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Fuzzy Evaluation of Process Capability for Bigger-the-Best Type Products

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Many industrial products can be characterised as of the bigger-the-best type. Quality characteristics and process yields of a bigger-the-best type product can be evaluated by using a process capability index. The process capability index C_{pl} is used to evaluate quality characteristics and process yields of the bigger-the-best type products because the formulae for this index are easy to understand and straightforward to apply. However, since sample data must be collected in order to calculate this index, a great degree of uncertainty may be introduced into the capability assessments due to sampling errors. The other shortcoming of the evaluation is that there exists a fuzzy condition for making a decision for rejecting a null hypothesis. In this paper, a method to incorporate the fuzzy inference with the process capability index in the bigger-the-best type quality characteristics assessments is presented, and a concise score concept is used to represent the grade of the process capability. An example shows that the proposed method is effective in application and thus confirms its feasibility.

Keywords: Bigger-the-best type quality characteristics; Fuzzy inference; Process capability index; Process yield

1. Introduction

Many industrial product characteristics such as tension strength, compression strength, and endurance of high temperature are desired to be the bigger-the-best. Industrial products generated through processes for giving such product characteristics are referred to as of the bigger-the-best type. Process yield is generally used to describe a process capability. Let a lower specification limit of the characteristics of the bigger-the-best type products be LSL. Then the process yield ($\%Yield$) can be represented as $\%Yield = P(X \geq LSL)$ where the random variable X represents the specification of products. Alternatively, Kane [1] proposes a process capability index C_{pl} to describe the process capability, i.e.

$$C_{pl} = \frac{\mu - LSL}{3\sigma} \tag{1}$$

where μ is the process mean and σ is the standard deviation. The process capability index is a unilateral specification and can be related to the process yield as

$$\%Yield = \Phi\,(3\,C_{pl}) \tag{2}$$

where Φ is the cumulative distribution function (CDF) of a standard normal distribution. We can observe that the function of process yield *vs.* C_{pl} is a strictly ascending curve, and there is a one-to-one relation between them, as shown in Fig. 1. We can also find, in contrast, that the higher the process capability index, the higher the process yield. The process capability index (C_{pl}) can not only reflect the process yield adequately, but can also take into consideration statistical information about the process such as the mean and standard deviations.

The process capability index can be used to evaluate a process capability in order to predict whether consumers will give an order for the associated bigger-the-best type products

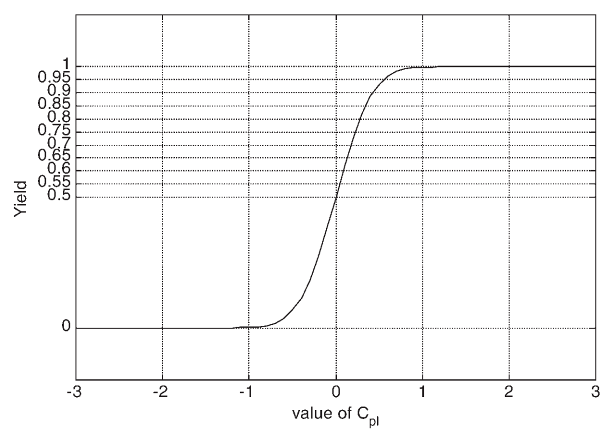

Fig. 1. The relationship between C_{pl} and process yield.

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or not. Let the minimum process capability required for the bigger-the-best type be C_{min}, then the process capability is satisfactory when $C_{pl} \geq C_{min}$. Consequently, a null hypothesis is made:

$$H_0: \quad C_{pl} \geq C_{min}$$

The process capability is sufficient when the hypothesis H_0 holds and the consumers will then give an order. Consumers will not give an order when the hypothesis is rejected. The process capability index C_{pl} of the whole population is normally not available since the process mean and standard deviation are generally unknown. Only an estimated capability index $\hat{C}_{pl}$ by using a sample can be obtained in practice. Instead of using $\hat{C}_{pl}$ and C_{min} directly for the evaluation of H_0, Cheng [2] proposed to use a *p-value* and an α*-risk* for the evaluation where the *p-value* is associated with $\hat{C}_{pl}$ and C_{min} and the α*-risk* represents the producer's risk. If *p-value* > α*-risk*, then H_0 exists.

Cheng [2] and Pearn and Chen [3] point out that the usually used α*-risk* values are 0.01, 0.025, and 0.05. Consequently, the *p-value* plays an important role in decision-making. It is normally said that we may reject the null hypothesis when 0.01 < *p-value* ≤ 0.05, and strongly reject the null hypothesis when the *p-value* ≤ 0.01. However, the description of the boundary between meeting the quality requirement and rejecting it is too abrupt by this route. To reduce the abruptness, a fuzzy statement is made to modify the crisp clearly defined boundary:

1. If *p-value* > α*-risk* is rather excessive enough *then* the sufficiency of the process capability is rather high.
2. If *p-value* > α*-risk* is enough *then* the sufficiency of the process capability is greater than the median.
3. If *p-value* > α*-risk* is not enough *then* the process capability is insufficient.
4. If *p-value* > α*-risk* is inadequate *then* the process capability is very insufficient.

The concept of fuzzy sets was first proposed by Zadeh [4] in 1965. Fuzzy theory has been applied in many fields in industry such as automatic control, manufacturing systems and decision-making [5–7]. In this paper, a fuzzy inference evaluation method is proposed for the fuzzy statement so that process capability for a bigger-the-best type product can be assessed. This fuzzy inference evaluation will consider a *p-value* as the input and obtain a score as the output. Both input and output are described by linguistic variables to account for the uncertain information associated with them. An approximate reasoning approach will then be developed for this evaluation, and a concise score concept will be used to represent the grade of the process capability. In addition, an evaluation procedure will be presented for ease of applications:

Formulation of *p*-Value as an Input

Consider that a sample of size n is obtained from a population of a bigger-the-best type product. An element selected from the sample has a product quality characteristic, X_i. The mean of this product quality characteristic is calculated as

$$\overline{X} = \frac{\sum_{i=1}^{n} X_i}{n}$$

and the standard deviation is obtained as

$$S = \sqrt{\left(\frac{\sum_{i=1}^{n} (X_i - \overline{X})^2}{n-1}\right)}$$

Then an estimated process capability index of the process is computed as

$$\hat{C}_{pl} = \frac{\overline{X} - LSL}{3S} \tag{3}$$

where *LSL* is the lower specification limit of the product characteristics. The expected value of $\hat{C}_{pl}$ can be stated as (refer to [8])

$$E[\hat{C}_{pl}] = b_n^{-1}\, C_{pl} \tag{4}$$

where

$$b_n = \sqrt{\left(\frac{2}{n-1}\right)} \times \left(\frac{\Gamma[(n-1)/2]}{\Gamma[(n-2)/2]}\right) \quad (n > 2) \tag{5}$$

We recognise that b_n is a function of the sample size n in Eq. (5), and the relative curve is shown in Fig. 2. From this curve, we can find that the value of b_n approaches unity when n is large enough. Thus, $\hat{C}_{pl}$ is the approximate estimation of C_{pl}. If we define

$$\hat{C}'_{pl} = b_n\, \hat{C}_{pl} \tag{6}$$

where $\hat{C}'_{pl}$ is a function of $(\overline{X}, S^2)$. Therefore, $\hat{C}'_{pl}$ is a uniformly minimum variance unbiased estimator (*UMVUE*) of C_{pl} for a normal distribution. The variance of $\hat{C}'_{pl}$ is then derived as [9]

$$Var[\hat{C}'_{pl}] = \left(\frac{\Gamma[(n-1)/2]\Gamma[(n-3)/2]}{\Gamma^2[(n-2)/2]}\right) [(1/9n) + (C_{pl})^2]] - (C_{pl})^2 \tag{7}$$

where $Var[\hat{C}'_{pl}]$ is also equal to the mean square error (*MSE*) of $\hat{C}_{pl}$ in Eq. (7). Furthermore, the *MSE* of $\hat{C}_{pl}$ is:

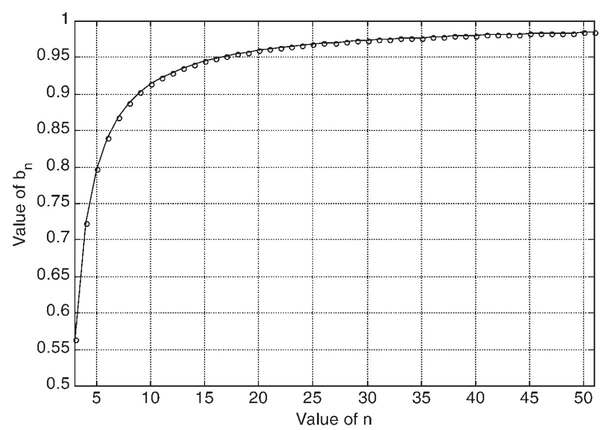

Fig. 2. The curve of sample n *vs.* the value of b_n.

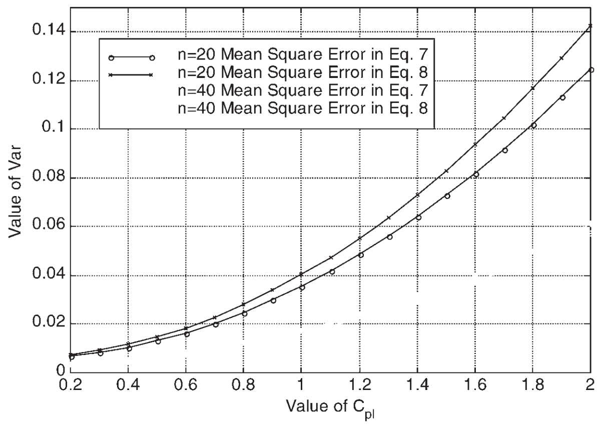

Fig. 3. The curves of $MSE(\hat{C}'_{pl})$ and $MSE(\hat{C}_{pl})$.

$$\begin{aligned} MSE[\hat{C}_{pl}] &= E[C_{pl} - C_{pl}]^2 \\ &= Var[\hat{C}_{pl}] + [E[\hat{C}_{pl}] - C_{pl}]^2 \\ &= \frac{Var[\hat{C}'_{pl}]}{b_n^2} + C_{pl}^2\left(\frac{1}{b_n} - 1\right) \end{aligned} \tag{8}$$

Since $b_n < 1$ and $\hat{C}'_{pl}$ is a unbiased estimator, we can determine that $MSE\ [\hat{C}'_{pl}] < MSE[\hat{C}_{pl}]$. The curves of $MSE[\hat{C}'_{pl}]$ and $MSE[\hat{C}_{pl}]$ with $n = 20$ and $n = 40$ are shown in Fig. 3. We observe that $MSE[\hat{C}'_{pl}]$ approaches $MSE[\hat{C}_{pl}]$ with a large n.

Let $y = b_n(\hat{C}'_{pl}/(3\sqrt{n})$, then y is distributed as a non-central t-distribution with $n - 1$ degrees of freedom and a non-centrality parameter $\delta = 3\sqrt{n}\ C_{pl}$ which is $t_{n-1}(\delta)$ [10]. Then the probability density function of $\hat{C}'_{pl}$ with $n - 1$ degrees of freedom is obtained as [11]

$$\begin{aligned} f_{\hat{C}'_{pl}}(y) &= \left(\frac{3\sqrt{n} \times 2^{-(n/2)}}{b_n\sqrt{((n-1)\pi)}\ \Gamma[(n-1)/2]}\right)\int_0^\infty t^{\left(\frac{n-2}{2}\right)} \exp \\ &\quad \{-0.5[t + (\frac{3\sqrt{(nt)}}{\sqrt{((n-1)b_n)}}\ y - \delta)^2]\}\ dt \end{aligned} \tag{9}$$

where $y \in R$, and the curves of this function are shown in Fig. 4 with $C_{pl} = 1$.

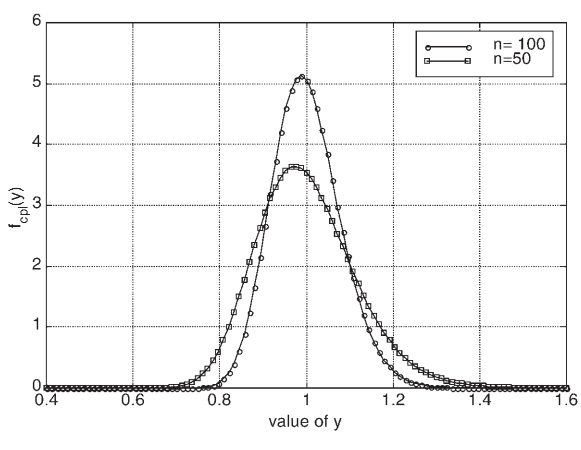

Fig. 4. The curves of probability density distribution.

Let the minimum process capability of a bigger-the-best type be required as C_{min}, then a null and opposite hypothesis can be stated as

$$H_0: \quad C_{pl} \geq C_{min} \text{ and } H_a: \quad C_{pl} < C_{min}$$

Consumers tend to give orders when (H_0) exists. The manufacturer should check defects in the manufacturing process in order to improve process capability when H_a exists. We can use the index of $\hat{C}'_{pl}$ to assess the process capability in order to predict whether the quality attains the required level or not. $(3\sqrt{n}/b_n)\hat{C}'_{pl}$ is a non-central distribution with $t_{n-1}(\delta)$. Let the estimated value $\hat{C}'_{pl} = V$, then *p-value* can be calculated as

$$\begin{aligned} p\text{-}value &= p\{\hat{C}'_{pl} \leq V \mid C_{pl} = C_{min}\} \\ &= p\{(3\sqrt{n}/b_n)\hat{C}'_{pl} \leq (3\sqrt{n}/b_n)V \mid C_{pl} = C_{min}\} \\ &= p\{t_{n-1}(\delta) \leq (3\sqrt{n}/b_n)V\} \end{aligned} \tag{10}$$

3. Fuzzy Inference

As mentioned in Section 2, the decision about an α-*risk* is an uncertain one, which also causes the problem of how large a *p-value* is enough to ensure the process capability? Thus, we will develop a method to incorporate fuzzy inference with the process capability index in the quality assessment. Generally, the procedure of fuzzy analysis consists of four steps: the definition of input/output fuzzy variables, of fuzzy rules, of fuzzy inference, and of defuzzification.

1. *The definition of input/output fuzzy variables*. In our study, the *p-value* represents an input variable and the output is a concise score value. Membership functions (MFs) of these input/output variables are defined by linguistic variables. There are four kinds of MF for representing fuzzification: triangular, trapezoid, Gaussian, and sigmoid. We adopt the triangular type as the input MF for the sake of simplicity and ease of describing the asymmetric property while the two sides use a trapezoid function. The output variable uses the Gaussian type MF. The triangular MF is specified by three parameters $\{a,b,c\}$ which determine the three corners of a triangle. If this function is $trimf(x;a,b,c)$ then

$$trimf\,(x;a,b,c) = \begin{cases} 0 & x < a \\ \dfrac{x-a}{b-a} & a \leq x \leq b \\ \dfrac{c-x}{c-b} & b \leq x \leq c \\ 0 & x > c \end{cases} \tag{11}$$

The input variables are composed of seven segments as $S3$, $S2$, $S1$ (small), ME (median), $B1$, $B2$ and $B3$ (big). The inference results of the output are related to the α-*risk* and the *allowance* in our analysis. In this paper, *allowance* is introduced to ensure the quality and the α-*risk* may be changed by the producer. The more quality is assured, the higher value of *allowance* is selected. Now, we define

$$ze = \alpha\text{-}risk + allowance \tag{12a}$$

$$ns = 0.8 \times ze \tag{12b}$$

$$nm = 0.5 \times ze \tag{12c}$$

$$nb = 0.2 \times ze \tag{12d}$$

$$ps = ze + 0.01 \tag{12e}$$

$$pm = ze + 0.025 \tag{12f}$$

$$pb = ze + 0.05 \tag{12g}$$

then these triangular MFs are *trimf*(x;nb,nm,ns), *trimf*(x;nm,ns,ze), *trimf*(x;ns,ze,ps), *trimf*(x;ze,ps,pm) and *trimf*(x;ps,pm,pb). Furthermore, the trapezoid MF is *trapmf*(x;a,b,c,d) which is specified by four parameters $\{a,b,c,d\}$, and then we define

$$trapmf\,(x;a,b,c,d) = \begin{cases} 0 & x < a \\ \dfrac{x-a}{b-a} & a \le x \le b \\ 1 & b \le x \le c \end{cases} \tag{13}$$

$$trapmf\,(x;a,b,c,d) = \begin{cases} \dfrac{d-x}{d-c} & c \le x \le d \\ 0 & x > d \end{cases}$$

Thus, the two side MFs $S3$ and $B3$ can be described as *trapmf*(x;-0.1,nb,nm) and *trapmf*(x;pm,pb,0.15,0.2). The input MFs are shown in Fig. 5 during the universe of input variables defined in {0 0.15} (which implies when *p-value* ≥ 0.15 when the inference score is equal to 100), the α-*risk* = 0.05 and the *allowance* = 0.

In addition, the output variables are composed of seven gaussian type MFs as $L3$ (low), $L2$, $L1$, ME, $H1$, $H2$ and $H3$ (high). A gaussian MF *gaussmf*(x;σ,m) is specified by two parameters: mean value m and standard deviation σ. This function is defined as

$$gaussmf(x;\sigma,m) = \exp\left(-\left(\frac{\mathrm{x-m}}{\sigma}\right)^2\right) \tag{14}$$

then the seven segments for $L3$, $L2$, $L1$, ME, $H1$, $H2$, and $H3$ are *gaussmf*(x;4.247,0), *gaussmf*(x;4.247,20), *gaussmf*(x;4.247,40), *gaussmf*(x;4.247,60), *gaussmf*(x;2.123,75), *gaussmf*(x;2.123,85) and *gaussmf*(x;4.247,100). Under α-*risk* = 0.05 and *allowance* = 0, the output MFs are as shown in Fig. 6.

2. *Fuzzy rules*. Fuzzy rules are important to obtain a successful inference result [12]. A rule base represents the experience and knowledge of the experts. In this study, the input variable has a *p-value* of one, thus the *if-then* rule will be straightforward. The concise concept can be stated as: if the *p-value* is larger than the α-*risk* plus the *allowance*, then the inference score is 60 or over (in Taiwan, a student is said to "pass" when his examination score is over 60, otherwise, "Fail"). The larger the *p-value*, the higher the score. On the otherhand, the process capability is insufficient when the inference score is less than 60, i.e. the smaller the *p-value*, the lower the inference score. Thus, seven *if-then* rules can be stated as:
   1. *If* (*p-value is* $S3$) *then* (*score is* $L3$)
   2. *If* (*p-value is* $S2$) *then* (*score is* $L2$)
   3. *If* (*p-value is* $S1$) *then* (*score is* $L1$)
   4. *If* (*p-value is* ME) *then* (*score is* ME)
   5. *If* (*p-value is* $B1$) *then* (*score is* $H1$)
   6. *If* (*p-value is* $B2$) *then* (*score is* $H2$)
   7. *If* (*p-value is* $B3$) *then* (*score is* $H3$)
3. *Fuzzy inference*. Fuzzy inference is an inference procedure for deriving a conclusion based on a set of *if-then* rules. In this paper, the Mamdani inference method [13] that employs the maximum–minimum product composition to operate fuzzy *if-then* rules is adopted. In the rule: *if* $x = A$ *then* $y = B$, let A, A', and B be the sets of X, X and Y, respectively. Suppose that the fuzzy implication $A \rightarrow B$ is stated as a fuzzy relation R on $X \times Y$. Then the MF of B'_i can be defined as

$$\mu_{B'}(y) = \max_X \{\min[\mu_{A'}(x),\mu_R(x,y)]\} \tag{15}$$

   and the fuzzy sets of B'_i can then be obtained from the fuzzy inference by the rules.
4. *Defuzzification*. The fuzzy sets of B'_i are obtained by step 3, then the defuzzification is used to find a crisp value

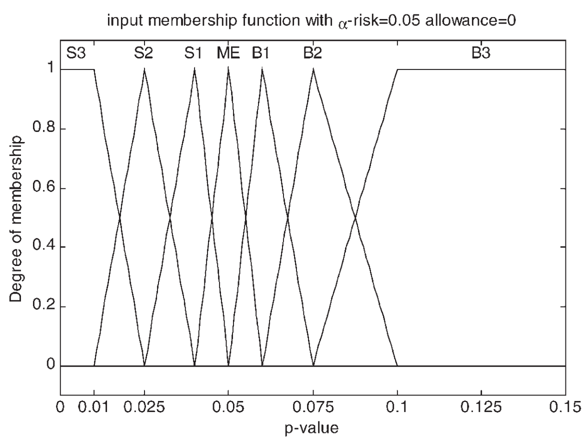

Fig. 5. The definition of input membership functions.

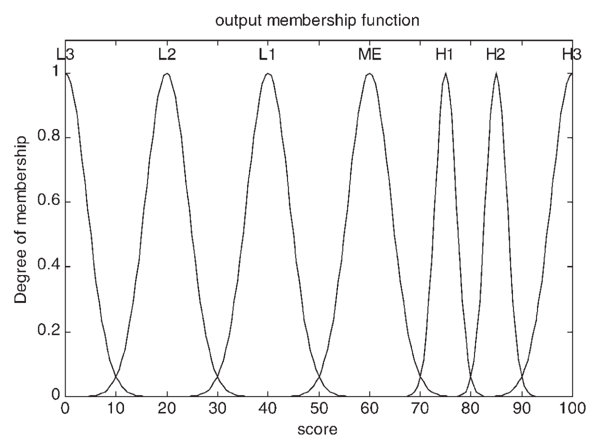

Fig. 6. The definition of output membership functions.

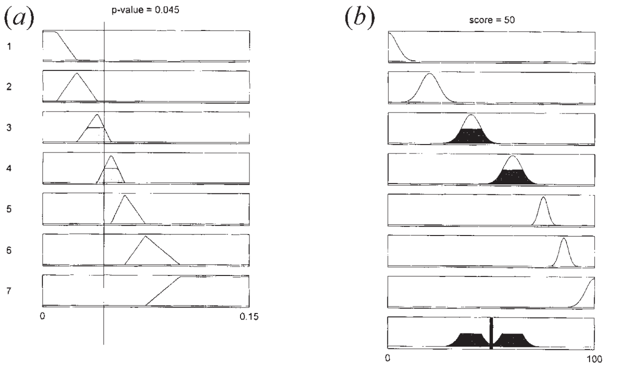

Fig. 7. The inference process and defuzzification. (*a*) *p-value* = 0.045; (*b*) score = 50.

$y^* \in Y$ which represents the fuzzy sets. Frequently used defuzzification methods are the weight, the area and the height method [14]. The weight defuzzification method is used in our study, and then we have

$$y^* = \frac{\int_Y y\, B(y)\mathrm{d}y}{\int_Y B(y)\mathrm{d}y} \tag{16}$$

For α-*risk* = 0.05, *allowance* = 0 and *p-value* = 0.045, the fuzzy inference and the result of defuzzification are shown in Fig. 7

From the above statement, we can recognise that the process capability is measured by the score of the inference results, which are determined by the α-*risk* test, and by the *allowance* and *p-value*. In this paper, we develop a Matlab program (see the Appendix) to perform the former using the Matlab Fuzzy Toolbox [15]. The curves of *p-value vs.* inference score are shown in Figs 8 to 10 with different α-*risk* = 0, 0.025, and 0.05 and for different *allowance* = 0, 0.01, and 0.02. From these curves, the relationship between the *p-value* (index of process capability observation) and the score are easily obtained. The higher the score, the higher the degree of process

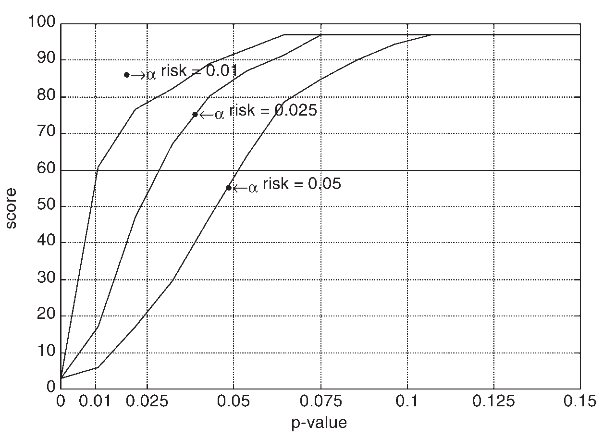

Fig. 8. The curves of *p-value vs.* score under *allowance* = 0.

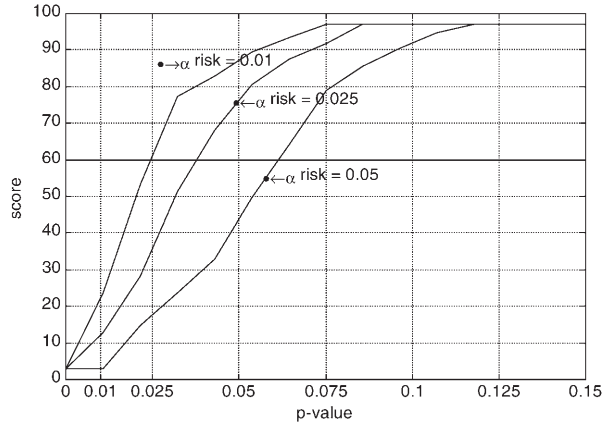

Fig. 9. The curves of *p-value vs* score under *allowance* = 0.01.

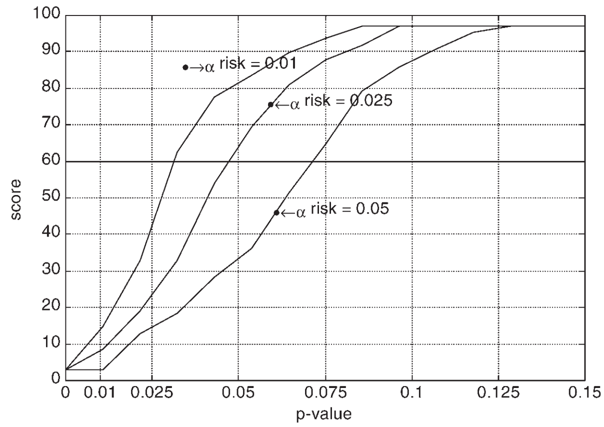

Fig. 10. The curves of *p-value vs* score under *allowance* = 0.02.

capability sufficiency. Thus, the process capability is easy to determine and then to apply in a proper strategy using this concise and easily comprehensible score. The strategy of the proposed fuzzy evaluation for process capability is given in Table 1.

4. Procedure of Fuzzy Evaluation for Process Capability

An assessment program for process capability is developed written in the Matlab language (see the Appendix) this evaluation program consists of the following steps:

Step 1: To assign the minimum capability index C_{min} and lower specification limit.

Step 2: To decide the test of α-*risk* and manufacturing *allowance*.

Step 3: To calculate the mean value $\overline{X} = \{(n)^{-1} (\sum_{i=1}^{n} X_i)\}$ and the standard derivation $S = \{(n-1)^{-1} \sum_{i=1}^{n} (X_i - \overline{X})^2\}^{1/2}$ from the selected n sample data.

Step 4: To compute the value of b_n from Eq. (5) and calculated $\hat{C}'_{pl}$ from Eq. (6). Now, $V = \hat{C}'_{pl}$, $\delta = 3\sqrt{n}C_{min}$ and degree of

Table 1. The strategy of fuzzy evaluation in process capability.

Item	Fuzzy evaluation		Strategy
1	$Score > 75$	The sufficiency of process capability is rather high	The consumer tends to give the order, and the producer does not change their process.
2	$60 \le Score \le 75$	The sufficiency of process capability is over median	The consumer still tends to give the order, but the producer should prevent their process capability slipping down.
3	$30 < Score < 60$	The process capability is insufficient	The consumer tends to reject the order unless he is willing to decrease the demand level
4	$Score \le 30$	The process capability is strongly insufficient.	The consumer tends to reject the order and the producer should check the defects in the production process in order to improve the process capability

freedom is equal to $n - 1$, thus the *p-value* can be obtained from Eq. (10) through the cumulative distribution function with a non-central t distribution.

Step 5: Once the *p-value* is obtained, the input/output MFs are defined using the developed program, then the crisp score value can be inferred by defuzzification.

Step 6: The concise value of the score is used to formulate the appropriate strategy following Table 1.

From the above procedure, we can easily perform the fuzzy evaluation of process capability and then to adopt the proper strategy simultaneously. Thus, this information reminds the producer to prevent his process capability slipping.

Illustrative Example

An example is given to illustrate the proposed procedure in detail. For a hook company in middle Taiwan, the item 8025–08 products should be safe at 2200 pounds load when working, and then be tested at 8800 pounds or until failure. Sixteen products are inspected using the above test as:

12.97	13.06	12.66	12.72	12.67	13.19	12.72	13.85
13.57	13.05	13.35	13.62	12.59	13.96	13.16	14.15

Step 1: $C_{min} = 1$ and $LSL = 12$ at constant time under tests, both are determined by the engineer.

Step 2: To decide the α-*risk* $= 0.05$ and *allowance* $= 0.01$.

Step 3: From these samples, we can compute $\overline{X} = 13.21$, $S = 0.50$.

Step 4: $n = 16$ then $b_{16} = 0.949$ is obtained, and thus $\hat{C}'_{pl} = (0.949) \times (13.21 - 12)/(1.5) = 0.7655$.

Step 5: Non-central parameter $\delta = 3\sqrt{16} \times 1 = 12$, degree of freedom $= 15$, $V = 0.7655$, thus *p-value* $= 0.1116$ is obtained.

Step 6: *p-value* $= 0.1116$, α-*risk* $= 0.05$ and *allowance* $= 0.01$, thus the linguistic variables are assigned and then $y^* = score = 97$ is obtained.

The above calculations are performed using the developed program (see the Appendix for detail). From Table 1, we can see that the consumer tends to give the order and the producers continue to maintain their process capability.

6. Conclusion

The index C_{pl} is one of the effective tools for the assessment of process capability of the bigger-the-best type quality characteristic. However, when the *p-value* is close to the α-*risk*, there exists a fuzzy condition for making a decision about rejecting a null hypothesis. In this paper, a method for incorporating the fuzzy inference with a process capability index is presented for a bigger-the-best type evaluation. A concise score concept is then used to represent the grade of the process capability. In addition, an evaluation procedure is also developed in order that the users can use the method efficiently and thus be able to control the quality to the required level over time. An example is given to demonstrate that the method presented is effective and feasibile. With little change, the methodology can be extended to assess the smaller-the-best type process capability for another unilateral specification.

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Appendix. A Matlab Program

```
%%% program of fuzzy evaluation for process capability
n=input('given sample No.    ');
cpl=input('given C=cpl    ');
lsl=input('given lower specification limit    ');
xbar=input('given sample mean    ');
s=input('given sample standard deviation    ');
alpha=input('given alpha-risk value    ');
alw=input('given allowance value    ');
disp('****** next to compute ')
aa1=gamma((n-1)/2);bb1=gamma((n-2)/2);
bn=sqrt(2./(n-1))*aa1/bb1
cpl_hei=bn*(xbar-lsl)/(3*s)
aa=gamma((n-1)/2)*gamma((n-3)/2);bb=gamma((n-2)/2)^2;
var=aa/bb*(1/9/n+cpl^2)-cpl^2
delta=3*sqrt(n)*cpl;
x=3*sqrt(n)/bn*cpl_hei;
p_value=nctcdf(x,n-1,delta)
disp('****** next to fuzzy inference')
% the input universe is between 0 to 0.15
ze=alpha+alw;
ns=ze*0.8;nm=ze*0.5;nb=ze*0.2;
ps=ze+0.01;pm=ze+0.025;pb=ze+0.05;
a=newfis('cfis');
a=addvar(a,'input','p-value',[0 0.15]);
a=addmf(a,'input',1,'S3','trapmf',[-0.1 0 nb nm]);
a=addmf(a,'input',1,'S2','trimf',[nb nm ns]);
a=addmf(a,'input',1,'S1','trimf',[nm ns ze]);
a=addmf(a,'input',1,'ME','trimf',[ns ze ps]);
a=addmf(a,'input',1,'B1','trimf',[ze ps pm]);
a=addmf(a,'input',1,'B2','trimf',[ps pm pb]);
a=addmf(a,'input',1,'B3','trapmf',[pm pb 0.15 0.2]);
a=addvar(a,'output','score',[0 100]);
a=addmf(a,'output',1,'L3','gaussmf',[4.247 0]);
a=addmf(a,'output',1,'L2','gaussmf',[4.247 20]);
a=addmf(a,'output',1,'L1','gaussmf',[4.247 40]);
a=addmf(a,'output',1,'ME','gaussmf',[4.247 60]);
a=addmf(a,'output',1,'H1','gaussmf',[2.213 75]);
a=addmf(a,'output',1,'H2','gaussmf',[2.213 85]);
a=addmf(a,'output',1,'H3','gaussmf',[4.247 100]);
rule=[ ...
    1 1 1 1
    2 2 1 1
    3 3 1 1
    4 4 1 1
    5 5 1 1
    6 6 1 1
    7 7 1 1];
a=addrule(a,rule);
score=evalfis(p_value,a);
if p_value >=0.15
    score=100;
end
score
disp('**** end of simulation ****')
```