Economic complexity as network complication: Multiregional input-output structural path analysis

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Abstract. This paper presents a description of some fundamental properties of networks of economic selfinfluence and transfer of economic influence within hierarchies of economic sub-systems using structural path analysis within a multiregional input-output system. In this fashion, exchange between sectors, activities and regions is viewed as a network that can be decomposed hierarchically; economic complexity is viewed as an emerging property of the process of network complication that accompanies the augmentation of inputs and the growing synergetic interactions between regional sub-systems. For the reasons of clarity, the cases of two and three regions are considered in detail. The treatment of the general case of n regions and the graph-theoretical description of the global augmentation process of the network complication is presented in two appendices, where the mathematical proofs can be found. It is expected that this analysis will provide a methodology that will be useful in understanding regional economic sustainability (i.e., spatial and temporal invariability), structural stability and structural changes in economic networks as well as providing insights into the role of internal and external trade between regions. To support this expectation, the detailed theoretical analysis of the block structural paths in the social accounting system is presented supplemented by economic analysis of the Indonesian social accounting matrices for 1975, 1980 and 1985.

1. Introduction

This paper argues that the modern notion of complexity which has emerged from non-linear dynamics innatural sciences, can be a useful conceptual framework for organizing consideration of economic development even in

the case of linear economic analysis using input-output systems. While a strict definition of the economic complexity will not be presented (see, for example, Waldrop 1992, or a discussion of economic complexity by Krugman 1995), the complexity will be considered conceptually as a result of a process of development that extends the multiplicity of economic interactions within the economic system. More precisely, this paper presents an attempt to consider economic complexity in multiregional economic systems as the result (or emerging property) of the process of gradual complication (sequential augmentation) of the network of hierarchical economic interdependencies between economic sectors, economic activities and all possible economic and spatial sub-systems. Thus, the main object of this study is a more complete description of *networks of economic influence* and *transfer of economic influence* within the multiregional input-output, social accounting or related system.

While network analysis has played a valued role in transportation systems research, applications in the field of input-output analysis have been less frequent (see some early applications by Campbell 1972, 1975); in large part, this stems from the fact that the applications often reduced the input-output structure to a Boolean zero-one matrix, thereby losing significant information about the nature and strength of flows along each linkage. However, new developments in network research have generated possibilities from a new set of approaches to the measure of economic structure complexity (see some recent initiatives by Kauffman 1988; Roy 1994, 1995; Roson 1994, 1996; Westin 1990, as well as Johansson et al. 1994 and Batten et al. 1995 who have proposed the notion of a network economy). The present paper offers some preliminary steps in this direction. As an economy expands, the issue arises as to what might be expected to happen to complexity; a single study of the Chicago region (Hewings et al. 1996) suggests that the process is far from monotonic and likely to vary according to the individual structure of the economy in question. Another important dimension of complexity concerns the hierarchical nature of interactions and exchange in an economy. Hierarchical considerations compel us to present our analysis not only on the micro-level of economic sectors and macro-level of overall economic system in question, but to introduce a meso-level of all possible economic sub-systems which include various combinations of sectors, activities, regions and national economies at all levels of aggregation. For the sake of convenience, we will usually use only the "regional" language, although in the empirical example we will concentrate on considerations of interdependencies between the economic activities within the social accounting system.

The paper relies on the insights of some recent elaboration of block structural path analysis for the three-block social accounting matrix and its application to the Indonesian economy (see Sonis et al. 1997b) using a simple three-division (with each division – factors, institutions and activities – serving as a proxy for region) of the economy. As a result, the process of gradual network complication is analyzed through penetration into the mathematical structure of the multiregional Leontief block inverse,

revealed through what may be referred to as generalized structural path analysis. In the next section, some antecedents are provided followed by a presentation of the network complication process in a two-region input-output system; this section reveals the important connection between the block structural path analysis with the celebrated Schur (1917) block-inverse matrix formula. Section 3 introduces the three-region system, while Sect. 4 includes the detailed theoretical analysis of the structural paths in the social accounting systems illustrated through an application to a set of three Indonesian SAMs. A concluding Sect. 5 draws the paper to a close. The transfer of these ideas to the *n*-region case and the graphical form of the network complication are discussed in two mathematical appendices where the interested reader can find full proofs and a detailed description of the process of augmentation of economic network.

2. Antecedents and the two-region case

Methodologically, multiregional input-output structural path analysis (Block SPA) is based on the unification and generalization of a long sequence of explorations into issues of structure in economies. The celebrated Schur (1917) block-inverse matrix formula and its extensions (see Henderson and Searle 1981) provided the basis for a great deal of subsequent work upon which Block SPA is based. In particular, the applications of structural path analysis to input-output and social accounting systems (see Lantner 1974; Crama et al. 1984; Defourny and Thorbecke 1984) highlighted the notion of important paths through the network of interactions in an economy. Miyazawa (1966, 1976) and Yamada and Ihara (1969) explored a complementary approach that focused on ideas of internal and external multipliers and an augmentation process that becomes important in multi-economy applications. This work has been extended in a series of papers (Sonis and Hewings 1990, 1991, 1993, 1995 b; Sonis et al. 1997 c).

However, it should be noted that Schur's (1917) main interest centered around the role of determinants in the 2×2 block matrix form; Banchiewicz (1937) transferred these results to the form of a 2×2 block matrix inversion while Miyazawa (1966) concentrated on the inner components of the 2×2 block matrix inverse and its economic interpretation. In the present paper, Miyazawa's language is placed in a broader Schur-Banachiewicz framework. The methodological base of Block SPA is an extension of the Schur-Banachiewicz formula (Schur 1917; Banachiewicz 1937) from the case of 2×2 blocks to any arbitrary number of blocks (regions). In the course of this development, it has been possible to propose a typology of synergetic interactions between regional sub-systems (see Sonis et al. 1996).

The presentation of the network complication process will begin with the two-region input-output system. The direct inputs can be represented by the following block matrix:

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{12} & A_{22} \end{bmatrix}, \tag{1}$$

where A_{11} and A_{22} are the square matrices of direct inputs within the first and second regions and A_{12} and A_{21} are the rectangular matrices showing the direct inputs purchased by the second region and vice versa. It is also possible to consider the case where the second region represents the rest of the economy. (It is important to stress that all considerations can be incorporated in the single region case in which sectors are divided into blocks presenting different sets of sectors, for example, resources, manufacturing and non-manufacturing sectors).

The corresponding Leontief inverse matrix, $B = (I–A)^{-1}$, has the following block form (the superscripts on the B's indicate the number of regions (two) in question):

$$B = \begin{bmatrix} B_{11}^2 & B_{12}^2 \\ B_{21}^2 & B_{22}^2 \end{bmatrix} \tag{2}$$

and this can be further elaborated with the help of the Schur-Banachiewicz formula (Schur 1917; Banachiewicz 1937; Miyazawa 1962; Sonis and Hewings 1993):

$$B = \begin{bmatrix} B_{11}^2 & B_{11}^2 A_{12} B_2 \\ B_{22}^2 A_{21} B_1 & B_{22}^2 \end{bmatrix} = \begin{bmatrix} B_{11}^2 & B_1 A_{12} B_{22}^2 \\ B_2 A_{21} B_{11}^2 & B_{22}^2 \end{bmatrix}, \tag{3}$$

where the matrices $B_1 = (I–A_{11})^{-1}$ and $B_2 = (I–A_{22})^{-1}$ represent the Miyazawa internal matrix multipliers for the first and second regions (revealing the interindustry propagation effects within the region) while the matrices $A_{21}B_1$, B_1A_{12}, $A_{12}B_2$, and B_2A_{21} show the induced effects on output or input between the two regions (Miyazawa 1966).

Further:

$$B_{11}^2 = (I - A_{11} - A_{12}B_2A_{21})^{-1}$$
$$B_{22}^2 = (I - A_{22} - A_{21}B_1A_{12})^{-1} \tag{4}$$

are the extended Leontief multipliers for the first and second regions, inverses of the so-called Schur complements:

$$S_1 = A_{11} + A_{12}B_2A_{21}$$
$$S_2 = A_{22} + A_{21}B_1A_{12} . \tag{5}$$

They include the direct inputs, A_{11}, A_{22} circulating within the regions and indirect inputs $A_{12}B_2A_{21}, A_{21}B_1A_{12}$ that represent the economic self-influence transactions of one of the regions through the other region. The Schur complements, S_1, S_2 were interpreted by Yamada and Ihara (1969) as the

Augmentation of Inputs S_1 into the First Region
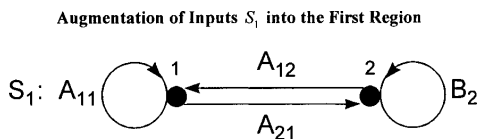
Fig. 1. Augmentation of inputs S_1 into the first region

augmented inputs (see Fig. 1). More generally, they have been referred to as the interregional feedback effects (see Miller and Blair 1985).

In this paper, the augmentation of inputs at the meso-level of regions is presented in terms of economic self-influence and the transfer of economic influence from region to region. Figure 1 represents a structural path of economic self-influence corresponding to the extended Leontief inverse; by using the Miyazawa decomposition, this extended Leontief inverse can be decomposed into the product of internal and external multipliers describing direct and induced self-influences (Miyazawa 1966, 1976):

$$B_{11}^2 = B_1 B_{11}^{2R} = B_{11}^{2L} B_1$$
$$B_{22}^2 = B_2 B_{22}^{2R} = B_{22}^{2L} B_2 \qquad (6)$$

where

$$B_{11}^{2L} = (I - B_1 A_{12} B_2 A_{21})^{-1} \; ; \quad B_{11}^{2R} = (I - A_{12} B_2 A_{21} B_1)^{-1}$$
$$B_{22}^{2L} = (I - B_2 A_{21} B_1 A_{12})^{-1} \; ; \quad B_{22}^{2R} = (I - A_{21} B_1 A_{12} B_2)^{-1} \qquad (7)$$

are the left and right Miyazawa external self-influence multipliers for the first and second region. It is easy to derive, analogous to Fig. 1, a structural path of self-influence corresponding to the analytical structure of the multipliers presented in (7).

The transfer of economic influence from one region to the other is presented by the block-components of the Leontief block inverse. The Miyazawa fundamental equations:

$$B_{12}^2 = B_{11}^2 A_{12} B_2 = B_1 A_{12} B_{22}^2$$
$$B_{21}^2 = B_{22}^2 A_{21} B_1 = B_2 A_{21} B_{11}^2 \qquad (8)$$

represent the structural path of transfer of influence (see Fig. 2).

Figures 1 and 2 represent the self-influence and transfer of influence augmentation process at the meso-level of regions; they represent the building blocks of the economic interactions between the economic sub-systems. For the case of two regions, a typology of 14 macro-level regional sub-systems was recently developed (Sonis et al. 1996). Each regional subsystem generates a decomposition of the Leontief block inverse into the product of partial Leontief inverses corresponding to the chosen regional sub-system.
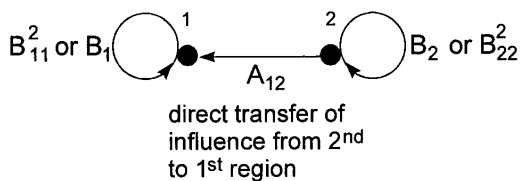
direct transfer of
influence from 2nd
to 1st region

Fig. 2. Transfer of influence B_{12}^2 from the second to first region

For example, the following formulae represent explicitly a separation of the direct and indirect self-influence and transfer of influence in the form of a triple decomposition that separates multiplicatively the effects of intra-regional economic relationships of isolated regional economies, $\begin{bmatrix} B_1 & 0 \\ 0 & B_2 \end{bmatrix}$, the intra-/interregional feedback effects on the level of direct inputs, $\begin{bmatrix} I - A_{11} & A_{12} \\ A_{21} & I - A_{22} \end{bmatrix}$ and the intra-regional economic dependencies of interact-ing regions $\begin{bmatrix} B_{11}^2 & 0 \\ 0 & B_{22}^2 \end{bmatrix}$:

$$
B = \begin{bmatrix} B_{11}^2 & 0 \\ 0 & B_{22}^2 \end{bmatrix} \begin{bmatrix} I - A_{11} & A_{12} \\ A_{21} & I - A_{22} \end{bmatrix} \begin{bmatrix} B_1 & 0 \\ 0 & B_2 \end{bmatrix} =
$$
$$
= \begin{bmatrix} B_1 & 0 \\ 0 & B_2 \end{bmatrix} \begin{bmatrix} I - A_{11} & A_{12} \\ A_{21} & I - A_{22} \end{bmatrix} \begin{bmatrix} B_{11}^2 & 0 \\ 0 & B_{22}^2 \end{bmatrix}. \tag{9}
$$

Equation (9) provides the block matrix analog of the decompositions (8) of the transfer of economic influence. The application of the Miyazawa decompositions (6) of the extended Leontief inverses into the product of external/internal multipliers provides further possibilities for construction of another block matrix analog of (9).

$$
B = \begin{bmatrix} B_{11}^L & 0 \\ 0 & B_{22}^L \end{bmatrix} \begin{bmatrix} I & B_1 A_{12} \\ B_2 A_{21} & I \end{bmatrix} \begin{bmatrix} B_1 & 0 \\ 0 & B_2 \end{bmatrix} =
$$
$$
= \begin{bmatrix} B_1 & 0 \\ 0 & B_2 \end{bmatrix} \begin{bmatrix} I & A_{12} B_2 \\ A_{21} B_1 & I \end{bmatrix} \begin{bmatrix} B_{11}^R & 0 \\ 0 & B_{22}^R \end{bmatrix} \tag{10}
$$

It is important to stress that, even for this simple two-region input-output system, there are three hierarchical levels: (i) the micro level of industries, (ii) the meso-level of region and (iii) the macro-level of the total economic system. In this hierarchy, expressions (4)–(8) reflect the meso-level of region and decompositions of type (9) and (10) illustrate what may be referred to as macro-level structural path analysis; they provide a vehicle to illustrate macro-level complexity that emerges from the meso-level aug-mentation of inputs. Another step could take the process to the level of

individual sectors and provide illustrations that would parallel the more traditional applications of structural path analysis to input-output and social accounting systems. At this micro-level, the paths through the network would trace flows from one sector to another but the proposed decompositions would be of a similar form.

In the next section, the case of network complication in three-region input-output systems will be illustrated, drawing again on the generalizations of the Schur-Banachiewicz (3) and Miyazawa (8) fundamental equations and external and internal multipliers (7).

3. The three-region input-output scheme

In this section, the augmentation process will be extended from the self-influence sub-network to the transfer of influence sub-network, drawing on the Yamada and Ihara (1969) concept of an augmentation process for the case of more than two regions. For the case of three regions, i, j, and s, the augmented inputs can be defined as $A_{ij}+A_{is}B_sA_{sj}$ (see Fig. 3).

In this paper, the augmentation of inputs at the meso-level of regions is presented in terms of economic self-influence and the transfer of economic influence from region to region: the economic self-influence of region i is described by the component B_{ii} of the Leontief inverse, B, while the transfer of influence from region j to region i is given by the component, B_{ij}. Using this form, it is possible to extend structural path analysis to the multiregional scheme.

Consider a three-region input-output system with the block matrix of direct inputs:

$$A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \tag{11}$$

and the corresponding Leontief inverse

$$B = (I - A)^{-1} = \begin{bmatrix} B_{11}^3 & B_{12}^3 & B_{13}^3 \\ B_{21}^3 & B_{22}^3 & B_{23}^3 \\ B_{31}^3 & B_{32}^3 & B_{33}^3 \end{bmatrix} . \tag{12}$$
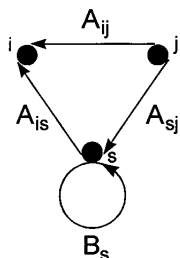

Fig. 3. Augmentation of inputs from region i to j through region s

To portray this inverse in the Schur-Banachiewicz form (3), the following partial block matrices of direct inputs for pairs of regions will be used; for example, the exclusion of the third region provides the following matrix of direct inputs:

$$A(3) = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$$

and the corresponding partial Leontief inverse:

$$B(3) = [I - A(3)]^{-1} = \begin{bmatrix} B_{11}^2(3) & B_{12}^2(3) \\ B_{21}^2(3) & B_{22}^2(3) \end{bmatrix} =$$

$$= \begin{bmatrix} B_{11}^2(3) & B_{11}^2(3)A_{12}B_2 \\ B_{22}^2(3)A_{21}B_1 & B_{22}^2(3) \end{bmatrix} = \begin{bmatrix} B_{11}^2(3) & B_1A_{12}B_{11}^2(3) \\ B_2A_{21}B_{22}^2(3) & B_{22}^2(3) \end{bmatrix}. \tag{13}$$

Analogous formulae can be written for a pair of first and third and for a pair of second and third regions.

The following generalization of the Schur-Banachiewicz inverse matrix formula holds for the three-region input-output system (the proofs are provided in Appendix I):

$$B = \begin{bmatrix} B_{11}^3 & B_{11}^3A_{12}^3B_{22}^2(1) & B_{11}^3A_{13}^3B_{33}^2(1) \\ B_{22}^3A_{21}^3B_{11}^2(2) & B_{22}^3 & B_{22}^3A_{23}^3B_{33}^2(2) \\ B_{33}^3A_{31}^3B_{11}^2(3) & B_{33}^3A_{32}^3B_{22}^2(3) & B_{33}^3 \end{bmatrix} \tag{14}$$

with the Yamada and Ihara augmented inputs:

$$A_{ij}^3 = A_{ij} + A_{is}B_sA_{sj} \quad i \neq j, \, i \neq s, \, j \neq s; \, i,j,s = 1,2,3 \tag{15}$$

and the extended regional Leontief inverses:

$$B_{ii}^3 = [I - A_{ii} - A_{ij}B_{jj}^2(i)A_{ji}^3 - A_{is}B_{ss}^2(i)A_{si}^3]^{-1}$$
$$i \neq j, \, i \neq s, \, j \neq s; \, i,j,s = 1,2,3 \, . \tag{16}$$

The corresponding augmented Schur complement presents the transregional economic self-influence at the meso-level of regions:

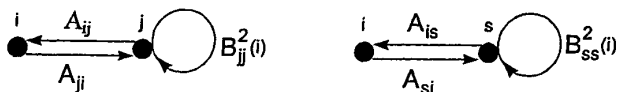$$S_i = A_{ii} + A_{ij}B_{jj}^2(i)A_{ji}^3 + A_{is}B_{ss}^2(i)A_{si}^3 \, . \tag{17}$$

The augmentation of inputs (15) leads to the detailed structure of augmentation in the Schur complement (17):

direct self-influence: A_{ii}

bilateral self-influence: $A_{ij}B^2_{jj}(i)A_{ij}$ and $A_{is}B^2_{ss}(i)A_{si}$

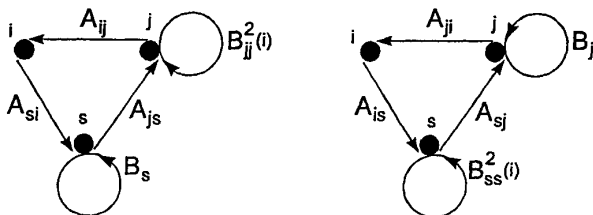three-lateral self-influence: $A_{ij}B^2_{jj}(i)A_{js}B_sA_{si}$ and $A_{is}B^2_{ss}(i)A_{sj}B_jA_{ji}$

Fig. 4. Structure of the augmented Schur complement for three regions

$$S_i = A_{ii} + A_{ij}B^2_{jj}(i)A_{ji} + A_{is}B^2_{ss}(i)A_{si} + A_{ij}B^2_{jj}(i)A_{js}B_sA_{si} +$$
$$+ A_{is}B^2_{ss}(i)A_{sj}B_jA_{ji} \quad i \neq j, i \neq s, j \neq s; i, j, s = 1, 2, 3 . \tag{18}$$

Thus, in the three-region system, the regional economic self-influence may be seen to comprise the superposition of (i) circulation (direct self-influence); (ii) self-influence generated through bilateral regional interdependencies and (iii) self-influence promoted by tri-lateral regional interdependencies (see Fig. 4). The expressions (17) and (18) reflect the existence of a nested hierarchy of different levels of augmentation represented in the recursive form in (14); in a sense, the process resembles the Matrioshka idea introduced by Sonis and Hewings (1991).

Furthermore, the generalization of the Miyazawa fundamental Eqs. (8) for the case of three regions also has a recursive form: the transfer of influence from region j to i is:

$$B^3_{ij} = B^3_{ii}A^3_{ij}B^2_{jj}(i) = B^2_{ii}(j)A^3_{ij}B^3_{jj} \quad i \neq j, i, j = 1, 2, 3 . \tag{19}$$

Moreover, the augmented Schur complement (17) can also be presented in a form:

$$S_i = A_{ii} + A^3_{ij}B^2_{jj}(i)A_{ji} + A^3_{is}B^2_{ss}(i)A_{si} . \tag{20}$$

The expressions (19) and (20) offer the option of presenting the Leontief inverse for the three-region system in an alternative form:

$$
B = \begin{bmatrix} B_{11}^3 & B_{11}^2(2)A_{12}^3 B_{22}^3 & B_{11}^2(3)A_{13}^3 B_{33}^3 \\ B_{22}^2(1)A_{21}^3 B_{11}^3 & B_{22}^3 & B_{22}^2(3)A_{23}^3 B_{33}^3 \\ B_{33}^2(1)A_{31}^3 B_{11}^3 & B_{33}^2(2)A_{32}^3 B_{22}^3 & B_{33}^3 \end{bmatrix} . \tag{21}
$$

Thus, the following generalization of the Miyazawa external and internal multipliers holds:

$$
B_{ii}^3 = B_i B_{ii}^{3R} = B_{ii}^{3L} B_i , \tag{22}
$$

where B_{ii}^{3R} and B_{ii}^{3L} are the right and left external self-influence multipliers for region i:

$$
\begin{aligned}
B_{ii}^{3R} &= [I - A_{ij}B_{jj}^2(i)A_{ji}^3 B_i - A_{is}B_{ss}^2(i)A_{si}^3 B_i]^{-1} \\
B_{ii}^{3L} &= [I - B_i A_{ij}B_{jj}^2(i)A_{ji}^3 - B_i A_{is}B_{ss}^2(i)A_{si}^3]^{-1} .
\end{aligned} \tag{23}
$$

The generalizations (22) and (23) can be transferred from the meso-level of regions to the higher macro-level of the inner and outer left and right block matrix multipliers. For example, for the left multipliers:

$$
\begin{aligned}
B = &\begin{bmatrix} B_{11}^3 & 0 & 0 \\ 0 & B_{22}^3 & 0 \\ 0 & 0 & B_{33}^3 \end{bmatrix} \begin{bmatrix} I & A_{12}^3 B_{22}^2(1) & A_{13}^3 B_{33}^2(1) \\ A_{21}^3 B_{11}^2(2) & I & A_{23}^3 B_{33}^2(2) \\ A_{31}^3 B_{11}^2(3) & A_{32}^3 B_{22}^2(3) & I \end{bmatrix} = \\
= &\begin{bmatrix} B_{11}^{3L} & 0 & 0 \\ 0 & B_{22}^{3L} & 0 \\ 0 & 0 & B_{33}^{3L} \end{bmatrix} \begin{bmatrix} B_1 & 0 & 0 \\ 0 & B_2 & 0 \\ 0 & 0 & B_3 \end{bmatrix} \\
&\begin{bmatrix} I - A_{11} & A_{12}^3 B_{22}^{2L}(1) & A_{13}^3 B_{33}^{2L}(1) \\ A_{21}^3 B_{11}^{2L}(2) & I - A_{22} & A_{23}^3 B_{33}^{2L}(2) \\ A_{31}^3 B_{11}^{2L}(3) & A_{32}^3 B_{22}^{2L}(3) & I - A_{33} \end{bmatrix} \begin{bmatrix} B_1 & 0 & 0 \\ 0 & B_2 & 0 \\ 0 & 0 & B_3 \end{bmatrix}
\end{aligned} \tag{24}
$$

Analogous expressions hold for the right multipliers.

4. Application of the generalized structural path analysis to social accounting systems

The one-economy form of structural path analysis has proven to be a popular instrument for the analysis of the structure of economies for which social accounting matrices (SAM) are available. Pyatt and Round (1979), and Round (1985, 1988) introduced a triple decomposition that has been combined with one-economoy structural path analysis by Defourny and

Thorbecke (1984) and Khan and Thorbecke (1988). In this section, the Pyatt and Round (1979) approach will be used to illustrate the three-region Block SPA (see also Sonis and Hewings 1988).

Consider the block form of the SAM, characterized by the following matrix and vectors:

$$A = \begin{bmatrix} 0 & 0 & A_{13} \\ A_{21} & A_{22} & 0 \\ 0 & A_{32} & A_{33} \end{bmatrix} ; \quad d = \begin{bmatrix} 0 \\ d_2 \\ d_3 \end{bmatrix} ; \quad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} , \tag{25}$$

where A represents the average expenditure propensities of the SAM, d the vector of exogenous injections and x the vector of endogenous outputs. The subsripts refer to the usual division of the SAM into (1) factors of production, (2) institutions and (3) production activities. In the analysis that follows, all three hierarchical levels are explored – the micro level of the structural components of each division, the meso level of the divisions themselves and finally the macro level of the overall SAM.

The following partial block matrices of direct inputs for the three pairs of blocks and their partial Leontief inverses have the form:

for the pair (*institutions, activities*)

$$A(1) = \begin{bmatrix} A_{22} & 0 \\ A_{32} & A_{33} \end{bmatrix} \tag{26}$$

with the corresponding partial Leontief inverse:

$$B(1) = [I - A(1)]^{-1} = \begin{bmatrix} B_2 & 0 \\ B_3 A_{32} B_2 & B_3 \end{bmatrix} \tag{27}$$

where $B_2 = (I - A_{22})^{-1}$ and $B_3 = (I - A_{33})^{-1}$;

for the pair (*factors, activities*)

$$A(2) = \begin{bmatrix} 0 & A_{13} \\ 0 & A_{33} \end{bmatrix} \tag{28}$$

with the corresponding partial Leontief inverse:

$$B(2) = [I - A(2)]^{-1} = \begin{bmatrix} I & A_{13} B_3 \\ 0 & B_3 \end{bmatrix} \tag{29}$$

for the pair (*factors, institutions*)

$$A(3) = \begin{bmatrix} 0 & 0 \\ A_{21} & A_{22} \end{bmatrix} \tag{30}$$

with the corresponding partial Leontief inverse:

$$B(3) = [I - A(3)]^{-1} = \begin{bmatrix} I & 0 \\ B_2 A_{21} & B_2 \end{bmatrix}. \tag{31}$$

The Yamada and Ihara augmented inputs, identified in (15), for the SAM are:

$$
\begin{aligned}
A_{12}^3 &= A_{13} B_3 A_{32}; & A_{13}^3 &= A_{13}; \\
A_{21}^3 &= A_{21}; & A_{23}^3 &= A_{21} A_{13}; \\
A_{31}^3 &= A_{32} B_2 A_{21}; & A_{32}^3 &= A_{32}.
\end{aligned}
\tag{32}
$$

The extended self-influence Leontief inverses at the meso level of the major divisions are (see 16):

$$
\begin{aligned}
B_{11}^3 &= [I - A_{13} B_3 A_{32} B_2 A_{21}]^{-1} \\
B_{22}^3 &= [I - A_{22} - A_{21} A_{13} B_3 A_{32}]^{-1} \\
B_{33}^3 &= [I - A_{33} - A_{32} B_2 A_{21} A_{13}]^{-1}.
\end{aligned}
\tag{33}
$$

The corresponding augmented complements:

$$
\begin{aligned}
S_1 &= A_{13} B_3 A_{32} B_2 A_{21} \\
S_2 &= A_{22} + A_{21} A_{13} B_3 A_{32} \\
S_3 &= A_{33} + A_{32} B_2 A_{21} A_{13}
\end{aligned}
\tag{34}
$$

have the economic network structure (see Fig. 5) associated with the blocks A_{22}, A_{33} and with the components of the quasi-permutation matrix of direct inputs:

$$
P = \begin{bmatrix} 0 & 0 & A_{13} \\ A_{21} & 0 & 0 \\ 0 & A_{32} & 0 \end{bmatrix}. \tag{35}
$$

This matrix represents the macro level feedback loop of the transfer of economic influence between factors, institutions and activities.

Drawing on (14), the Leontief inverse for this SAM has a form:

$$
\begin{aligned}
B &= \begin{bmatrix} B_{11}^3 & A_{13} B_3 A_{32} B_{22}^3 & A_{13} B_{33}^3 \\ B_2 A_{21} B_{11}^3 & B_{22}^3 & B_2 A_{21} A_{13} B_{33}^3 \\ B_3 A_{32} B_2 A_{21} B_{11}^3 & B_3 A_{32} B_{22}^3 & B_{33}^3 \end{bmatrix} = \\
&= \begin{bmatrix} I & A_{13} B_3 A_{32} & A_{13} \\ B_2 A_{21} & I & B_2 A_{21} A_{13} \\ B_3 A_{32} B_2 A_{21} & B_3 A_{32} & I \end{bmatrix} \begin{bmatrix} B_{11}^3 & 0 & 0 \\ 0 & B_{22}^3 & 0 \\ 0 & 0 & B_{33}^3 \end{bmatrix}. \tag{36}
\end{aligned}
$$

Factors self-influence $S_1 = A_{13}B_3 A_{32}B_2 A_{21}$

F $\quad A_{21}$ I
B_2

A_{13} $\quad A_{32}$

A B_3
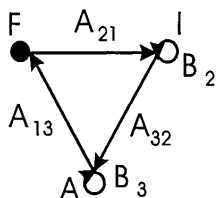
Institutions self-influence $S_2 = A_{22} + A_{21}A_{13}B_3 A_{32}$

F $\quad A_{21}$ I
A_{22}

A_{13} $\quad A_{32}$

A B_3
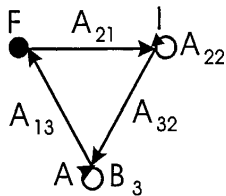
Activities self-influence $\quad S_3 = A_{33} + A_{32}B_2 A_{2\prime}A_{13}$

F $\quad A_{21}$ I
B_2

A_{13} $\quad A_{32}$

A A_{33}
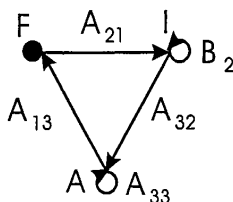
Fig. 5. Structure of the Schur complements for the social accounting matrix

Here, the diagonal matrix:

$$D_3 = \begin{bmatrix} B_{11}^3 & 0 & 0 \\ 0 & B_{22}^3 & 0 \\ 0 & 0 & B_{33}^3 \end{bmatrix} \tag{37}$$

represents the macro level of economic self-influence within the factors, institutions and activities, and the block multiplier:

$$M = \begin{bmatrix} I & A_{13}B_3 A_{32} & A_{13} \\ B_2 A_{21} & I & B_2 A_{21}A_{13} \\ B_3 A_{32}B_2 A_{21} & B_3 A_{32} & I \end{bmatrix} \tag{38}$$

represents the macro level transfer of incluence.

It is important to stress that the quasi-permutation matrix P represents the building block of this macro level transfer of influence:

$$
\mathrm{M} = \begin{bmatrix} I & A_{13}B_3A_{32} & A_{13} \\ B_2A_{21} & I & B_2A_{21}A_{13} \\ B_3A_{32}B_2A_{21} & B_3A_{32} & I \end{bmatrix} = \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix} +
$$

$$
+ \begin{bmatrix} I & 0 & 0 \\ 0 & B_2 & 0 \\ 0 & 0 & B_3 \end{bmatrix} \begin{bmatrix} 0 & 0 & A_{13} \\ A_{21} & 0 & 0 \\ 0 & A_{32} & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & A_{13} \\ A_{21} & 0 & 0 \\ 0 & A_{32} & 0 \end{bmatrix}
$$

$$
\begin{bmatrix} I & 0 & 0 \\ 0 & B_2 & 0 \\ 0 & 0 & B_3 \end{bmatrix} \begin{bmatrix} 0 & 0 & A_{13} \\ A_{21} & 0 & 0 \\ 0 & A_{32} & 0 \end{bmatrix} = I + D_2P + PD_2P , \qquad (39)
$$

where:

$$
D_2 = \begin{bmatrix} I & 0 & 0 \\ 0 & B_2 & 0 \\ 0 & 0 & B_3 \end{bmatrix} \qquad (40)
$$

is the diagonal block matrix of the direct self-influence of factors, institutions and activities. Thus, the SAM inverse has the following form, including the macro level direct and extended self-influence associated with the block diagonal matrices, D_2 and D_3, and the macro transfer of influence, P:

$$
B = MD_3 = [I + D_2P + PD_2P]D_3 . \qquad (41)
$$

At the meso level for the major divisions of the economy:

$$
Bd = \begin{bmatrix} B_{11}^3 & A_{13}B_3A_{32}B_{22}^3 & A_{13}B_{33}^3 \\ B_2A_{21}B_{11}^3 & B_{22}^3 & B_2A_{21}A_{13}B_{33}^3 \\ B_3A_{32}B_2A_{21}B_{11}^3 & B_3A_{32}B_{22}^3 & B_{33}^3 \end{bmatrix} \begin{bmatrix} 0 \\ d_2 \\ d_3 \end{bmatrix} =
$$

$$
= \begin{bmatrix} A_{13}B_3A_{32} \\ I \\ B_3A_{32} \end{bmatrix} B_{22}^3 d_2 + \begin{bmatrix} A_{13} \\ B_2A_{21}A_{13} \\ I \end{bmatrix} B_{33}^3 d_3 . \qquad (42)
$$

The expression (42) reveals the major paths of influence in the transmission of economic impulses at the meso level of the SAM; rather than drawing attention to the myriad micro level paths through the SAM, the focus is directed to the block paths. Of course, within these blocks, the individual paths are preserved. Figure 6 illustrates this meso perspective for the transmission of economic influence; it includes the multiplier complication pyramids and the corresponding networks of structural complication of the paths for institutions and activities. Table 1 provides a translation of the figure for Indonesian data for 1975, with the allocations across sectors/accounts shown in percentages.

Complication Pyramid
for Institutions

B^3_{22}

$B_3 A_{32} B^3_{22}$

$A_{13} B_3 A_{32} B^3_{22}$

F   I B^3_{22}

A_{13} A_{32}

A B_3
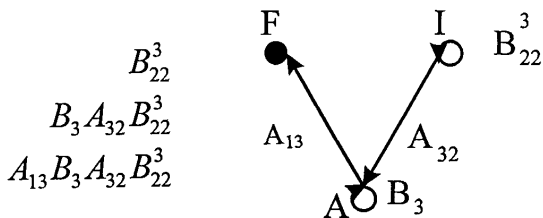
Complication Pyramid
for Activities

B^3_{33}

$A_{13} B^3_{33}$

$B_2 A_{21} A_{13} B^3_{33}$
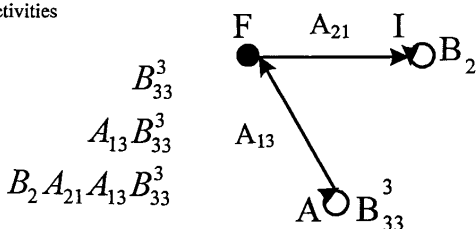
F A_{21} I B_2

A_{13}

A B^3_{33}

Fig. 6. Network complication for institutions and activities

The self-influence, $B^3_{22}d_2$, of the institution expenditure, d_2, on the institutional outputs and the influence of the institutional expenditures on factor outputs, $A_{13} B_3 A_{32} B^3_{22} d_2$, and the actitivies output, $B_3 A_{32} B^3_{22} d_2$, can be represented by the following complication chain:

$$d_2 \rightarrow B^3_{22} d_2 \rightarrow B_3 A_{32} B^3_{22} d_2 \rightarrow A_{13} B_3 A_{32} B^3_{22} d_2 \ . \tag{43}$$

The self-influence, $B^3_{33} d_3$, of the injections into the production activities, d_3, and the influence of these injections on factor outputs, $A_{13} B^3_{33} d_3$, and the institutions output, $B_2 A_{21} A_{13} B^3_{33} d_3$ is reflected by the following complication chain:

$$d_3 \rightarrow B^3_{33} d_3 \rightarrow A_{13} B^3_{33} d_3 \rightarrow B_2 A_{21} A_{13} B^3_{33} d_3 \ . \tag{44}$$

The existence of multiplier complication pyramids highlights one of the advantages of Block SPA since it can reflect the process of path building through the SAM and thus create an overall representation of the dominant economic relationships at the meso level.

These formulations will now be illustrated with reference to a set of three SAMs for the Indonesian economy for 1975, 1980 and 1985 (for a more comprehensive review of the economy, see Kim et al. 1992 and Sonis et al. 1997). The series of five-year plans, *Repelitas*, were promulgated be-

Table 1. (a) Interpretation of complication pyramid in Fig. 6 for institutions

Path	Description	Percentage allocation by account/sector 1975
B_{22}^3	Transformation of institutional injection (d_2) expenditures into institutional outputs	$\begin{pmatrix} 1 & 45.3 \\ 2 & 17.0 \\ 3 & 37.7 \end{pmatrix}$
$B_3 A_{32} B_{22}^3$	Further transmission of institutional outputs into activities and their transformation into activities output	$\begin{pmatrix} 1 & 43.2 \\ 2 & 4.1 \\ 3 & 16.8 \\ 4 & 10.9 \\ 5 & 24.1 \end{pmatrix}$
$A_{13} B_3 A_{32} B_{22}^3$	Transmission of activity output into factor inputs	$\begin{pmatrix} 1 & 46.3 \\ 2 & 53.7 \end{pmatrix}$

(b) Interpretation of complication pyramid in Fig. 6 for activities

Path	Description	Percentage allocation by account/sector 1975
B_{33}^3	Transformation of activity (d_3) injection into activity outputs	$\begin{pmatrix} 1 & 30.3 \\ 2 & 5.0 \\ 3 & 34.1 \\ 4 & 21.3 \\ 5 & 9.2 \end{pmatrix}$
$A_{13} B_{33}^3$	Further transmission of activity outputs into factor inputs	$\begin{pmatrix} 1 & 38.9 \\ 2 & 61.1 \end{pmatrix}$
$B_2 A_{21} A_{13} B_{33}^3$	Transmission of factor inputs into institution inputs and their transformation into institution outputs	$\begin{pmatrix} 1 & 63.9 \\ 2 & 25.4 \\ 3 & 10.7 \end{pmatrix}$

ginning in 1969. The first plan (1969–1973) emphasized food production and infrastructure development; most of the targets were met and, as a result, the GNP annual growth was 4.7% over this period. The second plan (1974–1979) focused on increasing welfare, especially job opportunities and income; in this period, GNP growth exceeded 7.7% per year. Issues of welfare distribution dominated the next plan with an increased emphasis on food self-sufficiency and of the provision of industrial raw materials. In constant prices, the share of GNP accounted for by manufacturing rose from 11% in 1975 to over 15% by 1980 and to 16% by 1985. The transportation and services sectors also recorded increases while the share of agriculture declined from 36% to 23% in 1975 (see Sundrum 1986, 1988).

The structural path analysis developed in this paper is used to evaluate the degree to which these changes were reflected in the structure of the

Table 2. Description of row/column labels for Indonesian social accounting matrices

	Entry	Description
Factors	1	Labor
	2	Capital
Institutions	1	Households
	2	Companies
	3	Government
Activities	1	Farm food crops, livestock, food manufacturing
	2	Estate crops, forestry, hunting
	3	Mining, non-food manufacturing, utilities, construction
	4	Trade, restaurants, hotels, transportation, communications
	5	Financial, real estate, government

Table 3. Block path for the institutions column account

		Institutions				Activities		Factors	
		d_2		$B_{22}^3 d_2$		$B_3 A_{32} B_{22}^3 d_2$		$A_{13} B_3 A_{32} B_{22}^3 d_2$	
		Volume	%	Volume	%	Volume	%	Volume	%
1975	1	0.0	0.0	447.5	45.3	384.1	43.2	272.4	46.3
	2	18.0	4.7	167.0	17.0	36.1	4.1	316.5	53.7
	3	363.0	95.3	372.5	37.7	149.0	16.8		
	4					96.7	10.9		
	5					223.3	24.1		
	Total	381.0	100.0	987.0	100.0	889.2	100.0	588.9	100.0
1980	1	166.5	25.0	747.6	48.9	492.3	40.0	392.1	48.1
	2	75.1	11.3	323.6	20.3	81.1	6.6	422.7	51.9
	3	424.5	63.7	524.3	32.8	219.5	17.9		
	4					152.7	12.4		
	5					284.3	23.1		
	Total	666.1	100.0	1 595.5	100.0	1229.9	100.0	814.8	100.0
1985	1	401.0	11.3	5 347.8	50.0	3649.7	36.8	3230.1	52.0
	2	313.3	8.9	1 912.1	17.9	495.9	5.0	2979.6	48.0
	3	2819.5	79.8	3 434.0	32.1	2008.8	20.2		
	4					1332.4	13.4		
	5					2445.0	24.6		
	Total	3533.8	100.0	10 693.9	100.0	9931.8	100.0	6209.7	100.0

economic transmissions within the Indonesian economy. The SAMs for each year were aggregated to a consistent set of entries, shown in Table 2.

Tables 3 and 4 illustrate some of the detailed structure of the complication of the Indonesian economy using the complication pyramids corresponding to institutions (67) and activities (68) column accounts (see Fig. 6). Since the SAMs were not price adjusted, the evaluation was made

Table 4. Block path for the activities column account

		Activities		$B_{33}^3 d_3$		Factors		Institutions	
		d_3				$A_{13}B_{33}^3 d_3$		$B_2 A_{21} A_{13} B_{33}^3 d_3$	
		Volume	%	Volume	%	Volume	%	Volume	%
1975	1	−948.0	−16.7	6143.0	30.3	4953.0	38.9	9278.3	63.9
	2	173.0	3.0	1019.0	5.0	7780.4	61.1	3689.7	25.4
	3	3842.0	67.6	6913.0	34.1			1553.6	10.7
	4	2619.0	46.1	4315.0	21.3				
	5	1.0	0.0	1864.0	9.2				
	Total	5687.0	100.0	20254.0	100.0	12733.4	100.0	14521.6	100.0
1980	1	−2456.0	−9.2	16272.0	21.6	18142.6	38.0	30387.3	52.8
	2	968.0	3.6	4954.0	6.6	29553.7	62.0	17562.8	30.5
	3	19545.0	73.3	32584.0	43.3			9577.1	16.7
	4	8661.0	32.5	14415.0	19.2				
	5	−59.0	−0.2	7030.0	9.3				
	Total	26659.0	100.0	75255.0	100.0	47696.3	100.0	57527.2	100.0
1985	1	−6469.0	−15.3	33665.0	21.8	39210.9	43.9	66223.9	61.4
	2	−129.0	−0.3	6399.0	4.1	50196.0	56.1	27182.2	25.2
	3	30778.0	72.8	65981.0	42.7			9577.1	
	4	17772.0	42.1	32379.0	21.0				
	5	314.0	0.7	16036.0	10.4				
	Total	42266.0	100.0	154160.0	100.0	89406.9	100.0	85686.9	100.0

in the context of the percentage allocations rather than concentrating on economic flows.

Table 3 presents the following features of the self-influence and the transfer of influence of the institution expenditures. In 1975, the institution expenditures, d_2, were concentrated in I3 (Government, 95.3%). In 1980, there was greater balance, with I2 (Companies) and I3 (Households) accounting for over 36%, although this percentage fell to 20% in 1985. The self-influence of the institution outputs were distributed almost equally between I1 and the rest of the institutions (I2+I3) in 1985. Factor 1's (labor) share of total factor output generated by institutions increased over the period 1975–1985. Institution expenditures generated direct impacts only on agricultural-related activity sectors A1 and A2; however, by the time the full system-wide effects are considered, the sector impacts are more widely distributed. Over all time periods, sector A1 (agriculture) received a smaller percentage of the total impact of institutional activities than it generated; the shares of sector A3 (mining and nonfood manufacturing) and A4 (trade and services) accounted for this reallocation.

In Table 3, there is some evidence of important structural changes in the Indonesian economy. The share of the components of institutional output generated by production activities exhibited an exchange between I1 and I2 in the interval 1975–1980 but, by 1985, the share distribution replicated

that for 1975. There are some marked changes in the activities self-influence; sector A1's share again decreased significantly from 1975–1980 but remained relatively stable through 1985.

5. Some concluding perspectives

This meso level structural path analysis can be complemented by more micro level analysis of individual paths using the procedures outlined in Defourny and Thorbecke (1984) and Khan and Thorbecke (1988). The brief empirical analysis of the network structure of the Indonesian SAM highlighted the perspective of application of the methodology of block structural path analysis to multi-account or multiregion economies, providing the opportunity to explore a variety of facets of the processes of economic development. Some earlier work (Hewings et al. 1996) proposed the notion of an evolutionary path along which economies might move; integral to this process was the notion of complexity, manifested through the expansion and deepening of the paths of interaction between sectors (in a single economy case) and between regions (in the multi-economy case). However, it is unlikely that the process will be monotonic; for example, Krugman (1990) has argued persuasively that through the strengthening of trading relationships, economies may tend to evolve towards greater similarities in economic structure. Trade will then be concentrated in intra-industry rather than inter-industry trade; however, Krugman makes no reference to what might happen within the regions. In network terms, the process would be manifested in changes in the structure of self-influence loops but with a concomitant increase in the volume of interactions manifested through the transfer of influence loops. However, the changes would result in a more specialized set of interregional interactions.

The methodology introduced here offers the opportunity to explore how regions, within a national economy, evolve in terms of changes in internal and external dependencies. The next step for enhancing understanding of the nature of economic complexity would be the introduction of nested hierarchies of economic sub-systems beginning at the level of sectors, and proceeding through clusters of industries, all economic activities within regions, all the way up to the national and even international economy. At the lowest level of analysis, the methodology enhances sectoral structural path analysis; at the highest (spatial) level, the analysis provides a strong linkage with multiregional feedback loop analysis (Sonis et al. 1995a, 1997a) and, at the intermediate level, the Matrioshka-type nested hierarchies (Sonis and Hewings 1991) together with the typologies of synergetic interactions within regional economic sub-systems (Sonis et al. 1996). Finally, it should be noted that the structure of influence and the transfer of influence is a universal process. The augmentation and complication of interactional networks in the transfer of economic flows, information, knowledge, technological and cultural innovations are the necessary components of the process of complexity in evolving systems.

Appendices

I. Derivation of the recursive generalization of the Schur-Banachiewicz formula for the case of n-regions

In this appendix, the generalization is extended to the case of *n*-regions, using mathematical induction. The proofs are as follows. For the case of *k* regions, $k = 1, 2, \ldots, n-1$, with the $k \times k$ block matrix of direct inputs:

$$A = \begin{bmatrix} A_{11} & A_{12} & \ldots & A_{1k} \\ A_{21} & A_{22} & \ldots & A_{2k} \\ \vdots & \vdots & \ldots & \vdots \\ A_{k1} & A_{k2} & \ldots & A_{kk} \end{bmatrix} \tag{A1}$$

the following formula for the Leontief $k \times k$ block inverse holds:

$$
\begin{aligned}
B &= (I - A)^{-1} \\
&= \begin{bmatrix} B_{11}^{k} & B_{11}^{k} A_{12}^{k} B_{22}^{k-1}(1) & \ldots & B_{11}^{k} A_{1k}^{k} B_{kk}^{k-1}(1) \\ B_{22}^{k} A_{21}^{k} B_{11}^{k-1}(2) & B_{22}^{k} & \ldots & B_{22}^{k} A_{2k}^{k} B_{kk}^{k-1}(2) \\ \vdots & \vdots & \ldots & \vdots \\ B_{kk}^{k} A_{k1}^{k} B_{11}^{k-1}(k) & B_{kk}^{k} A_{k2}^{k} B_{22}^{k-1}(k) & \ldots & B_{kk}^{k} \end{bmatrix} = \\
&= \begin{bmatrix} B_{11}^{k} & B_{11}^{k-1}(2) A_{12}^{k} B_{22}^{k} & \ldots & B_{11}^{k-1}(k) A_{1k}^{k} B_{kk}^{k} \\ B_{22}^{k-1}(1) A_{21}^{k} B_{11}^{k} & B_{22}^{k} & \ldots & B_{22}^{k-1}(k) A_{2k}^{k} B_{kk}^{k-1} \\ \vdots & \vdots & \ldots & \vdots \\ B_{kk}^{k-1}(1) A_{k1}^{k} B_{11}^{k} & B_{kk}^{k-1}(2) A_{k2}^{k} B_{22}^{k} & \ldots & B_{kk}^{k} \end{bmatrix}
\end{aligned} \tag{A2}
$$

where, for $i = 1, 2, \ldots, k$:

$$
\begin{aligned}
B_{ii}^{k} &= \left[I - A_{ii} - \sum_{\substack{j=1 \\ j \neq i}}^{k} A_{ij} B_{jj}^{k-1}(i) A_{ji}^{k} \right]^{-1} = \\
&= \left[I - A_{ii} - \sum_{\substack{j=1 \\ j \neq i}}^{k} A_{ij}^{k} B_{jj}^{k-1}(i) A_{ji} \right]^{-1}
\end{aligned} \tag{A3}
$$

and, for $i \neq j$, $i, j = 1, 2, \ldots, n$:

$$A_{ij}^k = A_{ij} + \sum_{\substack{s=1 \\ s \neq i,j}}^{k} A_{is} B_{ss}^{k-2}(i,j) A_{si}^{k-1} = A_{ij} + \sum_{\substack{s=1 \\ s \neq i,j}}^{k} A_{is}^{k-1}(j) B_{ss}^{k-2}(i,j) A_{sj} . \quad (A4)$$

Consider now the transfer from the case $k \leq n-1$ to $k=n$. For the $n \times n$ block matrix, A, of direct inputs, the corresponding Leontief $n \times n$ block inverse, B, satisfies the matrix equation $B(I-A) = I$ where

$$B = \begin{bmatrix} B_{11}^n & B_{12}^n & B_{13}^n & \cdots & B_{1n}^n \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ B_{21}^n & B_{22}^n & B_{23}^n & \cdots & B_{2n}^n \\ B_{31}^n & B_{32}^n & B_{33}^n & \cdots & B_{3n}^n \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ B_{n1}^n & B_{n2}^n & B_{n3}^n & \cdots & B_{nn}^n \end{bmatrix}$$

and

$$I - A = \begin{bmatrix} I - A_{11} & -A_{12} & -A_{13} & \cdots & -A_{1n} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ -A_{21} & I - A_{22} & -A_{23} & \cdots & -A_{2n} \\ -A_{31} & -A_{32} & I - A_{33} & \cdots & -A_{3n} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ -A_{n1} & -A_{n2} & -A_{n3} & \cdots & I - A_{nn} \end{bmatrix} .$$

The equation, $B(I-A) = I$, after separating the first row and column implies that:

$$B_{11}^n(I - A_{11}) + [B_{11}^n \ B_{12}^n \ \cdots \ B_{1n}^n] \begin{bmatrix} -A_{21} \\ -A_{31} \\ \cdots \\ -A_{n1} \end{bmatrix} = I \quad (A5)$$

and

$$B_{11}^n[-A_{12} \ -A_{13} \ \cdots \ -A_{1n}] + [B_{11}^n \ B_{12}^n \ \cdots \ B_{1n}^n] \cdot$$
$$\cdot \begin{bmatrix} I - A_{22} & -A_{23} & \cdots & -A_{2n} \\ -A_{32} & I - A_{33} & \cdots & -A_{3n} \\ \vdots & \vdots & \cdots & \vdots \\ -A_{n2} & -A_{n3} & \cdots & I - A_{nn} \end{bmatrix} = [0 \ 0 \ \cdots \ 0] . \quad (A6)$$

The next stage involves the search for a solution of the equation, $B(I–A)=I$, in a form:

$$B_{ij}^n = B_{ii}^n A_{ij}^* .\tag{A7}$$

Substitution of (A7) into (A6) provides the following:

$$[A_{12}^*\ A_{13}^*\ \dots\ A_{1n}^*]\begin{bmatrix} I - A_{22} & -A_{23} & \dots & -A_{2n} \\ -A_{32} & I - A_{33} & \dots & -A_{3n} \\ \vdots & \vdots & \dots & \vdots \\ -A_{n2} & -A_{n3} & \dots & I - A_{nn} \end{bmatrix} =$$

$$= [A_{12}\ A_{13}\ \dots\ A_{1n}]$$

or

$$[A_{12}^*\ A_{13}^*\ \dots\ A_{1n}^*]=[A_{12}\ A_{13}\ \dots A_{1n}\begin{bmatrix} B_{22}^{n-1}(1) & B_{23}^{n-1}(1) & \dots & B_{2n}^{n-1}(1) \\ B_{32}^{n-1}(1) & B_{33}^{n-1}(1) & \dots & B_{32}^{n-1}(1) \\ \vdots & \vdots & \dots & \vdots \\ B_{n2}^{n-1}(1) & B_{n3}^{n-1}(1) & \dots & B_{nn}^{n-1}(1) \end{bmatrix}\tag{9}$$

Therefore, the assumption (A2)

$$B_{ij}^{n-1}(1) = B_{ij}^{n-1} A_{ij}^{n-1} B_{jj}^{n-2}(1)$$

implies that

$$[A_{12}^*\ A_{13}^*\ \dots\ A_{1n}^*] = [A_{12}\ A_{13}\ \dots\ A_{1n}]\cdot$$

$$\begin{bmatrix} I & B_{22}^{n-2}(1,3)A_{23}^{n-1}(1) & \dots & B_{22}^{n-2}(1,n)A_{2n}^{n-1}(1) \\ B_{33}^{n-2}(1,2)A_{32}^{n-1}(1) & I & \dots & B_{33}^{n-2}(1,n)A_{3n}^{n-1}(1) \\ \vdots & \vdots & \dots & \vdots \\ B_{nn}^{n-2}(1,2)A_{n2}^{n-1}(1) & B_{n3}^{n-2}(1,3)A_{n3}^{n-1}(1) & \dots & I \end{bmatrix}.$$

$$\begin{bmatrix} B_{22}^{n-1} & 0 & \dots & 0 \\ 0 & B_{33}^{n-1} & \dots & 0 \\ \vdots & \vdots & \dots & \vdots \\ 0 & 0 & \dots & B_{nn}^{n-1} \end{bmatrix} = [A_{12}^n\ A_{13}^n\ \dots\ A_{1n}^n]\times$$

$$\begin{bmatrix} B_{22}^{n-1} & 0 & \dots & 0 \\ 0 & B_{33}^{n-1} & \dots & 0 \\ \vdots & \vdots & \dots & \vdots \\ 0 & 0 & \dots & B_{nn}^{n-1} \end{bmatrix} = [A_{12}^n B_{22}^{n-1}(1)A_{13}^n B_{33}^{n-1}(1)\dots A_{1n}^n B_{nn}^{n-1}(1)]$$

where

$$[A_{12}^n \ A_{13}^n \ \ldots \ A_{1n}^n] = [A_{12} \ A_{13} \ \ldots \ A_{1n}] \cdot$$

$$\begin{bmatrix} I & B_{22}^{n-2}(1,3)A_{23}^{n-1}(1) & \ldots & B_{22}^{n-2}(1,n)A_{2n}^{n-1}(1) \\ B_{33}^{n-2}(1,2)A_{32}^{n-1}(1) & I & \ldots & B_{33}^{n-2}(1,n)A_{3n}^{n-1}(1) \\ \vdots & \vdots & \ldots & \vdots \\ B_{nn}^{n-2}(1,2)A_{n2}^{n-1}(1) & B_{n3}^{n-2}(1,3)A_{n3}^{n-1}(1) & \ldots & I \end{bmatrix}.$$

In other words,

$$A_{1j}^n = A_{1j} + \sum_{\substack{s=2 \\ s \neq 2}}^{n} A_{1s} B_{ss}^{n-2}(1,j) A_{sj}^{n-1}(1) . \qquad (A10)$$

Thus,

$$A_{1j}^* = A_{1j}^n B_{jj}^{n-1}(1)$$

and

$$B_{1j}^n = B_{11}^n A_{1j}^n B_{jj}^{n-1}(1) \quad j = 2, \ldots, n \qquad (A11)$$

$$B_{11}^n = \left[I - A_{11} - \sum_{j=2}^{n} A_{1j}^n B_{jj}^{n-1}(1) A_{j1} \right]^{-1} . \qquad (A12)$$

Further, the substitution $1 \to i$ and $i \to 1$ implies that

$$A_{ij}^n = A_{ij} + \sum_{\substack{s=1 \\ s \neq i,j}}^{n} A_{is} B_{ss}^{n-2}(i,j) A_{sj}^{n-1}(i) \quad i \neq j \qquad (A13)$$

$$B_{ii}^n = \left[I - A_{ii} - \sum_{\substack{j=1 \\ j \neq i}}^{n} A_{ij}^n B_{jj}^{n-1}(i) A_{ji} \right]^{-1} \qquad (A14)$$

and

$$B_{ij}^n = B_{ii}^n A_{ij}^n B_{jj}^{n-1}(i) \quad i \neq j . \qquad (A15)$$

In analogous fashion, the solution of the matrix $(I–A)B = I$ may be found through the substitution of $B_{ij}^n = A_{ij}^{**} = B_{jj}^n$:

$$A_{ij}^n = A_{ij} + \sum_{\substack{s=1 \\ s\neq i,j}}^{n} a_{is}(j) B_{ss}^{n-2}(i,j) A_{sj} \quad i \neq j \tag{A16}$$

$$B_{ii}^n = \left[I - A_{ii} - \sum_{\substack{j=1 \\ j\neq i}}^{n} A_{ij} B_{jj}^{n-1}(i) A_{ji}^n \right] \tag{A17}$$

and

$$B_{ij}^n = B_{ii}^{n-1}(j) A_{ij}^n B_{jj}^n \quad i \neq j . \tag{A18}$$

This concludes the induction proof.

II. The augmentation process and multiregional structural path analysis

In this appendix, the generalization of the Schur-Banachiewicz system for *n*-regions is interpreted using multiregional structural path analysis as a way of reflecting and interpreting the gradual complication of the network of regional self-influence and transfer of influence. The one-region case has been elaborated in several sources (see Defourny and Thorbecke 1984; Khan and Thorbecke 1988; Thorbecke 1992). It is from this body of literature that the notion of economic influence has been derived; its application has been mainly directed towards the identification and interpretation of economic structure and to capture the transmission of influence within input-output and social accounting systems. However, the application has been mainly conducted at the micro-level; the translation to the regional meso-level requires the application of the concept of augmentation introduced by Yamada and Ihara (1969). The analytical basis of the meso-level structural path analysis, what has been referred to earlier as Block SPA, requires the sequential application of (A13) in a form that is analogous to a *telescopic* expression:

$$A_{ij}^n = A_{ij} + \sum_{\substack{j_1=1 \\ j_1 \neq i,j}}^{n} A_{ij_1} B_{j_1 j_1}^{n-2}(i,j) A_{j_1 j}^{n-1}(i) =$$

$$= A_{ij} + \sum_{\substack{j_1=1 \\ j_1 \neq i,j}}^{n} A_{ij_1} B_{j_1,j_1}^{n-2}(i,j) A_{j_1 j} +$$

$$+ \sum_{\substack{j_1=1 \\ j_1 \neq i,j}}^{n} \sum_{\substack{j_2=1 \\ j_2 \neq i,j,j_1}}^{n} A_{ij_1} B_{j_1 j_1}^{n-2}(i,j) A_{j_1 j_2} B_{j_2 j_2}^{n-3}(i,j,j_1) A_{j_2 j}^{n-2}(i,j) =$$

$$= A_{ij} + \sum_{\substack{j_1=1 \\ j_1 \neq i,j}}^{n} A_{ij_1} B_{j_1 j_1}^{n-2}(i,j) A_{j_1 j} +$$

$$+ \sum_{\substack{j_1=1 \\ j_1 \neq i,j}}^{n} \sum_{\substack{j_2=1 \\ j_2 \neq i,j,j_1}}^{n} A_{ij_1} B_{j_1 j_1}^{n-2}(i,j) A_{j_1 j_2} B_{j_2 j_2}^{n-3}(i,j,j_1) A_{j_2 j}^{n} +$$

$$+ \sum_{\substack{j_1=1 \\ j_1 \neq i,j}}^{n} \sum_{\substack{j_2=1 \\ j_2 \neq i,j,j_1}}^{n} \sum_{\substack{j_3=1 \\ j_3 \neq i,j,j_1 j_2}}^{n} A_{ij_1} B_{j_1 j_1}^{n-2}(i,j) A_{j_1 j_2} B_{j_2 j_2}^{n-3}(i,j,j_1) A_{j_2 j}^{n-3}(i,j_1,j_2) =$$

$$= \ldots = A_{ij} + \sum_{k=1}^{n} \sum_{\substack{j_1=1 \\ j_1 \neq i,j}}^{n} \sum_{\substack{j_2=1 \\ j_2 \neq i,j,j_1}}^{n} \sum_{\substack{j_3=1 \\ j_3 \neq i,j,j_1,j_2}}^{n} \cdots \sum_{\substack{j_k=1 \\ j_k \neq i,j,j_1,\ldots,j_{k-1}}}^{n}$$

$$A_{ij_1} B_{j_1 j_1}^{n-2}(i,j) A_{j_1 j_2} B_{j_2 j_2}^{n-3}(i,j,j_1) A_{j_2 j_3} \cdots$$

$$A_{j_{k-1} j_{k-1}} B_{j_k j_k}^{n-k+1}(i,j,j_1,\ldots,j_{k-1}) A_{j_k j} . \tag{A 19}$$

The augmented input, A_{ij}^n, enters the global transfer of influence, $B_{ij}^n = B_{ii}^n A_{ij}^n B_{jj}^{n-1}(i)$ where the matrices, B_{ii}^n and $B_{jj}^{n-1}(i)$, represent the self-influence of regions i and j through the networks connecting them with other regions.

The regional self-influence, B_{ii}^n, has the following recursive form:

$$B_{ii}^n = \left[I - A_{ii} - \sum_{\substack{s=1 \\ s \neq i}}^{b} A_{is}^n B_{ss}^{n-1}(i) A_{si} \right]^{-1} =$$

$$= \left[I - A_{ii} - \sum_{\substack{s=1 \\ s \neq i}}^{n} A_{is}^n B_{ss}^{n-1}(i) A_{si} - \sum_{\substack{s=1 \\ s \neq i}}^{n} \sum_{k=1}^{n} \sum_{\substack{j_1=1 \\ j_1 \neq i,s}}^{n} \sum_{\substack{j_2=1 \\ j_2 \neq i,s,j_1}}^{n} \cdots \right.$$

augmented
transfer of influence

$$A_{ij}^n$$

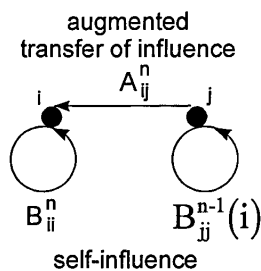i   j

$$B_{ii}^n$$ $$B_{jj}^{n-1}(i)$$

self-influence

Fig. 7. Aggregated graph of global transfer of influence B_{ij}^n

$$
\ldots \sum_{\substack{j_k=1 \\ j_k \neq i, s, j_1, \ldots j_{k-1}}}^{n} A_{is}^n B_{ss}^{n-1}(i) A_{sj_i} B_{j_1 j_1}^{n-2}(s, j_1) A_{j_1 j_2} B_{j_1 j_1}^{n-3}(i, s, j_1) A_{j_2 j_3} \ldots
$$

$$
\left. \ldots A_{j_{k-1} j_k} B_{j_k j_k}^{n-k+1}(i, s, j_1, \ldots, j_{k-1}) A_{j_k i} \right]^{-1} \tag{A20}
$$

The global transfer, B_{ij}^n, of economic influence from region j to region i can be presented by the aggregated graph illustrated in Fig. 7. Through the introduction into this scheme of the telescopic expansion of the augmentation process, provided by (A19) and (A20), one may obtain the global economic influence graph whose vertices correspond to regions and whose arcs are loaded by the components of augmented inputs of the following type:

$$
A\,\mathrm{ug}\,\mathrm{p}(i, j : k) = A_{ij_1} B_{j_1 i_1}^{n-1}(i, j_1) A_{j_1 j_2} B_{j_2 j_2}^{n-2}(i, j, j_1) A_{j_1 j_2} \ldots
$$

$$
\ldots A_{j_{k-1} j_k} B_{j_k j_k}^{n-k}(i, j, j_1, \ldots, j_{k-1}) A_{j_k j} . \tag{A21}
$$

Each such component corresponds to the appropriate elementary path, $p(i, j : k)$, that includes the vertices $i, j_1, j_2, \ldots, j_k, j$ and the arcs $(i, j_1)(j_1, j_2) \ldots (j_{k-1}, j_k)(j_k, j)$. Figure 8 describes the corresponding structure of the transfer

direct transfer of influence

$$A_{ij}$$ $$A_{j_1 j_2}$$ $$A_{j_{k-1} j_k}$$ $$A_{j_k j}$$

i $$j_1$$ $$j_2$$ $$j_k$$

$$B_{j_1 j_1}^{n-1}{}^{(i,j)}$$ $$B_{j_2 j_2}^{n-2}(i, j, j_1)$$ $$B_{j_k j_k}^{n-k}(i, j, j_1, \ldots, j_{k-1})$$
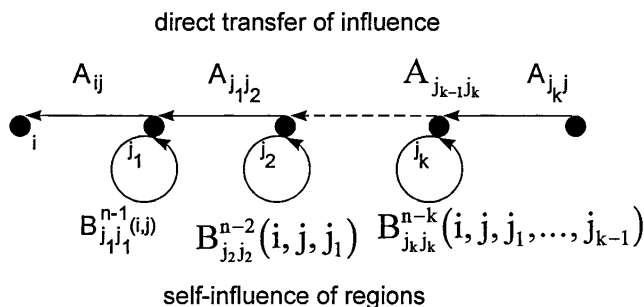
self-influence of regions

Fig. 8. Structure of transfer of influence through the elementary structural path

of influence for Aug $p(i,j)$ in the form of the set of direct transfers of influence, $A_{j_r j_{r+1}}, r=1,2,\ldots,k$, and the aggregated loops of regional self-influence corresponding to the partial Leontief block inverses, $B_{j_r j_r}^{n-r}(i,j,j_1,\ldots,j_{r-1})$, $r=1,2,\ldots,k$.

Thus, the components of the Leontief block inverse for the multi-regional input-output system can be represented as a total influence graph with the building blocks of the type revealed in Fig. 8. From this formulation, there exists the opportunity to view the myriad patterns of linkages and ripple effects – essentially, a formal methodology for unraveling the Leontief block inverse and decomposing its components into a set of partial paths of influence and self-influence.

An important example of this methodology provides for the transfer to the nested hierarchical level of regional sub-systems that can be achieved through the introduction of a generalization of the Miyazawa right and left external, self-influence multipliers for each region:

$$B_{ii}^{nR} = \left[I - \sum_{\substack{j=1 \\ j\neq i}}^{n} A_{ij} B_{jj}^{n-1}(i) A_{ji}^{n} B_i \right]^{-1}$$

$$B_{ii}^{nL} = \left[I - \sum_{\substack{j=1 \\ j\neq i}}^{n} B_i A_{ij} B_{jj}^{n-1}(i) A_{ji}^{n} \right]^{-1} . \qquad (A22)$$

This implies the generalized Miyazawa decompositions:

$$B_{ii}^{n} = B_i B_{ii}^{nR} = B_{ii}^{nL} B_i \qquad (A23)$$

At the macro level, (A22) and (A23) imply the decomposition of the Leontief block inverse with the use of the inner and outer left and right block multipliers. The following presentation illustrates an application using the inner and outer left block multipliers:

$$
B = \begin{bmatrix} B_{11}^{n} & 0 & \ldots & 0 \\ 0 & B_{22}^{n} & \ldots & 0 \\ \vdots & \vdots & \ldots & \vdots \\ 0 & 0 & \ldots & B_{nn}^{n} \end{bmatrix} \cdot
$$

$$
\cdot \begin{bmatrix} I & A_{12}^{n} B_{22}^{n-1}(1) & \ldots & A_{1n}^{n} B_{nn}^{n-1}(1) \\ A_{21}^{n} B_{11}^{n-1}(2) & I & \ldots & A_{2n}^{n} B_{nn}^{n-1}(2) \\ \vdots & \vdots & \ldots & \vdots \\ A_{n1}^{n} B_{11}^{n-1}(n) & A_{n2}^{n} B_{22}^{n-1}(n) & \ldots & I \end{bmatrix} =
$$

$$
= \begin{bmatrix} B_{11}^{nL} & 0 & \ldots & 0 \\ 0 & B_{22}^{nL} & \ldots & 0 \\ \vdots & \vdots & \ldots & \vdots \\ 0 & 0 & \ldots & B_{nn}^{nL} \end{bmatrix} \begin{bmatrix} B_{1} & 0 & \ldots & 0 \\ 0 & B_{2} & \ldots & 0 \\ \vdots & \vdots & \ldots & \vdots \\ 0 & 0 & \ldots & B_{n} \end{bmatrix} \cdot
$$

$$
\cdot \begin{bmatrix} I - A_{11} & A_{12}^{n} B_{22}^{(n-1)L}(1) & \ldots & A_{1n}^{n} B_{nn}^{(n-1)L}(1) \\ A_{21}^{n} B_{11}^{(n-1)L}(2) & I - A_{22} & \ldots & A_{2n}^{n} B_{nn}^{(n-1)L}(2) \\ \vdots & \vdots & \ldots & \vdots \\ A_{n1}^{n} B_{11}^{(n-1)L}(n) & A_{n2}^{n} B_{22}^{(n-1)L}(n) & \ldots & I - A_{nn} \end{bmatrix} \cdot
$$

$$
\cdot \begin{bmatrix} B_{1} & 0 & \ldots & 0 \\ 0 & B_{2} & \ldots & 0 \\ \vdots & \vdots & \ldots & \vdots \\ 0 & 0 & \ldots & B_{n} \end{bmatrix} . \tag{A24}
$$

An analogous presentation would apply for the right multipliers; the coresponding aggregated structural path for this decomposition is analogous to the one shown in Fig. 7.

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