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ORIGINAL ARTICLE

Tsai-Wai Chen · J. Y. Lin · K. S. Chen

Selecting a supplier by fuzzy evaluation of capability indices C_{pm}

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Abstract A process capability index C_{pm} that fits nominal-the-best type quality characteristics is an effective tool for assessing process capability since the index can adequately reflect a centring process capability and process yield. A valuable method using C_p estimators was developed by Chou [7] for practitioners to use to determine whether or not two processes have equal capability. However index C_p failed to measure process yield and process centring with bilateral specifications and in addition, more than two suppliers can be selected in an actual application. This study proposes a fuzzy inference method to select the best among the competing suppliers based on an estimated capability index of C_{pm} calculated from sampled data. This method has the advantages of fuzzy systems where a grade can be obtained instead of a more specific exact evaluation result. An illustrated example of colour STN displays demonstrates that the proposed method is effective and feasible for the evaluation of competing process capability.

Keywords Process capability indices · Process centring · Confidence interval · Fuzzy evaluation

K. S. Chen
Department of Industrial Engineering and Management,
National Chin-Yi Institute of Technology,
Taichung, Taiwan

J. Y. Lin
Department of Mechanical Engineering,
National Chung Hsing University,
Taichung, Taiwan

T.-W. Chen (✉)
Department of Mechanical Engineering,
National Chin-Yi Institute of Technology,
No. 35, Lane 215, Section 1,
Chung Shan Road, 411 Taiping,
Taichung, Taiwan
E-mail: cp@chinyi.ncit.edu.tw
Tel.: 886-4-23927423
Fax: 886-4-23930681

1 Introduction

Process capability indices (PCIs), are used to provide a numerical measure of whether a production process is capable of producing items satisfying the quality requirement preset in the factory that has received substantial research attention. Quantifying process potential and performance is important for any successful quality improvement activity and quality program implementation. The relationships between the actual process performance and the specification limits may be quantified using appropriate process capability indices. The two capability indices which have been widely used in manufacturing industry are C_p and C_{pk}. These two indices provide numerical measures of whether a manufacturing process meets the preset capability requirement, and they are defined as [1]:

$$C_p = \frac{USL - LSL}{6\sigma} = \frac{d}{3\sigma}, \tag{1}$$

$$C_{pk} = \min\left\{\frac{USL - \mu}{3\sigma}, \frac{\mu - LSL}{3\sigma}\right\} = \frac{d - |\mu - m|}{3\sigma}, \tag{2}$$

where USL and LSL are the upper and lower specification limits preset by the process engineers or product designers, μ is the process mean, σ is the process standard deviation. $d=(USL-LSL)/2$ is the half length of the specification interval (LSL, USL), and $m=(USL+LSL)/2$ is the mid-point of the specification interval. As noted by Boyles [2], C_p and C_{pk} are both yield-based indices and are independent of the target value T. They may fail to account for process yield and process centring with bilateral specifications. For this reason, Chan et al. [3] developed the index C_{pm} which takes the process centring into consideration. This index is defined as

$$C_{pm} = \frac{USL - LSL}{6\sqrt{\sigma^2 + (\mu - T)^2}} = \frac{d}{3\sqrt{\sigma^2 + (\mu - T)^2}}, \tag{3}$$

where T is the target value. By definition, C_{pm}, will obviously model the manufacturing process under the loss function approach. C_{pm} was not originally designed to provide an exact measure of the number of non-conforming items, but C_{pm} includes the process departure $(\mu - T)^2$ in the denominator of the definition to reflect the degree of process targeting. For instance, large process variance results in a lower C_{pm} value, and a large gap between the process mean and the target value results in a lower C_{pm} . The C_{pm} index attempts to take attention away from conformance to specifications and refocuses on optimal product quality, achieved only when critical dimensions are made according to target. Recently, many widely used statistical packages and quality researchers addressed the process capability of applying C_{pm} for cases in which the specification tolerances are asymmetrical [4, 5]. As noted by Kotz et al. [6], when the process is capable ($C_{pm} \geq 1$), the relationship between C_{pm} and process yield is % $Yield \geq 2\Phi(3C_{pm}) - 1$. For example, if the capability of the product is 1.0, then it guarantees that the total process yield is at least greater than 99.73%. Further, given $C_{pm} > c$, the bounds on $|\mu - T|$ can be calculated as

$$|\mu - T| < \frac{d}{3c}$$

On the other hand, a smaller value of C_{pm} implies a lower process yield, higher expected loss, and poor process capability. Therefore, index C_{pm} is suitable for nominal-the-best type processes (bilateral specifications).

Chou [7] developed an effective procedure that uses estimators of C_p, C_{pu} and C_{pl} for practitioners to determine whether or not two processes have equal capability. One of the purposes for which process capability indices can be used is selecting between the more capable process of two competing processes, as a result, the procedure can be used to select the better of two suppliers. Index C_p is failed to measure process yield and process centring with bilateral specifications, and also, in an actual application, more than two suppliers can be selected. Thus, a capability index $\hat{C}_{pm}$ is used calculated from sampled data to develop a similar procedure for practitioners to use in determining whether or not h ($h > 2$) processes have equal capability. The h processes are tested two at a time, there are ${}_hC_2 = h(h-1)/2$ possible paired comparisons. Furthermore, a great degree of uncertainty may be introduced into the capability assessment due to sampling errors. Consequently, critical value, p-value and confidence interval approaches are frequently used to assess the process capability more reliably during a hypothesis test. When using the confidence interval approach, the estimated indices of C_{pmi} and C_{pmj} calculated from sampled data are employed to determine whether or not two processes have equal capability. Nevertheless, as mentioned in [8], the described boundary between reaching a null hypothesis and rejecting a null hypothesis was too fine in previous methods. Thus, a fuzzy evaluation method that possesses the advantage of fuzzy systems where a grade can be obtained instead of an exact evaluation result is adopted to select the best among the h suppliers in this study.

The concept of fuzzy sets was first proposed by Zadeh [9] in 1965. A fuzzy/neuro-fuzzy approach has new been applied in many fields such as automatic control, optimal analysis, manufacturing systems and decision-making [10, 11, 12, 13] in industry. In this paper, a fuzzy evaluation method is proposed so that the capability of competing process can be assessed. This fuzzy inference evaluation will consider the normalizing indices δ and γ (introduced in Section 3) as inputs and obtain a result value as an output. Both input and output are described by linguistic variables to account for the uncertain information associated with them. Here, triangular and trapezoid membership functions are used to represent uncertain information about process variables. An approximate rule-based reasoning approach is also presented for quantitative analysis. By applying the value of the fuzzy inference result, a grade instead of exact evaluation is obtained in this paper. In addition, the evaluation procedure and an illustrated example will be presented for ease of applications.

2 Process capability confidence intervals

The process capability index C_{pm} for a whole population is not normally available since the process mean and the standard deviation are generally unknown. Only an estimated capability index $\hat{C}_{pm}$ by using a sample can be obtained in practice. Let $X_{i1}, X_{i2},..., X_{in}$, $i = 1, 2, ..., k$, be k sets of random samples of size n from each supplier (the sample size is selected to be the same for simplicity, however it is not necessary). Each product from a supplier has the same product specification and target value. The above calculations of average and variance are briefly summarized in Table 1.

Thus, the natural estimator of C_{pmi} can be written as follows:

$$\hat{C}_{pmi} = \frac{d}{3\sqrt{S_i^2 + \left(\overline{X}_i - T\right)^2}}, \quad i = 1, 2, ..., k, \tag{4}$$

where X_i and S_i^2 are the sample mean and sample variance of process i with sample size n_i. The probability density function of $\hat{C}_{pmi}$ (see Vannman and Kotz [14]) is

$$f_{\hat{C}_{pmi}}(x) = \frac{2^{1-n_i/2} C_i^{n_i}}{3^{n_i} x^{n_i+1}} \exp\left(-\frac{\lambda_i}{2} - \frac{C_i^2}{18x^2}\right) \times \sum_{j=0}^{\infty} \left\{ \left(\frac{\lambda_i C_i^2}{36x^2}\right)^j \Big/ \left(j!\Gamma\left(\frac{n_i}{2} + j\right)\right) \right\}, \tag{5}$$

where $x > 0$ and $C_i = \sqrt{n_i} d/\sigma_i$, $\lambda_i = \sqrt{n_i}(\mu_i - T)/\sigma_i$. As noted by Pearn et al. [15], under the normality assumption $\Sigma_{j=1}^{n_i}\left(X_{ij} - T\right)/\sigma_i$ is distributed as a non-central chi-square

Table 1 Sample data mean and variance

Sample	Mean	Variance
$X_{11}, X_{12},..., X_{1n}$	$\bar{X}_1 = \left(\Sigma_{j=1}^{n} X_{1j}\right)/n$	$S_1^2 = \Sigma_{j=1}^{n}\left(X_{1j} - \overline{X}_1\right)^2/(n-1)$
$X_{21}, X_{22},..., X_{2n}$	$\bar{X}_2 = \left(\Sigma_{j=1}^{n} X_{2j}\right)/n$	$S_2^2 = \Sigma_{j=1}^{n}\left(X_{2j} - \overline{X}_2\right)^2/(n-1)$
$\vdots$	$\vdots$	$\vdots$
$X_{k1}, X_{k2},..., X_{kn}$	$\bar{X}_k = \left(\Sigma_{j=1}^{n} X_{kj}\right)/n$	$S_k^2 = \Sigma_{j=1}^{n}\left(X_{kj} - \overline{X}_k\right)^2/(n-1)$

distribution with n_i degrees of freedom and a non-centrality parameter λ_i. The r-th moment about zero of $\hat{C}_{pmi}$ (see Pearn et al. [15] and Vannman [16]) is

$$E\left(\hat{C}_{pmi}\right)^r = \left(\frac{C_i}{3\sqrt{2}}\right)^r \times \sum_{j=0}^{\infty}\left\{\frac{e^{-\lambda_i/2}(\lambda_i/2)^j}{j\,!}\Gamma\left(\frac{n_i - r}{2} + j\right)/\Gamma\left(\frac{n_i}{2} + j\right)\right\}. \quad (6)$$

The quantity $v_i\left(C_{pmi}/\hat{C}_{pmi}\right)$ is approximately distributed as a chi-square distribution with v_i degrees of freedom which is denoted by $\chi^2(v_i)$. v_i can be estimated by calculating the value $\hat{v}_i$ from the sample as

$$\hat{v}_i = \frac{n_i\left(1 + \left[\left(\overline{X}_i - T\right)/S_i\right]^2\right)^2}{1 + 2\left[\left(\overline{X}_i - T\right)/S_i\right]^2} \quad (7)$$

It is well known that a great degree of uncertainty may be introduced into capability assessment due to sampling error. To achieve a more reliable capability assessment, the confidence interval approach is used below during the hypothesis test. Letting C_{li} and C_{ui} represent the lower and upper confidence limits of process i, it follows that

$$\begin{aligned} P\left\{C_{li} \le C_{pmi} \le C_{ui}\right\} &= P\left\{\hat{v}_i\left(\frac{C_{li}}{\hat{C}_{pmi}}\right)^2 \le \hat{v}_i\left(\frac{C_{pmi}}{\hat{C}_{pmi}}\right)^2 \le \hat{v}_i\left(\frac{C_{ui}}{\hat{C}_{pmi}}\right)^2\right\} \\ &= P\left\{\hat{v}_i\left(\frac{C_{li}}{\hat{C}_{pmi}}\right)^2 \le \chi^2(\hat{v}_i) \le \hat{v}_i\left(\frac{C_{ui}}{\hat{C}_{pmi}}\right)^2\right\} \\ &= 1 - \alpha. \end{aligned} \quad (8)$$

where α stands for the producer risk. Therefore

$$\hat{v}_i\left(\frac{C_{li}}{\hat{C}_{pmi}}\right)^2 = \chi^2_{\alpha/2}(\hat{v}_i) \text{ and } \hat{v}_i\left(\frac{C_{ui}}{\hat{C}_{pmi}}\right)^2 \chi^2_{1-\alpha/2}(\hat{v}_i), \quad (9)$$

and $\chi^2_\alpha(\hat{v}_i)$ denotes the $100\alpha\%$ lower-tail percentage points of $\chi^2(\hat{v}_i)$. Thus, the approximate $100(1-\alpha)\%$ confidence interval can be written as

$$[C_{li},\ C_{ui}] = \left[\sqrt{\chi^2_{\alpha/2}(\hat{v}_i)/\hat{v}_i} \times \hat{C}_{pmi}, \sqrt{\chi^2_{1-\alpha/2}(\hat{v}_i)/\hat{v}_i} \times \hat{C}_{pmi}\right]. \quad (10)$$

3 Fuzzy evaluation of supplier capability

In this study, the estimated indices of C_{pmi} and C_{pmj}, calculated from sampled data, are used to determine whether or not two processes have equal capability. To achieve a more reliable capability assessment, the confidence interval approach is used during the hypothesis test. From the previous section, one can see that the confidence interval of indices C_{pmi} and C_{pmj} are $[C_{li}, C_{ui}]$ and $[C_{lj}, C_{uj}]$, respectively. Using the statistical method, the comparison of these two indices can be represented as:

(1) If $[C_{li}, C_{ui}] \cap [C_{lj}, C_{uj}]\ ^{1}\ \phi$, then it is concluded that $C_{pmi} = C_{pmj}$.
(2) If $C_{li} > C_{uj}$, then it is concluded that $C_{pmi} > C_{pmj}$.
(3) If $C_{ui} < C_{lj}$, then it is concluded that $C_{pmi} < C_{pmj}$.

Nevertheless, it is rather ambiguous in rule one. In this rule, the indices are concluded to be equal whether the intersection is small or large. For instance:

Case A: When $[C_{li}, C_{ui}] = [0.5, 1.6]$ and $[C_{lj}, C_{uj}] = [1.5, 2.4]$ then $[C_{li}, C_{ui}] \cap [C_{lj}, C_{uj}] = [1.5, 1.6]$.

Case B: When $[C_{li}, C_{ui}] = [0.9, 1.8]$ and $[C_{lj}, C_{uj}] = [0.8, 1.7]$ then $[C_{li}, C_{ui}] \cap [C_{lj}, C_{uj}] = [0.9, 1.7]$.

One can recognise that case B is superior to case A for the possibility of "$C_{pmi} = C_{pmj}$" since the intersection in case B is larger than that of case A. In order to distinguish an equal grade of index C_{pm} (for i and j) in different intersections, a method of incorporating the fuzzy inference with a process capability index is now proposed. An approximating rule-based reasoning approach is used for quantitative analysis. In this study, supplier i is said to be superior to supplier j when the value of the inference result is positive. The larger the value result, the more capable supplier i is than supplier j. On the other hand, a negative value result implies that supplier i is inferior to supplier j. In other words, the supplier i is said to be an overall better-quality supplier than supplier j when the value of the inference result is equal to 1; supplier i is said to be completely of the same quality as supplier j when the value of the inference result is equal to 0 and supplier i is said to be completely of worse quality than supplier j when the value of the inference result is equal to -1. A result value of the inference within $\{0, 1\}$ or $\{-1, 0\}$ is used to represent the different grade of capability, as a result, a grade instead of an exact evaluation is obtained in this paper. Let the normalizing indices δ and γ be defined as

$$\delta = \frac{C_{li} - C_{uj}}{\max\left(C_{ui}, C_{uj}\right)}, \quad (11)$$

$$\gamma = \frac{C_{ui} - C_{lj}}{\max\left(C_{ui}, C_{uj}\right)}. \quad (12)$$

Then the fuzzy inference systems are composed of two inputs and one output as shown in Fig. 1.

Generally, the fuzzy analysis procedure consists of four steps: definition of input/output fuzzy

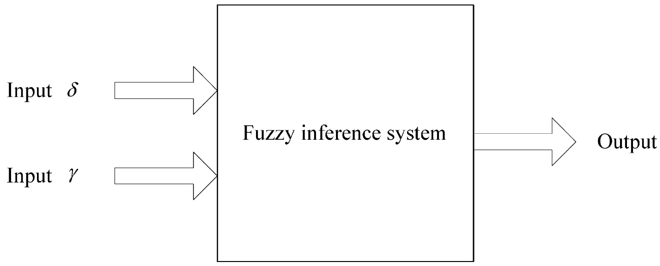

Fig. 1 Structure of fuzzy inference system

variables, fuzzy rules, fuzzy inference and defuzzification [17].

(1) Definition of input/output fuzzy variables. The membership functions (MFs) of input/output variables are defined by linguistic variables. There are four kinds of membership functions for representing fuzzification: in triangular, trapezoid, gaussian and sigmoid type. In this study, triangular and trapezoid types are adopted as MFs for the sake of simplicity and for describing the asymmetric property. The triangular MF is specified by three parameters $\{a,b,c\}$ which determine the three corners of triangle. If this function is denoted as *trimf* $(x;a,b,c)$ then

$$trimf(x;a,b,c) = \begin{cases} 0 & x<a \\ \frac{x-a}{b-a} & a \le x \le b \\ \frac{c-x}{c-b} & b \le x \le c \\ 0 & x > c \end{cases}. \quad (13)$$

Furthermore, the trapezoid MF is denoted *trapmf*$(x;a,b,c,d)$ which is specified by four parameters $\{a,b,c,d\}$, and then one has

$$trapmf(x;a,b,c) = \begin{cases} 0 & x<a \\ \frac{x-a}{b-a} & a \le x \le b \\ 1 & b \le x \le c \\ \frac{d-x}{d-c} & c \le x \le d \\ 0 & x > d \end{cases}. \quad (14)$$

The universe of input variables is defined in $\{-1, 1\}$ as shown in Fig. 2. Membership functions of input δ are defined as *trapmf*$(x;-1,-1,-0.4,-0.3)$, *trimf*$(x;-0.4,-0.3,-0.2)$, *trimf*$(x;-0.3,-0.2,-0.1)$, *trimf*$(x;-0.2,-0.1,0)$, *trimf*$(x;-0.1,0,0.1)$ and *trapmf*$(x;0,0.1,1,1)$ for representing *N4* (negative), *N3*, *N2*, *N1*, *Zero* and *Positive*, respectively. Also, input γ are defined as *trapmf*$(x;-1,-1,-0.1,0)$, *trimf*$(x;-0.1,0,0.1)$, *trimf*$(x;0,0.1,0.2)$, *trimf* $(x;0.1,0.2,0.3)$, *trimf*$(x;0.2,0.3,0.4)$ and *trapmf*$(x;0.3,0.4,1,1)$ for representing *Negative*, *Zero*, *P1* (positive), *P2*, *P3* and *P4*, respectively. In addition, the output variables are composed of seven triangular MFs for representing *L3* (inferior), *L2*, *L1*, *Equal*, *S1* (superior), *S2* and *S3*, as shown in Fig. 3.

(2) Fuzzy rules. Fuzzy rules are important for a successful inference result [10]. A rule base represents the experience and knowledge of experts. The fuzzy rules are similar to the intuitional thinking of a human. A fuzzy inference system, composed of two inputs and one output, could employ this kind of fuzzy rule as

If x_1 is A_{i1} and x_2 is A_{i2} *then* y is B_i (for $i=1$ to n),where x_1, x_2 and y are fuzzy system input and output variables; A_{i1}, A_{i2} and B_i are fuzzy subsets of their linguistic variables. In this study, the fuzzy inference system is applied to select the best supplier between competing processes using the confidence interval values of C_{li}, C_{ui} , C_{lj} and C_{uj}. Thirty-three *if-then* rules are employed in this study. They are:

$$\text{Rule 1 : if } (\delta \text{ is Positive}) \text{ and } (\gamma \text{ is P4}) \text{ then (result is S3).}$$
$$\text{Rule 2 : if } (\delta \text{ is Positive}) \text{ and } (\gamma \text{ is P3}) \text{ then (result is S3).}$$
$$\vdots$$
$$\text{Rule 33 : if } (\delta \text{is N4}) \text{ and } (\gamma \text{ is Negative}) \text{ then (result is L3).}$$

The fuzzy rules are listed in Table 2. Note that the fuzzy rules indicated in this table, *if* (δ is *Positive*) *and* (γ is *Zero*) or (γ is *Negative*) in addition to *if* (δ is *Zero*) *and*

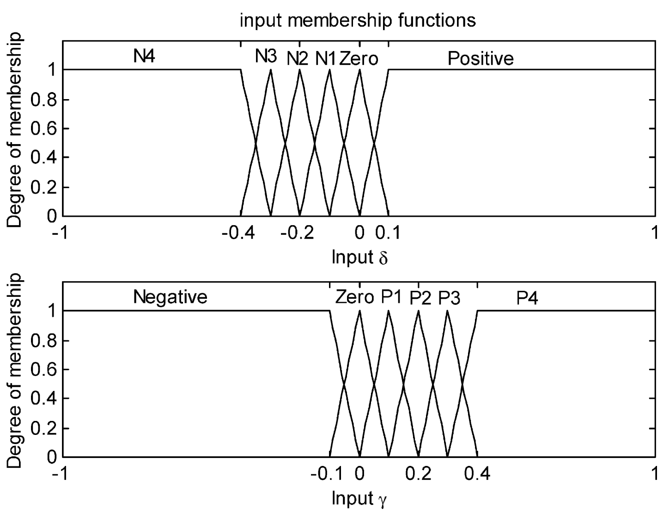

Fig. 2 Membership functions of input variables

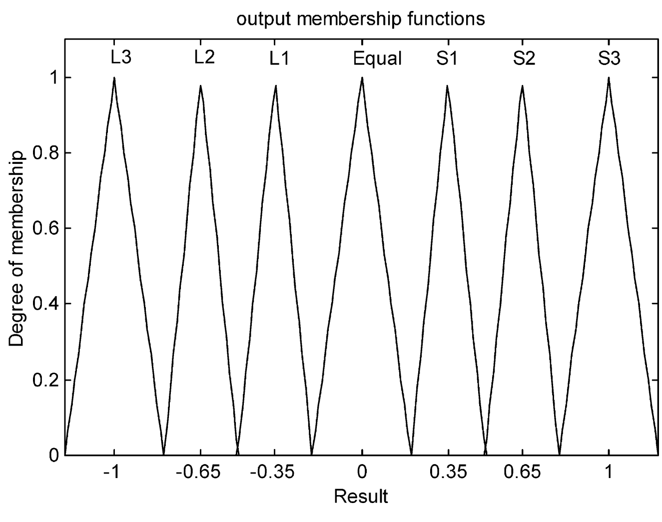

Fig. 3 Membership functions of output variables

Table 2 Selecting supplier fuzzy rules

δ	γ					
	P4	P3	P2	P1	Zero	Negative
Positive	S3	S3	S3	S3	–	–
Zero	S2	S2	S2	S1	Equal	–
N1	S2	S2	S1	Equal	L1	L3
N2	S1	S1	Equal	L1	L2	L3
N3	S1	Equal	L1	L2	L2	L3
N4	Equal	L1	L1	L2	L3	L3

(γ is *Negative*), never apply since the definition of two input variables always exists $\gamma \geq \delta$.

(3) Fuzzy inference. Fuzzy inference is an inference procedure for deriving a conclusion based on a set of *if-then* rules. In this paper, the Mamdani inference method [18] that employs the maximum-minimum product composition to operate fuzzy *if-then* rules is adopted. Let the rule be: *if* $x_1 = A_1$ *and* $x_2 = A_2$ *then* $y = B$, then the result of inference can obtain a fuzzy set with MF of B'_i as

$$\mu_{B'_i}(y) = \max_X \left\{ \min \left[\mu_{A'_{i1}}(x_1), \mu_{A'_{i2}}(x_2), \mu_{R_i}(x_1, x_2, y) \right] \right\}. \tag{15}$$

where

$$\mu_{R_i}(x_1, x_2, y) = \min \left[\mu_{A_{i1}}, \mu_{A_{i2}}, \mu_{B_i}(y) \right]. \tag{16}$$

(4) Defuzzification. The fuzzy sets of B'_i are obtained by step (3), then defuzzification is used to find a crisp value $y^* \in Y$ which represents the fuzzy sets. The frequently used defuzzification methods are the: weight, area and height method in [17]. The weight defuzzification method is used in this study, and then there is

$$y* = \frac{\int_Y y \cdot B(y) dy}{\int_Y B(y) dy}. \tag{17}$$

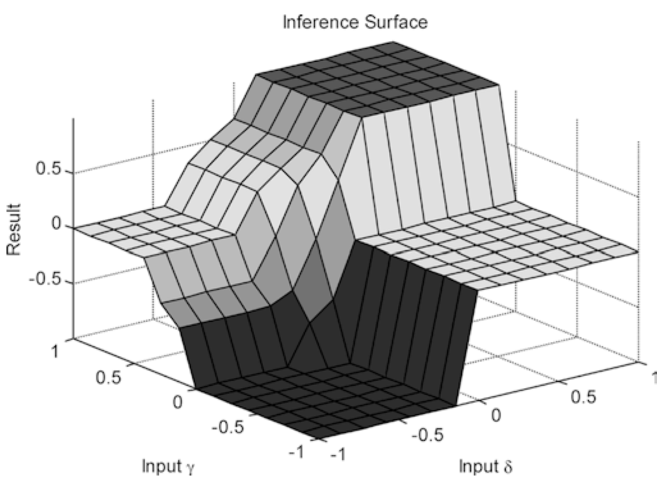

Fig. 4 Fuzzy inference surface

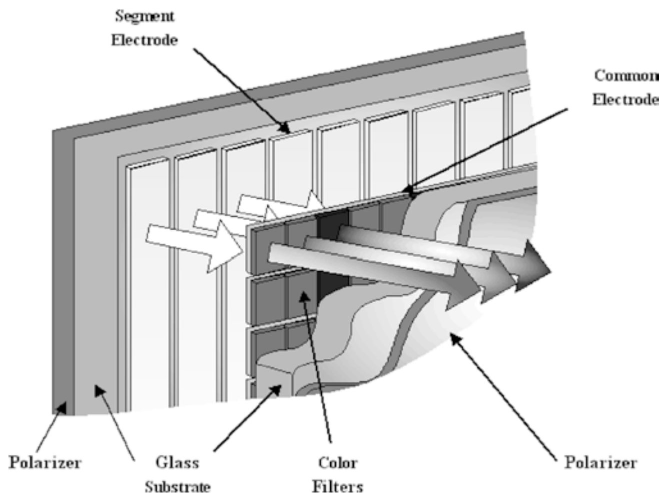

Fig. 5 Structure of colour STN displays

The result of fuzzy inference, performed by a *Matlab Logic Fuzzy Toolbox* [19], is shown in Fig. 4.

4 Procedure of fuzzy evaluation and illustrated example

An example is given to show the proposed procedure in detail. To illustrate how the testing procedure may be applied to the practical data collected from the factories, the following case on a colour STN display product was taken from a manufacturing industry located on central Taiwan. Colour STN displays are created by adding a colour filter to traditional monochrome STN displays. The structure of colour STN displays is depicted in Fig. 5 (cited from: www.winteck.com.tw). In colour STN displays; each pixel is divided into red, green, and blue sub-pixels. To control the light through the colour filter, different colours are made by a combination of these primary colours. The thickness of the membrane, which represents an important quality characteristic in this study, is measured for each pixel after finishing the post-baking process. The specification limits are set to $12{,}000 \pm 500$ Å (1 Å $= 10^{-7}$ *mm*), that is, the upper/lower specification limits are set to $USL = 12{,}500$, $LSL = 11{,}500$, and the target value is set to $T = 12{,}000$. If the thickness of the membrane for the colour STN does not fall within the tolerance (LSL, USL), the problem of chromatic aberration for colour STN displays will occur. Table 3 presents concise information about the four suppliers. The main purpose is to select the best

Table 3 Process capability value for four suppliers

Supplier	X_i	S_i	$\hat{C}_{pmi}$	$\hat{v}_i$	C_{li}	C_{ui}
SUP1	12,020	101	1.6187	61	1.3320	1.9049
SUP2	12,030	168	0.9766	61	0.8036	1.1493
SUP3	11,940	100	1.4292	65	1.1839	1.6740
SUP4	12,090	97	1.2596	77	1.0609	1.4579

Table 4 Fuzzy inference results

Pairs (i to j)	$[C_{li}, C_{ui}]$	$[C_{lj}, C_{uj}]$	δ	γ	Result
SUP1 to 2	[1.3320, 1.9049]	[0.8036, 1.1493]	0.0941	0.5860	+1.00
SUP1 to 3	[1.3320, 1.9049]	[1.1839, 1.6740]	−0.1795	0.3785	+0.43
SUP1 to 4	[1.3320, 1.9049]	[1.0609, 1.4579]	−0.0661	0.4431	+0.81
SUP2 to 3	[0.8036, 1.1493]	[1.1839, 1.6740]	−0.5200	−0.0207	−1.00
SUP2 to 4	[0.8036, 1.1493]	[1.0609, 1.4579]	−0.4488	0.0606	−0.82
SUP3 to 4	[1.1839, 1.6740]	[1.0609, 1.4579]	−0.1637	0.3662	+0.47

supplier among these four. The fuzzy evaluation procedure is stated as follows:

Step 1: Determine the sample size $n=60$ for all suppliers, then the value of mean and standard deviation are calculated as indicated in Table 3. In addition, the significant level (normally used 0.01, 0.025 and 0.05) is set to 0.05.

Step 2: Compute the value of $\hat{C}_{pmi}$ and $\hat{v}_i$.

Step 3: Compute the confidence interval values C_l and C_u for each supplier.

Step 4: Compute the normalising indices δ and γ for two-supplier pairs, and thus to obtain the fuzzy evaluation results using the proposed fuzzy inference system. The above calculations are performed using a specially developed program (see Appendix for detail). As indicated in Table 4, one can recognise that SUP1 is the best among these four suppliers since all values of inference results are positive for competing pairs (SUP1 to SUPj, for $j=2$, 3 and 4).

5 Conclusion

A capability index C_{pm} has been shown that can adequately reflect centring process capability and process yield and which is used to give a numerical measure about whether a production method is capable of producing items within the specification limits. Index C_p used in Chou [7] fails to measure process yield and process centring with bilateral specifications, and there are more than two suppliers which can be selected in actual application. In this paper, a fuzzy inference method has been adopted to evaluate the capability of competing processes based on an estimated index of C_{pm} calculated from sampled data. This fuzzy evaluation method considers the normalising indices δ and γ as inputs and obtains a result value as an output. Both input and output variables are defined in {−1, 1}. The presented method has the advantage of fuzzy systems, where a grade can be obtained instead of an exact evaluation. An illustrated example using colour STN displays is employed to demonstrate that the presented method is effective and thus supports its feasibility for assessment the capability of competing suppliers.

Appendix: A Matlab program

```
%Program of selecting supplier by fuzzy evaluation
  n = input ('given sample No. ');
  usl = input ('given upper specification limit ');
  lsl = input ('given lower specification limit ');
  T = input ('given target value ');
  alpha = input ('given alpha-risk value ');
  ns = input ('given No. of h suppliers ');
  for j = 1:ns
  xbar = input ('given supplier sample mean ');
  s = input ('given sample standard deviation ');
  d = (usl−lsl)/2;
  cpmhei = d/(3*sqrt(s^2 + (xbar−T)^2));
  V = cpmhei;
  zz = ((xbar−T)/s)^2;
  vhei = n*(1 + zz)^2/(1 + 2*zz);
  vhei = ceil (vhei);
  vl = chi2inv (alpha/2,vhei);
  vu = chi2inv (1−alpha/2,vhei);
  cl(j) = sqrt (vl/vhei)*cpmhei;
  cu(j) = sqrt (vu/vhei)*cpmhei;
  end%%%%%%
  a = newfis ('mmfis');
  a = addvar (a,'input','delta',[−1 1]);
  a = addmf (a,'input',1,'N4','trapmf',[−1.1 −1 −0.4
    −0.3]);
  a = addmf (a,'input',1,'N3','trimf',[−0.4 −0.3 −0.2]);
  a = addmf (a,'input',1,'N2','trimf',[−0.3 −0.2 −0.1]);
  a = addmf (a,'input',1,'N1','trimf',[−0.2 −0.1 0]);
  a = addmf (a,'input',1,'Zero','trimf',[−0.1 0 0.1]);
  a = addmf (a,'input',1,'Positive','trapmf',[0 0.1 1 1.1]);
  a = addvar (a,'input','gamma',[−1 1]);
  a = addmf (a,'input',2,'Negative','trapmf',[−1.1 −1
    −0.1 0]);
  a = addmf (a,'input',2,'Zero','trimf',[−0.1 0 0.1]);
  a = addmf (a,'input',2,'P1','trimf',[0 0.1 0.2]);
  a = addmf (a,'input',2,'P2','trimf',[0.1 0.2 0.3]);
  a = addmf (a,'input',2,'P3','trimf',[0.2 0.3 0.4]);
  a = addmf (a,'input',2,'P4','trapmf',[0.3 0.4 1 1.1])
  a = addvar (a,'output','Result',[−1.2 1.2]);
  a = addmf (a,'output',1,'I3','trimf',[−1.2 −1 −0.8]);
  a = addmf (a,'output',1,'I2','trimf',[−0.8 −0.65 −0.5]);
  a = addmf (a,'output',1,'I1','trimf',[−0.5 −0.35 −0.2]);
  a = addmf (a,'output',1,'Equal','trimf',[−0.2 0 0.2]);
  a = addmf (a,'output',1,'S1','trimf',[0.2 0.35 0.5]);
  a = addmf (a,'output',1,'S2','trimf',[0.5 0.65 0.8]);
  a = addmf (a,'output',1,'S3','trimf',[0.8 1 1.2]);
```

rule = [...

6	6	7	1	1
6	5	7	1	1
6	4	7	1	1
6	3	7	1	1
5	6	6	1	1
5	5	6	1	1
5	4	6	1	1
5	3	5	1	1
5	2	4	1	1
4	6	6	1	1
4	5	6	1	1
4	4	5	1	1
4	3	4	1	1
4	2	3	1	1
4	1	1	1	1
3	6	5	1	1
3	5	5	1	1
3	4	4	1	1
3	3	3	1	1
3	2	2	1	1
3	1	1	1	1
2	6	5	1	1
2	5	4	1	1
2	4	3	1	1
2	3	2	1	1
2	2	2	1	1
2	1	1	1	1
1	6	4	1	1
1	5	3	1	1
1	4	3	1	1
1	3	2	1	1
1	2	1	1	1
1	1	1	1	1];

a = addrule(a,rule);writefis(a,'cmmcpm');%comparing-pairsfor ii = 1:ns
– for j = ii + 1:ns
– cli = cl(ii);
– cui = cu(ii);
– clj = cl(j);
– cuj = cu(j);
– pv1 = (cli−cuj)/max(cui,cuj);
– pv2 = (cui−clj)/max(cui,cuj);
– score(1,1) = 0;
– score(ii,j) = evalfis([pv1,pv2],a);
– end
endclcuscoredisp('stop simulation')

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