimization problem that can be resolved by various methods. By resolving the problem, all machining parameters are determined together. These methods take a cutter motion statement/NC instruction as the smallest object to be optimized. That is, a cutting path segment will not be divided. If cutting path segments are divided into many small sub-segments, the problem space will be so greatly increased that the problem becomes unresolvable. Thus the results of these methods are not so 'optimal'.

Analytical methods [7, 8, 11, 12, 16, 20, 21, 23, 24] try to either establish a new direct analytical expression of the relationship between an optimization objective and an individual local machining parameter or use an existing one. This implies the overall optimization objective can be separated into a set of independent local optimization objectives. Unlike mathematical optimization methods, a local optimal machining parameter can be determined directly from the analytical expression (sometimes by comparison/iteration like [8, 11]) without consideration of any parameters at other positions. Thus optimization calculation becomes fairly simple generally. In the early years, a whole cutting path segment was considered as an object to be optimized like mathematical optimization methods. In the mid-1990s, optimal local position feedrates within a cutting path segment were studied [16, 22]. However, the optimization was done locally and individually for a single sub-segment. In contrast to mathematical optimization methods, the results of local and individual feedrate optimization are too 'optimal' to reach in practice because feedrate cannot be changed at very high frequencies. So it is necessary to consider multiple segments or sub-segments together in optimization. This paper presents a heuristic method for this kind of feedrate optimization with a consideration of various practical machining constraints. In [10], Li et al. reported a solid model-based milling process simulation and optimization system called BetterCut. This paper represents follow-up research on BetterCut, in which the optimization functions are greatly improved.

3 Optimization for milling VCP based on cutting force prediction

Most machining parameter optimization objectives that have been studied, whether technological or economical, are directly or indirectly related to cutting force; thus the optimization of cutting force is essential. Here we assume that, prior to optimization, cutting path segments have been divided into micro-segments of small length and the average cutting force at each micro-segment has been predicted.

Then we define three cutting force optimization objectives: upper force limit, lower force limit and force range. Upper limit optimization makes the cutting force as great as possible without exceeding an upper limit $F_{\max}$, lower limit optimization makes the cutting force as low as possible without dropping below a lower limit $F_{\min}$, and range optimization makes the cutting force fall into a range between $F_{\max}$ and $F_{\min}$.

Like most other research, we take feedrate as a variant to be changed for optimization.

3.1 Some concepts

Before discussing the optimization procedure and algorithm, some concepts should be explained/introduced.

3.1.1 Cutting segment

A cutting segment is a statement in CLD that causes a non-rapid feeding real cutting.

3.1.2 Computing step

A computing step is a small length used to disperse cutting segments. Here it is denoted by δ. It is obvious that not every cutting segment has a length that can be divided by δ exactly. So the value of the computing step could be changed to a new value as near as possible to the original one so that a cutting segment could be divided into micro-segments with the same length.

3.1.3 Feedrate change interval/distance

If a different feedrate is set for every micro-segment, the control system of a machine tool will change the feedrate with a high frequency. That would be unreasonable in practice. This constraint can be given by a feedrate change interval or feedrate change distance, which refers to the time interval or distance that a cutter goes through between two neighbouring feedrate changes. They are denoted by Δt and ΔD, respectively.

3.1.4 Short segment and long segment

If the cutting time of a segment is less than the feedrate change interval ΔT, or the length of the segment is less than the feedrate change distance ΔD, that is, the cutting time or the length of the segment is very short, then it is regarded as a short segment, and a long segment otherwise. The optimization of a short segment will involve other segments, while the optimization of a long segment may be done within its micro-segments.

3.1.5 Segment division ratio/length

When multiple segments or micro-segments are optimized together, the solution may indicate that the feedrate of the micro-segments of one segment has different values. Changing the feedrate of the micro-segments of a segment means the segment will be divided into several new segments. The original cutting statement will be changed into several new cutting statements and feedrate statements when outputting optimized CLD. If the length of CLD is increased too much, two constraint parameters, segment division ratio and length, are introduced. They are denoted by R and ΔL, respectively. Only when the ratio of the length of a new segment to the length of the original segment is greater than R or the length of a new segment is greater than ΔL is the division executed. Either of the two parameters controls the length of the optimized CLD, while the feedrate change interval or the distance as mentioned above is used to make the feedrate change acceptable to the machining system.

3.1.6 Tiny segment ratio/length

Extra micro-segments that cannot fit either feedrate change interval/distance or segment division ratio/length constraints may be found at the end of a segment. Since micro-segments of two different segments cannot be combined into a new segment, there is a problem on how to deal with the extra micro- segments. Here, a tiny segment ratio or length parameters are introduced to control whether extra micro-segments can form a new segment or should be merged into the last new segment. The parameters are denoted by r and Δl, respectively. If the ratio of the length of a candidate new segment to the length of the original segment is less than r, or the length of the candidate new segment is less than Δl, the candidate new segment is regarded as a tiny segment. If extra micro-segments do not cause a tiny segment, they can become a new segment, or they must be merged into the last new segment. That is, tiny segments should be avoided.

3.1.7 Feedrate range

Every machine tool has its feedrate allowance, generally a range limit. Here the upper feedrate limit is denoted by $\text{Feed}_{\max}$ and the lower feedrate limit by $\text{Feed}_{\min}$.

3.1.8 Feedrate value factor

When the optimal feedrate for a new segment is a range, for example between Feed_1 and Feed_2 ($\text{Feed}_1 < \text{Feed}_2$), BetterCut uses a feedrate value factor to determine the final value. This factor is denoted by $\lambda(0 \leq \lambda \leq 1)$, and the final value of feedrate is $(1-\lambda)\text{Feed}_1 + \lambda\text{Feed}_2$.

3.1.9 Force factor

Because the computing step δ is generally a small value (for example 1 mm), in a micro-segment the cutting force can be considered proportional to feedrate. Thus the concept of force factor is introduced to indicate the sensibility of cutting force to feedrate. It is defined as the ratio of cutting force to feedrate. Force factor is an important property of a micro-segment. It also indicates a difference between VCP and CCP. To a CCP, every micro-segment has the same force factor, while to a VCP, one micro-segment may have a different force factor than another micro-segment because the geometry at one cutter position may be different to that at another position. When the feedrates of several micro-segments are optimized together, not only cutting forces but also force factors should be considered.

3.1.10 Tip and bottom micro-segments

Since a CAM system uses a single feedrate for an operation and force factors are different at cutter positions, the curve of the cutting force along the cutting path may have a significant tip and valley, as shown in Fig. 1a. The corresponding micro-segments are called as tip micro-segments and bottom micro-segments, respectively. Under the constraints of feedrate change interval/distance, if the tip/bottom micro-segments are optimized together with neighbouring micro-segments, the cutting forces of the neighbouring micro-segments may be increased/reduced too much, as shown in Fig. 1b. Thus sometimes tip and bottom micro-segments should be optimized apart from other micro-segments and without considering the constraint of feedrate change interval/distance. Then there is the problem of how to identify tip and bottom micro-segments. Here a parameter called the tip-bottom ratio is introduced and denoted as rtb. If the ratio of the force factor of a micro-segment to the average force factor of the whole cutting path is greater than $(1 + rtb)$ or less than

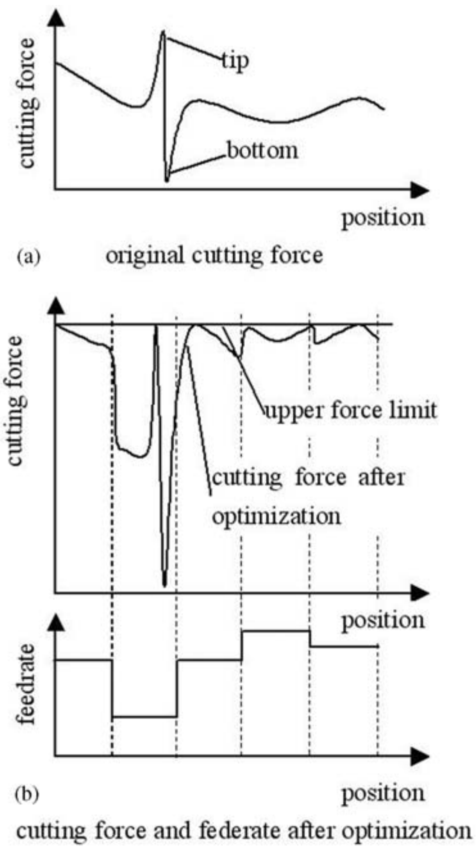


Fig. 1. Tip and bottom

$(1-rtb)$, the micro-segment is identified as a tip micro-segment or bottom micro-segment. To neighboring tip and bottom micro segments, whether they are optimized together or individually will produce different results. Theoretically, individual optimizations will get better results. However, it will cause longer CLD due to a greater feedrate change frequency, and the machining system may become unstable if it fails to follow the frequently changing feedrate. A switch is provided to set the optimization method for tip and bottom micro-segments. It is denoted as mtb. When mtb = TOGETHER, neighbouring tip and bottom micro-segments are optimized together; when mtb = INDIVIDUAL, they are optimized individually. Here the switch is provided for a user to make a tradeoff between possible impractical cutting parameters for tip/bottom micro-segments and a possible unintended cutting force of the neighbouring micro-segments.

3.1.11 Theoretically optimal feedrate

The theoretically optimal feedrate is the feedrate which makes the cutting force reach its objective. For upper limit optimization, the cutting force objective is the upper force limit denoted as $F_{\max}$, while for lower limit optimization, the lower force limit is denoted as $F_{\min}$. And for range optimization, a micro-segment has two theoretically optimal feedrates corresponding to $F_{\max}$ and $F_{\min}$.

3.2 Notations

In the previous section, some notations were given. They are listed here together for convenient reference:

δ	computing step
ΔT	feedrate change interval
ΔD	feedrate change distance
R	segment division ratio
ΔL	segment division length
R	tiny segment ratio
Δl	tiny segment length
$\text{Feed}_{\max}$	upper feedrate limit
$\text{Feed}_{\min}$	lower feedrate limit
λ	feedrate value factor
rtb	tip-bottom ratio
mtb	switch of optimization method for tip and bottom micro-segments
$F_{\max}$	upper force limit
$F_{\min}$	lower force limit

Here some other notations are introduced for the description of optimization procedures in the following sections. Suppose that a cutting path includes M cutting segments, which are denoted by $S_i (i = 1, 2, \ldots, M)$. For cutting segment S_i:

OldFeed_i	original feedrate
OldT_i	original cutting time
C_i^{ave}	average force factor
L_i	length
LS_i	property of short segment or long segment
TB_i	property indicating if any of its micro-segments are tip segment or bottom segment

During simulation, $S_i (i = 1, 2, \ldots, M)$ has been dispersed by computing step δ into n_i micro-segments $MS_{i,j} (j = 1, 2, \ldots, n_i)$. For micro-segment $MS_{i,j}$:

$L_{i,j}$	length
$\text{Old}F_{i,j}$	original cutting force
$\text{Opt}F_{i,j}$	optimal cutting force
$\text{Old}T_{i,j}$	cutting time
$\text{Opt}T_{i,j}$	optimal cutting time
$\text{OldFeed}_{i,}$	original feedrate
$\text{OptFeed}_{i,j}$	optimal feedrate
$C_{i,j}$	force factor
$TB_{i,j}$	property of tip or bottom micro-segment
$\text{TheFeed}_{i,j}^{\max}$	theoretically optimal feedrate corresponding to $F_{\max}$
$\text{The}T_{i,j}^{\max}$	cutting time according to $\text{TheFeed}_{i,j}^{\max}$
$\text{TheFeed}_{i,j}^{\min}$	theoretically optimal feedrate corresponding to $F_{\min}$
$\text{The}T_{i,j}^{\min}$	cutting time according to $\text{TheFeed}_{i,j}^{\min}$

During optimization, a common optimal feedrate is determined for several neighbouring micro-segments together. For micro-segments between $MS_{i,j}$ and $MS_{m,n}$:

$\text{OptFeed}^{\max}_{i\sim m,j\sim n}$	common optimal feedrate for upper limit optimization
$C^{\max}_{i\sim m,j\sim n}$	force factor used to determine $\text{OptFeed}^{\max}_{i\sim m,j\sim n}$
$\text{Opt}T^{\max}_{x,y}$	cutting time of $MS_{x,y}$ according to $\text{OptFeed}^{\max}_{i\sim m,j\sim n}$
$\text{OptFeed}^{\min}_{i\sim m,j\sim n}$	common optimal feedrate for lower limit optimization
$C^{\min}_{i\sim m,j\sim n}$	force factor used to determine $\text{OptFeed}^{\min}_{i\sim m,j\sim n}$
$\text{Opt}T^{\min}_{x,y}$	cutting time of $MS_{x,y}$ according to $\text{OptFeed}^{\min}_{i\sim m,j\sim n}$
$\text{OptFeed}^{\text{range}}_{i\sim m,j\sim n}$	common optimal feedrate for range optimization
$C^{\text{range}}_{i\sim m,j\sim k}$	force factor used to determine $\text{OptFeed}^{\text{range}}_{i\sim m,j\sim n}$
$F^{\text{range}}_{x,y}$	cutting force of $MS_{x,y}$ estimated by $\text{OptFeed}^{\text{range}}_{i\sim m,j\sim n}$
$\text{Opt}T^{\text{range}}_{x,y}$	cutting time of $MS_{x,y}$ according to $\text{OptFeed}^{\text{range}}_{i\sim m,j\sim n}$

3.3 Upper limit optimization

The objective of upper limit optimization is to make the cutting force along a cutting path as great as possible but not to exceed an upper limit. It can be used in rough machining to increase productivity under a safety condition (the upper limit).

3.3.1 Problem definition

Problem:

$$\text{Min}\sum_{i=1}^{M}\sum_{j=1}^{n_i}(F_{\max}-F_{i,j}).$$

Constraints:

1. Cutting force constraint: $F_{\max} \geq F_{i,j}$ if other constraints are not broken.
2. Feedrate constraint: $0 \neq \text{Feed}_{\min} \leq \text{OptFeed}_{i,j} \leq \text{Feed}_{\max}$.
3. Feedrate change constraint: For any group of neighbouring segments with the same feedrate, $S_p, S_{p+1}, S_{p+2}, \ldots, S_{p+q}$, it must be satisfied that $\sum_{m=p}^{p+q} T_m \geq \Delta T$ (or $\sum_{m=p}^{p+q} L_m \geq \Delta D$), unless the whole cutting path cannot satisfy this constraint.
4. Segment division constraint: Divide cutting segments as much as possible if for all new segments $NS_i (i = 1, 2, \ldots, N, N \geq 2)$ that an original cutting segment S_x is to be divided into, the condition that $L_i/L_x \geq R$ (or $L_i \geq \Delta L$) can be satisfied.
5. Tiny segment constraint: Suppose that an original cutting segment S_x is to be divided into N new segments $NS_i (i = 1, 2, \ldots, N, N \geq 2)$. For S_N, it must be satisfied that $L_N/L_x \geq r$ (or $L_N \geq \Delta l$).

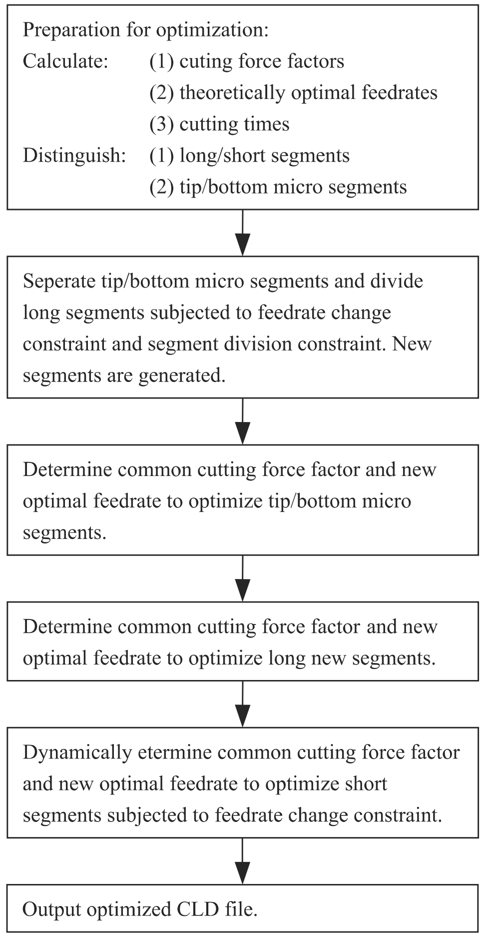


Fig. 2. Basic optimisation procedure

3.3.2 Basic optimization procedure

The algorithm to solve the optimization problem follows the basic procedure as shown in Fig. 2. Before optimization, the cutting force factor, theoretically optimal feedrate and cutting time of every micro-segment should be calculated. And tip/bottom micro-segments and long/short segments should be marked out. Then tip/bottom micro-segments are separated and long segments are divided. The original segments are transformed into a new list of segments. After that, the optimization is performed for new segments by the sequence of tip/bottom micro-segments, new long segments and short segments. During optimization, constraints are considered. Finally, an optimized CLD file is generated.

3.3.3 Algorithm details

Determination of $C^{\max}_{i\sim m,j\sim k}$ and $\text{OptFeed}^{\max}_{i\sim m,j\sim k}$: for upper limit optimization, a higher feedrate should be selected while keep the cutting force below the upper limit. $C^{\max}_{i\sim m,j\sim k}$ and $\text{OptFeed}^{\max}_{i\sim m,j\sim k}$ can be determined by the following steps:

1. Initiation: let $C_{\text{last}} = \boldsymbol{C}_{i,j}$, $\text{OptFeed}_{\text{last}} = \text{TheFeed}^{\max}_{i,j}$

2. Iteration:
   (a) Let $p = i$;
   (b) If $p = i$, let $q = j$; or let $q = 1$;
   (c) Let $C^{\max}_{i\sim p,j\sim q} = \mathrm{Max}(C_{\text{last}}, C_{p,q})$;
   (d) Let $\text{OptFeed}^{\max}_{i\sim p,j\sim q} = \text{TheFeed}^{\max}_{p,q}$, when

$$\begin{cases}\text{OptFeed}_{\text{last}} * C_{p,q} > \text{F}_{\max}\\ \text{TheFeed}^{\max}_{p,q} * C_{\text{last}} > \text{F}_{\max}\\ \text{OptFeed}_{\text{last}} * C_{p,q} > \text{TheFeed}^{\max}_{p,q} * C_{\text{last}}\end{cases}$$

(all are over the upper limit, select the lower one, as shown in Fig. 3a) or

$$\begin{cases}\text{OptFeed}_{\text{last}} * C_{p,q} > \text{F}_{\max}\\ \text{TheFeed}^{\max}_{p,q} * C_{\text{last}} \le \text{F}_{\max}\end{cases}$$

(select the one below the upper limit if another one is over the upper limit) as shown in Fig. 3b, or

$$\begin{cases}\text{OptFeed}_{\text{last}} * C_{p,q} \le \text{F}_{\max}\\ \text{TheFeed}^{\max}_{p,q} * C_{\text{last}} \le \text{F}_{\max}\\ \text{OptFeed}_{\text{last}} * C_{p,q} < \text{TheFeed}^{\max}_{p,q} * C_{\text{last}}\end{cases}$$

(all are below the upper limit, select the higher one, as shown in Fig. 3c);

   (e) Let $\text{OptFeed}^{\max}_{i\sim p,j\sim q} = \text{OptFeed}_{\text{last}}$, when

$$\begin{cases}\text{OptFeed}_{\text{last}} * C_{p,q} > F_{\max}\\ \text{TheFeed}^{\max}_{p,q} * C_{\text{last}} > F_{\max}\\ \text{OptFeed}_{\text{last}} * C_{p,q} \le \text{TheFeed}^{\max}_{p,q} * C_{\text{last}}\end{cases}$$

or

$$\begin{cases}\text{OptFeed}_{\text{last}} * C_{p,q} \le F_{\max}\\ \text{TheFeed}^{\max}_{p,q} * C_{\text{last}} \le F_{\max}\\ \text{OptFeed}_{\text{last}} * C_{p,q} \ge \text{TheFeed}^{\max}_{p,q} * C_{\text{last}}\end{cases}$$

or

$$\begin{cases}\text{OptFeed}_{\text{last}} * C_{p,q} \le F_{\max}\\ \text{TheFeed}^{\max}_{p,q} * C_{\text{last}} > F_{\max}\end{cases};$$

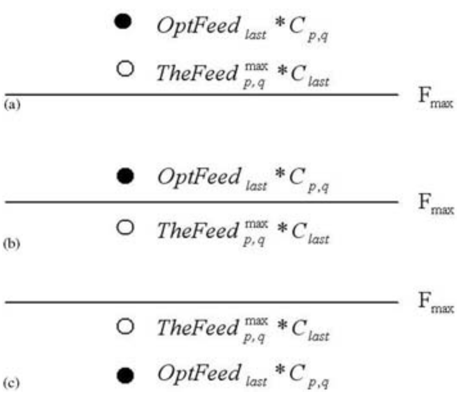


Fig. 3. Relation between $\text{TheFeed}^{\max}_{p,q}$ and $\text{OptFeed}_{\text{last}}$

   (f) Let $C_{\text{last}} = C^{\max}_{i\sim p,j\sim q}$, $\text{OptFeed}_{\text{last}} = \text{OptFeed}^{\max}_{i\sim p,j\sim q}$;
   (g) Let $q = q+1$; if $q > n_p$ or, $p = m$ and $q > k$, go to step (8), otherwise go to step (3);
   (h) Let $p = p+1$; if $p \le m$ then go to step (2); otherwise end the iteration.

Optimization procedure:

1. In the list of original segments, from S_1, for every cutting segment S_i and its micro-segments $MS_{i,j}(j = 1, 2, \ldots, n_i)$: (preparation for optimization)
   (a) Calculate force factors: let $C_{i,j} = \text{Old}F_{i,j}/\text{OldFeed}_{i,j}$;
   (b) Calculate average force factor: let

$$C^{\text{ave}}_i = \frac{\sum\limits_{i=1}^{M}\sum\limits_{j=1}^{n_i}(C_{i,j})}{\sum\limits_{i=1}^{M} n_i};$$

   (c) Calculate theoretically optimal feedrate $\text{TheFeed}^{\max}_{i,j}$:
      (i) If $C_{i,j} \neq 0$, then let $\text{TheFeed}^{\max}_{i,j} = F_{\max}/C_{i,j}$ (as mentioned above, in a micro-segment cutting force can be considered proportional to feedrate); otherwise, let $\text{TheFeed}^{\max}_{i,j} = \text{Feed}_{\max}$;
      (ii) If $\text{TheFeed}^{\max}_{i,j} > \text{Feed}_{\max}$, then let $\text{TheFeed}^{\max}_{i,j} = \text{Feed}_{\max}$;
      (iii) If $\text{TheFeed}^{\max}_{i,j} < \text{Feed}_{\min}$, then let $\text{TheFeed}^{\max}_{i,j} = \text{Feed}_{\min}$ (to meet the feedrate constraint);
   (d) Mark tip and bottom micro-segments:
      (i) If $C_{i,j} \ge C^{\text{ave}}_i(1+rp)$, then let $TB_{i,j} = \text{TOP}$;
      (ii) If $C_{i,j} \le C^{\text{ave}}_i(1-rp)$, then let $TB_{i,j} = \text{BOTTOM}$;
      (iii) Otherwise, $C^{\text{ave}}_i(1-rp) < C_{i,j} < C^{\text{ave}}_i(1+rp)$, then let $TB_{i,j} = \text{OTHER}$;
   (e) Calculate cutting time: let $\text{Old}T_i = L_i/\text{OldFeed}_i$, $\text{Old}T_{i,j} = L_{i,j}/\text{OldFeed}_{i,j}$;
   (f) Mark long and short segments: if

$$\sum_{j}^{\text{Old}T_{i,j}} < \Delta T \quad (\text{or } L_i < \Delta D),$$

   then let LS_i = SHORT, otherwise, let LS_i = LONG;
2. Set up an empty list for new segments;
3. In the list of original segments, from S_1, to any cutting segment S_i: (separate tip and bottom micro-segments, and divide long segments)
   (a) If LS_i = SHORT, then
      (i) Let $j = 1$;
      (ii) Let $k = j$;
      (iii) If $TB_{i,k} \neq TB_{i,j}$, then: Copy $MS_{i,m}(j \le m < k)$ to the end of the new segment list as a new short segment S_x, let $TB_x = TB_{i,j}$, its number of micro-segments $n_x = k - j$; let $j = k$, go to (ii);

(iv) If $k = n_i$, then: Copy $MS_{i,m}(j \leq m \leq n_i)$ to the end of the new segment list as a new short segment S_x, let $TB_x = TB_{i,j}$, its number of micro-segments $n_x = n_i - j + 1$; process next segment;
(v) Let $k = k + 1$; go to (iii);

(b) If LS_i = LONG, then

(i) Let $j = 1$;
(ii) Let $k = j$;
(iii) If $TB_{i,k} \neq TB_{i,j}$, then: Copy $MS_{i,m}(j \leq m < k)$ to the end of the new segment list as a new short segment S_x, let $TB_x = TB_{i,j}$, its number of micro-segments $n_x = k - j$; let $j = k$, go to (ii);
(iv) If $k = n_i$: If $\sum_{m=j}^{n_i} L_{i,m}/L_i \geq r$ or $\sum_{m=j}^{n_i} L_{i,m} \geq \Delta l$, then copy $MS_{i,m}(j \leq m \leq n_i)$ to the end of the new segment list as a new short segment S_x, let $TB_x = TB_{i,j}$, its number of micro-segments $n_x = n_i - j + 1$, process next segment; otherwise, copy $MS_{i,m}(j \leq m \leq n_i)$ to the end of last segment S_y in the new segment list, its number of micro-segments $n_y = n_y + n_i - j + 1$, if TB_y = OTHER, then $TB_y = TB_{i,j}$;
(v) Determine $C^{\max}_{i\sim i,j\sim k}$, $\text{OptFeed}^{\max}_{i\sim i,j\sim k}$ as described at the beginning of the algorithm;
(vi) Calculate $\text{Opt}T^{\max}_{i,m}$: $OptT^{\max}_{i,m} = \text{Old}T_{i,m}/\text{OldFeed}_{i,m} * \text{OptFeed}^{\max}_{i\sim i,j\sim k}$, $j \leq m \leq k$;
(vii) If $\sum_{m=j}^{k} \text{Opt}T^{\max}_{i,m} \geq \Delta T$ or $\sum_{m=j}^{k} L_{i,m} \geq \Delta D$, and $\sum_{m=j}^{k} L_{i,m}/L_i \geq R$ or $\sum_{m=j}^{k} L_{i,m} \geq \Delta L$, then: Copy $MS_{i,m}(j \leq m \leq k)$ to the end of the new segment list as a new long segment S_x, let $TB_x = TB_{i,j}$, its number of micro-segments $n_x = k - j + 1$; let $j = k$, go to (ii);
(viii) Let $k = k + 1$; go to (iii);

4. In the new segment list, from S_1, to every cutting segment S_i that its TB_i = TOP or TB_i = BOTTOM (optimize tip and bottom segments):

(a) If mtb = INDIVIDUAL, then:

(i) For every micro-segment $MS_{i,j}(j = 1, 2, \ldots, n_i)$: let $\text{OptFeed}_{i,j} = \text{TheFeed}^{\max}_{i,j}$, $\text{Opt}F_{i,j} = C_{i,j} * \text{OptFeed}_{i,j}$; copy $MS_{i,j}$ as a new segment S_x to before S_i in the new segment list, its number of micro-segments $n_x = 1$;
(ii) Delete S_i from the new segment list;

(b) If mtb = TOGETHER, then:

(i) Determine $C^{\max}_{i\sim i,1\sim n_i}$;
(ii) Determine $\text{OptFeed}^{\max}_{i\sim i,1\sim n_i}$;
(iii) For every micro-segment $MS_{i,j}(j = 1, 2, \ldots, n_i)$: let $\text{OptFeed}_{i,j} = \text{OptFeed}^{\max}_{i\sim i,1\sim n_i}$, $OptF_{i,j} = C_{i,j} * OptFeed_{i,j}$;

5. In the new segment list, from S_1, for every cutting segment S_i in which its TB_i = OTHER and LS_i = LONG: (optimize long segment)

(a) Determine $C^{\max}_{i\sim i,1\sim n_i}$, $\text{OptFeed}^{\max}_{i\sim i,1\sim n_i}$;
(b) For every micro-segment $MS_{i,j}(j = 1, 2, \ldots, n_i)$: let $\text{OptFeed}_{i,j} = \text{OptFeed}^{\max}_{i\sim i,1\sim n_i}$, $\text{Opt}F_{i,j} = C_{i,j} * \text{OptFeed}_{i,j}$;

6. In the new segment list, suppose the number of cutting segments is M', from S_1, for every group of neighbouring non-tip/bottom short cutting segments (TB_i = OTHER and LS_i = SHORT) S_p, $S_{p+1}, S_{p+2}, \ldots, S_{p+q}$ (optimize short segments):

(a) Let $l = p$;
(b) Let $k = l$;
(c) Determine $C^{\max}_{l\sim k,1\sim n_k}$, $\text{OptFeed}^{\max}_{l\sim k,1\sim n_k}$;
(d) Calculate: $\text{Opt}T^{\max}_{m,n} = \text{Old}T_{m,n}/\text{OldFeed}_{m,n} * \text{OptFeed}^{\max}_{l\sim k,1\sim n_k}$, $l \leq m \leq k$, $1 \leq n \leq n_m$;
(e) If $\sum_{m=l}^{k} \sum_{n=1}^{n_m} \text{Opt}T^{\max}_{m,n} \geq \Delta T$ (or $\sum_{m=l}^{k} L_m \geq \Delta D$), then:

(i) For every micro-segment $MS_{i,j}(i = l, l+1, \ldots, k,\ j = 1, 2, \ldots, n_i)$: let $\text{OptFeed}_{i,j} = \text{OptFeed}^{\max}_{l\sim k,1\sim n_k}$, $\text{Opt}F_{i,j} = C_{i,j} * \text{OptFeed}_{i,j}$;
(ii) Let $l = k + 1$, go to (b);

(f) If $k \neq p + q$, then $k = k + 1$, go to (c);
(g) If $k = p + q$, then:

(i) If there are no non-tip/bottom cutting segments before S_l and after S_k, then: For every micro-segment $MS_{i,j}$ $(i = l, l+1, \ldots, k,\ j = 1, 2, \ldots, n_i)$, let $\text{OptFeed}_{i,j} = \text{OptFeed}^{\max}_{l\sim k,1\sim n_k}$, $\text{Opt}F_{i,j} = C_{i,j} * \text{OptFeed}_{i,j}$; process next group of non-tip/bottom short cutting segments;
(ii) If there are no non-tip/bottom cutting segments before S_l, $k < M'$ and TB_{k+1} = OTHER, there must be LS_{k+1} = LONG, then: Determine $C^{\max}_{l\sim k+1,1\sim n_{k+1}}$, $\text{OptFeed}^{\max}_{l\sim k+1,1\sim n_{k+1}}$; for every micro-segment $MS_{i,j}$ $(i = l, l+1, \ldots, k+1,\ j = 1, 2, \ldots, n_i)$, let $\text{OptFeed}_{i,j} = \text{OptFeed}^{\max}_{l\sim k+1,1\sim n_{k+1}}$, $\text{Opt}F_{i,j} = C_{i,j} * \text{OptFeed}_{i,j}$; process next group of non-tip/bottom short cutting segments;
(iii) If $l > 1$ and TB_{l-1} = OTHER, and there are no non-tip/bottom cutting segments after S_k, then: search backward from l for a minimal x where $\text{OptFeed}_{x,1} = \text{OptFeed}_{x\ll 1,1} = \ldots = \text{OptFeed}_{l-1,1}$; determine $C^{\max}_{x\sim k,1\sim n_k}$, $\text{OptFeed}^{\max}_{x\sim k,1\sim n_k}$; for every micro-segment $MS_{i,j}$ $(i = x, x+1, \ldots, k,\ j = 1, 2, \ldots, n_i)$, let $\text{OptFeed}_{i,j} = \text{OptFeed}^{\max}_{x\sim k,1\sim n_k}$, $OptF_{i,j} = C_{i,j} *$ $\text{OptFeed}_{i,j}$; process next group of non-tip/bottom short cutting segments;
(iv) If $l > 1$ and TB_{l-1} = OTHER, $k < M'$ and TB_{k+1} = OTHER, then: Determine $C^{\max}_{l\sim k+1,1\sim n_{k+1}}$, $\text{OptFeed}^{\max}_{l\sim k+1,1\sim n_{k+1}}$; search backward from l for a minimal x where $\text{OptFeed}_{x,1} = \text{OptFeed}_{x+1,1} = \ldots = \text{OptFeed}_{l-1,1}$; Determine $C^{\max}_{x\sim k,1\sim n_k}$, $\text{OptFeed}^{\max}_{x\sim k,1\sim n_k}$;
If $C^{\max}_{l\sim k+1,1\sim n_{k+1}} * \text{OptFeed}^{\max}_{l\sim k+1,1\sim n_{k+1}} > C^{\max}_{x\sim k,1\sim n_k} * \text{OptFeed}^{\max}_{x\sim k,1\sim n_k}$, then for every micro segment $MS_{i,j}$ $(i = l, l+1, \ldots, k+1,\ j = 1, 2, \ldots,$

n_i), let $\text{OptFeed}_{i,j} = \text{OptFeed}^{\max}_{l\sim k+1,1\sim n_{k+1}}$, $\text{Opt}F_{i,j} = C_{i,j} * \text{OptFeed}_{i,j}$; otherwise, for every micro-segment $MS_{i,j}$ ($i = x, x+1, \ldots, k$, $j = 1, 2, \ldots, n_i$), let $\text{OptFeed}_{i,j} = \text{OptFeed}^{\max}_{x\sim k,1\sim n_k}$, $\text{Opt}F_{i,j} = C_{i,j} * \text{OptFeed}_{i,j}$; process next group of non-tip/bottom short cutting segments;

7. Generate optimized CLD according to the new segment list.

3.4 Lower limit optimization

The objective of lower limit optimization is to make the cutting force along a cutting path as low as possible but not drop below some lower limit. It can be used in semi-finishing or finishing machining to assure quality productivity while maintaining a given productivity (the lower limit). The problem definition and algorithm are similar to those for upper limit optimization.

3.5 Range optimization

The objective of range optimization is to make the cutting force along a cutting path fall into a range with as few segment divisions and feedrate changes as possible so that the size of CLD may not be increased very much. The control effect of the cutting force depends on the given range.

3.5.1 Problem definition

Problem: $\text{Min} \sum_{i=1}^{M} \sum_{j=1}^{n_i} G_{i,j}$, where

$$G_{i,j} = \begin{cases} F_{i,j} - F_{\max} & \text{when } F_{i,j} > F_{\max} \\ 0 & \text{when } F_{\min} \le F_{i,j} \le F_{\max} \\ F_{\min} - F_{i,j} & \text{when } F_{i,j} < F_{\min}. \end{cases}$$

Constraints:

1. Feedrate constraint: Try not to change feedrate and keep $0 \neq \text{Feed}_{\min} \le \text{OptFeed}_{i,j} \le \text{Feed}_{\max}$.
2. Feedrate change constraint: Same as upper limit optimization.
3. Segment division constraint: Try not to divide segments; for all new segments NS_i ($i = 1, 2, \ldots, N$, $N \ge 2$) which an original cutting segment S_x is to be divided into, the condition that $L_i/L_x \ge R$ (or $L_i \ge \Delta L$) can be satisfied.
4. Tiny segment constraint: Same as upper limit optimization.
5. Feedrate value constraint: When the above constraints are satisfied and the feedrate of a micro-segment $MS_{i,j}$ can be selected from Feed_{opt} to $\text{Feed}'_{\text{opt}}$ ($\text{Feed}_{\text{opt}} \ge \text{Feed}'_{\text{opt}}$), its feedrate should be selected as $\text{Feed}_{i,j} = \lambda \text{Feed}_{\text{opt}} + (1 - \lambda)\text{Feed}'_{\text{opt}}$.

3.5.2 Algorithm

The determination of $C^{\min}_{i\sim m,j\sim k}$ and $\text{OptFeed}^{\min}_{i\sim m,j\sim k}$ is similar to $C^{\max}_{i\sim m,j\sim k}$ and $\text{OptFeed}^{\max}_{i\sim m,j\sim k}$ as described in Sect. 3.3.3.

Optimization procedure: Similar to upper limit optimization but $C^{\max}_{i\sim m,j\sim k}$, $\text{OptFeed}^{\max}_{i\sim m,j\sim k}$, $C^{\min}_{i\sim m,j\sim k}$ and $\text{OptFeed}^{\min}_{i\sim m,j\sim k}$ are all considered and the feedrate value factor is used to determine the optimal feedrate.

3.6 Evaluation of optimization effectiveness

Some criteria are introduced to evaluate optimization effectiveness.

Optimization effect OE: Indicate how close the optimization result is to the ideal result (Fig. 4). For upper limit optimization, this is defined as

$$OE = \frac{\sum_{m=1}^{M'} \sum_{n=1}^{n_m} \text{Opt}F_{m,n}}{\sum_{m=1}^{M'} \sum_{n=1}^{n_m} F_{\max}};$$

for lower limit optimization

$$OE = \frac{\sum_{m=1}^{M'} \sum_{n=1}^{n_m} F_{\min}}{\sum_{m=1}^{M'} \sum_{n=1}^{n_m} \text{Opt}F_{m,n}};$$

and for range optimization

$$OE = 1 - \left(\sum_{m=1}^{M'} \sum_{n=1}^{n_m} \left(\left| F_{\max} - \text{Opt}F_{m,n} \right| + \left| \text{Opt}F_{m,n} - F_{\min} \right| - (F_{\max} - F_{\min}) \right) \right) \Bigg/ \left(2 \sum_{m=1}^{M'} \sum_{n=1}^{n_m} (F_{\max} - F_{\min}) \right).$$

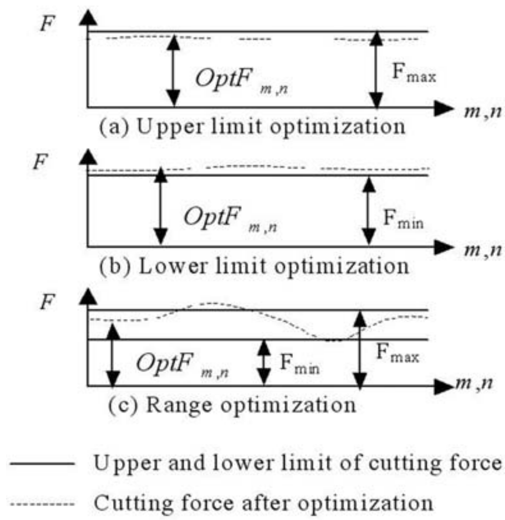


Fig. 4. Definition of optimization effect

The optimization effect is between 0 and 1.

Original cutting time OT and current (optimized) cutting time CT : $OT = \sum_{m=1}^{M} \sum_{n=1}^{n_m} \text{Old}T_{m,n}$, $CT = \sum_{m=1}^{M'} \sum_{n=1}^{n_m} \text{Opt}T_{m,n}$.

Productivity ratio PR: $PR = OT/CT$.

Length ratio of CLD length DLR: DLR = current length of CLD/original length of CLD.

Segment change number (division number) SCN: $SCN = M' - M$.

Feedrate change number FCN: Total change number of OptFeed_i ($1 \leq i \leq M'$).

4 Case study

4.1 The case

The part to be machined is shown in Fig. 5. A cylindrical hole with a 60-mm diameter, 4-mm greatest depth and 1-mm smallest depth is to be made on a cylindrical face. A slot-milling cutter with a diameter of 10 mm is to be used in the machining. The cutting path, a set of homocentric circles and several lines, is generated by the Manufacturing application of Unigraphics. The distance of two neighbouring circles, as well as the length of the line connecting the two circles, is 5 mm. When milling the hole along the cutting path, the materials to be removed and the geometrical cutting parameters change with the cutter positions. Thus it is a VCP. The cutting force will change with the cutter positions.

Since lower limit optimization is similar to upper limit optimization, we only demonstrate examples of upper limit optimization and range optimization in the following discussion. Parameters to be used in cutting, simulation and optimization are listed in Table 1. These parameters are set in BetterCut before simulation and optimization.

4.2 Cutting force prediction

In this case study, the Material Removing Rate (MRR) model is used to predict the average cutting force. The result is shown in Fig. 5. The cutting force is a continuously changing curve where tips such as A are at the positions on the line connecting two neighbouring circles of the cutting path, while bottoms such as B are at the end of a circle. This curve is dynamically drawn when BetterCut is running a simulation.

4.3 Upper limit optimization

Figure 6 shows the result of upper limit optimization. It can be seen that, after optimization, the cutting force curve is very close to the line of the upper force limit $F_{\max}$. At most positions its waviness is very small. So it is nearly constant cutting force machining. Because of the constraints of $\text{Feed}_{\max}$ and $\text{Feed}_{\min}$, at some positions where the original cutting forces are very large such as A, cutting forces cannot be optimized to below $F_{\max}$. Similarly, for some positions where the original cutting forces are very small such as B, cutting forces after optimization are still very small. The optimization effect reaches 0.92. So the cutting force is well controlled. And productivity is increased by $2.48/(9.69 - 2.48) \approx 34\%$. However, the length of CLD is increased by $136/(151 - 136) \approx 907\%$.

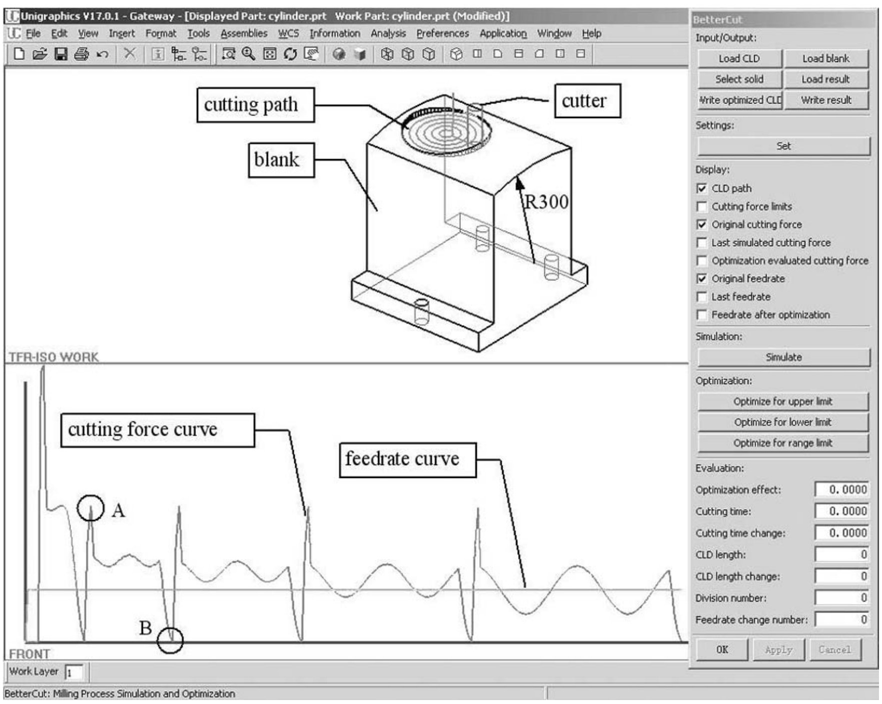


Fig. 5. Cutting force simulation result

Table 1. Parameters to be used in cutting, simulation and optimisation

Parameters	Upper limit optimization	Range optimization
Workpiece material	Steel(HB120)	Steel(HB120)
Cutter status	Sharp	Sharp
Cutter material	High speed steel	High speed steel
Spindle speed (rpm)	800	800
Original feedrate (mm/min)	40	40
Upper force limit F_{max} (N)	130	40
Lower force limit F_{min} (N)	-	20
Upper feedrate limit $Feed_{max}$ (mm/min)	200	20
Lower feedrate limit $Feed_{min}$ (mm/min)	20	5
Feedrate value factor λ	0.5	0.5
Feedrate change distance ΔD (mm)	4	4
Segment division length ΔL (mm)	3	3
Tiny segment length Δl (mm)	2	2
Computing step δ (mm)	2	2
Tip-bottom ratio rtb	0.5	0.5
Optimization method for tip/bottom micro segments mtb	TOGETHER	TOGETHER

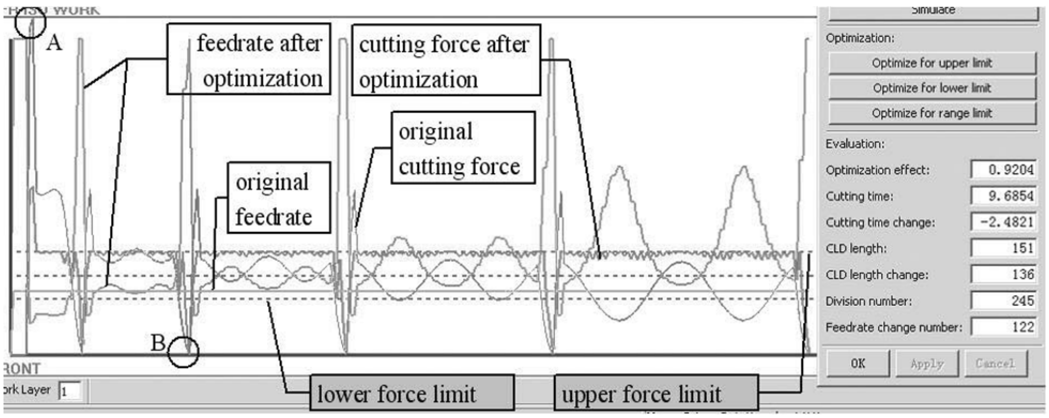


Fig. 6. Result of upper limit optimisation

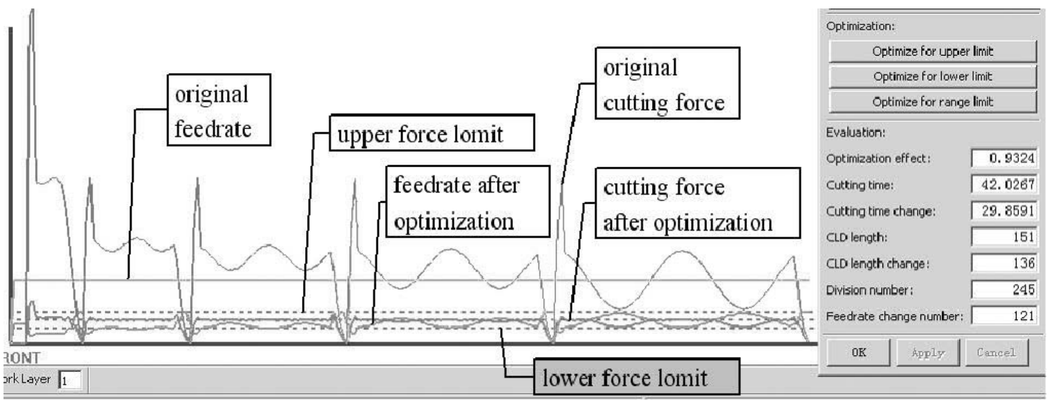
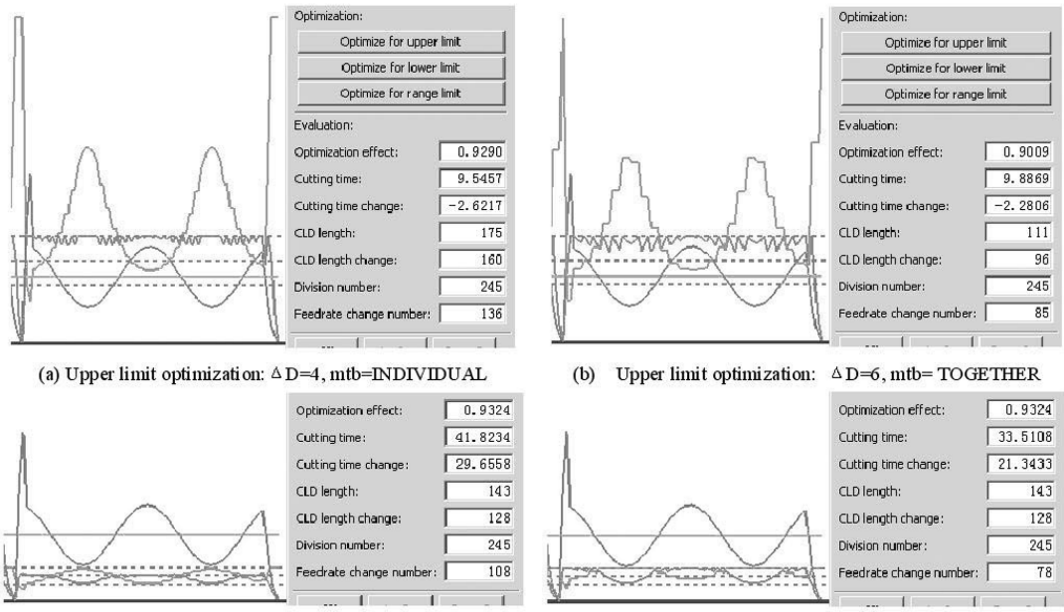


Fig. 7. Result of range optimisation

4.4 Range optimization

Figure 7 shows the result of range optimization. Because the feedrate value factor is set at 0.5, at most positions cutting forces after optimization are close to $(F_{max} - F_{min})/2$ and between F_{max} and F_{min}. The waviness is larger than that of upper limit optimization. Because of the constraints of $Feed_{max}$ and $Feed_{min}$, too, at some positions cutting forces go beyond (F_{min}, F_{max}). The optimization effect reaches 0.93, but productivity is decreased by $29.86/(42.03 - 29.86) \approx 245\%$, while the length of CLD is also increased by $136/(151 - 136) \approx 907\%$.

It can be seen that upper limit optimization always decreases its waviness, while the resulting waviness of range optimization depends on the given range, that is, the narrower the range, the smaller the waviness. Both optimizations lengthen CLD. The influence on productivity depends on the setting of the upper force



(a) Upper limit optimization: ΔD=4, mtb=INDIVIDUAL (b) Upper limit optimization: ΔD=6, mtb= TOGETHER

(c) Range optimization: ΔD=6, λ =0.5, mtb= INDIVIDUAL (d) Range optimization: ΔD=6, λ =1, mtb= INDIVIDUAL

Fig. 8. Comparison of optimisations with different parameters

limit and range. It can also be seen that the cost of cutting force optimization is the waviness of feedrate. This cost can be adjusted by Feed_{max} and Feed_{min}, and the change frequency can be adjusted by feedrate change interval Δt or feedrate change distance ΔD.

4.5 Comparison of optimizations with different parameters

To illustrate the influence of parameters on the optimization result, some tests are done as shown in Fig. 8. The influence of mtb can be seen from Fig. 6 and Fig. 8a. When mtb = TOGETHER, the cutting force curve generally will not have the horizontal lines which can be found when mtb = INDIVIDUAL. At the positions corresponding to these horizontal lines, the feedrate of every micro-segment is equal to its theoretical optimal feedrate, and thus the feedrate is changed frequently. This may be impossible in real machining. From Fig. 8a and 8b, the influence of different feedrate change distances can be compared. The greater ΔD, the fewer the feedrate changes, but the lower the optimization effect, and the greater the cutting force waviness. There is a problem about how to select ΔD in practice. An engineer's practical experience will be very helpful. For an unfamiliar machining system, cutting experiments can be designed and executed to determine a proper ΔD. Figures 8c and d show a comparison of influence of different feedrate value factors in range optimization. A greater λ makes the cutting force closer to the upper force limit and closer to the lower force limit otherwise. It can be seen that there is a significant difference in the feedrate change number when $\lambda = 1$ and when $\lambda = 0.5$ (from 108 to 78). This is interesting. The reason is that the two neighbouring groups of micro-segments, $\text{OptFeed}^{\text{max}}_{i\sim m, j\sim n}$ or $\text{OptFeed}^{\text{min}}_{i\sim m, j\sim n}$ may be the same (equal to Feed_{max} or Feed_{min}). In this case, using $\lambda = 1$ or $\lambda = 0$ may reduce the feedrate change. This would be useful in practical applications.

5 Conclusions

VCP is ubiquitous in production. Not only is the machining of free-form surfaces done by VCP, but the machining of a regular surface often introduces VCPs. Thus the optimization of VCP is important to industrial practice. The optimization methods proposed in this paper are based on cutting force prediction. The optimization is done for segments or micro-segments together so that optimization effectiveness can be very high while various practical constraints are met if possible. These heuristic algorithms are very efficient. For the case studies presented in Sect. 4, it takes a Pentium IV 1.4-GHz PC less than 1 s to finish an optimization. The case studies show the effectiveness to be very good.

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