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## Quick decision-making support for inspection allocation planning with rapidly changing customer requirements

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**Abstract** Manufacturing products with multiple quality characteristics is always one of the main concerns for an advanced manufacturing system. To assure product quality, finite automatic inspection systems should be available and employed. Inspection planning to allocate inspection stations should then be performed to manage limited inspection resources.

Since product variety in batch production or job shop production is increased to satisfy the changing requirements of various customers, the specified tolerance of each quality characteristic varies from time to time. Except for a finite inspection resource constraint, therefore, manufacturing capability, inspection capability, and tolerance specified by customer requirement are considered concurrently in this research. A unit cost model is constructed to represent the overall performance of an advanced manufacturing system. Both the internal and external costs of a multiple quality characteristic product are considered. The inspection allocation problem can then be solved to reflect customer requirements.

Since determining the optimal inspection allocation plan seems to be impractical as the problem size gets larger, in this research, two decision criteria (i.e., sequence order of workstation and tolerance interval) are employed concurrently to develop a heuristic method. The performance of the heuristic method is measured in comparison with an enumeration method that generates the optimal solution. The result shows that a feasible inspection allocation plan to meet changing customer requirements can be determined efficiently.

**Keywords** Inspection allocation · Inspection resource constraint · Multiple quality characteristics · Internal cost · External cost

### 1 Introduction

Since the manufacture of products with multiple quality characteristics is always one of the concerns of an advanced manufacturing system (AMS) that integrates successive automatic manufacturing stages (i.e. processes or workstations) to manufacture products. Automatic inspection stations (AIS) are employed to assure product quality. If there is only a finite inspection resource available, an inspection allocation problem occurs in a multistage manufacturing system. To solve the inspection allocation problem, two basic concerns are involved: at which workstation should an inspection activity be conducted, and which type of inspection station class should be used if an inspection activity is needed [1]. The solutions involve determining whether and what kind of inspection station class should be allocated immediately following each workstation in a multistage manufacturing system. Lee and Unnikrishnan pointed out this could occur in an electronic manufacturing industry that performs precision testing on package circuits or chips [1]. In small and medium industries, precision inspection is needed and only limited inspection resources are available [2, 3].

Depending on varying inspection capabilities and applications, inspection stations can be categorised into several inspection station classes. That is, each inspection station class consists of several inspection stations that have the same application usage and capability. To solve the inspection allocation problem, the solutions involve determining whether and what kind of inspection station class should be allocated immediately following each workstation in a multistage manufacturing system [1, 2, 3, 4]. There could be a finite number of inspection station classes available that are suitable for monitoring one or more workstations. Lee and Unnikrishnan (1998) solved the inspection allocation problem with this kind of inspection resource constraint [1]. The application of a limited number of inspection stations for each inspection station class has also been considered [2, 3].

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Product variety in batch production or job shop production increases to satisfy the changing requirements of various customers, and the tolerances specified vary from time to time. Therefore, manufacturing capability, inspection capability, and tolerance should be integrated to evaluate and improve the inspection performance [5, 6]. An inspection error model that deals with inspection capability, manufacturing capability, and tolerance was therefore applied to solve the inspection allocation problem that reflects the rapidly changing customer requirements [2, 3]. An AMS that fabricates only a single quality characteristic product was also studied [2] and an AMS that fabricates multiple quality characteristic products was also looked at, but it considered only the internal costs that occur in the system [3]. It is necessary to extend the research to consider the overall performance of an AMS that can depict not only product quality, but also the costs which occur in/after the manufacturing system, that is, both internal and external costs of an AMS should be analysed to solve the inspection allocation problem.

The objective of this paper is to solve the inspection allocation problem in an AMS that fabricates a multiple quality characteristic product while assuring product quality and establishing relative costs. Based on finite inspection resource constraints, a unit cost model is constructed to represent the overall performance of an advanced manufacturing system. Both the internal cost and external costs of a multiple quality characteristic product are considered. The inspection allocation problem can then be solved to reflect customer requirements. Since determining the optimal inspection plan seems to be impractical as the problem size becomes quite large, in this research, two decision criteria (i.e., sequence order of workstation and tolerance interval) are employed concurrently to develop a heuristic method. The performance of the heuristic method is measured in comparison with an enumeration method that generates the optimal solution. The result shows that a feasible inspection allocation plan to meet changing customer requirements can be efficiently determined. A feasible manufacturing plan can then be concurrently evaluated by solving the inspection allocation problem.

## 2 System and model analysis

The assumption that both manufacturing capability and inspection capability are normally distributed is made throughout this research. This is a common practice in using statistical methods (i.e., statistical quality control and process capability study) to solve real manufacturing problems, and also applies to the study of inspection capability [2, 3, 5, 6, 7, 8]. However, the probability density function in the following models is modified to use the actual validated distribution if necessary.

### 2.1 Notation

For system and model analysis, the following notations are used:

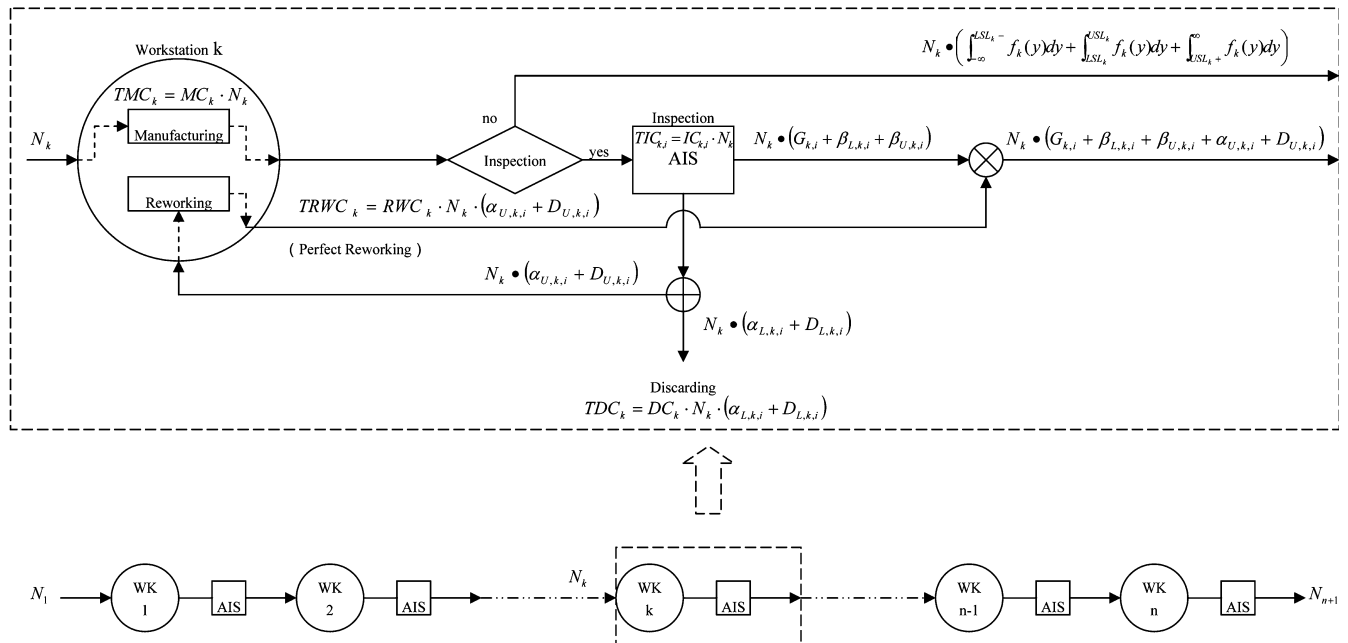
$k$ :	the $k$ th quality characteristic fabricated on workstation $k$ in the manufacturing system, $k = 1, \dots, n$ .
$f_k(y)$ :	manufacturing capability of workstation $k$ which is normally distributed as $f_k(y) \sim N(\mu_k, \sigma_k)$ , where $y$ is the true value of the $k$ th quality characteristic.
$m_k$ :	the target value of the $k$ th quality characteristic.
$LSL_k$ :	lower specification limit specified for quality characteristic $k$ (i.e., $m_k - K_{Lk}\sigma_k$ ).
$USL_k$ :	upper specification limit specified for quality characteristic $k$ (i.e., $m_k + K_{Uk}\sigma_k$ ).
$i$ :	inspection station class $i$ available for the manufacturing system, $i = 1, \dots, r$ .
$g_i(x/y)$ :	inspection capability of inspection station class $i$ and is normally distributed as $g_i(x/y) \sim N(x, \sigma_i)$ , where $x$ represents the mean value of the measurement data on a part with true value $y$ .
$NI_i$ :	available number of inspection station class $i$ for the manufacturing system.
$N_k$ :	number of parts entering workstation $k$ for manufacturing quality characteristic $k$ .
$N_{n+1,LSL}^+$ :	the expected number of products for which quality characteristics are all larger than $LSL_k$ .
$N_{n+1,Replacement}$ :	the expected number of products with at least one quality characteristic less than $LSL_k$ that will be sold and sent back for replacement with a new product.
$N_{n+1,k,Repair}$ :	the expected number of products that will be sold and sent back for repair of the $k$ th quality characteristic.
$N_{n+1,LSL\_USL}$ :	the expected number of conforming products for which quality characteristics are all within $[LSL_k, USL_k]$ .
$G_{k,i}$ :	expected probability that conforming units are correctly classified as good units by inspection station class $i$ after the workstation $k$ process.
$D_{L,k,i}$ :	expected probability that nonconforming units with quality characteristics less than $LSL_k$ are correctly classified as bad units by inspection station class $i$ after the workstation $k$ process.
$D_{U,k,i}$ :	expected probability that nonconforming units with quality characteristics larger than the $USL_k$ are correctly classified as bad units by inspection station class $i$ after the workstation $k$ process.

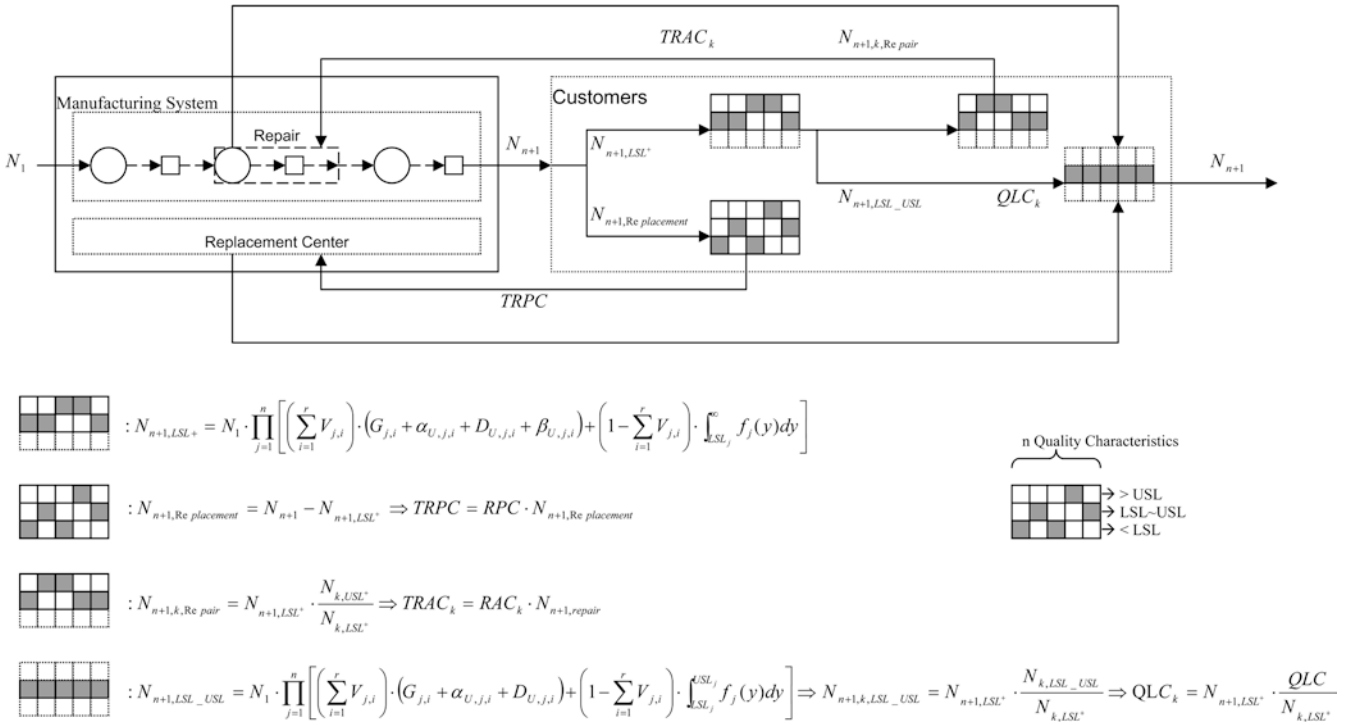
$\alpha_{L,k,i}$ :	expected type I error that conforming units are incorrectly classified as bad units with quality characteristics less than the $LSL_k$ by inspection station class $i$ after the workstation $k$ process.	$RWC_k$ :	unit rework cost of quality characteristic $k$ .
$\alpha_{U,k,i}$ :	expected type I error that conforming units are incorrectly classified as bad units with quality characteristics larger than the $USL_k$ by inspection station class $i$ after the workstation $k$ process.	$TRWC_k$ :	expected total rework cost of quality characteristic $k$ .
$\beta_{L,k,i}$ :	expected type II error that nonconforming units with quality characteristics less than the $LSL_k$ are incorrectly classified as good units by inspection station class $i$ after the workstation $k$ process.	$DC_k$ :	unit discard cost of quality characteristic $k$ .
$\beta_{U,k,i}$ :	expected type II error that nonconforming units with quality characteristics larger than the $USL_k$ are incorrectly classified as good units by inspection station class $i$ after the workstation $k$ process.	$TDC_k$ :	expected total discard cost of quality characteristic $k$ .
$MC_k$ :	unit manufacturing cost of quality characteristic $k$ .	$RPC$ :	unit replacement cost of a nonconforming product sold to a customer.
$TMC_k$ :	expected total manufacturing cost of quality characteristic $k$ .	$TRPC$ :	expected total replacement cost for nonconforming products sold to customers.
$IC_{k,i}$ :	unit inspection cost of inspection station class $i$ for quality characteristic $k$ .	$RAC_k$ :	unit repair cost of a nonconforming product sold to a customer for quality characteristic $k$ .
$TIC_{k,i}$ :	expected total inspection cost of inspection station class $i$ for quality characteristic $k$ .	$TRAC_k$ :	expected total repair cost for the $k$ th quality characteristic of nonconforming product sold to customers.
		$K_{q,k}$ :	coefficient of quality loss function for the $k$ th quality characteristic.
		$A_{c,k}$ :	the quality loss cost per part for the $k$ th quality characteristic and
		$QLC_k$ :	expected quality loss cost for the $k$ th quality characteristic of conforming products sold to customers.
		$UC$ :	expected unit cost of a product that sold to a customer.

Fig. 1 Internal cost analysis of a multiple quality characteristic manufacturing system

### 2.2 System description

There are several types of multistage manufacturing systems, such as, serial, non-serial, and a re-entrant hybrid of the serial or non-serial type. Each kind of manufacturing system should be solved considering its





**Fig. 2** External cost analysis of a multiple quality characteristic manufacturing system

different system characteristics and limitations. One serial multistage manufacturing system that fabricates products with multiple quality characteristics was studied in this research. As shown in Figs. 1 and 2, the system characteristics and limitations under consideration are as follows:

1. The manufacturing system integrates several successive workstations (i.e.,  $n$ ) to fabricate products serially in batches,  $N_1$ . None or only one inspection system can be assigned after each workstation to perform a 100% inspection.
2. Each workstation is responsible for manufacturing to a specific quality characteristic and has its own manufacturing capability,  $f_k(y)$ , that is normally distributed. The probability of producing nonconforming quality characteristics depends not only on the manufacturing capability, but also the tolerance specified,  $[LSL_k, USL_k]$ .
3. There are  $i$  finite inspection station classes available for this manufacturing system. Depending on usage and capability, each inspection station class could be suitable for performing inspection operations for one or more workstations. Each inspection station class also has its own specific capability,  $g_k(x/y)$ , that is normally distributed.
4. The number of  $NI_i$  available inspection stations in each class is finite. Each inspection station is assigned once and cannot be re-assigned to monitor other workstations in the middle of batch production. The inspection time for each inspec-

tion station can be represented by the inspection cost.

5. There are two kinds of inspection errors when applying any inspection station. Type I error,  $\alpha$ , occurs when a conforming part is rejected. Type II error,  $\beta$ , occurs when a nonconforming part is accepted. The inspection error of an inspection station is not a constant or has a specified probability. This variation depends not only on inspection capability and manufacturing capability, but also on specified tolerance [2, 3].
6. In case of  $\alpha_{L,k,i}$  and  $D_{L,k,i}$ , a part will be discarded if its  $k$ th quality characteristic is measured and found to be less than the lower specification limit.
7. In case of  $\alpha_{U,k,i}$  and  $D_{U,k,i}$  a part is considered perfectly reworked if its  $k$ th quality characteristic is measured and found to be larger than the upper specification limit.
8. In case of  $G_{k,i}$ ,  $\beta_{L,k,i}$  and  $\beta_{U,k,i}$ , a part will be sent to next workstation for further manufacturing.
9. In case of  $N_{n+1,Replacement}$ , a product sold to a customer is said to be in nonconformance when there is one or more nonconforming quality characteristic. The nonconforming product is then sent back for replacement with a new product when it has one or more nonconforming quality characteristic which is less than the relative lower specification limit and which cannot be repaired.
10. In the case of  $N_{n+1,k,Repair}$ , the nonconforming product will be sent back for repairing the  $k$ th quality characteristic when its nonconforming quality characteristics are all larger than the relative upper specification limits.

**Table 1** Analysis of manufacturing capability, inspection capability and tolerance [2, 3]

Measurement true dimension	$X < LSL_k$	$LSL_k \leq x \leq USL_k$	$X > USL_k$	Manufacturing capability
$y < LSL_k$	$D_{L,k,i}$	$\beta_{L,k,i}$	–	$\int_{-\infty}^{LSL_k} f_k(y)dy$
$LSL_k \leq y \leq USL_k$	$\alpha_{L,k,i}$	$G_{k,i}$	$\alpha_{U,k,i}$	$\int_{LSL_k}^{USL_k} f_k(y)dy$
$y > USL_k$	–	$\beta_{U,k,i}$	$D_{U,k,i}$	$\int_{USL_k}^{\infty} f_k(y)dy$

$$G_{k,i} = P(LSL_k \leq x \leq USL_k, LSL_k \leq y \leq USL_k) = \int_{LSL_k}^{USL_k} f_k(y) \int_{LSL_k}^{USL_k} g_i(x|y) dx dy$$

$$D_{L,k,i} = P(x < LSL_k, y < LSL_k) = \int_{-\infty}^{LSL_k} f_k(y) \int_{-\infty}^{LSL_k} g_i(x|y) dx dy$$

$$D_{U,k,i} = P(x > USL_k, y > USL_k) = \int_{USL_k}^{\infty} f_k(y) \int_{USL_k}^{\infty} g_i(x|y) dx dy$$

$$\alpha_{L,k,i} = P(x < LSL_k, LSL_k \leq y \leq USL_k) = \int_{LSL_k}^{USL_k} f_k(y) \int_{-\infty}^{LSL_k} g_i(x|y) dx dy$$

$$\alpha_{U,k,i} = P(x > USL_k, LSL_k \leq y \leq USL_k) = \int_{LSL_k}^{USL_k} f_k(y) \int_{USL_k}^{\infty} g_i(x|y) dx dy$$

$$\beta_{L,k,i} = P(LSL_k \leq x \leq USL_k, y < LSL_k) = \int_{-\infty}^{LSL_k} f_k(y) \int_{LSL_k}^{USL_k} g_i(x|y) dx dy$$

$$\beta_{U,k,i} = P(LSL_k \leq x \leq USL_k, y > USL_k) = \int_{USL_k}^{\infty} f_k(y) \int_{LSL_k}^{USL_k} g_i(x|y) dx dy$$

$$G_{k,i} + D_{L,k,i} + D_{U,k,i} + \alpha_{L,k,i} + \alpha_{U,k,i} + \beta_{L,k,i} + \beta_{U,k,i} = 1$$

$$\int_{-\infty}^{LSL_k} f_k(y) dy + \int_{LSL_k}^{USL_k} f_k(y) dy + \int_{USL_k}^{\infty} f_k(y) dy = 1$$

11. In the case of  $N_{n+1,LSL\_USL}$ , a product sold to a customer is said to be in conformance when there is no nonconforming quality characteristic. However, there may still be a quality loss due to functional variation that could cause customer dissatisfaction.

2.3 Cost model analysis

Generally, designers determine the upper and lower specification limits (USL/LSL) to establish the functional ability of products to satisfy the requirements of different customers. The specification limits are then directly applied to inspect and monitor manufacturing quality. Table 1 shows all possible situations when considering manufacturing capability, inspection capability and tolerance concurrently as specified [2, 3, 5, 6]. As shown in Fig. 1, the expected number of parts entering into a workstation depends on whether or not an automatic inspection station is used. Let  $\sum_{i=1}^r V_{k,i} = 1$  represent that there is an inspection station of the  $i$ th inspection class applied for monitoring the workstation  $k$ , otherwise,  $\sum_{i=1}^r V_{k,i} = 0$ . The number of parts entering the workstation  $k$ , where  $2 \leq k \leq n$ , then can be expressed as [2, 3]:

$$N_k = N_{k-1} \cdot \left[ \left( \sum_{i=1}^r V_{k-1,i} \right) \cdot (G_{k-1,i} + \alpha_{U,k-1,i} + D_{U,k-1,i} + \beta_{L,k-1,i} + \beta_{U,k-1,i}) + \left( 1 - \sum_{i=1}^r V_{k-1,i} \right) \right]$$

$$= N_1 \cdot \prod_{j=1}^{k-1} \left[ \left( \sum_{i=1}^r V_{j,i} \right) \cdot (G_{j,i} + \alpha_{U,j,i} + D_{U,j,i} + \beta_{L,j,i} + \beta_{U,j,i}) + \left( 1 - \sum_{i=1}^r V_{j,i} \right) \right] \tag{1}$$

The expected number of parts in a batch that sold to customers can be interpreted as:

$$N_{n+1} = N_1 \cdot \prod_{j=1}^n \left[ \left( \sum_{i=1}^r V_{j,i} \right) \cdot (G_{j,i} + \alpha_{U,j,i} + D_{U,j,i} + \beta_{L,j,i} + \beta_{U,j,i}) + \left( 1 - \sum_{i=1}^r V_{j,i} \right) \right] \tag{2}$$

The costs concerned in this research can be divided into internal costs and external costs. The internal costs, which occur inside the manufacturing system, include costs such as manufacturing, inspection, reworking, and discarding. An external cost occurs after the products have been sold to customers, such as the cost of replacement, repair, and quality loss. Based on Eqs. 1

**Table 2** Internal/external cost of a multiple quality characteristic manufacturing system

Internal cost	External cost
$TMC_k = MC_k \cdot N_k$	$TRPC = RPC \cdot N_{n+1,Replacement}$
$TIC_k = \left( \sum_{i=1}^r V_{k,i} \right) \cdot IC_{k,i} \cdot N_k$	$TRAC_k = RAC_k \cdot N_{n+1,k,Repair}$
$TRWC_k = \left( \sum_{i=1}^r V_{k,i} \right) \cdot RWC_k \cdot N_k \cdot (\alpha_{U,k,i} + D_{U,k,i})$	$QLC_k = N_{n+1,LSL^+} \cdot \frac{QLC}{N_{k,LSL^+}}$
$TDC_k = \left( \sum_{i=1}^r V_{k,i} \right) \cdot DC_k \cdot N_k \cdot (\alpha_{L,k,i} + D_{L,k,i})$	

and 2, the possible costs can be analysed for each quality characteristic of a product.

2.3.1 Internal cost analysis

As shown in Fig. 1, manufacturing cost occurs in every workstation and depends on  $N_k$ . Inspection cost depends not only on  $N_k$ , but also on whether and what kind of inspection station has been used. The rework cost occurs after workstation  $k$  only when the  $k$ th quality characteristic of a part is measured and found to be larger than  $USL_k$ . The discard cost occurs after workstation  $k$  only when the  $k$ th quality characteristic of a part is measured and found to be less than  $LSL_k$ . Table 2 lists the cost models of all possible internal costs [2, 3].

2.3.2 External cost analysis

2.3.2.1 Replacement cost and repair Cost To study the costs of replacement and repairing, the expected number of products for which quality characteristics are all larger than  $LSL_k$  should be examined and can be expressed as:

$$N_{n+1,LSL^+} = N_1 \cdot \prod_{j=1}^n \left[ \left( \sum_{i=1}^r V_{j,i} \right) \cdot (G_{j,i} + \alpha_{U,j,i} + D_{U,j,i} + \beta_{U,j,i}) + \left( 1 - \sum_{i=1}^r V_{j,i} \right) \cdot \int_{LSL_j}^{\infty} f_j(y) dy \right] \quad (3)$$

That is, the expected number of products with at least one quality characteristic which is less than  $LSL_k$  that will be sold and sent back for replacement can be expressed as:

$$N_{n+1,Replacement} = N_{n+1} - N_{n+1,LSL^+} \quad (4)$$

An estimate of the expected number products that will be sent back for repair of the  $k$ th quality characteristic,  $N_{n+1,k,Repair}$ , which is the expected number of the  $k$ th quality characteristic that is larger than  $USL_k$  and cannot be tested after workstation  $k$ , can be obtained as follows:

$$N_{k,USL^+} = N_k \cdot \left[ \left( \sum_{i=1}^r V_{k,i} \right) \cdot \beta_{U,k,i} + \left( 1 - \sum_{i=1}^r V_{k,i} \right) \cdot \int_{USL_k}^{\infty} f_k(y) dy \right] \quad (5)$$

Also, the expected number of the  $k$ th quality characteristic that is larger than  $LSL_k$  can be expressed as:

$$N_{k,LSL^+} = N_k \cdot \left[ \left( \sum_{i=1}^r V_{k,i} \right) \cdot (G_{k,i} + \alpha_{U,k,i} + D_{U,k,i} + \beta_{U,k,i}) + \left( 1 - \sum_{i=1}^r V_{k,i} \right) \cdot \int_{LSL_k}^{\infty} f_k(y) dy \right] \quad (6)$$

That is, the expected number of the products that will be sold and sent back for repair of the  $k$ th quality characteristic can be expressed as:

$$\begin{aligned} N_{n+1,k,Repair} &= N_1 \cdot \left( \frac{N_{1,LSL^+}}{N_1} \cdot \frac{N_{2,LSL^+}}{N_2} \cdot \dots \cdot \frac{N_{k,USL^+}}{N_k} \right. \\ &\quad \left. \dots \cdot \frac{N_{n,LSL^+}}{N_n} \right) \cdot \left( \frac{N_{k,LSL^+}}{N_k} \cdot \frac{N_k}{N_{k,LSL^+}} \right) \\ &= N_1 \cdot \left( \frac{N_{1,LSL^+}}{N_1} \cdot \frac{N_{2,LSL^+}}{N_2} \cdot \dots \cdot \frac{N_{k,LSL^+}}{N_k} \right. \\ &\quad \left. \dots \cdot \frac{N_{n,LSL^+}}{N_n} \right) \cdot \left( \frac{N_{k,USL^+}}{N_{k,LSL^+}} \right) \\ &= N_{n+1,LSL^+} \cdot \frac{N_{k,USL^+}}{N_{k,LSL^+}} \end{aligned} \quad (7)$$

Be aware that the repair cost occurs only when every nonconforming quality characteristic of a product is larger than  $USL_k$ , otherwise, a replacement cost will be incurred. A nonconforming product could still be replaced with a new product even if its  $k$ th quality characteristic is larger than  $USL_k$ , since its other quality characteristics could be less than the relative  $LSL_k$ . Table 2 also shows the models for replacement and repair costs.

2.3.2.2 Quality loss cost After studying the expected number of products that need to be replaced or repaired, the quality loss cost for the conforming products should be further interpreted. The quality loss defined by Taguchi is applied for the  $k$ th quality characteristic:

$$L_k(y) = K_{q,k} \cdot (y_k - m_k)^2 = \frac{A_{c,k}}{\Delta_k^2} (y_k - m_k)^2 \quad (8)$$

where  $y_k$  is the true value of the  $k$ th quality characteristic and  $\Delta_k = (USL_k - LSL_k)/2$ . Other kinds of quality loss function, however, should be developed for certain applications if necessary.

Let the expected number of the  $k$ th conforming quality characteristic that are manufactured in workstation  $k$  be:

$$\begin{aligned} N_{k,LSL-USL} &= N_k \cdot \left[ \left( \sum_{i=1}^r V_{k,i} \right) \cdot (G_{k,i} + \alpha_{U,k,i} + D_{U,k,i}) \right. \\ &\quad \left. + \left( 1 - \sum_{i=1}^r V_{k,i} \right) \cdot \int_{LSL_k}^{USL_k} f_k(y) dy \right] \end{aligned} \quad (9)$$

Be aware that a product could still be replaced or repaired even its  $k$ th quality characteristic is within the specification limits, since its other quality characteristics could be outside the limits. However, the expected number of conforming products for which their  $k$ th quality characteristics that are all within  $[LSL_k, USL_k]$  can be estimated by:

$$N_{n+1,k,LSL-USL} = N_{n+1,LSL^+} \cdot \frac{N_{k,LSL-USL}}{N_{k,LSL^+}} \quad (10)$$

As shown in Eq. 8, the quality loss function defined by Taguchi is employed to construct the quality loss cost model, however, it should be modified by considering manufacturing capability, inspection capability, and tolerance concurrently [3]. According to Eqs. 9 and 10, the quality loss cost model of the  $k$ th quality characteristic can be seen in Table 2. The value of  $QLC$  depends on whether an inspection station is applied after a workstation. If  $\sum_{i=1}^r V_{k,i} = 1$ , then

$$QLC = K_{q,k} \cdot \left[ \int_{LSL_k}^{USL_k} f_k(y)(y-m)^2 \times \int_{LSL_k}^{USL_k} g_i(x|y) dx dy + (\alpha_{U,k,i} + D_{U,k,i}) \cdot \int_{LSL_k}^{USL_k} f_k(y)(y-m)^2 dy \right] \quad (11)$$

Otherwise,

$$QLC = K_{q,k} \cdot \int_{LSL_k}^{USL_k} f_k(y)(y-m)^2 dy \quad (12)$$

As shown in Table 2, the unit cost model of a conformance product with multiple quality characteristics that sold to customer can be expressed as:

$$\text{Min. } UC = \sum_{k=1}^n (TMC_k + TIC_k + TRWC_k + TDC_k + TRAC_k + QLC_k) + TRPC / .N_{n+1} \quad (13)$$

subject to:

$$\sum_{i=1}^r V_{k,i} \leq 1 \quad (14)$$

$$\sum_{k=1}^n V_{k,i} \leq NI_i \quad (15)$$

Equations 14 and 15 show the inspection resource limitations. Equation 14 represents the finite inspection class resource constraint and that none or only one inspection station can be assigned after each workstation. Equation 15 shows that there are limited inspection stations available for each inspection station class.

### 3 Decision criteria and heuristic method

The inspection allocation problem can be solved since the unit cost model has been established. Early researchers applied optimal techniques, i.e., a

dynamic programming approach and non-linear integer programming to solve their own objective models [9, 10], however, these kinds of optimisation techniques become impractical as the problem size becomes large. It was proposed that a heuristic approach was more attractive to practitioners [1, 2, 3].

Based on identifying a nonconforming product as early as possible to reduce unnecessary successive costs, the sequence order of a workstation can be used as one of the criteria in developing a heuristic method. That is, the earlier a workstation is in a manufacturing stage, the higher the priority for it to be monitored by a suitable

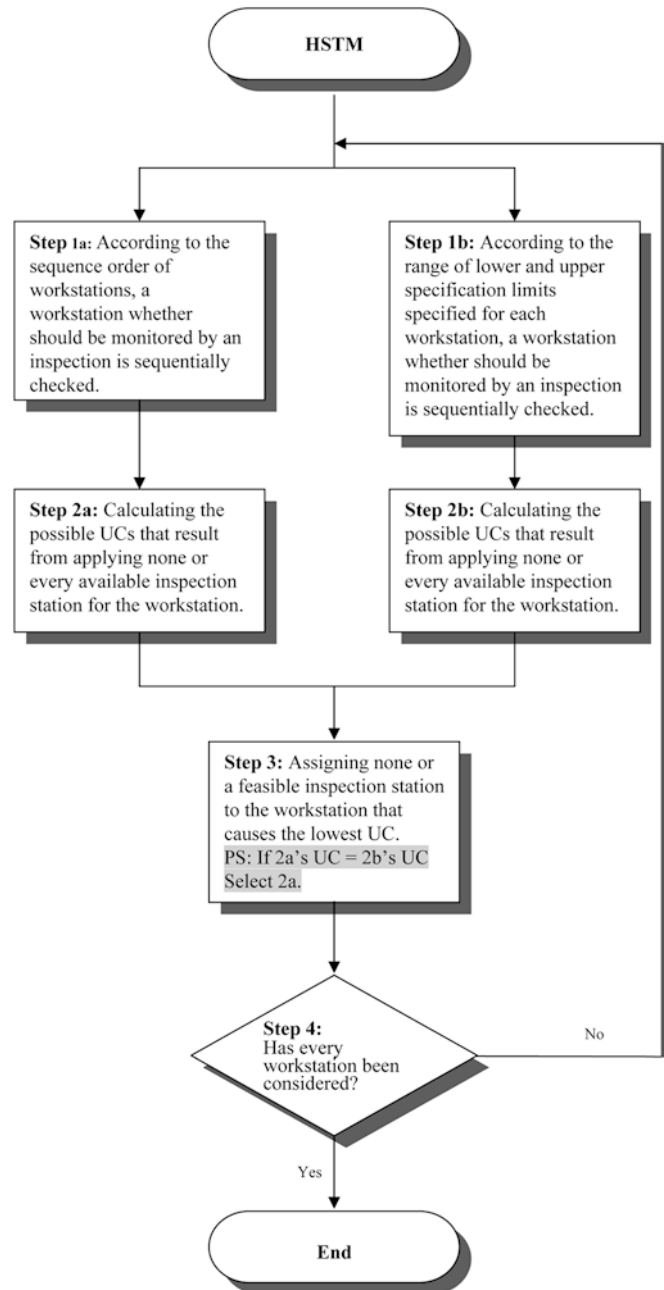


Fig. 3 Flowchart of the HSTM

**Table 3** Performance of the HSTM in comparison to EM

STM	Optimal cost similarity	Processing time savings
Performance	99.33%	98.63%
Parameter	Range	Deviation
$\sigma_k$	0.1–1.0	0.02
$[LSL_k, USL_k]$	$2.0-3.0 \sigma_k$	0.1
$\sigma_i$	$P/T = 6\sigma_i/USL - LSL$ $\Rightarrow 0.01-0.1$	0.001
$MC_k$	10–250	10
$RWC_k$	10–250	10
$DC_k$	10–250	10
$RAC_k$	10–400	10
$K_{q,k}$	10–400	10
$RPC$	1500–3000	10
$IC_{k,i}$	10–30	1

inspection station. Consequently, the higher the defective rate of workstation  $k$ , the more necessary it is to assign an inspection station for monitoring workstation  $k$ . A nonconforming product can then be screened out before entering the next workstation to reduce unnecessary successive costs. Since the defective rate is affected not only by the manufacturing capability of a workstation, but also by the tolerance specified for meeting the requirement of customers, the range of lower and upper of specification limits,  $K_{Lk} + K_{Uk}$ , can be the other criteria. Therefore, two decision criteria (i.e., sequence order of workstation and tolerance interval) are employed concurrently to develop the hybrid sequence/tolerance method (HSTM) to solve the inspection allocation problem. Figure 3 shows the procedure for the heuristic method. Heuristic performance is compared using an enumeration method (EM) that generates the optimal solution.

#### 4 Discussion and conclusion

The heuristic method was written in VBA and run using Microsoft Excel to measure its performance. A computer with Intel Pentium III 1 GHz CPU and 496 MB RAM was used. As mentioned in Sec. 2.3, there are many parameters that affect the unit cost model, such as manufacturing capability, inspection capability, tolerance, and the relative costs. A multistage manufacturing system with seven successive workstations and three inspection station classes was utilised to study cases with different parameters. The batch size was set to be 1000 for each case. It still takes time to run the EM with all possible parameter combinations. Therefore, thirty cases were generated randomly and run to evaluate the per-

formance of the heuristic method. As shown in Table 3, the parameter values of thirty cases were randomly generated using a uniform random number generator. Since most gauge capability studies were conducted to see if  $\sigma_i^2$  is small relative to  $\sigma_k^2$ , the precision-to-tolerance (P/T) ratio,  $6\sigma_i/USL_k - LSL_k$ , is applied and is at least less than 0.1. Therefore, the value of  $\sigma_i$  is randomly generated to provide a P/T ratio between 0.01 and 0.1. The inspection usage and resource constraint of each case are also generated randomly.

Table 3 shows that the expected unit cost of the inspection allocation plan determined by the HSTM is close to that of the optimal inspection allocation plan determined by the EM. The HSTM has a better processing time efficiency than the EM. The HSTM has an acceptable performance for production of expected unit cost and processing time in comparison with the EM. The result shows that a feasible inspection allocation plan can be determined efficiently for meeting customers' changing requirements. A feasible manufacturing plan can then be evaluated by concurrently solving the inspection allocation problem.

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