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ORIGINAL ARTICLE

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Research on an algorithm for an NC machining hypoid pinion

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Abstract Based on the flexibility characteristics of the NC machine tool, which means that various motions can be performed arbitrarily on NC machine tools, a new algorithm for manufacturing a hypoid pinion is proposed. This gets rid of the limitation of traditional mechanical machine tools and their algorithms. When the cutter tilt method is combined with the modification method, a method of manufacturing the pinion with a prescribed-size cutter on an NC machine tool can be realised. This helps to simplify cutter specifications and realise the error compensation of the cutter size. In this paper the algorithm is derived. According to the equations of cutting, the machine settings are calculated.

Keywords Spiral bevel and Hypoid gears · Hypoid pinion finishing · Cutting algorithm · Machine settings · NC machining

1 Introduction

This research is aimed at hypoid gears, which represent the most common kind of gearing. According to the Gleason method, the point radius of a pinion finishing cutter is calculated from the cutting process of the hypoid pinion [1,2,3,4,5]. This causes too many types of pinion finishing-cutters. Due to the great difference between the cutter radii for cutting the convex side and for cutting the concave side, two different cutters are needed for pinion finishing. If a cutter's size can be determined to be in a certain range, it helps to simplify cutter types and realise the error compensation for the cutter size. Furthermore, in the pinion finishing process, the convex and concave sides of the pinion teeth, respectively, can also be cut by the same cutter with spread blades during an index cycle on the CNC milling or grinding machine tool. How to make a new calculation model of the machine tool settings for hypoid pinion manufacture comprises the research content in this paper.

The limitations caused by the traditional mechanical machine tool can be dissolved by the NC machine tool. Therefore, we propose a new cutting algorithm by using the methods of the cutter tilt and the variable ratio of roll simultaneously. According to the principle of line contact, if the lengthwise curvature of the tooth surface at the mean point can be satisfied, the profile curvature cannot be ensured simultaneously when the cutter size is prescribed in a certain range. If the transmission ratio between the generating gear and work piece is variable, the change of the profile curvature of the tooth surface at its mean point can be compensated so as to satisfy the local conjugation after calculating a modified roll constant. Through this method, the imaginary conical generating gear, which has a pitch angle of less than 90°, is used to manufacture the hypoid pinion. The algorithm mainly applies to the pinion finishing on an NC milling or grinding machine tool. The error of the cutter radius can be compensated by modifying the given value of the cutter size.

To ensure a favorable meshing quality, a single side cutting method was used. That is, the pinion was manufactured by generation, each tooth side was cut separately and the calculation of the machine settings was made in the same two steps. To improve the meshing characteristic, the authors used the local conjugate method. This means the surfaces of hypoid gears were modified with a tooth profile and crowning in the manufacturing process. With the equation of meshing, the parameters, including the normal vector, the normal curvature and the geodesic curvature of the mean point could be worked out. Based on the abovementioned method, a new cutting calculation model for hypoid

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pinions could be made. The milling or grinding machine settings for hypoid pinion with teeth of tapered depth could be calculated by using the cutter whose size is prescribed accordingly.

2 The curvature and normal vector of the pinion tooth surface at the mean point

The calculated mean contact point M (also called the mean point) for the manufacture of hypoid gears is selected at the middle of the tooth lengthwise. Generally, the pinion tooth surface is assumed to be in contact with the gear generating surface at the point M, and the instantaneous transmission ratio for gear and pinion should be equal to the prescribed value [3]. The generating gear with uniform teeth is used to manufacture the hypoid gear. Usually, the cutting process of the gear tooth can be carried out either by the forming method or by generation one [4]. The generating gear makes a hypoid drive together with the produced gear. The mean point is the pitch point. The pressure angle at pitch point α_f is equal to the pressure angle of the gear root. The gear pitch angle δ_{f2} is equal to its root cone angle. The lengthwise curvature A_{f2}, profile curvature B_{f2} and torsion C_{f2} on the gear tooth surface at the mean point M can be worked out in the cutting calculation for the gear. The pitch cone of the gear cutting is also called the gear technological pitch cone [1]. For the pinion cutting calculation, the curvature and normal vector on the pinion tooth surface at the mean point must be calculated according to the conjugation relationship between the gear and the pinion. Two defined technological pitch cones illustrate the meshing relation between the gear and the pinion, as shown in Fig. 1. These pitch cones contact each other at the mean point M, and the pitch plane T contacts each cone along the generatrices H_1M and H_2M. The technological pitch cone angle $\delta_{\alpha 1}$ of the pinion is equal to its face angle. The coordinate system $S_{m2}(M,e_{\alpha 1},e_{\alpha 2},e_{\alpha 3})$ whose origin coincides with point M is made. The axis $e_{\alpha 1}$ coincides with the direction of the relative sliding velocity vector, which is tangential to the gear tooth trace at M. Axis $e_{\alpha 3}$ coincides with the common normal vector n through point M that is perpendicular to pitch plane T. Axes $e_{\alpha 1}$ and $e_{\alpha 2}$ are located in pitch plane T. Then axis $e_{\alpha 2}$ is obtained by the formula $e_{\alpha 2}=e_{\alpha 3}\times e_{\alpha 1}$. The common normal vector of the gear surface at point M is expressed as $\mathbf{n}_{M2}=[0\ \cos\alpha_f\ \sin\alpha_f]^T$. The coordinate system $S_1(A_1,\boldsymbol{i}_1,\boldsymbol{j}_1,\boldsymbol{k}_1)$ is rigidly connected to the pinion as shown in Fig. 2. Axis $\mathbf{i}_1$ coincides with axis of the pinion and axis $\mathbf{k}_1$ is perpendicular to the axis of the pinion through point M; thus, axis $\mathbf{j}_1$ is obtained by formula $\mathbf{j}_1=\mathbf{k}_1\times\mathbf{i}_1$. According to the principle of conjugation, the common normal vector to the pinion tooth surface at point M coincides with that of the gear tooth surface. So, in the coordinate system S_1, the common normal vector to the pinion tooth surface at point M is expressed as:

$$\mathbf{n}_M=\begin{bmatrix} n_{a1} \\ n_{a2} \\ n_{a3} \end{bmatrix}=\begin{bmatrix} \cos\beta_{a1} & \sin\beta_{a1} & 0 \\ -\sin\beta_{a1} & \cos\beta_{a1} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\delta_{a1} & 0 & \sin\delta_{a1} \\ 0 & 1 & 1 \\ -\sin\delta_{a1} & 0 & \cos\delta_{a1} \end{bmatrix}\begin{bmatrix} 0 \\ \cos\alpha_f \\ \sin\alpha_f \end{bmatrix} \quad (1)$$

where β_{a1} is the nominal spiral angle of gear, which is known.

According to the conjugate relation between gear and pinion, we can calculate the relative profile curvature $\tilde{B}_{12}$ of the gear tooth in a normal section and the angle ω_{12} that characterises the direction a tangent to the line of contact between the generated gear surface and pinion tooth surface. Let us denote the tooth lengthwise with a normal curvature of the pinion technological pitch cone as A_{a1}, the geodesic curvature as C_{a1}, the profile curvature as B_{a1} and the profile lateral relative curvature as $\tilde{B}_{12}$, then the equations for the curvatures of the pinion technological pitch cone at the mean point M are obtained as follows:

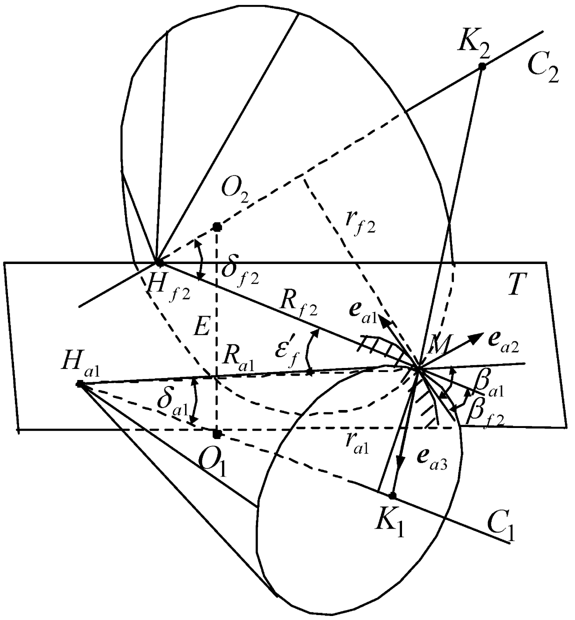

Fig. 1 The technological pitch cones of the pinion and the gear

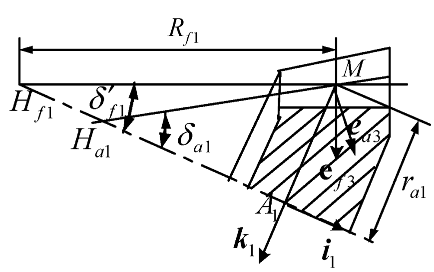

Fig. 2 The schematic of the pinion pitch cone

$$\begin{cases} A_{a1} = A_{f2} - \tilde{B}_{12} \tan^2 \omega_{12} \\ B_{a1} = B_{f2} - \tilde{B}_{12} \\ C_{a1} = C_{f2} + \tilde{B}_{12} \tan \omega_{12} \end{cases} \tag{2}$$

Thus, the common normal vector and the curvature of the pinion at point M can be calculated.

3 The determination of the pinion's finishing cutter radius

Applying the algorithm proposed in this paper, the point radius of the finishing cutter can be determined. Analogously to the way that the point radius of the pinion roughing cutter is determined, the pinion finishing cutter radius is given but not obtained by the cutting calculation. The point radius of cutter r_{01} can be expressed by the following:

$$r_{01} = r_0 \mp W_1 \tag{3}$$

where $W_1 = W_{L1}$−(0.7 ~ 1.0)

The upper minus sign applies to the inside blade, and the lower plus sign applies to the outside. W_{L1}, W_1 characterise the narrowest width of the pinion tooth bottom and pinion finishing cutter point width, respectively. The r_0 represents the mean radius of the pinion cutter which is selected according to the diameter of the gear blank, and the blade angle of the cutter is selected from the design standard. In addition, the point radius of cutter r_{01} can also be given directly in a certain range.

4 The algorithm for the manufacturing pinion

The generating gear with teeth of a uniform depth is used to manufacture the hypoid pinion. It makes a hypoid drive together with the produced pinion. In the pinion's cutting calculation, the calculated mean contact point is the pitch point. In the Gleason method, the point radius of the cutter and the forming radius r_M that is the projection of the mean curvature radius of the generating surface in the section coinciding with the pitch plane are determined by calculating. However, here the radius of the cutter is prescribed in a certain range. Therefore, the forming radius r_M is a certain value as shown in Fig. 3. Thus, the lengthwise curvature of tooth surface at the mean point can be ensured in the pinion finishing process according to the principle of conjugation, but the profile curvature cannot be ensured simultaneously. We can find out the change of the profile curvature, since the profile curvature can be modified in the cutting process by the variable transmission ratio of the angular velocity between the produced pinion and the generating gear. Therefore, the requirement of the profile curvature can be satisfied by solving the modified roll constant. As a result, we can manufacture the hypoid pinion with the prescribed size cutter. The algorithm for the pinion finishing is given as follows.

To cut the pinion tooth surface and the root surface at the same time, the pitch plane of the generating gear (parallel to the cutter plane) should be parallel to the pinion root cone generatrix. The pinion cutting pitch cone which passes through the pitch point M and is parallel to its root cone is different from the pinion technological pitch cone. Through their relationship, the normal vector and curvature of the pinion tooth surface at the mean point for the cutting pinion can be obtained.

Usually, pinion cutting pitch cones are characterised by pitch angle δ_{f1}, radius r_{f1} and pitch cone distance R_{f1}. When the pinion is cut with a generating cone gear, the real pinion root surface is different from the theoretical one [1]. We can then use $\Delta\theta$ as a modification of δ_{f1}. The formula can be described as:

$$\delta'_{f1} = \delta_{f1} - \Delta\theta \tag{4}$$

where the initial value of $\Delta\theta$ is zero.

The coordinate system $S_{m1}(M, e_{f1}, e_{f2}, e_{f3})$ whose origin coincides with point M is made. The axis e_{f1} coincides with the direction of the relative sliding velocity, which is tangential to the gear tooth trace at M. Axis e_{f3} coincides with the common normal vector **n** through point M, which is perpendicular to the pitch plane, and axes e_{f1}

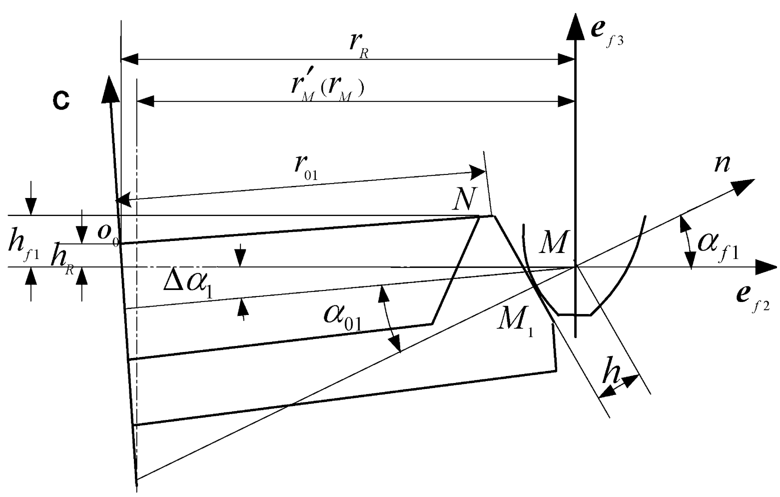

Fig. 3 The normal section of the pinion tooth at the mean point

and e_{f3} are located in the pitch plane. Thus, the axis e_{f2} is determined by the formula $e_{f2}=e_{f3}\times e_{f1}$. The common normal vector $\mathbf{n}_{1M}$ of the gear surface at point M is expressed as:

$$\mathbf{n}_{1M}=\begin{bmatrix} n_{f1} \\ n_{f2} \\ n_{f3} \end{bmatrix}=\begin{bmatrix} \cos\beta_{f1} & \sin\beta_{f1} & 0 \\ -\sin\beta_{f1} & \cos\beta_{f1} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\delta'_{f1} & 0 & \sin\delta'_{f1} \\ 0 & 1 & 1 \\ -\sin\delta'_{f1} & 0 & \cos\delta'_{f1} \end{bmatrix} \begin{bmatrix} 0 \\ \cos\alpha_{f1} \\ \sin\alpha_{f1} \end{bmatrix} \tag{5}$$

where α_{f1} is the pressure angle and β_{f1} is the spiral angle. Since Eq. 1 and Eq. 5 express the same common normal vector in S_1, then

$$\mathbf{n}_M=\mathbf{n}_{1M} \tag{6}$$

The spiral angle β_{f1} and the pressure angle α_{f1} can be obtained from Eq. 6:

$$\sin\alpha_{f1}=\sin\alpha_f\cos\left(\delta_{a1}-\delta'_{f1}\right) - \cos\alpha_f\sin\left(\delta_{a1}-\delta'_{f1}\right)\sin\beta_{a1} \tag{7}$$

$$\cos\beta_{f1}=\frac{\cos\alpha_f\sin\beta_{a1}}{\cos\alpha_{f1}} \tag{8}$$

The pitch cone of pinion cutting is not the technological pitch cone because the hypoid gears, with teeth of a tapered depth, are cut by the generating gear with teeth of a uniform depth. Therefore, the pinion tooth lengthwise direction e_{a1} of the technological pitch cone does not coincide with the tooth lengthwise direction e_{f1} of the pinion cutting pitch cone, but they are located in tangent plane T_p at point M as shown in Fig. 4. And the angle Δ between them is expressed by the following trigonometric function [5]:

$$\sin\Delta=\frac{e_{a1}\times e_{f1}}{t_{a1}t_{f1}}=\frac{\cos\beta_{a1}}{\cos\alpha_{f1}}\sin\left(\delta_{a1}-\delta'_{f1}\right) \tag{9}$$

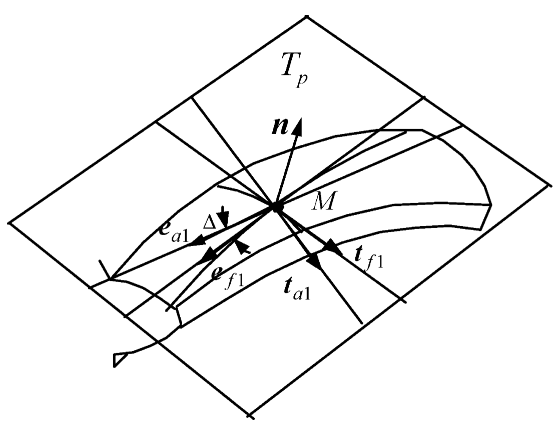

Fig. 4 A schematic of the tangent plane at the mean point

The curvature parameters A_{a1}, B_{a1},C_{a1} of the technological pitch cone at point M have been calculated before. The vector e_{a1} and t_{a1} represent the tooth lengthwise and the profile directions of the technological pitch cone on the tangent plane T_p at the point M. The vector t_{a1} is determined by the formula $t_{a1}=\boldsymbol{n}\times e_{a1}$. The normal curvature and the geodesic curvature on the pinion tooth surface at the mean point along e_{f1} direction are, respectively, denoted by A_{f1}, C_{f1}. Furthermore, the normal curvature along t_{f1} direction that is determined by formula $t_{f1}=\boldsymbol{n}\times e_{f1}$ is represented by B_{f1}, Now the equations can be obtained from Euler's formula as the following:

$$\begin{cases} A_{f1}=A_{a1}\cos^2\Delta+B_{a1}\sin^2\Delta-C_{a1}\sin 2\Delta \\ B_{f1}=A_{a1}\sin^2\Delta+B_{a1}\cos^2\Delta+C_{a1}\sin 2\Delta \\ C_{f1}=(A_{a1}-B_{a1})\sin\Delta\cos\Delta+C_{a1}\left(\cos^2\Delta-\sin^2\Delta\right) \end{cases} \tag{10}$$

Because the lengthwise and profile direction vectors $\mathbf{e}_{f1}$, $\boldsymbol{e}_{f1}$ are the two principal directions on the generating gear tooth surface at the mean point, the normal curvature along direction $\mathbf{e}_{f1}$ of cutter crown is expressed by:

$$A_{01}=-\frac{\cos\alpha_{f1}}{r_M} \tag{11}$$

Since the cutter size is prescribed, the forming radius is also a certain value. The forming radius of the mean point can be obtained from the geometric relationship shown in Fig. 3:

$$r_M=\frac{r_{01}\cos\alpha_{f1}+h\cos\left(\alpha_{f1}-\alpha_{01}\right)+h_{f1}\sin\alpha_{01}}{\cos\alpha_{01}} \tag{12}$$

where α_{01} is the blade angle of the pinion head cutter.

Because of the straight blade cutting edges, the profile tooth curvature of the generating gear B_{01} is equal to zero, and the geodesic curvature C_{01} is equal to zero. In addition, the relative normal curvature of the contact line along the lengthwise direction is zero [6]. The lengthwise curvature of the tooth surface at the mean point can be ensured in the pinion finishing process according to the principle of conjugation, but the profile curvature cannot be ensured simultaneously. To avoid bias contact, the torsion along the length direction of teeth is held to the line. Therefore, we can get the following identity from the conjugate principle of line contact [7]:

$$\left(A_{01}-A_{f1}\right)\left(B_{01}-B'_{f1}\right)=\left(C_{01}-C_{f1}\right)^2 \tag{13}$$

or

$$\left(-\frac{\cos\alpha_{f1}}{r_M}-A_{f1}\right)\left(-B'_{f1}\right)=\left(-C_{f1}\right)^2 \tag{14}$$

Here the B'_{f1} is the profile curvature. Then,

$$B'_{f1} = \frac{C_{f1}^2}{A_{f1} - A_{01}} \quad (15)$$

$$\mathrm{tg}\omega_2 = -\frac{C_{f1}}{B'_{f1}} \quad (16)$$

The angle ω_2 characterises the direction of a tangent to the line of contact between the generating surface and the pinion produced surface at the point M. The profile curvature B'_{f1} is unequal to the required curvature B_{f1}. Therefore, the requirement of the profile curvature of tooth surface does not need to be satisfied. The change is expressed by the following:

$$\Delta\left(\frac{1}{B_{f1}}\right) = \frac{1}{B'_{f1}} - \frac{1}{B_{f1}}; \quad (17)$$

Since the angular velocity of the generating gear can be changed by the numerical control in the machining, the value of the ratio of roll may be nonconstant. According to the relative curvature equation, the modification method can be used for modifying the profile curvature; the angular velocity of the generating gear can be deonoted as ω_{01}, and the angular acceleration as ϵ_{01}. The following equation is defined as:

$$2c = \frac{\varepsilon_{01}}{\omega_{01}^2} \quad (18)$$

The value of $2c$ is called the modified roll constant. According to the principle of the infinitesimal geometric, the following equation is expressed through differentiating to $1/B_{f1}$:

$$\Delta\left(\frac{1}{B_{f1}}\right) = -(2c)\frac{\cos\alpha_{01}}{R_{01}\cos\delta_{01}\cos\beta_{01}\left(\frac{1}{R_{f1}tg\delta_{f1}} + \frac{1}{R_{01}tg\delta_{01}}\right)^2} \quad (19)$$

where the R_{01}, δ_{01} and β_{01} are the pitch cone mean distance, pitch cone angle and spiral angle of the generating gear, respectively. These parameters can be determined by the following equations.

As a result:

$$(2c) = -\Delta\left(\frac{1}{B_{f1}}\right)R_{01}\sin\delta_{01}\cos\beta_{01}\left(\frac{1}{R_{f1}tg\delta_{f1}} + \frac{1}{R_{01}tg\delta_{01}}\right)^2 \times \frac{1}{\cos\alpha_{01}} \quad (20)$$

Due to obtaining $2c$, the revolution angle of the pinion is expressed by the following equation:

$$\varphi = i_{01}\left(\Delta q_1 - c\cdot\Delta q_1^2\right) \quad (21)$$

where Δq_1 is the increment of the revolution angle of the generating gear, and i_{01} is the theoretical ratio of the roll at the contact point. Thus, the change of profile curvature may be removed through the variable ratio of roll. Knowing the parameters r_M and B'_{f1}, ω_2 on the conjugate tooth surfaces of the generating gear and the pinion at pitch point M, it is easy to determine the equations of pinion cutting for calculating machine settings according to the principle of meshing.

5 The determination of the generating gear pitch cone and the machine tool settings

Since the generating gear pitch cone is conical, the pitch angle δ_{01} is different from 90º. Based on the differential geometry, the curvature parameters: $\mathrm{tg}\omega_2$, $\tilde{B}_{01} = -B'_{f1}$ of the generating gear tooth surface and pinion tooth surface at pitch point M can be described in the following equations [6]:

$$\begin{cases} \mathrm{tg}\omega_2 = \frac{R_{f1}\mathrm{tg}\delta'_{f1}R_{01}\mathrm{tg}\delta_{01}}{R_{f1}\mathrm{tg}\delta'_{f1}+R_{01}\mathrm{tg}\delta_{01}}\left[\left(\frac{\mathrm{tg}\beta_{f1}}{R_{f1}\mathrm{tg}\delta'_{f1}}+\frac{\mathrm{tg}\beta_{01}}{R_{01}\mathrm{tg}\delta_{01}}\right)\sin\alpha_{f1} - \left(\frac{1}{R_{f1}\cos\beta_{f1}}-\frac{1}{R_{01}\cos\beta_{01}}-\frac{\mathrm{tg}\beta_{f1}-\mathrm{tg}\beta_{01}}{r_M}\right)\cos\alpha_{01}\right] \\ \frac{1}{B'_{f1}}\left(\frac{1}{R_{01}\mathrm{tg}\delta_{01}}+\frac{1}{R_{f1}\mathrm{tg}\delta'_{f1}}\right)^2 = \left(\frac{tg\beta_{f1}}{R_{01}\cos\beta_{01}}-\frac{tg\beta_{01}}{R_{f1}\cos\beta_{f1}}\right)\cos\alpha_{f1} + \left[\left(1+tg\beta_{01}tg\beta_{f1}\right)\sin\alpha_{f1}+\left(tg\beta_{f1}-tg\beta_{01}\right)tg\omega_2\right]\left(\frac{1}{R_{01}\mathrm{tg}\delta_{01}}+\frac{1}{R_{f1}\mathrm{tg}\delta_{f1}}\right) \end{cases} \quad (22)$$

We can then get the equations of the pinion cutting which the parameters of generating gear pitch cone R_{01}, β_{01} and δ_{01} must satisfy. Since there are 3 unknowns, but only 2 equations, the solution is uncertain. Before assigning the solution, we can define $\beta_{01} = \beta_{f1}$, and assume the mean generating gear cone with a mean distance of $R_{01} = R'_{01}$, pitch angle $\delta_{01} = \delta'_{01}$ in this case. Then we can get its special solution R'_{01}, $tg\delta'_{01}$. If we introduce a third order of modifying value Δk, then we can assume:

$$\frac{1}{R_{01}\mathrm{tg}\delta_{01}} = \frac{1}{R'_{01}\mathrm{tg}\delta'_{01}} + \Delta k \quad (23)$$

In the initial calculation, we can also grant Δk as zero.

The generating gear is assumed to be meshing at pitch point M with the pinion. The geometry relation can be described in Fig. 5. The shaft angle between generating gear and pinion is $90° + \delta_{M1}$, where:

$$\sin\delta_{M1} = \sin\delta'_{f1}\sin\delta_{01} - \cos\delta'_{f1}\cos\delta_{01}\cos\varepsilon'_{01} \quad (24)$$

Here

$$\varepsilon'_{01} = \beta_{f1} - \beta_{01} \quad (25)$$

The value of the machine root angle δ_{M1} is used as a modification constant. Parameter Δk can be modified by an iterative method in the calculation of the cutting process to agree with Eq. 24. By solving the above equations, the pitch cone parameters R_{01}, δ_{01}, β_{01} of the generating gear for the pinion can be calculated, and then the machine-tool settings, such as radial setting S_1,

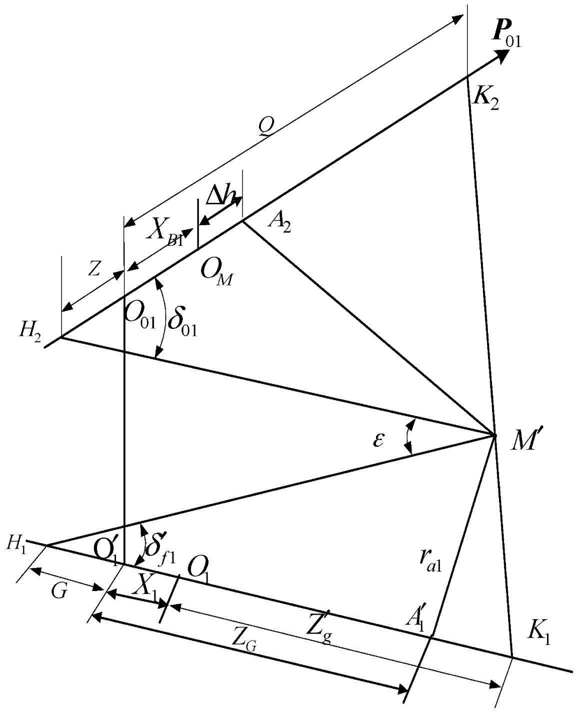

Fig. 5 The relative geometric position of the generating gear and the pinion in meshing

cradle angle q_1, the tilt angle i, the basic swivel angle j and the parameters of pinion blank position δ_{M1}, blank offset E_{01}, machine center to back X_1 and sliding setting X_{B1}, can also be determined by the geometric relation of the generating gear and pinion as shown in Fig. 5 [6]. Through the pinion's machine tool settings, we can calculate the tool trace on the NC machine tool for the manufacturing pinion.

6 A numerical example

The design parameters of the gear drive are represented in Table 1. The data for the pinion head-cutter that generates the pinion's concave side and convex side are represented in Table 2. The pinion machine-tool settings calculated by the above algorithm are represented in Table 3. By using simulation software we have developed by a method called 'Z-Buffer', the simulation results of the generated pinion are shown in Fig. 6. To evaluate the meshing quality of the sets calculated by this new method, the contact pattern and the transmission error as shown in Fig. 7 and 8 can be obtained by using the tooth contact analysis [8, 9]. The results are satisfactory.

Table 1 Blank data

	Pinion	Gear
Number of teeth	9	35
Face width	31.45	26.95
Pinion offset	24	
Shaft angle	90°	
Root cone angle	14°0'	69°16'
Pitch cone angle	14°51'	75°17''
Face cone angle	19°47'	75°17'
Spiral angle	48°20'	30°10'
Hand of spiral	L.H.	R.H.

Table 2 Parameters of the pinion head-cutter

Cutter diameter	190.50
Inside blade angle	28°30'
Outside blade angle	14°0'
Point width	0.80

Table 3 The pinion machine tool settings

	Concave	convex
Point diameter of cutter	190.90	190.10
Cutter blade angle	14°0'	28°30'
Radial setting	84.12	91.16
Cradle angle	161°15'	150°31'
Basic tilt angle	76°45'	70°17'
Swivel angle	225°11'	234°58'
Blank offset	−18.91	−32.62
Machine center to back	−0.85	2.28
Machine root angle	357°32'	357°53'
Sliding setting	11.17	15.35
Ratio of roll	3.625	3.988
2c	0.056	−0.063

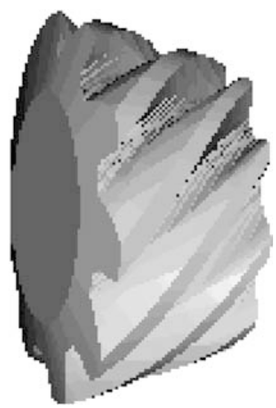

Fig. 6 The simulation result of the generated pinion

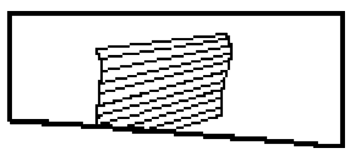

Fig. 7 The contact pattern on the gear convex tooth surface

7 Conclusions

In this paper a new algorithm for finishing a hypoid pinion on an NC machine tool is proposed, based on the

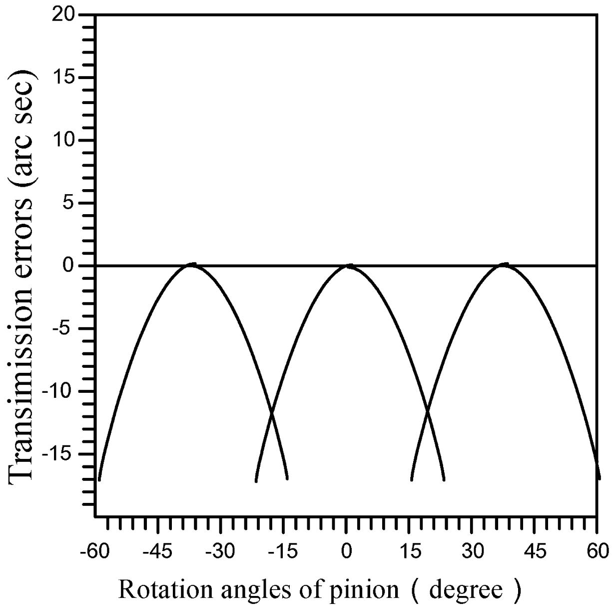

Fig. 8 The calculated transmission error for the dive side

simultaneous cutter tilt and the variable ratio of the angular velocities between the pinion being produced and the generating gear. Using the line contact conjugate principle, the requirement of the lengthwise curvature is satisfied by the position of the cutter for hypoid pinion finishing. The change of the profile curvature is removed by calculating the modified roll constant. In this paper some equations are obtained for calculating the machine settings for hypoid pinion finishing by the abovementioned cutting method. The methods can predict the reducing of the cutter specification and the error compensation of the cutter size. To ensure the favorable property of meshing, the solution is based on a local conjugate method. Numerical examples of calculation and simulation have been illustrated. In the pinion finishing process, the convex and concave teeth sides can also be cut, respectively, by the same cutter with spread blades during an index cycle on the CNC milling or grinding machine, which helps reduce the cost and enhance productivity.

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