Ann Reg Sci (2000) 34:55–68

The Annals of
Regional Science

The models and economics of carpools

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Received: August 1997/Accepted: October 1998

Abstract. For studying carpooling problems, this paper presents two models, namely deterministic and stochastic, and gives the economic explanations to the model solutions. We investigate the jockeying behavior of work commuters between carpooling and driving alone modes through solving each model for both no-toll equilibrium and social optimum. The logit-based stochastic model involves the consideration on preference option of mode choice. Under some assumptions, the paper explains how the amount of carpooling is affected by fuel cost, assembly cost, value of time, preferential or attitudinal factors and traffic congestion. It is found that carpooling is sensitive to traffic congestion reduction only when a congestion externality-based tolling scheme is implemented.

1. Introduction

A carpooler, as defined in this paper, is anyone who shares transportation to work in a private vehicle with another worker. To individuals, carpooling is usually cheaper than either using a car alone or using mass transit because of the splitting of expenses between two or more riders and no walk to or wait for scheduled public transportation. Surveys (Bureau of Census 1979) show that the average transit commuter trip requires about 70% more time than the average carpool trip. The comfort level of carpooling is basically the same as that of the private vehicle, but the need to own a special car for regular travel to work, with all the appending costs, is greatly reduced (Oppenheim 1979). In terms of the 1975–1976 national surveys made by U.S. Bureau of the Census (1979) in 41 urban areas, 20 to 23% of American workers who commute in a vehicle are carpoolers (also see, ITE Committee 6A11, 1981). In 1990, the share of carpooling to work declined but still remained at 13.4% nationwide (FHWA 1993).

All correspondence to Hai-Jun Huang. I gratefully acknowledge the Transport Operations Research Group of Newcastle University for providing research conditions, where I worked for one year as a visiting scholar supported by the State Education Commission of China.

Carpooling is also a well developed mode of transportation in the United Kingdom where 12% of total trip miles are travelled by passengers in non-household cars, while only 10% travelled on local stage buses (Bonsall 1981; Vincent and Wood 1979). In Canada, an estimated 1.1 million persons ride to work as automobile passengers every working day, while 1.3 million choose some form of public transit (McCoomb and Steuart 1981). In Australia, carpooling has also been accepted as an important urban transportation mode (Richardson and Young 1982).

Historically, the interest in carpooling and vanpooling has been greatest at times of fuel shortage or crisis (e.g., the 1973–1974 energy crisis). The current interest has outlasted this stimulus to include considerable reductions in traffic congestion, air pollution, noise levels and parking space by increasing the amount of carpooling and other forms of commuter ridesharing. In the late 1970s, many important empirical studies of carpooling were conducted (e.g., Brunso et al. 1979; Hartgen 1977; Kendall 1975; Margolin et al. 1978; McCoomb and Steuart 1981; Richardson and Young 1982; Vincent and Wood 1979). These researches focused on understanding who are most likely to carpool, how carpooling takes place and why commuters carpool. Teal (1987) did a comprehensive and definitive analysis on answering these questions based on data from the 1977–1978 Nationwide Personal Transportation Survey of America.

A remarkable fact is that in most countries the level of using carpooling as a travel mode is still relatively low in comparison with our expectation. The reason is that in some ways carpooling is inferior to both driving alone and public transit riding. Furthermore, for people working flexible hours, carpooling is not convenient. With the spread of flextime, this will be an increasing obstacle to the spread of carpooling. Carpooling requires an increase in travel time due to the need to pick up and deliver carpool members. Carpoolers should suffer from the inconvenience caused by assembling riders and the loss of independence and privacy (Horowitz and Sheth 1977; Teal 1987). The service quality of mass transit has been being improved, which undoubtedly enhances its competitive force. In addition, for many people the anonymity of transit riding may be a more comfortable social climate than carpooling (Teal 1987). In order to increase the proportion of commuters who carpool, some organized ridesharing programs have been conducted (Collura 1994; Giuliano et al. 1990). Among these programs, the employer-based one (for example, employers reduce or remove parking subsides) was found to be an efficient approach (Ferguson 1990; Willson and Shoup 1990).

The approaches used to study carpooling problems can be classified into three types in terms of their starting points (Kostyniuk 1982). The first type is to estimate the ridesharing potential of an area by considering the process and conditions of formation of a group who will travel together in a common vehicle (e.g., Berry 1975; Kendall 1975). The second approach of predicting ridesharing demand is based on utility maximization principle by viewing ridesharing as an individual or household travel decision. Consequently, a series of logit models and microsimulation models were developed (e.g., Ben-Akiva and Atherton 1977; Bonsall and Kirby 1979; Cambridge Systematics 1977).

The third approach considers demand and supply effects to obtain equilibrium traffic flows, in which ridesharing plays a role of reducing traffic congestion through commuters' modal shift (see a review by Charles River Associates 1980). The economic simulation model proposed by Small (1977) is a representative one in which the conventional logit demand model and the simple deterministic queuing model of uniform traffic flow are iteratively used. The equilibrium model developed by Daganzo (1982) can treat carpooling problems on general urban networks, but the demand for carpools is assumed to be fixed or independently elastic so that the model fails to capture the important phenomenon of commuter jockeying among modes.

This paper presents two models for studying carpooling problems and gives the economic explanations to the model solutions. In Sect. 2, we investigate the jockeying (shifting) behavior of work commuters between carpooling and driving alone modes by using a simple deterministic model for both no-toll equilibrium and social optimum. In Sect. 3, a logit-based model, also for both no-toll equilibrium and social optimum, is developed to involve the stochastic character of modal choice and the preference option. A numerical example is used to validate our study in Sect. 4. Conclusions and extensions for incorporating elastic trip demand are given in Sect. 5.

The model solutions derived in this paper provide us clear and theoretically rigorous explanations about how the amount of carpooling is affected by fuel cost, assembly cost, value of time, preferential or attitudinal factors and traffic congestion. With the simplified but reasonable assumptions we find that, in no-toll competitive equilibrium circumstance which implies no external interference exists, the amount of carpooling is not related to traffic congestion and consequently the traffic volume cannot be reduced as anticipated. A congestion externality-based tolling scheme can help to reach a social optimum and then realize the potential of carpooling.

2. Deterministic model

Suppose that each working day in the morning rush hour N identical individuals must travel to work from a residential area. Without loss of generality, the workplace of individuals is assumed to be a point (i.e., it has no spatial dimension) and is connected to the residential area by a single road. For simplicity, two travel modes are assumed, i.e., people could either drive a car alone to work or share a car with another to form a *two-person* carpool. It is straight forward to incorporate carpool with three or more occupants. Lee (1984) has studied the optimal size of a carpooling vehicle. Let x and y denote the numbers of carpoolers and solo driving persons, respectively, $x + y = N$; then the number of vehicles is $v = y + x/2 = N - x/2$. Clearly, $N \geq v \geq N/2$ which corresponds to $0 \leq x \leq N$.

There are many factors (some interrelated) affecting carpooling propensity and modal choice. Teal (1987) categorized the measurable factors into three groups, i.e., socio-demographic, transportation and locational variables. We start our analysis from the most simple transportation cost equilibrium. The travel costs for *each* carpooler and a solo driver, respectively, can be defined as

$$c_x = \beta t(v) + (f + a)/2, \tag{1}$$

$$c_y = \beta t(v) + f, \tag{2}$$

where β is the value of time; $t(v)$ is the travel time spent on the line haul part of working trip, which is assumed to be a continuous, monotonically increasing and strictly convex function of traffic volume v (this is a commonly used function form in practice); f is the fuel cost consumed on the line haul part; a is the assembly cost for forming a carpooling vehicle, which includes the travel cost generated by picking up the companion in a residential area and the coordination cost resulting from unpredicted mutual waiting. The differences in costs other than line-haul between the two modes are assumed to be negligible or are counted in the parameter a.

2.1. No-toll equilibrium

According to classical traffic equilibrium theory, with the no-toll condition we have

$$\begin{cases} x = N \quad \text{and} \quad y = 0, & \text{if } c_x < c_y \text{ (i.e., } a < f) \\ x = 0 \quad \text{and} \quad y = N, & \text{if } c_x > c_y \text{ (i.e., } a > f) \\ x \text{ and } y \text{ take arbitrary nonnegative} & \\ \text{values, s. t. } x + y = N, & \text{if } c_x = c_y \text{ (i.e., } a = f). \end{cases} \tag{3}$$

Obviously, the above equilibrium solution is not generally supported by empirical observations. In reality, driving alone is the first most popular mode of commuting to work when no powerful factors motivate carpooling. Note that the equilibrium state given by (3) is not related to the traffic congestion function $t(v)$ and an explicit modal split can not be obtained when $c_x = c_y$. The reasons for the former are: carpoolers can not divide the travel time as all commuters experience the same travel time on the line haul part of the trip; and the mode choice behavior in this model is assumed to be not affected by any other factors relating to traffic congestion. So, without any external intervention, very little can be expected for traffic congestion reduction through self-forming of carpools. The latter is because the gasoline consumption for each vehicle is assumed to be traffic flow-independent, i.e., travel time-independent in our study, and f is then a constant. Otherwise, the modal split can be obtained from the equation $a = f(t(v))$ where the fuel cost f takes a functional form of the travel time t.

2.2. Social optimum

We now consider a social optimum-based equilibrium solution through minimizing the total social cost of the system.

$$\text{Minimize } TSC = xc_x + yc_y, \tag{4}$$

subject to

$$x + y = N, \tag{5}$$

as well as $x \geq 0$ and $y \geq 0$. The first-order optimality conditions include

$$c_x + N\beta t'(v)/2 = \mu, \tag{6}$$

$$c_y + N\beta t'(v) = \mu, \tag{7}$$

where μ is the Lagrange multiplier associated with (5), and $t'(v) = dt(v)/dv$. Equations (6) and (7) represent a kind of equilibrium for which a tolling scheme must be implemented. *Each* member in a carpooling vehicle should pay $[N\beta t'(v)/2]$, so a *two-person* vehicle is charged with a toll amounting to $[N\beta t'(v)]$ which is exactly the same with that paid by a solo driver as shown in (7). Then, the tolling is *anonymous* to all vehicles.

We give a theoretical explanation to the second terms of the left hand sides in (6)–(7). Each additional commuter will impose a congestion cost (known as an *externality*) on other trips by increasing travel time (the index used in this paper) or intensifying air pollution and noise level (the index representing public benefits). However, people will not consider this cost conscientiously. By setting a toll exactly equal to the externality we can ensure that the optimal private choices of commuters will also be the optimal social choices. Hence, the traffic congestion effect is now reflected through the congestion externality-based toll $[N\beta t'(v)]$ although the commuters still have identical travel time. This tolling scheme represents a kind of external intervention on the jockeying behavior of commuters between two modes, under which each carpooler needs to pay only half of that each solo driver does. In fact, the tolling scheme considered here, is a variety of the well-known marginal-cost pricing principle for optimizing congested road traffic flows. According to the theory, road users should pay a toll equal to the difference between the marginal social cost and the marginal private cost in order to achieve social optimum.

The explicit modal split can be uniquely determined from (6) and (7). With the definitions of c_x and c_y, combining (6) and (7) yields

$$a = f + N\beta t'(v). \tag{8}$$

In terms of the property of function $t(v)$ and the feasible region of v-values, $N \geq v \geq N/2$, the minimum and maximum values of $t'(v)$ are $t'(v)|_{v=N/2}$ and $t'(v)|_{v=N}$, respectively. As a result, that (8) holds must imply $f + N\beta t'(v)|_{v=N/2} < a < f + N\beta t'(v)|_{v=N}$. Hence, the equilibrium solution under social optimum becomes

$$\begin{cases} x = N \quad \text{and} \quad y = 0, & \text{if } a \leq f + N\beta t'(v)|_{v=N/2} \\ \text{solution of (8) with } v = N - x/2, & \text{if } f + N\beta t'(v)|_{v=N/2} \\ & \qquad < a < f + N\beta t'(v)|_{v=N} \\ x = 0 \quad \text{and} \quad y = N, & \text{if } a \geq f + N\beta t'(v)|_{v=N}. \end{cases} \tag{9}$$

Equation (9) says: when $a \leq f + N\beta t'(v)|_{v=N/2}$, all commuters will join carpools since the assembly cost a is so small that the marginal social carpooling cost is always less than the marginal social solo driving cost, i.e., $c_x + N\beta t'(v)/2 \leq c_y + N\beta t'(v)$ always holds; when $a \geq f + N\beta t'(v)|_{v=N}$, all commuters will drive alone as $c_x + N\beta t'(v)/2 \geq c_y + N\beta t'(v)$ always holds; a positive-flow equilibrium state (i.e., both x and y are positive) exists if and only if the assembly cost a is in a certain range, i.e., $f + N\beta t'(v)|_{v=N/2} < a < f + N\beta t'(v)|_{v=N}$ which clearly requires a greater than f. Comparing (9) and (3), we find that the frontier value of the assembly cost which initiates carpool commuting (i.e., let $x > 0$) is raised from f to $f + N\beta t'(v)|_{v=N}$, and then the carpooling propensity increases. This, of course, is the result of conducting tolling scheme.

Consider an example with the commonly used travel time function $t(v) = A + Bv^4$, here A is the free-flow travel time of the single road and B is a constant negatively proportional to the road capacity. For $f + \beta BN^4/2 < a < f + 4\beta BN^4$, the positive-flow equilibrium solution is $x = 2\left[N - \left(\frac{a-f}{4\beta BN}\right)^{1/3}\right]$ and $y = 2\left[\left(\frac{a-f}{4\beta BN}\right)^{1/3} - N/2\right]$, the traffic volume is $v = \left(\frac{a-f}{4\beta BN}\right)^{1/3}$, and the toll for each vehicle is $(a-f)$. From this solution, it can be seen that the amount of carpooling x, is positively proportional to the fuel cost f and the value of time β (if $a > f$) because carpoolers can divide these out-of-pocket costs (the β-related externality has been transferred into toll); negatively proportional to the assembly cost a because this cost is borne by carpoolers only. Similar analyses can be done for y and v.

Although the above equilibrium solution is more reasonable than that in no-toll equilibrium, it still is not realistic since the marginal social travel costs are equalized on both solo drivers and carpoolers. In real situations, this method of equalizing the costs of both sides may not be appropriate (for example, driving alone is the first most popular mode although its cost is also the most) and other modal split models which support empirical observations should be used. In next section, we develop a more general and more realistic model which considers the effects caused by random factors and preference option.

3. Stochastic model with preference option

Mode choice is a very complex decision process. Many influence factors are difficult to quantify and measure. To account for these factors in practice, a special "modal split" function must be developed. This function should take into account the measurable costs but allow for situations in which these costs are not equal at equilibrium. We use *generalized* utility function to characterize each mode as below

$$U_x = U - c_x + \xi_x, \tag{10}$$

$$U_y = U - c_y + \varphi + \xi_y, \tag{11}$$

where U is a constant term representing the utility received through a working trip, it could be related to individual's daily income; $\xi_x(\xi_y)$ represents the uncertainty in specifying the utility of carpooling mode (driving alone mode). In (11), the parameter φ (positive valued) has special meaning, it is the summation of all the attitudinal or psychological factors that make commuters have a subjective preference for driving alone mode (Duecker et al. 1977; Horowitz and Sheth 1977). This subjective preference is mainly caused by the desire for self reliance, independence and privacy. Speaking rigorously, subjective preference should not be regarded as a kind of measurable utility. In (11) it just plays a role in enhancing the probability of choosing driving alone mode.

3.1. No-toll equilibrium

Suppose the random terms (ξ_x, ξ_y) in (10) and (11) be identically and independently distributed Gumbel variables with mean zero, then at equilibrium the modal split at aggregate demand level is governed by a logit formula specified below (Anderson et al. 1992; Oppenheim 1995)

$$x = N/[1 + \exp(\theta U_y - \theta U_x)] = N/[1 + \exp(\theta\varphi + 0.5\theta(a - f))], \tag{12}$$

$$y = N/[1 + \exp(\theta U_x - \theta U_y)] = N/[1 + \exp(-\theta\varphi - 0.5\theta(a - f))], \tag{13}$$

where θ is a positive parameter relating to the standard deviation of random terms. The values of φ and θ can be estimated from survey data. Note that $\varphi > 0$ implies the share of solo drivers is greater than the share of carpoolers in the case of $a = f$, even in the case of $a < f$ sometimes (it depends on the values of φ and θ). Most surveys support this modal share.

It is easy to examine that modal split (12)–(13) correctly describes the relationships between carpooling share and cost parameters (a, f), perception accuracy on travel costs (θ), as well as subjective preference to solo driving (φ). This model solution is more flexible and more realistic than the model solutions presented in preceding section. Meanwhile, we should note that this solution is not sensitive to the change in traffic congestion and consequently the changes in public and social benefits since nothing relating to travel time is included in (12)–(13). Therefore, as done in Subsect. 2.2, the social optimum-based equilibrium must be applied.

3.2. Social optimum

Using the "representative commuter" concept introduced by Oppenheim (1995)[1], we can show that the gross direct utility of the representative commuter at aggregate demand level is

$$DUR = -(x \ln x + y \ln y)/\theta + (N \ln N)/\theta, \tag{14}$$

subject to $x + y = N$ (see a proof in the Appendix of Huang and Yang 1995). The DUR can be interpreted as a measure of commuter welfare from working trip at an aggregate level, in the absence of income effects (the daily income is assumed to be identical to all commuters, so its effects can be omitted).

On the other hand, the total social cost is $[xc_x + y(c_y - \varphi)]$, here the parameter φ is considered as a measure that the preference "decreases" the cost

[1] The "representative commuter" concept is based on the discrete choice theory of imperfect competition (see, Anderson et al. 1992).

of solo driving. Hence, the net social benefit (or net social welfare) can be measured as

$$NSB = [-(x \ln x + y \ln y)/\theta + (N \ln N)/\theta] - [xc_x + y(c_y - \varphi)]. \tag{15}$$

Maximizing the NSB subject to $x + y = N$ generates the first-order optimality conditions as

$$(\ln x + 1)/\theta + c_x + N\beta t'(v)/2 = \mu, \tag{16}$$

$$(\ln y + 1)/\theta + c_y - \varphi + N\beta t'(v) = \mu, \tag{17}$$

where μ is the Lagrange multiplier associated with constraint $x + y = N$, and $t'(v) = dt(v)/dv$ with $v = y + x/2 = N - x/2$. As explained in Sect. 2, the term $N\beta t'(v)$ should be regarded as a toll to be imposed on each vehicle. The first terms in the left hand sides of (16) and (17) represent the effects on costs perceived by commuters, caused by random factors.

Combining (16) and (17) with $x + y = N$, we get the modal share formula as follows

$$x = N/[1 + \exp(\theta\varphi + 0.5\theta(a - f) - 0.5\theta N\beta t'(v))], \tag{18}$$

$$y = N/[1 + \exp(-\theta\varphi - 0.5\theta(a - f) + 0.5\theta N\beta t'(v))], \tag{19}$$

with $v = N - x/2$. The accurate solution can be obtained by solving the above nonlinear equations[2]. Here, we first employ (18)–(19) to provide an economic explanation of the commuters' jockeying or shifting behavior between two modes.

Equation (18) shows that the portion of carpoolers in all commuters increases when the fuel cost consumed on the line haul part of working trip goes up, declines when the assembly cost for forming a carpooling vehicle increases. This is the same with that observed from the *deterministic* social optimum solution (9), see the example used in Subsect. 2.2. Equation (18) also shows that the greater the φ-value (i.e., the preference to solo driving), the less the number of carpoolers. Certainly, this is intuitively reasonable.

On the other hand, investigating the relation between the share of carpoolers and the value of time in (18) is not as easy as in the deterministic social optimum solution. For the example used in Subsect. 2.2, we have $t'(v) = 4Bv^3 = 4B(N - x/2)^3$. Substituting this into (18) and finding the derivation of x with respect to β, we get

$$\frac{dx}{d\beta} = \frac{mnv^3 \exp(-n\beta v^3)}{N/x^2 + 1.5mn\beta v^2 \exp(-n\beta v^3)}, \tag{20}$$

where $m = \exp(\theta\varphi + 0.5\theta(a - f))$ and $n = 2\theta NB$. Clearly, $dx/d\beta > 0$ holds since $N > v > N/2, 0 < x < N, m > 0$ and $n > 0$. Note that the x-solution given by (18) generally does not take its boundary-values. Hence, the amount of carpooling is still positively proportional to the value of time, the reason of

[2] Equation (18) has a unique x-solution since its left hand side is a linear increasing function of x and its right hand side is a monotonically decreasing function of x, hence only one crossing point exists for these two curves.

which is the same with that explained in Sect. 2: when β-value increases, the β-related externality goes up, more commuters will shift to select carpooling mode for reducing individual cost by sharing the toll.

Comparing (18) and (12), we find that $N/[1+\exp(\theta\varphi+0.5\theta(a-f)-0.5\theta N\beta t'(v))] > N/[1+\exp(\theta\varphi+0.5\theta(a-f))]$ always holds since $0.5\theta N\beta t'(v) > 0$. This means there are more carpoolers in social optimum-based equilibrium than in non-toll equilibrium, i.e., more commuters will switch to carpooling mode to reduce individual cost by sharing the toll, thus resulting in a traffic volume reduction. This confirms again that the tolling scheme, as an organized external interference, plays a role in increasing the amount of carpooling. More importantly, the decline in traffic volume will lead to reductions in energy consumption, air pollution, noise levels and parking space requirements. Of course, these social and public benefits are not felt by individuals, but are reached by individuals' private choices under the tolling scheme. Meanwhile, the tolling scheme can raise additional revenue for traffic system improvements.

4. Numerical example

Now we provide a numerical example to compare the x-solutions obtained by the two models presented in this paper, each with both no-toll equilibrium and social optimum. The BPR (U.S. Bureau of Public Roads) travel time function is used, $t(v) = 25 + 0.3 \times 10^{-6}v^4$ where the free-flow travel time is 25 min. Other parameters are: $N = 100$ commuters, $f = 10$ (\$), $\beta = 0.1$ (\$/min), $\theta = 0.25$, and $\varphi = 4$. The x-solutions given by (3), (9), (12) and (18) against different assembly costs are displayed by four curves in Fig. 1. Evidently, they

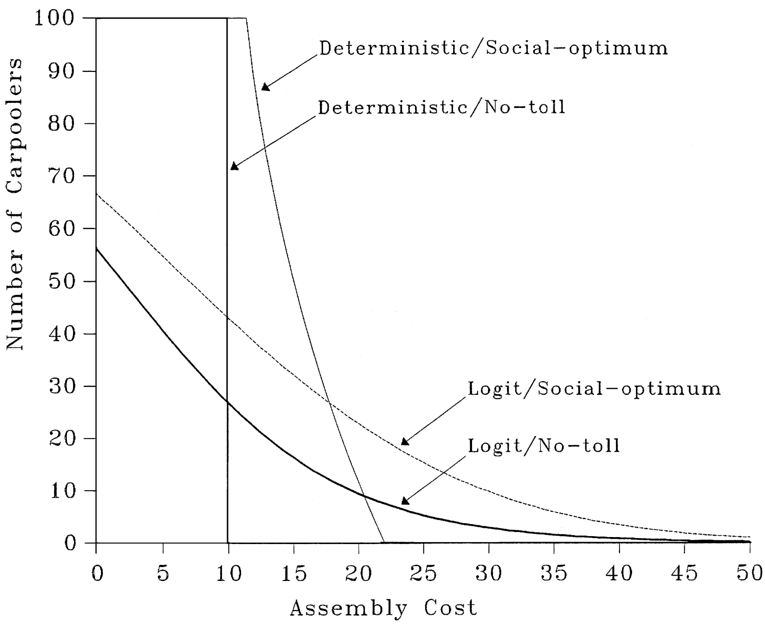

Fig. 1. The x-solutions against assembly cost

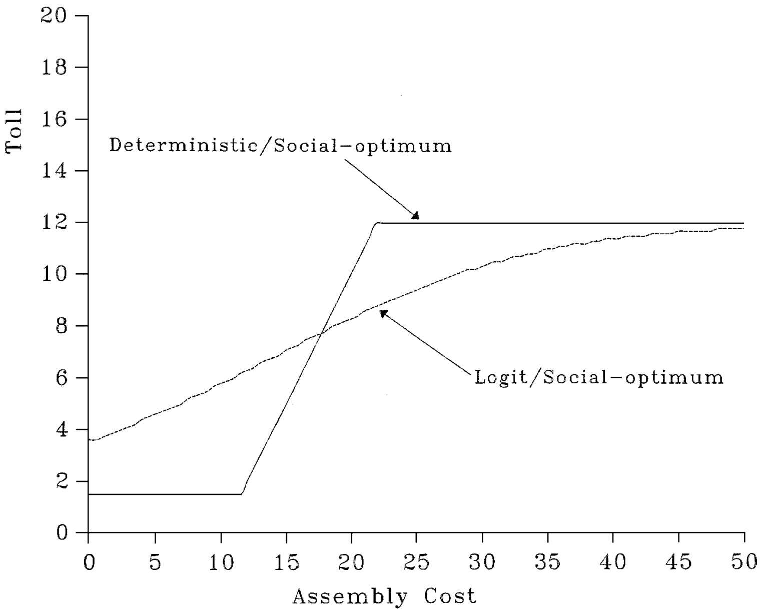

Fig. 2. The vehicle-based tolls by the models for social optimum against assembly cost

coincide with the analyses on these model solutions presented in this paper. In the logit/no-toll equilibrium, i.e., the solution (12), the proportion of carpoolers is still less than 50% even in the case of a being a lot less than f $(= 10)$, which is supported by most empirical surveys. It is expected that the logit/no-toll or social optimum model can give correct prediction to the number of carpooling commuters when φ and θ are finely calibrated from observed data. We also note that the curve representing the x-solutions of (18) is above that of (12). This definitely demonstrates the positive role of tolling scheme in advocating carpool commuting. Similar numerical analyses for x-, y- and v-solutions can be carried out by changing one parameter and fixing the others.

The tolls generated by the two models for social optimum are shown in Fig. 2. The curve of tolls by the deterministic social optimum model turns two corners at points $a = 11.5$ and 22, because there exist corner solutions as shown in (9) for the optimization problem (4). The tolls equal $(a - f)$, forming a oblique line in Fig. 2, when $11.5 < a < 22$. The tolls by the logit-based stochastic social optimum model, with respect to assembly costs, construct a nonlinear, monotonically and gently increasing curve.

5. Conclusions and extensions

5.1. Conclusions

In this paper, the jockeying behavior of work commuters between carpooling and driving alone modes is investigated by using two models, deterministic and stochastic, each for both no-toll equilibrium and social optimum equilibrium. The logit formula of modal split with the consideration of preference *ceteris paribus* for solo driving is suitable for estimating the proportion of carpoolers in all commuters. By analyzing the model solutions, clear explanations are obtained for the amount that carpooling is affected by fuel cost, assembly cost, value of time, preferential or attitudinal factors and traffic congestion. In no-toll competitive equilibrium circumstance (this implies no external interference exists), traffic congestion can not be reflected in the resultant modal split. A congestion externality-based tolling scheme can help to reach a social optimum, realize the potential of carpooling and then reduce the traffic volume further.

It should be pointed out that the models presented in this paper are simple from the view point of practice. They are based on a number of assumptions: e.g., only one single road is considered, the value of time is the same between carpooling and driving alone, and the fuel cost is independent of traffic congestion. Their value therefore lies in the insights they offer. In addition, the methodology used in this paper can be extended to deal with the cases in which mass transit, heterogeneous commuters, multi-origin and -destination network, priority lanes for high-occupancy vehicles, elastic trip demand, or combinations of some of these elements, are incorporated into carpooling problems. If so, more complex models must be developed, but the analyses and explanations would be yet more illuminating. As an example, in the next subsection, we extend the models for considering the elasticity on total trip demand.

5.2. Extensions

So far we have studied the internal demand elasticity which reflects the extent to which commuters divide themselves efficiently between carpooling and driving alone modes for travel. It is tempting to try to deal with both external (or overall) demand elasticity and internal demand elasticity within one model. Let $D^{-1}(N)$ denote the marginal trip benefit or the inverse of demand function of the total commuting trips, which is assumed to decline with commuting demand, $dD^{-1}(N)/dN \leq 0$. The question now is how to find the implemented total demand, N, at various equilibria involved in this paper.

For the deterministic no-toll equilibrium discussed in Subsect. 2.1, the total demand is determined by equalizing marginal private travel cost and marginal trip benefit, i.e.,

$$\begin{cases} \text{solution of } \beta t\left(\dfrac{N}{2}\right) + \dfrac{f+a}{2} = D^{-1}(N), & \text{if } a < f \\ \text{solution of } \beta t(N) + f = D^{-1}(N), & \text{if } a > f \\ \text{no unique solution of } N, & \text{if } a = f. \end{cases} \tag{21}$$

For the deterministic social optimum equilibrium discussed in Subsect. 2.2, we can get the total demand by solving a maximization problem of the total benefit. Here, the total benefit of the system is given by the area under curve $D^{-1}(N)$, minus the total social cost. The maximization problem becomes

$$\text{maximize } TB = \int_0^N D^{-1}(w)\,dw - (xc_x + yc_y), \text{s. t. } x + y = N \quad \text{and}$$
$$x, y, N \geq 0. \tag{22}$$

The first-order optimality conditions are

$$c_x + N\beta t'(v)/2 = D^{-1}(N), \tag{23}$$

$$c_y + N\beta t'(v) = D^{-1}(N), \tag{24}$$

and $x + y = N$. If the maximization occurs at an interior point of the feasible region, then the equilibrium solution (N^*, x^*, y^*) can be uniquely obtained from above equations. The corner solution for (22) can be checked as made in Subsect. 2.2.

For the logit-based stochastic no-toll equilibrium presented in Subsect. 3.1, the total demand, N, should be determined by solving the following nonlinear equations

$$(\ln x + 1)/\theta + c_x = D^{-1}(N), \tag{25}$$

$$(\ln y + 1)/\theta + c_y - \varphi = D^{-1}(N), \tag{26}$$

with $x + y = N$. The left hand sides of (25) and (26) are the marginal private trip costs perceived by carpoolers and solo drivers, respectively, which include the effects caused by random factors. The x- and y-solution of (25)–(26) are that given by (12) and (13), respectively.

For the logit-based stochastic social optimum equilibrium considered in Subsect. 3.2, the total demand can be determined by solving a maximization problem of the total net social benefit, i.e.,

$$\text{maximize } TNSB, \text{s. t. } x + y = N \quad \text{and} \quad x, y, N \geq 0 \quad \text{where}$$

$$TNSB = \int_0^N D^{-1}(w)\,dw + [-(x\ln x + y\ln y)/\theta + (N\ln N)/\theta]$$
$$- [xc_x + y(c_y - \varphi)]. \tag{27}$$

The x-, y- and N-solution can be obtained from the following first-order optimality conditions

$$(\ln x + 1)/\theta + c_x + N\beta t'(v)/2 = D^{-1}(N) + (\ln N + 1)/\theta, \tag{28}$$

$$(\ln y + 1)/\theta + c_y - \varphi + N\beta t'(v) = D^{-1}(N) + (\ln N + 1)/\theta, \tag{29}$$

with $x + y = N$. In (28) and (29), the term $N\beta t'(v)$ should be regarded as a toll to be imposed on each vehicle.

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