

Forecasting industrial employment figures in Southern California: A Bayesian vector autoregressive model

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Abstract. In this paper, we construct a Bayesian vector autoregressive model to forecast the industrial employment figures of the Southern California economy. The model includes both national and state variables. The root mean squared error (RMSE) and the Theil's U statistics are used in selecting the Bayesian prior. The out-of-sample forecasts derived from each model and prediction of the turning points show that the Bayesian VAR model outperforms the ARIMA and the unrestricted VAR models. At longer horizons the BVAR model appears to do relatively better than alternative models. A prior that becomes increasingly looser produces more accurate forecasts than a tighter prior in the BVAR estimations.

1. Introduction

Recent demographic shifts in the U.S. and particularly in the Southern California region have put a strain on the availability and allocation of resources. As such, accurately forecasting regional employment figures has become an issue of increasing importance to both researchers and policymakers in economic development. In this paper, we construct a Bayesian vector autoregressive model (BVAR) for the Southern California economy to forecast the employment figures for the region's major industries. The out of sample forecasts obtained from the BVAR model are then compared with the forecasts from unrestricted VAR and best fit ARIMA models. Regional and national variables are included in the model to capture the economic interactions of the Southern California economy with the nation.

Econometric forecasting models for regional and nationwide economies are generally formulated as simultaneous-equations structural models. As such, for correct identification of individual equations, some variables have to be excluded from certain equations. According to Cooley and LeRoy (1985) the exclusion is often carried out with little theoretical justifications. Structural

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models, although useful for policy simulations, are not only poorly suited for forecasting but also the exogenous variables in these models have to be projected first (Diebold 1997).

A vector autoregression model (VAR) provides an alternative approach that is particularly suited for forecasting purposes. Although the VAR model is less theoretical, it can be viewed as an approximation to the reduced form of a structural system of simultaneous equations (Zellner 1979 and Zellner and Palm 1974). The VAR model is based on regularities in the historical data of the variables being forecast. The out-of-sample accuracy of the Bayesian vector autoregression (BVAR) forecasts is compared with that of forecasts from an unrestricted VAR model and from a univariate ARIMA model. The root mean squared error and Theil's U statistic are used to evaluate forecasting accuracy.

After testing for the optimal lag-length and performing block exogeneity tests on the model, the findings show that the Bayesian VAR model produces more accurate forecasts than the unrestricted VAR and the ARIMA models. At longer horizons, the BVAR model appears to produce better forecasts than the alternative VAR and ARIMA models. Further, a prior that becomes increasingly looser produces more reliable forecasts than a prior becoming increasingly tighter in the BVAR estimations, suggesting that in forecasting regional economic variables BVAR models outperform ARIMA and VAR.

The better performing BVAR models can be used by policy makers and individuals who use such forecast information in their decision making. Accurate employment forecasts can promote regional stability and growth. For agents to form correct expectations, it is important that they obtain as reliable estimates as possible while minimizing the data constraints, time and expense requirements. Thus, we show that the BVAR model delivers on all of these accounts.

The remainder of the paper is organized as follows. Section 2 presents the theoretical framework, Sect. 3 discusses VAR, BVAR and benchmark ARIMA models. Section 4 compares the out-of-sample accuracy of the forecasts generated from such alternative models and discusses implications of the findings for forecasting. Section 5 provides some concluding remarks.

2. Theoretical framework

We begin by assuming that there exists an employment level, which corresponds to the equilibrium level of GDP at the income-expenditure equilibrium. Further, there must also be employment levels – which we name autonomous and induced employment – corresponding to the autonomous and induced components of real GDP. For instance, when a given firm in a region hires a certain number of workers and pays income for their services, these workers in turn generate induced employment when they spend their income in other goods. Thus, the levels of autonomous and induced employment can be represented as:

$$AE_t = A_t + \delta E_t \quad (1)$$

$$A_t = \alpha_0 + \alpha_1' Y_t + \alpha_2' Z_t + \alpha_3 E_{t-1} \quad (2)$$

where AE is aggregate employment in the region at time t , and the first and second terms on the right hand side of Eq. (1) represent autonomous and induced employment, respectively. Y_t is a $(N \times 1)$ vector of national variables such as the industrial production index, and the composite index of leading indicators, α_1 is a $(1 \times N)$ vector of parameters and Z_t is a $(K \times 1)$ vector of regional variables such as the regional industrial production index α_2 is a $(1 \times K)$ vector of parameters where K can be equal to N . After substituting (2) into (1) the equations above can be represented as,

$$AE_t = \alpha_0 + \alpha'_1 Y_t + \alpha'_2 Z_t + \delta E_t + \alpha_4 E_{t-1} \tag{3}$$

The parameters of this equation can be forecasted using conventional econometric methods. Solving for E_t at the equilibrium where total expenditure (TE) equals real GDP¹, the multipliers corresponding to a change in each component of A_t are given by:

$$\partial E_t / \partial Y_{1t} = \alpha_{11} / (1 - \delta) \tag{4}$$

$$\partial E_t / \partial Z_{1t} = \alpha_{21} / (1 - \delta) \tag{5}$$

$$\partial E_t / \partial E_{t-1} = \alpha_3 / (1 - \delta) \tag{6}$$

Thus, a one unit exogenous increase in local employment due to an increase in a national variable – like a national indicator – would change employment in the region by a magnitude of $\alpha_{11} / (1 - \delta)$, which is essentially an employment multiplier. The model with the better forecasting accuracy also produces more accurate multipliers. In this study, however, we focus on forecasting performance rather than on the calculation of the multipliers.

3. Data and econometric methodology

The specification above can be estimated using univariate and multivariate econometric methodologies where more than one lag of each variable can be included to approximate the true data generating process. We compare autoregressive integrated moving average (ARIMA), vector autoregression (VAR) and Bayesian VAR models and briefly discuss each below.

Autoregressive moving average models use only the past observations of a given series and they can approximate the data generating process more parsimoniously than purely autoregressive or moving average models. Following Box and Jenkins (1970), the ARIMA($p = 1, d = 1, q = 1$) model where p stands for the number autoregressive terms, d the number of differences, q the number of moving average terms can be represented as:

$$\Delta y_t = \alpha_0 + \beta_1 \Delta y_{t-1} + \varepsilon_t + \phi \varepsilon_{t-1} \tag{7}$$

where Δy_t is the first difference of y_t and $\phi \varepsilon_{t-1}$ is the moving average term.

¹ Here, we make use of the fact that employment corresponding to TE must be equal to the employment corresponding to RGDP at the equilibrium point.

Because the error term in the moving average process is not observable, non-linear optimization techniques are used to obtain parameter estimates.

A multivariate extension of the ARIMA model is the VAR model. The advantage of the VAR model over a structural specification is that a VAR model is a suitable approach to uncover dynamic interactions of variables while minimizing the restrictive assumptions of a structural model (Sims 1980). As Zellner (1979), and Zellner and Palm (1974) show, any linear structural model can be expressed as a VAR moving average (VARMA) model and, under some conditions, a VARMA can be expressed as a VAR model. Thus, a VAR model resembles a typical large-scale structural model. Due to the fact that in regional modeling the lack of data precludes researchers from using large-scale structural models, VARs become especially attractive in dynamic policy simulations and forecasting.

The VAR model can be expressed as:

$$Z_t = C + \sum_{s=1}^m A_s Z_{t-s} + e_t \tag{8}$$

where Z_t is a $N \times 1$ column vector of N variables, C is the deterministic component comprised of a constant, and A_s are respectively, $N \times 1$ and $N \times N$ matrixes of coefficients, m is the lag length, and e_t is the $N \times 1$ innovation vector. By construction, e_t is uncorrelated with all the past Z_s .

One disadvantage of this model and ARIMA is the problem of over-parameterization, which may lead to inefficient estimates and large out-of-sample forecasting errors commonly solved by excluding statistically insignificant lags.

An alternative approach is to use a Bayesian VAR (BVAR) model (Litterman 1981; Doan et al. 1984; Todd 1984; Litterman 1986, and Spencer 1993). Instead of entirely including or excluding a lag of a variable, restrictions are introduced on the coefficients that tend to be less important based on statistical tests and economic theory. For instance, if longer lags are less important, a restriction can be imposed on these lags that are more likely to be zero than the coefficients on shorter lags. Specifying normal prior distributions with zero means, small and decreasing standard deviations on increasing lags can impose such restrictions. When the coefficient associated with the first lagged dependent variable in each equation in the VAR has a mean value of unity and a zero value as the mean for all other coefficients in the equation, the prior is commonly referred to as the ‘‘Minnesota Prior,’’ first developed and used mainly by economists associated with the University of Minnesota and Federal Reserve Bank of Minneapolis.

The standard deviation $[S(i, j, m)]$ of the prior distribution for lag m of variable j in equation i , for all i and j , and m is expressed as

$$S(i, j, m) = \{wg(m)f(i, j)\}s_i/s_j \tag{9}$$

$$f(i, j) = 1 \quad \text{if } i = j \text{ and } k_{ij} \text{ otherwise } (0 \leq k_{ij} \leq 1) \tag{10}$$

$$g(m) = m^{-d}, \quad d > 0 \tag{11}$$

where s_j is the standard error of the univariate autoregression for variable i . The s_i/s_j term enables for the specification of the prior without having to

consider the magnitudes of the variables. The term w is the tightness parameter where a decreasing value of w means a tighter prior. The parameter $g(m)$ gives the tightness on lag m relative to lag 1. The decay parameter d is used to determine the rate of decay of increasing lags. A larger value of d enables the lags to decay faster. The parameter $f(i, j)$ represents the tightness of variable j in equation i relative to variable i . The lower the value of k_{ij} , the tighter is the prior. The larger a model is with respect to the number of observations, the more important the prior becomes.

Typical estimation situations are models with 9 parameters per equation for 40 data points, 30 for 120, and 70 for 400 (Doan 1990). The larger a model for a given sample size, the greater is the tightness or the closer the restriction toward zero. It is usually better to include extra lags and use a decaying lag prior than to truncate at an early lag. Thus, tightness directly controls the important own lags.

The Minnesota prior, based on the random walk hypothesis, makes use of the statistical observation that some economic variables seem to behave as though the changes in their values are completely unpredictable. For these variables, the best forecast is that variable's current value. As Litterman (1986) and Todd (1984) suggest, BVAR models with a Minnesota prior can also be used to better forecast the movement of housing prices or risky assets, real estate and precious metals, as one can express more realistically the researcher's true state of knowledge and uncertainty about the structure of the economy relative to other existing models.

Using quarterly data obtained from the Bureau of Labor Statistics, a BVAR model is estimated for 1983:1 to 1994:3. We concentrate on the three largest industries – manufacturing, retail, services – that make up approximately three-fourths of the total employment figures for five counties (Los Angeles, Orange, Riverside, San Bernardino and Ventura) in the Southern California region². Out-of-sample one through four quarters ahead forecasts are computed for 1994:4 to 1996:4. Both state and national variables are included in the model. The state variables include total employment for Southern California and for each selected industry, whereas the national variables included are a leading indicator composite index and an industrial production index. The model is estimated with four lags of each variable (i.e., 17 parameters including the constant). All variables are measured in logs as pointed out by Sims et al. (1990, p. 136).

The optimal Bayesian prior is selected by examining the Theil's U values for the out-of-sample forecasts. The combination of the parameters in the prior that produces the lowest U values, on average, is chosen. The Kalman (1960) filtering algorithm is used for updating of coefficient estimates to generate optimal forecasts for the Theil's U statistics four quarters ahead. The Kalman filtering algorithm provides a convenient way to compute the new coefficient vector, which would be obtained using one more or one less observation. The Kalman filter for a standard linear model is expressed as

$$Y_t = X_t B_t + u_t \quad (12)$$

$$B_t = B_{t-1} + v_t \quad (13)$$

² The forecast accuracy statistics of the three models estimated for sub industries are available on request from the authors.

where B_t is the vector of states at time “ t ,” u_t are the residuals from the Eq. (12) and v_t are the residuals from the process in Eq. (13). Both u_t and v_t are assumed to be independent. When the variance of v_t is equal to zero there is said to be no time variation in the parameters.

As recommended by Doan (1990), the initial values of overall tightness, w , and harmonic lag decay, d , are set at 0.2 and 1, respectively whereas the initial values for $f(i, j)$, the relative weights of variables j in equation i , are taken from Kinal and Ratner (1986) and Shoesmith (1992). After some specification search, the weights of a national variable in a national equation, k_{nn} , and the weight of a national variable in a state equation, k_{sn} , are selected to be 0.9. The weight in state equations for national variables, k_{ns} , is 0.6 and the weight of a state variable in other state equations, k_{ss} , varies from 0.01 to 0.9 depending on which variable is being estimated. The weight in own equations is 1. These weights conform to Litterman’s circle-star structure. Star (national) variables affect both star and circle (state) variables whereas the circle variables influence primarily other circle variables. The priors selected here are looser than the on used by Kinal and Ratner (1986) and Dua and Ray (1995).

In all, the prior structure varies from one estimation to another but in most cases the own lags of the variable being forecasted are weighted more heavily than the lags of the other variables. The decay parameter, which controls the influence of longer lags in the system, is set at 0.01 producing the most accurate out-of-sample forecasts in most of the estimations. That is as lags increase their effect on the dependent variables decreases gradually so that the effect of the first lags is greater than that of longer lags.

4. Evaluation of accuracy

The benchmark forecasts are estimated from the best-fit univariate ARIMA models. The stationarity, autocorrelation, and partial autocorrelation functions, significance of coefficients, and the Akaike (1974) Information Criterion (AIC) are used in selecting the best model for each industry category. The AIC penalizes the addition of right-hand-side variables and is expressed as

$$AIC = \exp(2k/N) \sum_{n=1}^N e_t^2/N \tag{14}$$

where k is the number of parameters estimated per sample observation N , and e is the residual series from the regression. The model that minimizes AIC is selected.

The out-of-sample forecast accuracy is evaluated using the Theil’s U statistic (1966) to compare the accuracy of a forecast to that of a “naive” competitor. The statistic is the ratio of the 1-step-ahead mean squared error for a given forecast to that of a random walk forecast $y_{t+1} = y_t$, expressed as

$$U_t = \sum_{t=1}^T (y_{t+1} - y_{t+1,t})^2 / \sum_{t=1}^T (y_{t+1} - y_t)^2 \tag{15}$$

where T is the number of periods being forecasted and $y_{t+1,t}$ is the forecast

of y one period ahead at time t . A U value of 1 indicates that the model forecasts are as good as naive (no change) forecasts. A U statistic that is greater than 1 indicates that the naive forecasts are better than the forecasts that come out of the model. If U is less than 1, the forecasts from the model are better than naive forecasts. Both Theil's U -statistic – a scale free measure –, and the RMSE measure of forecast accuracy – a scale dependent measure – are reported.

The VAR model is estimated in levels with four lags to make it comparable with the BVAR model. The Theil's U statistics and the RMSEs from the ARIMA and the VAR are compared with the three versions of the Bayesian VAR. The first version of the BVAR model is with a tight prior ($w = 0.2, d = 1$) – BVAR-1. The second version is with a tighter prior ($w = 0.1, d = 2$) – BVAR-2 and the third are with a looser prior ($w = 0.6, d = 0.01$) – BVAR-3. Tables 1–4 report the Theil's U statistics and the RMSE for the three biggest industries and the total non-farm employment. Based on these results the following inferences can be made:

When the RMSEs are compared with the Theil's U statistics, the U statistics follow a consistent pattern with the RMSEs for all variables. This result is

Table 1. Accuracy of out-of-sample forecasts (1993 : 2–1994 : 2): Manufacturing (in logs)

Quarter ahead	N	ARIMA (1, 1, 0)	VAR (unrestricted)	BVAR-1 ($w=0.2, d=1$)	BVAR-2 ($w=0.1, d=2$)	BVAR-3 ($w=0.6, d=0.01$)
<i>U</i>						
1	5	0.608	0.479	0.623	0.799	0.570
2	4	0.364	0.426	0.446	0.699	0.389
3	3	0.384	0.444	0.430	0.656	0.391
4	2	0.250	0.377	0.358	0.601	0.339
<i>RMSE</i>						
1	5	0.007	0.006	0.007	0.010	0.007
2	4	0.008	0.009	0.009	0.015	0.008
3	3	0.012	0.013	0.013	0.020	0.012
4	2	0.009	0.014	0.014	0.023	0.013

Table 2. Accuracy of out-of-sample forecasts (1993 : 2–1994 : 2): Retail trade (in logs)

Quarter ahead	N	ARIMA (2, 0, 3)	VAR (unrestricted)	BVAR-1 ($w=0.2, d=1$)	BVAR-2 ($w=0.1, d=2$)	BVAR-3 ($w=0.9, d=0.01$)
<i>U</i>						
1	5	1.06	0.271	0.791	0.934	0.307
2	4	1.14	0.283	0.707	0.910	0.259
3	3	0.642	0.503	0.788	0.900	0.461
4	2	1.93	1.32	1.85	1.84	1.21
<i>RMSE</i>						
1	5	0.023	0.005	0.014	0.017	0.006
2	4	0.029	0.005	0.014	0.018	0.005
3	3	0.012	0.008	0.012	0.014	0.007
4	2	0.028	0.012	0.017	0.017	0.008

Table 3. Accuracy of out-of-sample forecasts (1993 : 2–1994 : 2): Services (in logs)

Quarter ahead	<i>N</i>	ARIMA (2, 0, 0)	VAR (unrestricted)	BVAR-1 (<i>w</i> =0.2, <i>d</i> =1)	BVAR-2 (<i>w</i> =0.1, <i>d</i> =2)	BVAR-3 (<i>w</i> =0.4, <i>d</i> =0.01)
<i>U</i>						
1	5	1.89	0.670	0.713	0.736	0.745
2	4	1.17	0.668	0.625	0.760	0.574
3	3	1.52	1.00	0.767	0.910	0.887
4	2	0.906	0.755	0.934	1.03	0.795
RMSE						
1	5	0.012	0.004	0.004	0.005	0.005
2	4	0.008	0.005	0.004	0.006	0.004
3	3	0.011	0.008	0.006	0.007	0.007
4	2	0.010	0.009	0.010	0.011	0.009

Table 4. Accuracy of out-of-sample forecasts (1993 : 2–1994 : 2): Total Non-Farm Employment (in logs)

Quarter ahead	<i>N</i>	ARIMA (2, 0, 1)	VAR (unrestricted)	BVAR-1 (<i>w</i> =0.2, <i>d</i> =1)	BVAR-2 (<i>w</i> =0.1, <i>d</i> =2)	BVAR-3 (<i>w</i> =0.4, <i>d</i> =2)
<i>U</i>						
1	5	1.21	0.750	0.775	0.847	0.742
2	4	1.86	1.52	0.740	1.21	0.438
3	3	1.54	2.74	1.13	1.75	0.913
4	2	0.18	3.63	2.41	3.43	1.90
RMSE						
1	5	0.004	0.002	0.002	0.003	0.002
2	4	0.005	0.004	0.002	0.004	0.001
3	3	0.005	0.009	0.004	0.005	0.003
4	2	0.000	0.009	0.006	0.008	0.004

not consistent with the findings of Dua and Ray (1995) where RMSEs are not found to follow a consistent pattern with the Theil’s *U* statistics.

When the results from the BVAR models are compared with the ARIMA models, the BVAR-3 performs better than the corresponding ARIMA models. Thus, the BVAR forecasts are preferred to ARIMA forecasts. When the BVAR models are compared with the VAR models, the BVAR model outperforms the unrestricted VAR models. The VAR model, however, outperforms the ARIMA models in all estimations.

When the three BVAR models are compared with each other, BVAR-3 (with the looser prior) produces the most accurate forecasts, and the BVAR-2 (with the tighter prior) produces the least accurate forecasts. The results clearly show that the BVAR model has superior performance in forecasting employment series for the Southern California economy. In the class of BVAR models, the model with a looser prior produces more accurate forecasts than the VAR and ARIMA models.

The stronger performance of the looser prior model (BVAR-3) suggests that the empirical model may hunger for more degrees of freedom and so does not want to be “cut off” from the lagged variables that might offer such.

Nevertheless, the number of lags was kept to four to eliminate the possibility that BVAR models may perform better due to the inclusion of more lags rather than the model's true forecasting ability. As is the case in any BVAR estimation, the accuracy of the forecasts seems to be sensitive to the specification of the prior.

Another way to evaluate the performance of alternative models is to examine their ability in predicting a turning point. We focus on the performance of the optimal BVAR model (BVAR-3) with that of the ARIMA and VAR models.

Figures 1 through 4 plot the out of sample forecasts from the ARIMA, unrestricted VAR and the BVAR-3 for manufacturing, retail sales, services and total non-farm employment. Both VAR and BVAR-3 models in general correctly predict the direction of change. On the other hand, the ARIMA model does not perform well in predicting the turning points for manufacturing (in Fig. 1 for example, a declining trend is forecasted whereas the actual series exhibits a rising trend). However, based on the closeness of the forecasted turning points to the actual turning points, the BVAR-3 model performs better in predicting the business cycles rather than the VAR models.

5. Concluding remarks

In this study three econometric models – ARIMA, BVAR, and VAR – are estimated and their forecasting performance is compared with the purpose of finding the regional model that produces the most accurate out-of-sample employment forecasts for the five counties in the Southern California region. The results show that the Bayesian VAR model outperforms the unrestricted VAR models and the best-fit univariate ARIMA models. At longer horizons, the BVAR model appears to do relatively better than the other models considered in this study. In the class of BVAR models, the models with a looser prior generally outperform the tighter BVAR models. Further, the Bayesian VAR model is able to predict the direction of change better than the ARIMA and VAR models.

The relatively poor performance of the ARIMA model can be linked to its univariate feature, which uses only the past performance of a given economic variable to forecast its future. ARIMA models require tedious numerical optimization and many of the extensions of the ARIMA models can be conveniently applied to a VAR framework. Thus, the VAR model is multivariate and captures cross-variable relationships where all variables are endogenous. BVAR model is an improvement over VAR and ARIMA models simply because the BVAR model is a less restrictive, simpler alternative.

Perhaps the most important practical implication of this study is that practitioners and policy makers in regional economies can utilize BVAR models to obtain superior forecasts, become better informed and, therefore, generate correct expectations about where the economy is likely to be headed in the future.

Others interested in undertaking similar research need to take into account the model selection and specification issues carefully. The accuracy of the forecasts is sensitive to the specification of the prior, which is selected on the basis of “minimizing” out of sample forecast errors. Thus, these priors may not be optimal beyond the forecasting period.

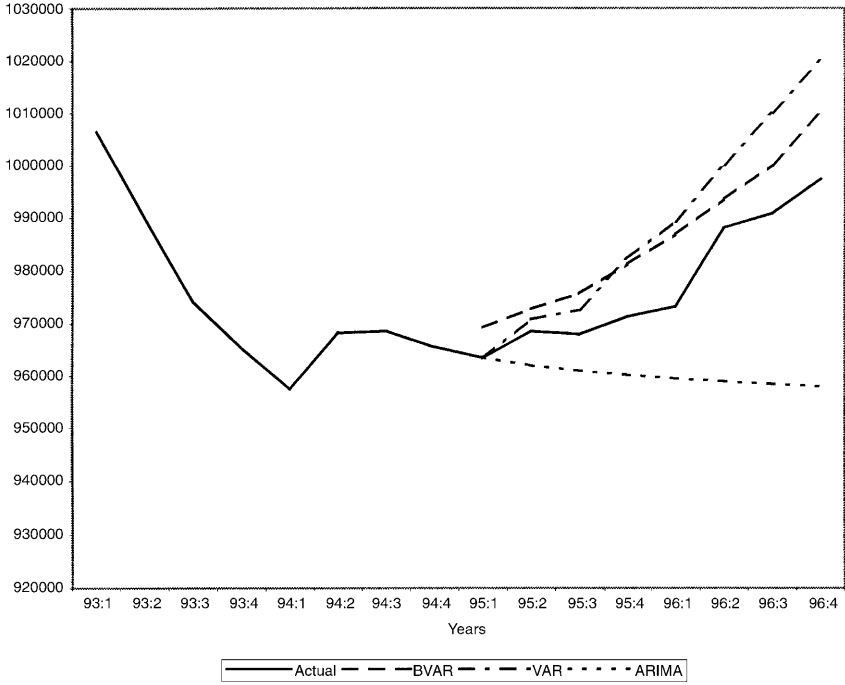


Fig. 1. Manufacturing

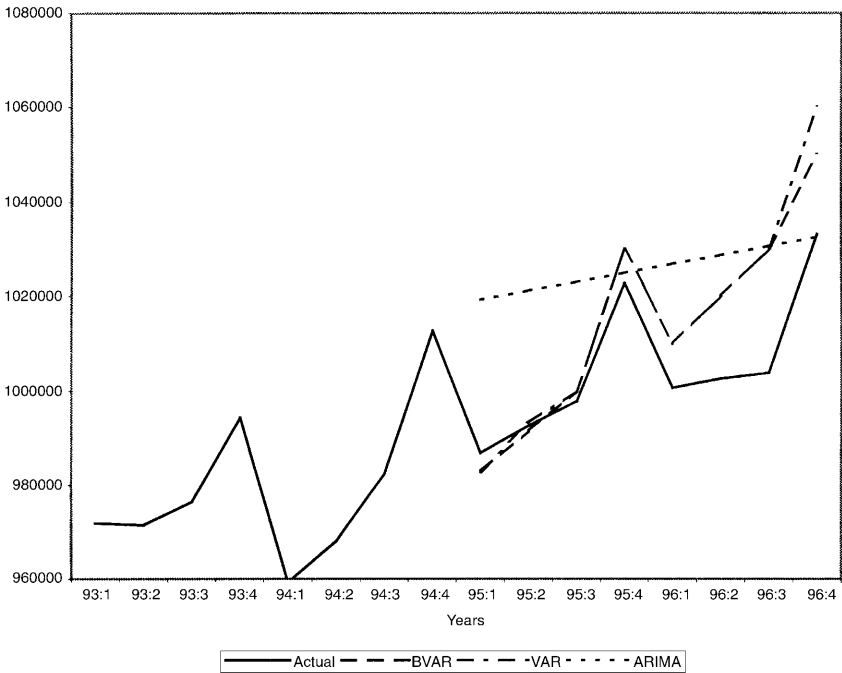


Fig. 2. Retail sales

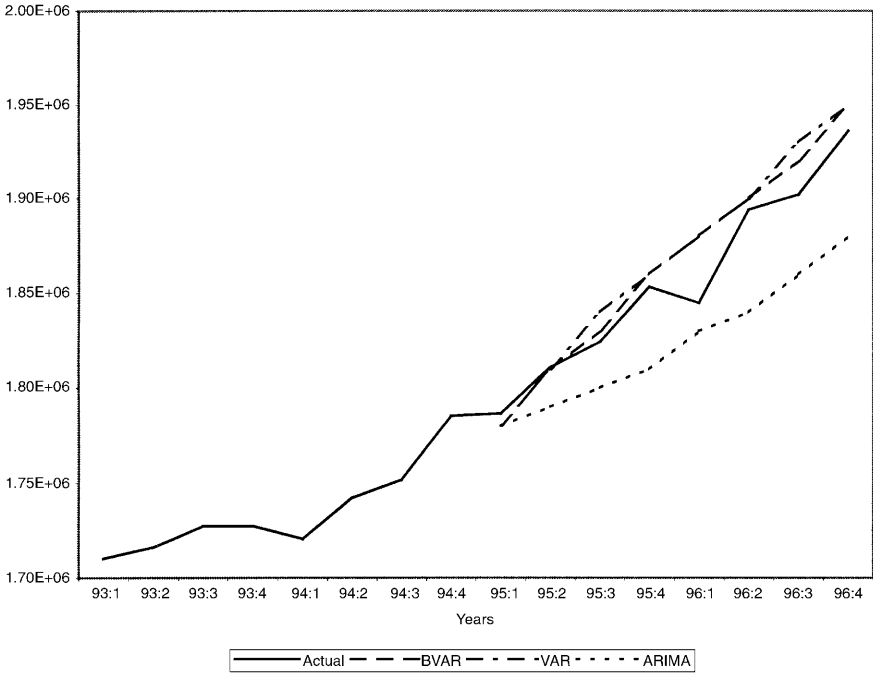


Fig. 3. Services

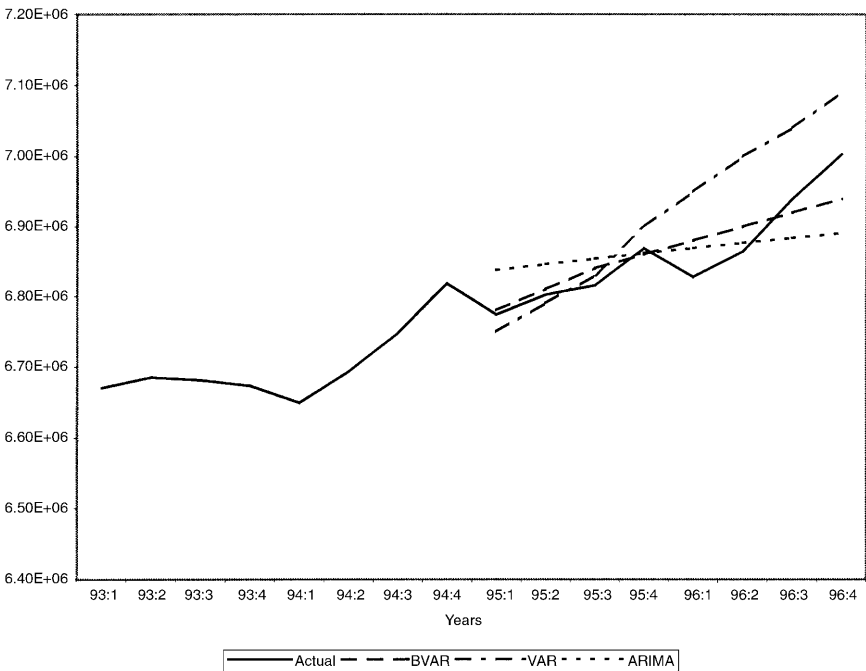


Fig. 4. Total employment

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