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An Integrated Machining Approach for a Centrifugal Impeller

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When a 3-axis CNC machining centre is used for producing an impeller, great difficulties, i.e. collisions between the cutting tool and the impeller, can occur. The blade of an impeller is usually designed with a ruled surface. As the surface is normally twisted in design to achieve the required performance, it can cause overcut and collision problems during machining. The hub of the impeller is usually designed with an irregular surface, and is machined within a narrow and deep groove. The issues – how to satisfy the quality requirements of the part, reduce the machining time, and avoid the occurrence of collision – become an integral problem. This work develops an integrated 5-axis machining module for a centrifugal impeller by combining related machining technologies. As a result, cutter location (CL) data based on the geometric model of blade and hub are generated. Finally, the CL data are confirmed through software simulation. The results of verification show that the machining methodology and procedure adopted are successful.

Keywords: CAM; 5-axis CNC; 5-axis Machining; Impeller; Tool-path Planning

1. Introduction

Five-axis computer numerical controlled (CNC) machines are widely used to produce aerospace parts, turbine impellers, and machining dies. These parts usually have complex geometry which is represented by parametric or freeform surfaces. As an improvement over 3-axis machining, 5-axis machining offers advantages such as higher productivity and better machining quality. In 5-axis machining, the tool axis has two additional degrees of freedom, allowing a more efficient tool path. The centrifugal impeller is a good demonstration of the efficient designing and manufacturing capabilities of 5-axis machining.

The surface model of the impeller is designed with an extremely twisted surface and its blades overlap with each other substantially. If traditional 3-axis machining is employed, there will be serious collisions between the cutting tool and blades. For such complex shapes, it is general practice to adopt 5-axis machining. The major difference between 3-axis machining and 5-axis machining is that the tool axis in 5-axis machining can be rotated to adapt to the curvature change on the cutting surface to avoid collision or interference between the cutter and the workpiece. Furthermore, 5-axis machining can improve the degree of freedom, precision, efficiency and quality for the machined surface to satisfy the various requirements of product design. Because of the high cost of 5-axis machines and the trend for increasingly complicated relative positioning between the cutter and workpiece, it is important to understand how to plan effective tool paths and how to obtain correct cutter location data (CL data). These are very important tasks for manufacturing a product, and form a major focus for the present study.

To shorten the product development process and become more competitive, computer-aided design (CAD) and computer-aided manufacturing (CAM) are implemented in mechanical manufacturing. Based on the required performance for 3-axis machining, the corresponding CAM system is available and ready for use, but compared to 3-axis machining, there is room for improvement of the CAM system for 5-axis machining. Our goal is to design and develop a 5-axis machining module for centrifugal impeller manufacturing. For impellers with similar geometric characteristics, the module developed here can be used to plan the tool path effectively, shorten the lead time of pre-manufacturing, and reduce the manufacturing cost.

2. Theoretical Model

The centrifugal impeller shown in Fig. 1 is a circular revolving entity. It is composed of fifteen identical blades and a hub. The angle variation between two blades is 24°. Usually, by preliminary aerodynamics and fluid mechanics calculation, the hub curve is obtained by using an optimisation algorithm. Rotating the hub curve around the axis of rotation forms the hub surface. The blade of an impeller is composed of a suction surface, a pressure surface, a leading edge and a trailing edge.

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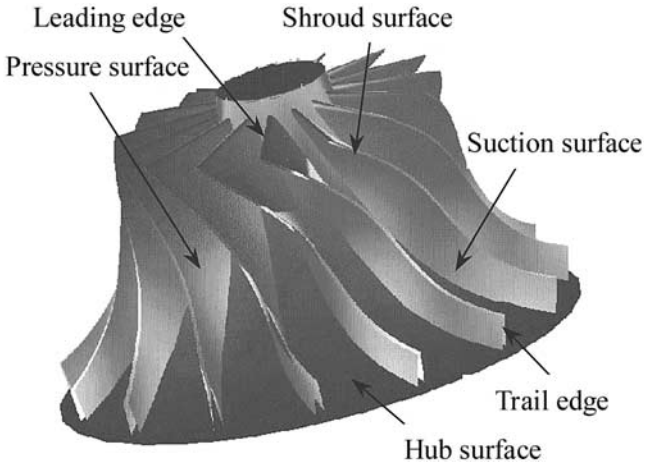

Fig. 1. Surface model of a centrifugal impeller [1].

The shroud surface identifies the outer shape of the blade. The surface of the blade will be closed between the shroud surface, the suction surface and the pressure surface. Rotating the shroud curve around the axis of rotation forms the shroud surface.

There are a few related studies on the flank milling of ruled surfaces. Sielaff [1] avoids the interference problem by using the midpoint of the ruling line as the base point, and using the surface normal at this point as the axis of rotation, and then rotating the cutter until the line of contact on the curve surface is achieved. Lai et al. [2] focused on seriously distorted non-developable ruled surface machining, and used the enclosed surface of a circular surface or cone surface to approximate a ruled surface in their work on ruled surface machining. Lai investigated this issue using kinematics to make the cutter tangent on one edge of the ruled surface, and located the intersection on the other edge to minimise the error. Related work on machining on the leading edge is very limited. At present, information regarding leading-edge machining still relies heavily on data and experience collected from industrial practice.

An iso-parametric method is frequently used to plan the tool path on the hub surface. Each tool path is generated along the direction of each parameter. More often than not the planned tool paths will be highly concentrated at the narrow area of the curved surface. The effectiveness of machining must improved in this case. To reduce the amount of tool-path data, Suresh and Yang [3] calculate tool path by controlling the residual scallop-height, and the interval between tool paths is obtained and expressed by cutter diameter, scallop-height, and radius of curvature. Suresh and Yang find that this method can reduce the amount of tool-path data, but there are still problems to be resolved in real applications. Lin and Koren [4] propose a planning method for effective tool paths. They use an offset of the tool path, which guarantees that the cutter will move in a new area of the part surface, without redundant machining. Their work also involves the investigation of an accurate tool-path interval and its conversion into a parametric interval. The derivation of the maximum CC path interval is based on a ball-mill cutter for 3-axis machining. There will be errors on the curved surface caused by the residual scallop-shaped area between the tool paths. There will also be a chord error along the moving direction of cutter. The chord error will affect the number of cutting points needed for each tool path. To control the error, it is necessary to evaluate the distance between the straightline portion of the tool path and the curve of the workpiece. In dealing with this, Loney and Ozsoy [5] use a numerical analysis method to calculate the parametric value and based on the result to determine whether it is necessary to add extra cutting points. The above work focuses on algorithms for a single surface or individual machining technology. No provision is made for as an integrated machining methodology. Our work focuses on the machining of various geometric models to explore problems encountered and includes flank milling of a twisted ruled surface on the blade, machining on the thin leading edge, tool-path planning on the hub surface, and the planning of tool paths for narrow and deep machining areas. The machining technology for a centrifugal impeller and 5-axis machining are integrated in this work. The complete manufacturing process planning for the impeller will also be discussed.

3. Process Planning for Impeller

Process planning for the centrifugal impeller is discussed in this section. First to be discussed are problems encountered in blade machining, and then an algorithm is proposed for the tool axis adopted to obtain cutter location data during blade machining. Major focuses are cutter contact data correction at the area between the blade and hub surface, and selection of the machining procedure. For machining planning on the hub surface, the tool-path algorithm is thoroughly discussed with consideration of the machining requirement. The result is a complete manufacturing process for impeller machining.

3.1 Algorithm on Tool Axis

For machining on the leading edge, a straight line is used to approximate two boundary curves of the ruled surface. Bohez et al. [6] describes a new tool-path generation algorithm, from which the undercut can be further diminished by moving the tool away from the surface that has to be machined and along the surface normal at a point P on the isoparametric line, until the tool is made tangent to the inner and outer edge of the blade surface. Point P is defined such that the angle between the surface normal at point P and the surface normals at both the inner and outer edge of the blade surface are the same. The cutter radius is R, and the angle between the two normals is ϕ. The tool axis is then obtained by moving along the normal of the curved surface from point P at a distance of $R/\cos(\phi/2)$.

We adopt the "line approximation method" for the machining of the leading edge. The boundary normal at one end of the curved surface is used to determine the initial tool axis. Based on the characteristics of a twisted ruled surface, the major error occurs at the other end of the curved surface. The tool axis needs to be modified, as shown in Fig. 2, to avoid overcut.

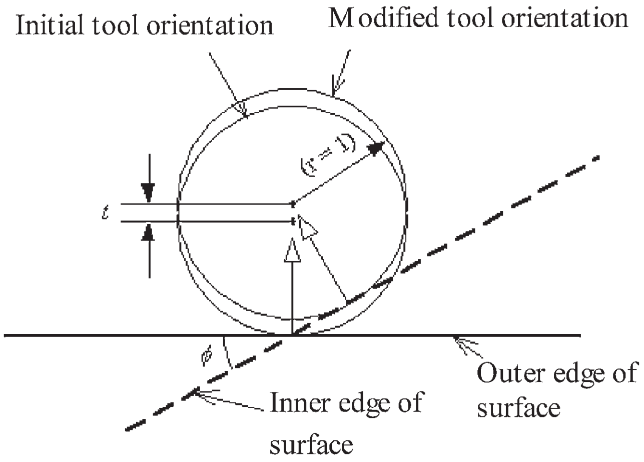

Fig. 2. Line approximation method.

The initial tool axis is based on the boundary normal at the outer edge of the ruled surface. There will be maximum error at the inner edge of the ruled surface. To make the cutter tangent to the inner edge and outer edge of the ruled surface, a cutter that is positioned at the inner edge of the ruled surface should be moved a distance of t along the normal at the outer edge of the ruled surface. The amount of displacement is $t = R(\sec\phi - 1)$ where the cutter radius is R and the angle between the two boundary normals is ϕ.

The algorithm for flank milling on the blade surface adopts the "maximum vector displacement method". The initial tool axis is selected to make the tool axis parallel to the ruling line on the ruled surface. That means the cutter must be offset at a distance equal to the cutter radius. A taper cutter is often used for increased strength. The method for determining the tool axis is now described. The cutter is offset along the same direction at both the inner and outer edge of the ruled surface, but the displacement will not be the same owing to the taper. Since the initial tool axis is parallel to the ruling line, there will be an overcut on the ruled surface. The maximum overcut occurs at the outer edge of the ruled surface [7] as the angle, between the normal at the other edge of the ruled surface and the direction the cutter shifted, is the maximum.

To further study the maximum error, the tool axis is used as the normal of the examination plane. To evaluate the amount of error, the ruled surface is divided into a collection of several ruling lines. The intersections between the examination plane and each ruling line are made and the distance from each intersection to the tool axis is calculated. After comparing all the values, the location of the shortest distance between the tool axis and the intersection is then identified. If the distance is smaller than the cutter radius, an overcut is generated. Otherwise, there is an undercut where the distance is greater than the cutter radius. The cutter is then moved to the tangent ruled surface to obtain the final tool axis, as shown in Fig. 3.

3.2 Cutter Contact Data Correction

Generally, the blade and hub surface should be connected by a fillet based on the geometric model of blade, but from the original geometric data, the blade is a single ruled surface, and the bottom of the blade and hub surface is simply connected. If the tool-path calculation is based on a single blade without considering the relationships between blade surface and hub surface, there will be an overcut at the upper part of the hub surface. It is necessary to consider the blade surface together with the hub surface when planning the tool path on the blade surface.

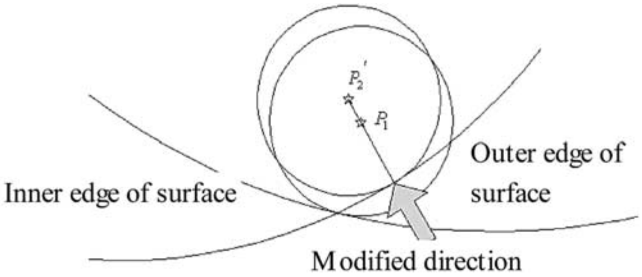

Fig. 3. Maximum vector displacement method.

There are two ways to avoid the overcut. One is to trim the blade surface and construct a fillet between the blade surface and the hub surface, and then incorporate the new surface data into the algorithm to avoid an overcut on the hub surface. The other is to plan, based on the original blade surface data, while moving the cutter away from the hub surface till the cutter is tangent to the hub surface. This study adopts the latter method.

3.3 Tool-path Planning on Blade

The general machining procedure usually includes rough milling, semi-finishing, and fine milling. A blade is a thin component, especially the leading edge. If the normal machining procedure is adopted on the leading edge of blade, the leading edge is easily damaged owing to potential machine vibration. This is not caused by a tool-path problem or an error of the tool axis.

It is difficult to control vibration during machining. To avoid this problem, it is necessary to rearrange the machining procedure. This work shows that it is important to machine the leading edge first, to keep more material in the blade to maintain the strength of the blade. This procedure substantially reduces the damage caused by vibration. The leading edge of the blade is machined prior to fine milling of the blade surface. The sequence of machining is to machine the pressure surface of the blade from the lower part towards the upper part, and then to proceed to machine the suction surface of the blade from the upper part towards the lower part. This sequence should use down-milling to minimise the effect of vibration.

3.4 Tool-Path Planning on the Hub Surface

It is necessary to calculate the tool path on one machining section when planning the tool path on the hub surface. The so-called machining section is the portion of the hub surface between the two blades. Before carrying out tool-path planning, the machining section must be clearly identified to obtain the working boundaries. The upper edge and lower edge of the hub surface are regarded as the working boundaries. The left and right working boundaries must be found by using the blade surface. There are two ways to calculate the boundary. One is to construct a fillet between the blade surface and the hub surface. The boundary curve can be obtained and modified by having the new surface trimmed by the fillet. The new curve can be treated as the new right and left boundary of the machining section. The other way is more precise. This study uses the tool path of the flank milling on the left and right blade as the boundary of the machining section. This will reduce the redundant tool paths between blade machining and hub surface machining, but the tool path for flank milling on the blade is the prerequisite.

The machining section formed by tool paths of blade machining and the hub surface boundary is rather irregular. There is an extremely narrow area in the middle and upper part of the machining section and the narrow opening has a funnel shape. The lower portion is an area with a large difference in width between the upper and lower part. The iso-parametric method is not applicable to this area for the tool-path calculation because the tool paths are very dense in the narrow opening. Using the iso-parametric method produces an uneven surface and low efficiency, therefore, the non-constant parametric method is a better choice for tool-path planning.

For hub surface machining, each tool path is calculated for intersection by the left and right working boundaries individually. If there is more than one intersection on the working boundaries, the path is trimmed by the working boundary curve, in which an odd-numbered curve path is picked as the tool path and the even-numbered curve path will be eliminated. The even-numbered curve path is outside the working boundary, as shown in Fig. 4. For convenience of judging the connecting path and the boundary, the machining section is divided into upper and lower subsections by defining the location which is the shortest distance between the left and right boundary on the parametric plane. This will make the shapes of the two subsections simpler and easier to handle. A one-way method is adopted in this work for tool-path planning on a hub surface, which means that one of the working boundaries is used as the initial path. First of all, the working boundary of the machining section is defined. The tool path is generated based on the constant chord error method and the next tool path is calculated until this path runs into the other working boundary. Because of the irregular shape of the machining section, once the tool path is available, it is necessary to make sure that it does not exceed the working boundary. Bad contact points should be eliminated at the end of the process.

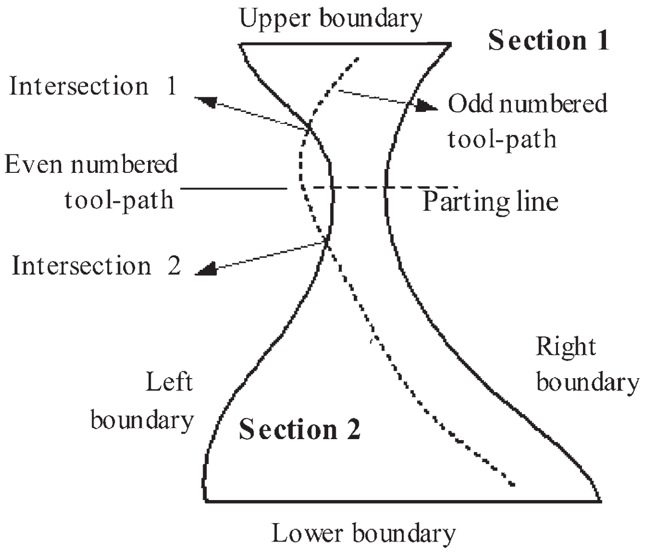

Fig. 4. Expression for boundary check method.

For the planning of the tool axis on the hub surface, an interpolation scheme is adopted. The bottom of the blade is thicker than the top. When defining a plane to cross two blades, the cross-section of the blades and the hub surface is shown as a "V" shape. Since the blade itself is a revolving component, it should be a geometrically feasible shape if the interpolated method for the planning of the tool axis is adopted. Therefore, the initial tool axis for hub surface machining is given by using prior flank milling on blades at the two sides using the interpolated tool axis. An interference check is then made to find the final tool axis.

3.5 Complete Manufacturing Process of an Impeller

The complete manufacturing process of a centrifugal impeller includes shroud surface machining, hub surface rough milling, blade flank milling, and hub surface fine milling. The following section gives details of each machining step and the result of machining data calculation. Machining simulation is also shown.

First, the outer profile of the shroud surface of the impeller is machined by turning. After turning is completed, rough milling will proceed on the hub surface between the two blades. Rough milling on the hub surface produces a rough profile of the impeller. The rough tool path is obtained from the tool path of flank milling at a specific angular interval around the axis of rotation between blades. At the same time, the tool tip position needs to move away from the hub surface along the tool axis to provide the fine milling allowance. The remaining material will be removed when hub surface fine milling takes place.

The leading edge of the blade is of a relatively thin geometrical shape. To avoid damaging the leading edge by machine vibration and to improve the material strength, flank milling on the leading edge of the blade proceeds immediately after rough milling of the hub surface. Because the geometrical shape of the leading edge is constructed by a ruled surface with a negative curvature, this study adopts straight lines to approximate the surface boundary. The tool axis for flank milling on the leading edge is planned by this method. This method theoretically causes an undercut, and its machining is proceeded by six sub-sequences to minimise these errors.

The procedure for blade surface machining is to machine the pressure surface of the blade first, then the suction surface of blade is machined. Using the "maximum vector displacement method" for planning the flank milling, the associated tool axis is slightly different from that generated on the leading edge.

In order to avoid a cutter mark caused by the different tool axis at the joint between the blade pressure surface, the blade suction surface and the blade leading edge, it is found necessary to interpolate for the tool tip position and the tool axis at several machining positions. This makes tool axis transition smooth. The tool path for blade machining is shown in Fig. 5. Figure 5(*a*) shows the tool tip path linked by the tool tips. There are six machining sequences for the leading edge of the blade, and six separate tool paths on the leading edge of the blade are displayed. The tool axis at each cutting point is shown in Fig. 5(*b*). To show the tool axis clearly, the tool axis for leading-edge machining is represented by the first machining sequence. In actual machining, the tool axis of each sequence is not the same.

The constant scallop-height method is used to plan the tool path for fine milling on the hub surface. The machining section is divided into upper and lower subsections, and the upper subsection is machined first. The first tool path uses the right boundary of the machining section and the constant scallop-height method is used to control the desired feed between each tool path from the right to the left. The tool path is then linked in a zigzag style. The difference between the wide geometrical shape and the narrow one on the hub surface is clearly seen. It is found that there is no overlap on each tool path and machining efficiency is achieved as a result. The complete machining tool path of the impeller is shown in Fig. 5.

4. Machining Simulation and Verification

The surface model of the impeller is shown in Fig. 1 where the hub and blades are displayed. The angle between two blades is 24° (there are 15 blades for a complete impeller). The cutter used is a taper ball-mill of 0.05 in, with a free length of 50 mm and the taper angle of 3°. The desired chord error and scallop height on the hub surface is 0.01 mm. The following machining simulation is made with the software package Anvil Verify [8].

The machining simulation on the leading edge of the blade by Anvil Verify is shown in Fig. 6. Since every sequence is shown in a different colour during machining, the difference between each sequence can be seen. Machining on the blade surface is shown in Fig. 6. The flank milling is done from the lower part towards the upper on the pressure surface of blade. After the milling is completed on the pressure surface, the cutter is moved to the suction surface of the blade, and flank milling is carried out from the upper part towards the lower.

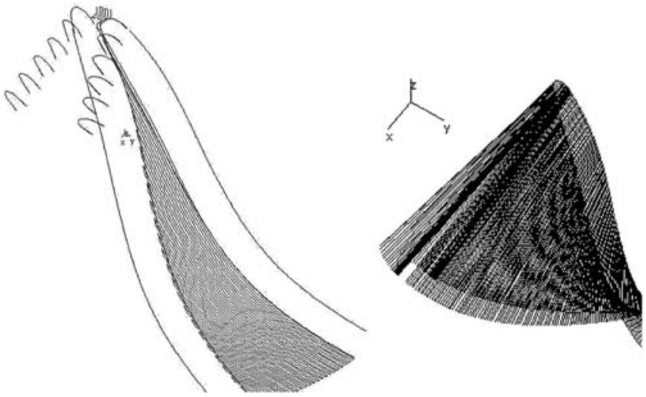

Fig. 5. The complete tool path for machining the impeller.

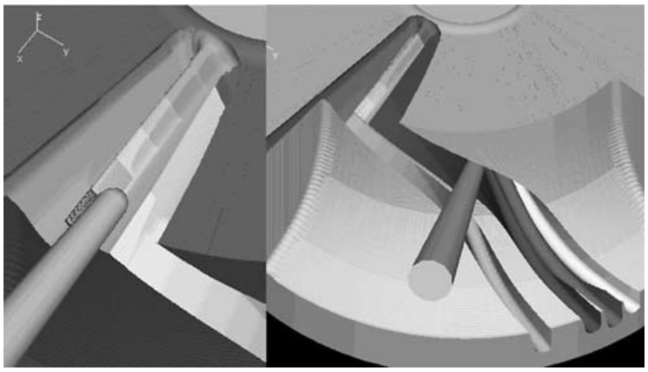

Fig. 6. A machining simulation of the leading edge and blade.

In Fig. 7, simulation of the hub surface machining is shown. The tool does not interfere with the two blades during machining. This confirms the feasibility of interpolating the tool axis. The complete machining of the impeller is obtained by rotating each tool path though an appropriate angular interval around the axis of rotation. In this case, there are 15 blades; therefore the angle between each pair of blades is 24°. Based on the sequence of rough milling on the hub surface, machining on the leading edge, fine milling on the blade, and fine milling on the hub surface, input is made to the program to simulate the complete machining.

5. Error Analysis

The purpose of error analysis is to compare the surface after machining with the original design model. Usually, a CMM is used to measure the freeform surface. The machined surface is compared with the original surface model and the difference

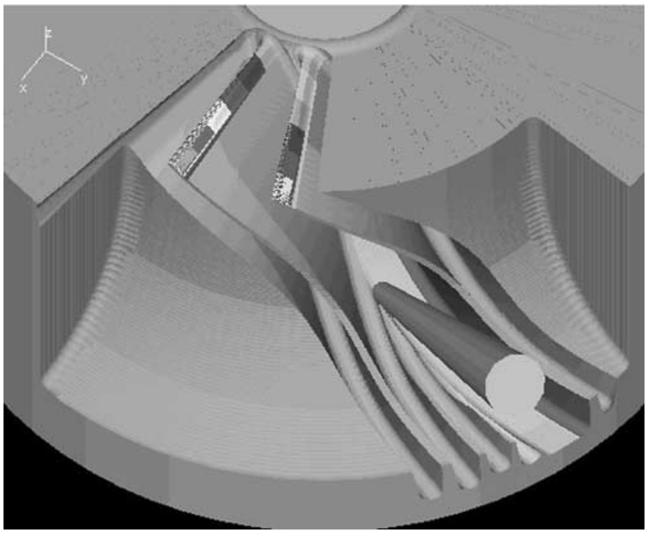

Fig. 7. A machining simulation of the hub surface.

will be identified. The residual scallop height or chord error caused by the machining on the surface can be measured by special equipment that is designed to measure surface roughness. As the CMM is limited to 3-axis, this prevents it from measuring the machined surface. Therefore, the CMM cannot be used for error inspection. The roughness measurement on the surface also encounters similar problems.

Our error analysis methodology uses software to simulate the machined entity after machining, and compare it with original surface model. Detailed error inspection procedures are given as follows: establish the original surface model, obtain physical data of the entity after machining simulation, and then conduct error analysis.

5.1 Original Surface Model

Since the original geometrical data are all surfaces, it is necessary to form an enclosed surface for constructing the surface model. Helix software is used to build the original surface model, shown in Fig. 8. There is no fillet at the joint between the blade bottom and the hub surface on the original surface model. For the impeller, flank milling on the blade makes a smooth joint between the blade bottom and hub surface by the semi-hemispherical shape of the cutting tool. This provides the fillet and it is not necessary to have the fillet on the original surface model.

5.2 Simulated Physical Machining Data

The simulation function of Vericut [9] is used to conduct machining simulation to obtain the physical data after machining. Vericut can generate the physical machining data during machining simulation. The physical data are recorded in STL format. The physical data recorded in STL format are only limited to the surface data which are divided into several triangle patches. Each patch includes the three corners of the triangles and the normal of the patch, but, as these data are in STL format, a real surface model cannot be rebuilt without topology data.

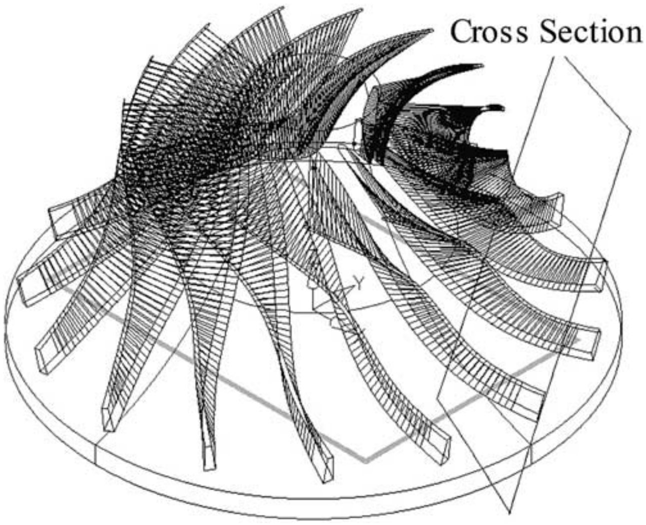

Fig. 8. A cross-section along the direction of the ruling line.

Vericut has Auto-Difference and Slice modules that are related to error comparison. Auto-Difference is used to compare the STL file of the surface model with the physical machining data. The difference can be displayed on a monitor screen, but the analysis is more visual in nature and is not a numerical result. Slicing is used to make the intersections by inputting the physical machining data and the user's defined plane. The result of the intersection can be output as an IGES format. In this work, a Slice module is use to output physical machining data that is compared with the original surface model for error analysis.

5.3 Error Comparison

Comparison is made of the error in terms of contour dimension. The way to calculate the contour error is to use the same plane to make a cross-section at physical machining data and original surface model, and make the comparison between the two cross-section contours. In this work, the Slice function of Vericut is used to convert the cross-section of physical machining data via the IGES interface into Helix software. The cross-section of the original surface model is computed in the Helix environment. Because of computational limitation, only one machining section is selected. This means that there will be only the shape of two blades in cross-section and the shape of the hub between blades. The rest is the unmachined portion of the workpiece.

Table 1 shows the location data of the cross-plane. There are two ways to define the cross-plane: one is with three points; the other is to define the plane normal and its distance from the origin. These data are for Vericut software. Cross-section 1 and cross-section 2 are made along the direction the ruling line. The main purpose is to observe the difference the between machined contour of the blade and the original contour of the blade, and the machining situation on the hub surface. One of the cross-sections obtained in this way is shown in Fig. 8. The cross-section obtained by Vericut is a Polyline, and not a continuous line, and straight lines can be used to connect broken lines. Because the cross-section and the coordinate definition are the same for these two models, the difference can be observed by appropriate software to match these two intersections. The error comparison on cross-section 1 is shown in Fig. 9 which shows the overlap between the two cross-sections. The lighter line is the contour of the original surface

Table 1. Location data of cross-section.

Cross-section number	Cross-section data
Cross-section 1 (along ruling line direction of blade)	Cross-section normal (0.09917, 0.85438, −0.51009), vertical distance between cross-section and original point = 27.3894
Cross-section 2 (along ruling line direction of blade)	Cross section normal (0.09787, 0.61223, −0.78460), vertical distance between cross-section and original point = 6.42038

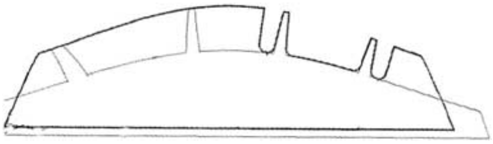

Fig. 9. Overlap of cross-section 1.

model. It is shown that the positions and shapes of the two contours match very well. To further investigate the fine differences, the contour of the blade is enlarged. Reference grid lines are used to measure the amount of error, as shown in Fig. 10. From the enlarged drawing, the position of the maximum error is identified. The amount of error can be obtained with the aid of the grid lines. In Fig. 10, the dimension of each grid line cell is 1 mm. It is shown that the maximum error is located at the midsection of the blade. The error is about 0.04 mm after proper evaluation. This is a good result based on general machining requirements. In the drawing, at the joint between the left blade and the hub surface, there is a circular shape difference between the machined contour and the original contour. This is because the fillet is not constructed on the original surface model. From the contour of the circular shape, it can be seen that the radius is almost equal to that (1.27 mm) of the cutter used.

The same methodology is used to process other cross-sections and estimate the errors. Because the error is very small, it is necessary to enlarge the portions compared for error evaluation. The result shows that the maximum error occurs again at the midsection of blade. The estimated error is an undercut of 0.05 mm. (Fig. 10).

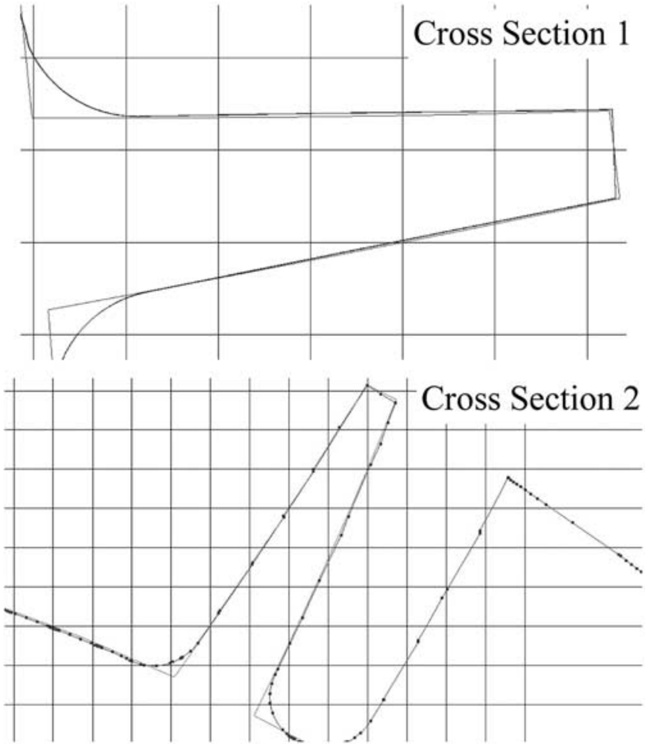

Fig. 10. Enlargement of part of the cross-section.

5.4 Discussion on Error

Based on the result of the contour error, it is found that most error is undercut error and that maximum error is mostly located at the midsection of the blade. This shows that the algorithm for flank milling of the blade generates a maximum undercut at the midsection. The error can be reduced if multiple machining tool paths are used.

From the contour comparison, some overcut at the joint between hub surface and blade is observed. This is caused by the fact that the tool tip is modified during flank milling. As shown in Fig. 11, although the tool tip can be made tangent to the hub surface by retracting the tool tip, the semi-hemi-spherical shape of the cutter may cut into the hub surface because of the different curvature of cutter and hub surface. For a taper tool, retracting the cutting tool along the tool axis away from the machined surface will increase the undercut during flank milling. The difference is shown as a shaded area in the drawing. Because the tool axis is already modified, the exact location where the cutter contacts the blade surface cannot be accurately estimated. Moreover, there is no guarantee that the retraction of the cutter will not generate other errors in the flank milling. The issue remains to be resolved in the future.

6. Conclusion

The objective of this study is to propose a method for improving the machining of impellers using 5-axis machining. The main module developed in the C++ language, automatically generates a tool path of the impeller model, and avoids blade deformation and chatter, and shortens the lead-time of pre-manufacturing, and reduces the manufacturing cost.

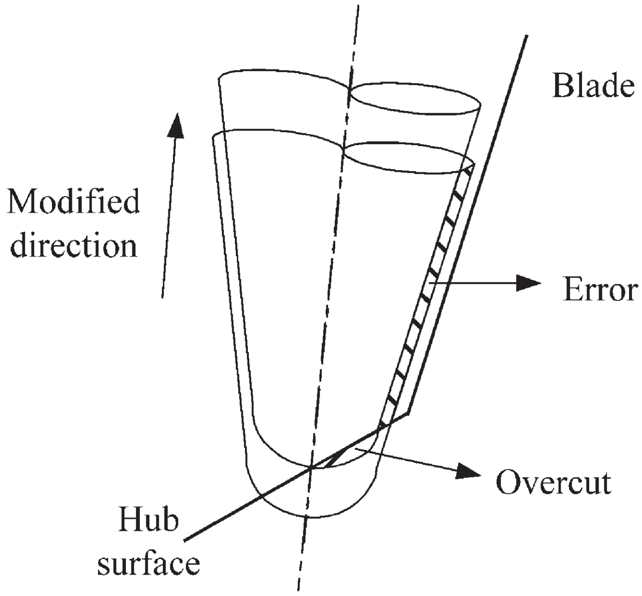

Fig. 11. Error generated by raising tool.

In this work, the machining area is divided roughly into two areas: blade and hub. Their geometrical characteristics are studied and suitable machining methodology is introduced. The production technique for 5-axis machining technology related to a centrifugal impeller are integrated for complete production process planning. The CL data generated are confirmed through software simulation. For error analysis, a comparison between the physical machining data and the original surface model is made. Finally, the results of the verification prove the machining methodology and the procedure applied is useful and successful.

References

1. W. Sielaff, “Fuenfachsiges NC-Umfangsfraesen verwundener Regelflaechen”, Springer-Verlag, Berlin, 1981.
2. Tien-Chin Lai, Feng Lee, De-Jr Kong, Shiau-Chuen Wang and Shiu-Tang Wu, “A new method for machining ruled surface by computers”, Journal of Xi’an Jiao Tong University, 30(2), pp. 61–67, 1996.
3. K. Suresh and D. C. H. Yang, "Constant scallop-height machining of free-form surfaces", Transactions ASME Journal of Engineering for Industry, 116, pp. 253–259, May 1994.
4. Rong-Shine Lin and Y. Koren, "Efficient tool-path planning for machining free-form surfaces", Transactions ASME Journal of Engineering for Industry, 118, pp. 20–28, February 1996.
5. Gregory C. Loney and Tulga M. Ozsoy, "NC machining of free form surfaces", Computer-Aided Design, 19(2), pp. 85–90, 1987.
6. Erik L. J. Bohez, S. D. Ranjith Senadhera, Ketan Pole, Joost R. Duflou and Tsau Tar, "A geometric modeling and five-axis machining algorithm for centrifugal impellers", Journal of Manufacturing Systems, 16(6), pp. 422–436, 1997.
7. B. K. Choi, J. W. Park and C. S. Jun, “Cutter-location data optimization in 5-axis surface machining”, Computer-Aided Design, 25(6), pp. 377–386, 1993.
8. MCS Inc, “ANVIL EXPRESS 5.0”, 1999.
9. CGTech, “VERICUT Version 4.1”, 1999.