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ORIGINAL PAPER

Regional subsidies and interregional labor movement

Daisuke Matsuzaki[1] · **Yoshiyasu Ono**[2]

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Abstract

Japan distributes lump-sum grants and subsidies to the vast majority of local regions. Each region makes a decision regarding expenditure and can choose between non-distortionary direct transfers to the region's natives and subsidies to stimulate the local economy. Considering a two-region economy with interregional labor migration, we compare the welfare effects of direct transfers and economic stimulation subsidies including those intended to support production, employment, wages, and residents. The results show that under full employment, replacing direct transfers with stimulation subsidies benefits (harms) natives if the recipient region specializes in labor-intensive (labor-saving) activities. However, such replacements can be detrimental to natives if the region suffers unemployment due to wage rigidity. For example, wage and resident subsidies may cause harm as they promote immigration, without stimulating production.

JEL Classification H71 · R23 · R51

1 Introduction

A majority of Japan's local regions, including prefectures, cities, towns, and villages, receive lump-sum grants and subsidies from the central government. These grants can be allocated as direct lump-sum transfers to the region's natives or as subsidies that stimulate local economic activities and immigration. A typical example

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✉ Daisuke Matsuzaki
matsuzaki@toyo.jp

Yoshiyasu Ono
ono@iser.osaka-u.ac.jp

1 Faculty of Economics, Toyo University, 5-20-28 Hakusan, Bunkyo, Tokyo 112-8606, Japan

2 Institute of Social and Economic Research, Osaka University, 6-1, Mihogaoka, Ibaraki, Osaka 567-0047, Japan

of such grants and subsidies is tax allocation grants (*chiho kofu zei*). In 2019, the Japanese government distributed approximately 16.2 trillion yen of tax allocation grants across numerous local regions. Each region is given discretion to determine how the funds are spent.[1] The choice between lump-sum transfers to the regions' natives and stimulation subsidies has a critical impact upon local welfare. Therefore, in this study, we examine if native households in a recipient region are better off when lump-sum transfers are replaced by stimulation subsidies.

We adopt a two-region, two-factor, two-commodity model, in which a subsidy-recipient region produces only one commodity, while in a donor region, two commodities are produced. Each region may experience full employment or some unemployment, with workers able to migrate between regions in pursuit of higher utility. The welfare of native households in each region is contingent on the factor intensity of the local industry and the employment conditions of the two regions. Welfare is affected by the subsidies through changes in terms of trade, interregional migration, and local employment. We examine the welfare effects of a subsidy replacement on various combinations of employment conditions and factor intensity as both factors vary depending on region and period in Japan. This analysis may also apply to interregional subsidies and transfers that exist in many other countries.

The equalization subsidies literature has explored the welfare effects of regional subsidies in the context of interregional migration. For example, Flatters et al. (1974), Boadway and Flatters (1982), and Wildasin (1986) focus on negative externalities attributable to the congestion of local public goods and explore migration subsidies that promote the optimal interregional allocation of households. Albouy (2012) extends Boadway and Flatters' model by introducing household income heterogeneity. However, these studies ignore unemployment. By contrast, our study examines the welfare effects of replacing direct transfers with subsidies promoting local employment, production, and immigration on the recipient and donor regions under a variety of employment conditions.

Harris and Todaro (1970) were the first to model the combination of local unemployment and interregional migration. Their model is based on an economy with a rural and an urban area. Both areas specialize in different commodities. The rural area achieves full employment, whereas the urban area faces unemployment due to wage rigidity. Their results indicate that urban job creation leads to higher urban unemployment (the Todaro paradox). The authors further analyze the effects of urban employment subsidies and immigration restrictions on social welfare. Several studies have since extended the Harris–Todaro model by introducing factors including perfect or imperfect capital mobility, labor market monopsony, land market, urban pollution, corruption, and microeconomic foundations on wage rigidity (i.e., efficiency wage and search matching) and have explored the validity and extensions of the model (e.g., Bhagwati and Srinivasan 1974; Corden and Findlay 1975; Nakagome 1989; Raimondos 1993; Brueckner and Kim 2001; Sato 2004; Basu 2004; Choi and Yu 2010; Tawada and Sun 2010; Pi and Zhou 2015).

[1] See https://www.soumu.go.jp/main_content/000599203.pdf in the website of the Ministry of Internal Affairs and Communications for this figure.

Our analysis differs from the Harris–Todaro model literature in three ways. First, while the existing models focus on social welfare, our model considers local development and regional welfare distribution.[2] Second, we account for unemployment in both the recipient and donor regions because in Japan, unemployment in recipient regions tends to be more severe than that which exists in donor regions. We consider all combinations of unemployment and full employment in the two regions. Finally, rather than assuming perfect specialization in both regions, we hypothesize that the recipient region produces only one commodity, while the donor region produces two commodities, and hence, the Heckscher–Ohlin mechanism works. In this setting, replacing a non-distortionary lump-sum transfer with subsidies to stimulate economic activities creates a terms-of-trade effect in addition to an employment expansion effect in one or both regions depending on the employment conditions.

2 Model

Our model economy has two commodities (commodity 1 and 2) and two regions (subsidy-recipient region R and donor region D). The government imposes an equal lump-sum tax amounting to T on all households in the economy and distributes a land subsidy amounting to S in region R. Land in region j ($j = R, D$) is uniformly owned by all native households. Households are able to move freely between the two regions while retaining land ownership in their native region.[3] Thus, the land subsidy is equivalent to a lump-sum transfer exclusively to the region's native households. Our analysis remains the same even when we use capital instead of land, provided that capital is immobile. Region R produces only commodity 1, while region D produces both commodities.[4] Both commodities are produced using labor and land (or capital) as inputs.

The government considers replacing lump-sum transfer S to region R's native households with the following stimulation subsidies: production subsidy ϵ, employment subsidy z, wage subsidy s_w, and resident subsidy s_r (or equivalently, a reduction in the resident tax). These subsidies are expected to promote production and employment in region R as well as increasing immigration. Note that S and s_r are both given to the region's households; however, the former is provided exclusively to the region's native households, while the latter is provided

[2] Fields (2005) analyzes the effects of various policies on the wage distribution of rural and urban regions. Chang et al. (2009) and Gilbert et al. (2015) examine the effect of trade liberalization on economic growth and regional welfare. Li and Wang (2015) explore the influence of migrant remittances on urban welfare. All of them use the Harris–Todaro framework.

[3] Even if they sell land when they emigrate after a change in the land subsidy, the result is the same because the land price reflects the change and hence they receive the surplus (or incur the cost) due to it.

[4] In the $2\times2\times2$ Heckscher–Ohlin model with factor mobility, any difference in relative productivity between two countries and any policy that differentiates relative productivity make the smaller country perfectly specialized, while the larger country produces both commodities if there is a large size difference. We apply this property in our model. In the case of a specialized smaller country, a unique equilibrium arises even in the presence of such differences in parameters and policies.

to all residents in the region, including immigrants. Thus, the former yields no economic effect other than interregional redistribution, while the latter induces immigration.

2.1 Households

The population of native households in region j, denoted as household j, is given by L_o^j, while the current population of households living in region j is given by L_n^j $(j = R, D)$. Accordingly,

$$L = L_o^R + L_o^D = L_n^R + L_n^D, \tag{1}$$

where L is total population, which is constant. For simplicity, we assume that both regions initially have no immigrants, and thus,

$$L_o^j = L_n^j \quad \text{for } j = R, D. \tag{2}$$

Further, all households have the same utility function:

$$u^j = \hat{u}\left(c_1^j, c_2^j\right) \quad \text{for } j = R, D,$$

where c_i^j $(i = 1, 2; j = R, D)$ is household j's consumption of commodity i. Assuming that the utility function is homothetic, we have

$$p_1(\omega)c_1^j = \varphi(\omega)Y^j, \quad p_2(\omega)c_2^j = (1 - \varphi(\omega))Y^j, \tag{3}$$

where $p_i(\omega)$ is the price of commodity i measured in terms of the composite of the two commodities; ω is the price of commodity 2 relative to commodity 1; Y^j is household j's disposable income; and $\varphi(\omega)$ is the ratio of consumption expenditure on commodity 1, which satisfies

$$0 < \varphi(\omega) < 1, \quad \varphi'(\omega) \geqq 0. \tag{4}$$

Because the utility function is homothetic, the value of $\hat{u}(c_1^j, c_2^j)$, into which we substitute the optimal levels of c_1^j and c_2^j given in (3), is independent of the relative price ω. Thus,

$$\hat{u}\left(\frac{\varphi(\omega)}{p_1(\omega)}Y^j, \frac{1-\varphi(\omega)}{p_2(\omega)}Y^j\right) = \phi\left(Y^j\right)\hat{u}\left(\frac{\varphi(\omega)}{p_1(\omega)}, \frac{1-\varphi(\omega)}{p_2(\omega)}\right) \equiv u\left(Y^j\right), \tag{5}$$

where $\hat{u}(.,.)$ satisfies

$$\frac{\mathrm{d}\hat{u}\left(\frac{\varphi(\omega)}{p_1(\omega)}, \frac{1-\varphi(\omega)}{p_2(\omega)}\right)}{\mathrm{d}\omega} = \hat{u}_1\frac{\mathrm{d}\left(\frac{\varphi(\omega)}{p_1(\omega)}\right)}{\mathrm{d}\omega} + \hat{u}_2\frac{\mathrm{d}\left(\frac{1-\varphi(\omega)}{p_2(\omega)}\right)}{\mathrm{d}\omega} = 0.$$

Because $\hat{u}_2/\hat{u}_1 = \omega = p_2(\omega)/p_1(\omega)$ under rational household behavior, the above equation yields

$$p_1'(\omega) = -\frac{(1-\varphi(\omega))p_1}{\omega} < 0, \quad p_2'(\omega) = \frac{\varphi(\omega)p_2}{\omega} = \varphi(\omega)p_1 > 0. \tag{6}$$

Household j's disposable income, Y^j $(j = R, D)$, depends on location choice. Locations with a higher disposable income yield higher utility because the two commodities are freely traded and commodity prices are the same across both regions. Thus, interregional migration in pursuit of higher utility renders each household's disposable income equal across the two locations:

$$\begin{aligned} Y^R &= \left(w^R + s_w\right)\gamma^R + \frac{S + r^R K^R}{L_o^R} + s_r - \frac{T}{L} = w^D\gamma^D + \frac{S + r^R K^R}{L_o^R} - \frac{T}{L}, \\ Y^D &= \left(w^R + s_w\right)\gamma^R + \frac{r^D K^D}{L_o^D} + s_r - \frac{T}{L} = w^D\gamma^D + \frac{r^D K^D}{L_o^D} - \frac{T}{L}, \end{aligned} \tag{7}$$

where γ^j is the employment rate in region j.[5] From the first equation, it is obvious that the present analysis remains valid when S is a lump-sum transfer to household R rather than a land subsidy. The second and third expressions of each equation in (7) give the expected income when the household is located in regions R and D, respectively.

From (7), we obtain

$$\Lambda\left(w^D, \gamma^D, \gamma^R, w^R + s_w, s_r\right) = w^D\gamma^D - \left(w^R + s_w\right)\gamma^R - s_r = 0, \tag{8}$$

which we term the migration function. This implies that expected labor income is equal between the two locations because each household retains land ownership in its native region.[6]

2.2 Firms

Region D produces both commodities, and the production function of sector i $(i = 1, 2)$ displays constant returns to scale with respect to the two production factors, land K_i and labor L_i. The production function is represented by

$$f_i\left(k_i\right)L_i, \quad \text{where } f_i'(\cdot) > 0,\ f_i''(\cdot) < 0,\ k_i = K_i/L_i. \tag{9}$$

Because the government offers no subsidy to region D, profit maximization in the two sectors yields

$$\begin{aligned} r^D &= p_1(\omega)f_1' = p_2(\omega)f_2', \\ w^D &= p_1(\omega)\left[f_1 - f_1'k_1\right] = p_2(\omega)\left[f_2 - f_2'k_2\right], \end{aligned} \tag{10}$$

where r^j and w^j are, respectively, the land rental rate and wage level in region j. From (6) and (10), we derive

[5] Here, we assume random turnover in the labor market.

[6] See footnote 3.

$$
\begin{aligned}
& r^D = r^D(\omega), \quad r^{D'}(\omega) = \frac{p_2(\omega) f_2 - (1-\varphi) r^D \left(k_2 - k_1\right)}{\omega(k_2 - k_1)}, \\
& w^D = w^D(\omega), \quad w^{D'}(\omega) = -\frac{p_2(\omega) f_2 k_1 + (1-\varphi) w^D \left(k_2 - k_1\right)}{\omega(k_2 - k_1)}, \\
& k_1 = k_1(\omega), \quad k_1'(\omega) = \frac{f_2}{\left(k_2 - k_1\right) f_1''}, \\
& k_2 = k_2(\omega), \quad k_2'(\omega) = \frac{f_1}{\left(k_2 - k_1\right) \omega^2 f_2''}.
\end{aligned}
\tag{11}
$$

Region R has only sector 1, and the government offers production subsidy ϵ and employment subsidy z to firms in this region. Therefore, optimal firm behavior yields

$$
\begin{aligned}
& r^R = (1+\epsilon) p_1(\omega) f_1'\left(k_1^R\right), \\
& \frac{w^R - z}{1+\epsilon} = p_1(\omega)\left[f_1\left(k_1^R\right) - f_1'\left(k_1^R\right) k_1^R\right] \to k_1^R = k_1^R\left(\omega, \frac{w^R - z}{1+\epsilon}\right),
\end{aligned}
\tag{12}
$$

where k_1^R is the land–labor input ratio in region R.

2.3 Government

In region R, the government initially offers a land subsidy S and replaces it with a production subsidy ϵ and an employment subsidy z provided to firms, a wage subsidy s_w provided to employed households, and a resident subsidy s_r for all households including immigrants. The subsidies are financed by a lump-sum tax T, which is uniformly imposed on all households. Accordingly, the government's budget equation is expressed as

$$
\left\{\left[z + s_w + \epsilon p_1 f_1\left(k_1^R\right)\right]\gamma^R + s_r\right\} L_n^R = T - S. \tag{13}
$$

We assume that stimulation subsidies s_r, s_w, z, and ϵ are initially zero for simplicity, and that T is fixed because we examine the effects of replacing S with stimulation subsidies:

$$
s_r = 0, \quad s_w = 0, \quad z = 0, \quad \epsilon = 0, \quad T = \text{const.} \tag{14}
$$

2.4 Markets

Because region R includes only sector 1, the factor market satisfies

$$
L_n^R = \frac{K^R}{\gamma^R k_1^R}, \tag{15}
$$

where K^j is the land endowment of region j ($j = R, D$). Region D has two sectors; hence, the factor markets satisfy

$$k_1 L_1 + k_2 L_2 = K^D, \quad L_1 + L_2 = \gamma^D L_n^D. \tag{16}$$

From (1) and (16), we obtain

$$L_1 = \frac{k_2 \gamma^D \left(L - L_n^R\right) - K^D}{k_2 - k_1}, \quad L_2 = \frac{K^D - k_1 \gamma^D \left(L - L_n^R\right)}{k_2 - k_1}. \tag{17}$$

From (3), (9), and (15), we obtain the following market equilibrium conditions for commodities 1 and 2:

$$\begin{aligned} &p_1 f_1 L_1 + p_1 f_1\left(k_1^R\right) \frac{K^R}{k_1^R} = \varphi(\omega) Y, \quad p_2 f_2 L_2 = (1 - \varphi(\omega)) Y, \\ &Y \equiv Y^R L_o^R + Y^D L_o^D. \end{aligned} \tag{18}$$

These yield the market function Θ, representing the excess supply of commodity 1:

$$\begin{aligned} &\Theta\left(\omega, \gamma^D, \gamma^R, \frac{w^R - z}{1 + \epsilon}\right) = f_1\left(k_1\right) L_1 + f_1\left(k_1^R\right) \frac{K^R}{k_1^R} - \delta(\omega) f_2\left(k_2\right) L_2 = 0, \\ &\text{where} \quad \delta(\omega) \equiv \frac{\omega \varphi(\omega)}{1 - \varphi(\omega)}, \ \delta'(\omega) > 0, \end{aligned} \tag{19}$$

and $k_i(\omega)$ ($i = 1, 2$), L_n^R, L_i ($i = 1, 2$), and k_1^R are given in (11), (15), (17), and (12), respectively. The property that $\delta'(\omega) > 0$ is derived from (4). Naturally, we assume that an increase in the relative price of commodity 2, ω, decreases the excess supply of commodity 1:

$$\Theta_\omega < 0. \tag{20}$$

Because wage rigidity leads to unemployment in the present setting, either $w^D(\omega)$ or γ^D in region D and either w^R or γ^R in region R are flexible. Thus, the market function in (19) and the migration function in (8) fully determine the equilibrium.

3 Welfare analysis of subsidy replacements

Using the established model, we obtain the welfare effects of subsidy replacements. Given the total amount of lump-sum tax T, the government originally pays land subsidy S to household R and then replaces it with a resident, wage, employment, or production subsidy (s_r, s_w, z, or ϵ). This motivates workers to migrate to the region or firms to increase local production.

While the four subsidies raise the wage income, their effects on employment differ significantly. Resident subsidy s_r and wage subsidy s_w motivate household D to move into region R by raising region R's resident income without stimulating local production. This decreases the local wage w^R if region R achieves full employment, or employment γ^R if region R experiences unemployment. These properties are reflected in the migration function in (8). Employment subsidy z and production subsidy ϵ motivate local firms to hire more workers in region R, change region D's endowment ratio, and alter the relative commodity price ω if region D achieves full employment or region D's employment γ^D if region D experiences unemployment. This is shown in the market function in (19).

3.1 Effects on the two regions' income

By substituting r^D and w^D in (11), r^R and $k_1^R = k_1^R\left(\omega, \frac{w^R - z}{1+\epsilon}\right)$ in (12), S in (13), and $L_n^R\left(= \frac{K^R}{\gamma^R k_1^R}\right)$ in (15) to Y^R and Y^D in (7); totally differentiating the results; and taking account of (2), (6), (8), (14), (17) and the second equation of (18); we obtain

$$\begin{aligned} \mathrm{d}Y^R &= -\frac{(1-\varphi)\left(w^R + p_1 f_1'\left(k_1^R\right)k_1^R\right)\gamma^R}{\omega}\mathrm{d}\omega + w^R \mathrm{d}\gamma^R, \\ \mathrm{d}Y^D &= \frac{(1-\varphi)\left(w^R + p_1 f_1'\left(k_1^R\right)k_1^R\right)\gamma^R L_o^R}{\omega L_o^D}\mathrm{d}\omega + w^D \mathrm{d}\gamma^D. \end{aligned} \tag{21}$$

This depicts the welfare effects of the subsidy replacement given that the utility of household j (for $j = R, D$) is $u\left(Y^j\right)$ in (5).

Note that neither $\mathrm{d}Y^R$ nor $\mathrm{d}Y^D$ in (21) directly depends on the policy variables S, z, s_w, s_r, and ϵ. In other words, the policy variables only affect Y^R and Y^D indirectly, through changes in the terms of trade and each region's employment rate. We call these the terms-of-trade effect and the employment expansion effect, respectively. In addition, replacement policies create a redistribution effect, from landowners to firms or workers. When both $L_o^R = L_n^R$ in (2) and $s_r = 0$, $s_w = 0$, $z = 0$, and $\epsilon = 0$ in (14) are valid, the redistribution does not yield any interregional redistribution, as explained below.

When a land subsidy is replaced with resident and wage subsidies, subsidy payments are transferred from landowners to workers in region R. As shown in (14), the initial resident and wage subsidies are zero, and thus, an increase in immigrants does not create a transfer. As per (2), the number of immigrants is negligible, and therefore, almost all subsidies are given to native workers. Thus, subsidy transfers resulting from the replacement of the land subsidy with resident and wage subsidies are almost fully absorbed by native workers. Employment and production subsidies for local firms are allocated to local landowners and native and immigrant workers. However, assuming that there are no immigrants initially, all increases in employment and production subsidies at the cost of a reduced land subsidy go to local landowners and native workers. Thus, the four subsidy replacements create no interregional subsidy redistribution; that is, we find only the terms-of-trade and employment expansion effects. The mathematical expressions $\mathrm{d}Y^R$ and $\mathrm{d}Y^D$ in (21) clarify this property, which is summarized as follows:

Lemma 1 *When (2) and (14) hold, there is no interregional redistribution effect; there are only the terms-of-trade and employment expansion effects.*

If region D's wage $w^D(\omega)$ is fixed, ω is constant, as is evident from (10). Thus, the terms-of-trade effect does not occur. By contrast, if full employment prevails in a region, there is no employment expansion in the region. These properties can be restated as follows:

Lemma 2 *If region D faces unemployment, the terms-of-trade effect does not occur. If region j ($j = R, D$) is fully employed, the employment expansion effect is zero for household j.*

Changes in the terms of trade ω, region R's wage w^R, and employment rates γ^D and γ^R, which determine the effects on income and welfare, as shown in (21), are derived from the migration function $\Lambda\left(w^D(\omega), \gamma^D, \gamma^R, w^R + s_w, s_r\right)$ in (8) and the market function $\Theta\left(\omega, \gamma^D, \gamma^R, \frac{w^R - z}{1+\epsilon}\right)$ in (19). Thus, we totally differentiate the two functions to obtain

$$\begin{aligned} \mathrm{d}\Lambda = 0: &\quad w^{D'}\gamma^D \mathrm{d}\omega + w^D \mathrm{d}\gamma^D - \left(\gamma^R \mathrm{d}w^R + w^R \mathrm{d}\gamma^R\right) = \gamma^R \mathrm{d}s_w + \mathrm{d}s_r, \\ \mathrm{d}\Theta = 0: &\quad \frac{\Theta_\omega}{\Theta_{w^R}}\mathrm{d}\omega + \frac{\Theta_{\gamma^D}}{\Theta_{w^R}}\mathrm{d}\gamma^D + \left(\mathrm{d}w^R + \frac{\Theta_{\gamma^R}}{\Theta_{w^R}}\mathrm{d}\gamma^R\right) = w^R \mathrm{d}\epsilon + \mathrm{d}z, \end{aligned} \tag{22}$$

where the coefficients satisfy

$$\begin{aligned} & w^{D'}\gamma^D \lesseqgtr 0 \text{ and } \frac{\Theta_\omega}{\Theta_{w^R}} \lesseqgtr 0 \text{ if } k_1 \lesseqgtr k_2; \\ & \frac{\Theta_{\gamma^D}}{\Theta_{w^R}} > 0, \quad \frac{\Theta_{\gamma^R}}{\Theta_{w^R}} > 0. \end{aligned} \tag{23}$$

See Appendix 1 for proofs of the sign conditions.

3.2 Welfare effects

First, we consider the case in which $w^D(\omega)$ is constant, and thus, region D faces unemployment. When w^R is also fixed (i.e., there is unemployment in both regions), the employment expansion effect is the only welfare effect on both households, as per Lemmas 1 and 2. Figure 1 illustrates the migration curve ($\Lambda = 0$) and the market curve ($\Theta = 0$) on the plane of $\left(\gamma^D, \gamma^R\right)$ in this case. From the sign conditions given in (23), the migration curve is positively inclined, while the market curve is negatively inclined. Increases in resident subsidy s_r and wage subsidy s_w induce immigration without stimulating production in region R, which worsens γ^R, improves γ^D, and shifts the migration curve ($\Lambda = 0$) in a downward-rightward direction, while the market curve ($\Theta = 0$) is unaffected. Rises in production subsidy ϵ and employment subsidy z stimulate production in region R, which increases γ^R, induces migration that raises γ^D, and moves the market curve in an upward-rightward direction, while

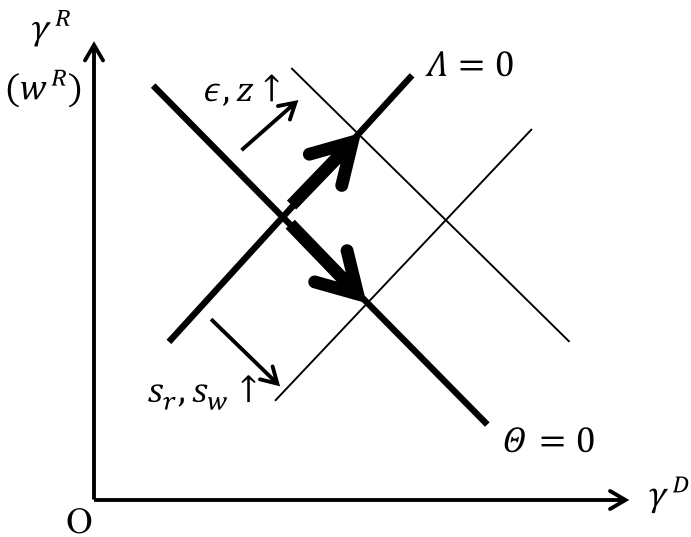

Fig. 1 Unemployment in region D (ω = constant)

the migration curve remains unchanged. These movements are mathematically derived from (22).

When full employment prevails in region R (i.e., $\gamma^R = 1$) but unemployment exists in region D (i.e., $w^D(\omega)$ is constant), there is no welfare effect on household R, while the employment expansion effect is the only welfare effect on household D, as per Lemmas 1 and 2. From the sign conditions of the coefficients for the two curves given in (23), in this case, both curves on the plane of (γ^D, w^R) are similar in shape and the policy parameters yield similar effects to those in the previous case (Fig. 1). Increases in s_r and s_w induce migration without stimulating production in region R, which decreases w^R, improves γ^D, and shifts the migration curve in a downward-rightward direction, while the market curve is unaffected. Increases in ϵ and z stimulate production in region R, which raises w^R, improves γ^D, and shifts the market curve in an upward-rightward direction, while the migration curve is unaffected.

The results in the case where region D faces unemployment are summarized as follows:

$$\begin{aligned} &s_r \uparrow,\ s_w \uparrow \Rightarrow \gamma^R \text{ (or } w^R) \downarrow; \quad \epsilon \uparrow,\ z \uparrow \Rightarrow \gamma^R \text{ (or } w^R) \uparrow; \\ &s_r \uparrow,\ s_w \uparrow,\ \epsilon \uparrow,\ z \uparrow \Rightarrow \gamma^R \uparrow. \end{aligned} \tag{24}$$

Next, we consider the case in which region D achieves full employment. From Lemmas 1 and 2, the terms-of-trade effect is the only welfare effect on household D in this case. The direction of the terms-of-trade effect depends on the factor intensities $\left(k_1 \lesseqgtr k_2\right)$, which is evident from the sign conditions of the coefficients of $d\omega$ in the migration and market functions in (23). The two curves slope in the opposite direction in the cases of $k_1 < k_2$ and $k_1 > k_2$, as illustrated in Fig. 2.

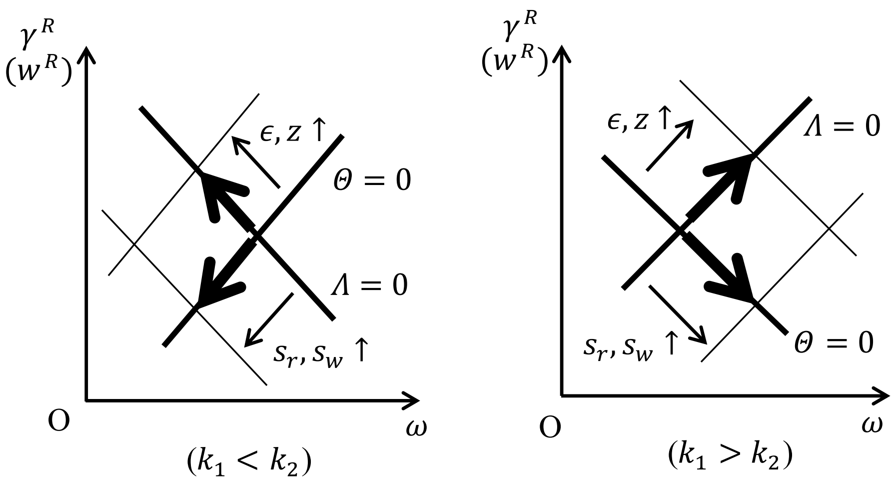

Fig. 2 Full employment in region D ($\gamma^D = 1$)

Similar to the case in which region D faces unemployment (Fig. 1), increases in s_r and s_w induce immigration without stimulating production in region R, which decreases γ^R (if unemployment exists in region R) or w^R (if full employment prevails in region R) and shifts the migration curve ($\Lambda = 0$) downward, while the market curve ($\Theta = 0$) is unaffected. Increases in ϵ and z stimulate production in region R, raise γ^R or w^R, and decrease labor in region D, which in turn, increases (decreases) the production of commodity 2 and lowers (raises) ω if $k_1 < k_2$ ($k_1 > k_2$). Thus, the market curve ($\Theta = 0$) shifts in the upward-leftward (upward-rightward) direction if $k_1 < k_2$ ($k_1 > k_2$), while the migration curve ($\Lambda = 0$) is unaffected. Thus, from Fig. 2, if region R faces unemployment, the employment expansion effect is positive under ϵ and z and negative under s_r and s_w. The terms-of-trade effect appears irrespective of whether region R faces unemployment, and its direction depends on the labor intensities of the two sectors. Thus, in the case where region D achieves full employment, we obtain

$$\begin{aligned} &s_r \uparrow,\ s_w \uparrow \Rightarrow \gamma^R \text{ (or } w^R) \downarrow; \quad \epsilon \uparrow,\ z \uparrow \Rightarrow \gamma^R \text{ (or } w^R) \uparrow; \\ &s_r \uparrow,\ s_w \uparrow,\ \epsilon \uparrow,\ z \uparrow \Rightarrow \omega \downarrow \text{ (resp. } \uparrow) \text{ if } k_1 < k_2 \text{ (resp. } k_1 > k_2). \end{aligned} \tag{25}$$

From (21), (24), (25), and Lemmas 1 and 2, we obtain the following proposition:

Proposition 1 *The following are in the context of the employment expansion effect:*

(i) *Irrespective of whether region R faces unemployment, the four subsidies improve region D's employment and household D's welfare if region D faces unemployment.*
(ii) *Irrespective of whether region D faces unemployment, if region R faces unemployment, resident subsidy s_r and wage subsidy s_w decrease region R's employment, while production subsidy ϵ and employment subsidy z increase it. A decrease (increase) in employment negatively (positively) impacts household*

R's welfare. This is the only welfare effect on household R if there is unemployment in region D. The effect disappears if full employment prevails in region R.

The following applies to the terms-of-trade effect:

(iii) *When region D achieves full employment, the four subsidies improve (worsen) region R's terms of trade if it specializes in the labor-intensive (labor-saving) sector. If region R also achieves full employment, this is the only welfare effect on both households.*

We discuss the welfare effect on households D and R from the perspective of Proposition 1. There is a straightforward welfare effect on household D. If region D faces unemployment, the terms of trade remain fixed. The four subsidies promote labor immigration to region R and improve employment in region D irrespective of the employment conditions in region R. Thus, household D is better off because of the four subsidies. If region D achieves full employment, the terms-of-trade effect is the only welfare effect on household D. Owing to labor outflow, the relative price of the labor-intensive commodity increases, and thus, the four subsidies render household D worse off if $k_1 < k_2$ and better off if $k_1 > k_2$.

As for the welfare effect on household R, if region R achieves full employment, the employment expansion effect does not occur on household R, as mentioned in Lemma 2. If region D faces unemployment, there is no terms-of-trade effect. Therefore, no welfare effect occurs on household R in this case. If region R faces unemployment, increases in s_r and s_w lower γ^R, but increases in ϵ and z raise γ^R, irrespective of whether region D faces unemployment. This is the only welfare effect on household R if region D also faces unemployment. Thus, household R is made worse off by increases in s_r and s_w but benefits from increases in ϵ and z.

If region D achieves full employment and region R faces unemployment, the terms-of-trade effect occurs in addition to the employment expansion effect on household R. Because the four subsidies decrease labor in region D and increase (decrease) the production of commodity 2 if it is labor-saving (labor-intensive), its relative price declines (rises). Thus, the terms-of-trade effect on household R is positive if $k_1 < k_2$ and negative if $k_1 > k_2$. This is the only welfare effect on household R if full employment prevails in region R. However, if region R faces unemployment, the employment expansion effect occurs. Therefore, when $k_1 < k_2$, increases in ϵ and z improve the terms of trade and expand employment, rendering household R better off. Increases in s_r and s_w worsen employment while improving the terms of trade, and thus, the welfare effect on household R is ambiguous. When $k_1 > k_2$, increases in s_r and s_w worsen the terms of trade and employment, making household R worse off. Increases in ϵ and z improve employment while worsening the terms of trade, and therefore, the welfare effect on household R is ambiguous.

3.3 Employment expansion versus terms-of-trade effects

The previous subsection reveals an ambiguous welfare effect on household R when region D achieves full employment and region R faces unemployment. We show

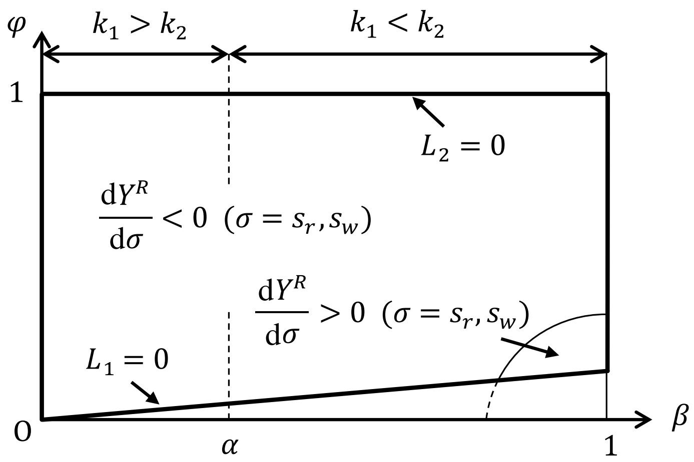

Fig. 3 Sign of $\mathrm{d}Y^R/\mathrm{d}\sigma$ $(\sigma = s_r, s_w)$

that the welfare effect can be positive and negative in this case using the following Cobb–Douglas utility and production functions:

$$u = c_1^{\varphi} c_2^{1-\varphi},\ f_1(k_1) = k_1^{\alpha},\ f_2(k_2) = k_2^{\beta} \text{ where } \alpha, \beta, \varphi \in (0, 1). \tag{26}$$

Figure 3 illustrates the areas in which $\mathrm{d}Y^R/\mathrm{d}\sigma$ $(\sigma = s_r,\ s_w)$ is positive and negative (see Appendix 2 for the derivation). From the figure, $\mathrm{d}Y^R/\mathrm{d}\sigma$ $(\sigma = s_r,\ s_w)$ is likely to be negative but is positive when φ is low and β is significantly higher than α. Intuitively, if φ is low, the ratio of consumption expenditure on commodity 2 is high, and thus, the benefit due to the decline in ω is large. Moreover, if β is close to 1 but α is not, the production of commodity 1 requires far more labor than the production of commodity 2 does. Thus, an increase in wage leads to a marginal rise in the production cost of commodity 2 but a significant increase in the cost of commodity 1, causing ω to decline significantly. Since the benefit and magnitude of the decline in ω are both large, the terms-of-trade effect is significantly positive and dominates the negative employment expansion effect.

Figure 4 illustrates the areas in which $\mathrm{d}Y^R/\mathrm{d}\sigma$ $(\sigma = \epsilon,\ z)$ is positive and negative (see Appendix 2 for the derivation). It shows that $\mathrm{d}Y^R/\mathrm{d}\sigma$ $(\sigma = \epsilon,\ z)$ is likely to be positive but is negative when φ is low and β is considerably lower than α. Intuitively, if φ is low, the ratio of consumption expenditure on commodity 2 is high, and thus, the detrimental impact of a rise in ω is large. Moreover, if β is significantly lower than α, the production of commodity 2 requires far more labor than that of commodity 1; thus, an increase in wage causes a significantly higher rise in the production cost of commodity 2 compared with that of commodity 1, which considerably raises ω. In other words, both the detrimental impact and the magnitude of a rise in ω are large, and the negative terms-of-trade effect is so substantial that it dominates the positive employment expansion effect, rendering household R worse off.

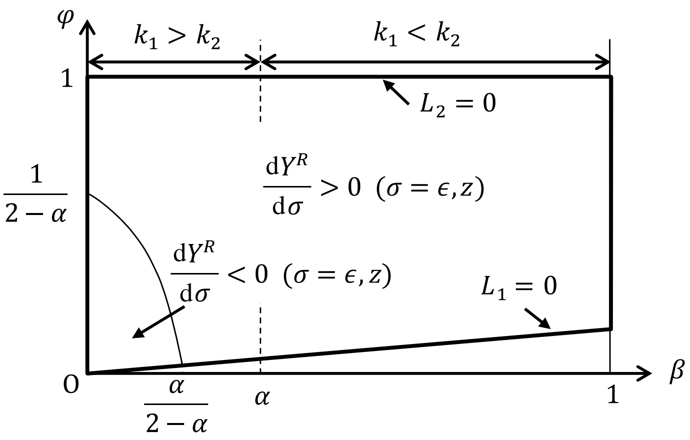

Fig. 4 Sign of $\mathrm{d}Y^R/\mathrm{d}\sigma\,(\sigma = \epsilon, z)$

4 Policy implications

We apply Proposition 1 to select Japanese prefectures and consider the policy implications. Since the policy implications depend on the employment conditions in the two regions and the labor intensities of the two sectors, as mentioned in the proposition, we focus on four prefectures where these properties vary: Okinawa, Yamaguchi, Fukui, and Fukushima (Table 1). Okinawa and Yamaguchi have allowed the United States to set up military bases in both prefectures, while Fukui and Fukushima have permitted the establishment of nuclear power plants in the regions. The government provides the prefectures with compensation subsidies in addition to nation-wide tax allocation grants.

We first consider the implications for Okinawa. One of the major industries in Okinawa is tourism, which is a labor-intensive sector. In fact, the labor intensity of Okinawa has always been considerably higher than that of Tokyo, a typical donor region (Table 1). Okinawa's unemployment rate has also been much higher than that of Tokyo. Thus, as per Proposition 1(ii) and (iii), replacing direct lump-sum transfers to Okinawa's natives with production and employment subsidies benefits them. Further, resident and wage subsidies may have a detrimental impact on Okinawa's natives, although their effect is ambiguous.

During 2009–2010, following the financial crisis of 2007–2008, Tokyo's unemployment rate increased to 5.5%, which was much higher than before, while that of Okinawa was 7.5% (Table 1). This exemplifies a case in which both the recipient and donor regions experience unemployment. Thus, in line with Proposition 1(ii), production and employment subsidies will improve the recipient region's employment and welfare, whereas resident and wage subsidies will worsen them. Thus, applying the insight from our results, in such a case, irrespective of whether Tokyo achieves full employment, Okinawa should opt for production and employment subsidies rather than resident and wage subsidies.

Table 1 Unemployment rate and labor intensity

Prefecture	Unemployment rate												$\frac{\text{Employed}}{\text{GDP}}$
	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	Average 2006–2016	Average 2006–2016
Okinawa	7.6	7.4	7.4	7.5	7.5	6.9	6.8	5.7	5.4	5.1	4.4	6.5	1.65
Yamaguchi	3.0	2.9	3.2	4.2	3.9	3.5	3.5	3.4	3.0	2.8	2.4	3.3	1.16
Fukui	2.5	2.7	2.7	3.5	3.3	3.0	2.6	2.6	2.4	1.8	1.9	2.6	1.26
Fukushima	4.5	4.3	4.4	5.5	5.2	4.8	4.0	3.6	3.1	3.1	2.7	4.1	1.30
Tokyo	4.2	3.9	3.9	4.7	5.5	4.8	4.5	4.2	3.8	3.6	3.2	4.2	0.70

Data sources: Employed workers and unemployment rate in each region: http://www.stat.go.jp/data/roudou/index.htm

Regional GDP: https://www.esri.cao.go.jp/jp/sna/data/data_list/kenmin/files/contents/tables/h28/syuyo1.xlsx

As shown in Table 1, Fukui has consistently reported high employment. Tokyo particularly suffered high unemployment from 2009 to 2011. During the period, the unemployment rates in Fukui and Tokyo were 3.5% and 4.7% in 2009, 3.3% and 5.5% in 2010, and 3.0% and 4.8% in 2011. This reflects a case in which the donor region suffers unemployment, while the recipient region achieves full employment. Therefore, as stated in Proposition 1(ii), the four subsidies yield no welfare effect on the recipient region's natives because neither the terms-of-trade effect nor the employment expansion effect occurs.

If both regions achieve full employment, the four subsidy policies benefit the recipient region through the term-of-trade effect if its industry is labor-intensive, as is clear from Proposition 1(iii). This case is indicative of Yamaguchi, Fukui, and Fukushima in 2014 and later, when Tokyo reported a recovering employment rate.

Finally, we treat the case in which the recipient region is small, considering that many small cities, towns, and villages in Japan also receive lump-sum transfers from the government. As is evident from (27) and (28) in Appendix 1, in this case, the coefficients of $d\omega$ and $d\gamma^D$ in the second equation of (22) are considerably high if region R is much smaller than region D (i.e., $L_n^D \gg L_n^R$), rendering the $\Theta = 0$ curve vertical at a fixed level of γ^D in Fig. 1 or ω in Fig. 2. On the other hand, the coefficients take finite values in the first equation of (22), keeping the $\Lambda = 0$ curve inclined. Thus, irrespective of whether region D achieves full employment, changes in ϵ and z do not affect the $\Theta = 0$ curve and neither ω nor γ^D changes. By contrast, changes in s_r and s_w move the $\Lambda = 0$ curve as illustrated in Figs. 1 and 2. These findings are summarized as follows:

$$s_r \uparrow,\ s_w \uparrow \Rightarrow \gamma^R \text{ (or } w^R) \ \downarrow; \quad \epsilon \uparrow,\ z \uparrow \Rightarrow \text{no effect.}$$

If region R suffers unemployment, the resident subsidy s_r and wage subsidy s_w directly induce workers to migrate to region R and decrease employment rate γ^R, irrespective of region D's employment conditions. Furthermore, neither the terms-of-trade effect (because region R is small) nor the redistribution effect (from Lemma 1) occurs. Thus, region R is worse off. If region R achieves full employment, the employment expansion effect does not occur, either. Thus, household R's welfare does not change. Production subsidy ϵ and employment subsidy z result in the movement of labor from region D to region R, causing a change in ω or γ^D and thereby affecting w^R or γ^R. However, when region R is small, the effects on ω or γ^D are negligible, and thus, neither w^R nor γ^R changes, creating no welfare effect on household R.

These findings are summarized in the following proposition:

Proposition 2 *If the recipient region is much smaller than the donor region, production subsidy ϵ and employment subsidy z yield no welfare effect on the region's natives. Resident subsidy s_r and wage subsidy s_w have a detrimental impact on the region's native if the region suffers unemployment, but generate no welfare effect if the region achieves full employment.*

Therefore, small local regions aiming to promote immigration should avoid resident and wage subsidies, even though they directly attract workers. Production and employment subsidies yield the same economic effect as lump-sum transfers to the regions' native households.

5 Concluding remarks

When offering grants and subsidies to a local region, a government can choose between a direct lump-sum transfer to the region's natives and various subsidies that affect local economic activities and immigration. Using a two-region, two-commodity, two-factor model in which the subsidy-recipient region produces only one of the two commodities, we analyze the welfare effects of replacing a lump-sum transfer to the recipient region's natives with production, employment, resident, and wage subsidies on the recipient and donor regions' natives under different employment conditions.

If full employment prevails in both regions, the replacement policies incite workers to migrate to the recipient region because labor income in that region increases. The donor region's wage also increases because of emigration and the relative price of the labor-intensive commodity rises. Thus, if the recipient region specializes in the labor-intensive (labor-saving) commodity, the terms of trade in the region improve (deteriorate), and the native households are better (worse) off. If unemployment exists in the donor region while the recipient region achieves full employment, the relative price does not vary and the replacement policies have no welfare effect on the native households of the recipient region.

If unemployment exists in the recipient region, replacing a lump-sum transfer to the region's native households with production and employment subsidies expands employment and benefits them. However, resident and wage subsidies do not stimulate local production but cause workers to move into the region, thus worsening local employment conditions and detrimentally impacting native households. If the donor region also suffers unemployment, there is no change in the relative commodity price. Therefore, only the employment effect occurs. In other words, the native households of the recipient region are made better off by production and employment subsidies but worse off by resident and wage subsidies.

If the donor region achieves full employment and the recipient region suffers unemployment, the four subsidies increase the wage level in the donor region, which causes the relative price of the labor-intensive commodity to rise. Thus, if the recipient region's local product is labor-intensive, the region's terms of trade improve. Moreover, production and employment subsidies increase local employment, and thus, the native households of the recipient region are undoubtedly better off. Resident and wage subsidies, however, worsen the region's local employment. They reduce, or even negate, the benefit due to improvement in the terms of trade. This may be the case of Okinawa, where the unemployment rate is among the highest in Japan and where the major industry is labor-intensive (i.e., tourism). The Okinawa government provides land and employment subsidies.[7] These findings suggest that Okinawa would benefit from the replacement of land subsidies with industry-promoting subsidies such as production and employment subsidies.

The Japanese government also provides grants and subsidies to numerous small cities, towns, and villages, many of which have expressed concerns about the declining population. We find that replacing lump-sum transfers to the native households with resident and wage subsidies in small regions directly induces immigration but worsens unemployment and thus, causes native households to be worse off. Replacing lump-sum transfers with production and employment subsidies, which also induces immigration, yields no welfare effect on the native households.

Our study focuses on the welfare effects of replacing a direct transfer to the region's native households with subsidies that stimulate local economic activities and immigration. Nevertheless, it would be worth analyzing the welfare effects of replacing certain stimulation policies with others. A relevant example is the replacement of a resident subsidy with production and employment subsidies. Determining an optimal combination of these stimulation subsidies may also worth exploring. We suggest this as a topic for future work.

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Appendix 1: Proof of (23)

Because Θ in (19) represents the excess supply of commodity 1 and satisfies $\Theta_\omega < 0$ as mentioned in (20), we obtain

$$\Theta_\omega \equiv \frac{\partial \Theta}{\partial \omega} = \frac{(\omega+\delta)f_2^2 L_1}{\left(k_2-k_1\right)^2 f_1''} + \frac{(\omega+\delta)f_1^2 L_2}{\left(k_2-k_1\right)^2 \omega^3 f_2''} - \delta' f_2 L_2 - \frac{(\omega+\delta)f_2 k_1 \gamma^D L_n^R w^R (1-\varphi)}{p_2\left(k_1^R\right)^2 f_1''\left(k_1^R\right)\left(k_2-k_1\right)} < 0. \tag{27}$$

Totally differentiating Θ in (19),

$$\Theta_\omega \mathrm{d}\omega + \Theta_{\gamma^D}\mathrm{d}\gamma^D + \Theta_{\gamma^R}\mathrm{d}\gamma^R = \Theta_{w^R}\left(w^R \mathrm{d}\epsilon - \mathrm{d}w^R + \mathrm{d}z\right),$$

where

[7] See the website of the Okinawa Prefecture (https://www.pref.okinawa.jp) for details on subsidies in Okinawa.

$$
\begin{aligned}
&k_1 \lesseqgtr k_2 \Leftrightarrow \\
&\Theta_{\gamma^D} \equiv \frac{\partial \Theta}{\partial \gamma^D} = \frac{\left(f_1 k_2 + \delta f_2 k_1\right) L_n^D}{k_2 - k_1} \gtreqless 0 \\
&\Theta_{\gamma^R} \equiv \frac{\partial \Theta}{\partial \gamma^R} = \frac{\gamma^D \left(f_1 k_2 + \delta f_2 k_1\right) L_n^R}{\gamma^R \left(k_2 - k_1\right)} \gtreqless 0 \\
&\Theta_{w^R} \equiv \frac{\partial \Theta}{\partial \left(\frac{w^R - z}{1+\epsilon}\right)} = -\frac{(\omega + \delta) f_2 k_1 \gamma^D L_n^R}{p_1 \left(k_1^R\right)^2 f_1'' \left(k_1^R\right) \left(k_2 - k_1\right)} \gtreqless 0.
\end{aligned}
\tag{28}
$$

From the two equations in (10), we obtain $p_2 f_2 = r^D k_2 + w^D$, and thus, $w^{D'}$ given in (11) satisfies

$$
w^{D'} = -\frac{(p_2 f_2 - (1-\varphi) w^D) k_1 + (1-\varphi) w^D k_2}{\omega (k_2 - k_1)} \lesseqgtr 0 \quad \text{if } k_1 \lesseqgtr k_2. \tag{29}
$$

The sign conditions in (23) are derived from (27), (28), and (29).

Appendix 2: Boundary curves in Figs. 1 and 2

In the case of full employment in region D (i.e., $\mathrm{d}\gamma^D = 0$) and unemployment in region R (i.e., $\mathrm{d}w^R = 0$), $\mathrm{d}Y^R$ in (21), to which we apply the solution of $\mathrm{d}\omega$ and $\mathrm{d}\gamma^R$ obtained from (22), satisfies

$$
\begin{aligned}
&\mathrm{d}Y^R = \frac{\Phi_1}{\Theta_\omega - \Omega} \left(\mathrm{d}s_r + \gamma^R \mathrm{d}s_w\right) + \left[\frac{f_2 k_1 L_n^R}{\left(k_1^R\right)^2 f_1'' \left(k_1^R\right) \left(k_2 - k_1\right)^2 \left(\Theta_\omega - \Omega\right)}\right] \Phi_2 \left(\bar{w}^R \mathrm{d}\epsilon + \mathrm{d}z\right), \\
&\Phi_1 = -\left[\frac{(1-\varphi)\left(w^R + p_1 f_1'\left(k_1^R\right) k_1^R\right)}{\omega} \frac{\left(f_1 k_2 + \delta f_2 k_1\right) L_n^R}{\bar{w}^R \left(k_2 - k_1\right)} + \Theta_\omega\right], \\
&\Phi_2 = \left(k_2 - k_1\right) f_1 \left(k_1^R\right) \gamma^R + \left(\delta f_2 k_1 + \omega \left(f_2 - f_2' \left(k_2 - k_1\right)\right) k_2\right), \\
&\Theta_\omega - \Omega = \Theta_\omega - \frac{\left(f_1 k_2 + \delta f_2 k_1\right) \left(\left(\varphi f_2 + (1-\varphi) f_2' k_2\right) p_2 k_1 + (1-\varphi) w^D k_2\right) L_n^R}{\omega w^D \left(k_2 - k_1\right)^2} < 0,
\end{aligned}
\tag{30}
$$

where $\Theta_\omega < 0$ in (27). Thus, $\mathrm{sgn}\left(\mathrm{d}Y^R / \mathrm{d}\sigma\right) = -\mathrm{sgn}(\Phi_1)$ for $\sigma = s_r, s_w$ and $\mathrm{sgn}\left(\mathrm{d}Y^R / \mathrm{d}\sigma\right) = \mathrm{sgn}\left(\Phi_2\right)$ for $\sigma = \epsilon, z$. To explicitly obtain the signs of Φ_1 and Φ_2, we apply the specific utility and production functions in (26) and assume that the employment rate in region R is almost unity: $\gamma^R \approx 1$.

Applying the utility and production functions in (26) to the two equations in (10) yields

$$
\frac{f_1}{k_1} \Big/ \frac{f_2 \omega}{k_2} = \frac{\beta}{\alpha}.
$$

Because $L_2 = (K^D - k_1 L_1)/k_2$, in line with the first equation in (16), substituting L_2 and the above-mentioned equation into (19) when $\gamma^R \approx 1$ yields

$$L_1 = \frac{\frac{\varphi}{1-\varphi}K^D - \frac{\beta}{\alpha}K^R}{\left(\frac{\beta}{\alpha} + \frac{\varphi}{1-\varphi}\right)k_1}, \quad L_2 = \frac{(1-\beta)(K^R + K^D)}{\left(\frac{\beta}{\alpha} + \frac{\varphi}{1-\varphi}\right)(1-\alpha)k_1}. \tag{31}$$

Since both commodities are produced in region D, L_1 must be positive, and thus,

$$\varphi > \frac{\beta K^R}{\alpha K^D + \beta K^R} \equiv \varphi^c, \tag{32}$$

where φ^c generates the line of $L_1 = 0$ in Figs. 3 and 4.

Substituting Θ_ω of (27) into $\boldsymbol{\Phi}_1$ of (30) yields

$$\begin{aligned}\boldsymbol{\Phi}_1 = -\frac{\left(f_1 k_2 + \delta f_2 k_1\right)}{\omega\left(k_2 - k_1\right)^2 \bar{w}^R}\Bigg[&\left(p_1 f_1^R\left(k_1^R\right) - \frac{(\omega+\delta) f_2 k_1 \left(\bar{w}^R\right)^2}{p_1\left(k_1^R\right)^2 f_1''\left(k_1^R\right)\left(f_1 k_2 + \delta f_2 k_1\right)}\right)(1-\varphi)\left(k_2 - k_1\right)L_n^R \\ &+ \frac{\omega\left(k_2 - k_1\right)^2 \bar{w}^R}{\left(f_1 k_2 + \delta f_2 k_1\right)}\left(\frac{(\omega+\delta) f_2^2}{\left(k_2 - k_1\right)^2 f_1''}L_1 + \left(\frac{(\omega+\delta) f_1^2}{\left(k_2 - k_1\right)^2 \omega^3 f_2''} - \delta' f_2\right)L_2\right)\Bigg].\end{aligned}$$

Applying the equations in (26) and L_1 and L_2 in (31) to $\boldsymbol{\Phi}_1$ when $\gamma^R \approx 1$ yields

$$\boldsymbol{\Phi}_1 = -\frac{k_1^{\alpha-1}}{\omega(1-\varphi)(\beta-\alpha)^2}F(\varphi, \beta), \tag{33}$$

where

$$\begin{aligned}F(\varphi, \beta) = &(1-\varphi)(\beta-\alpha)(\beta(1-\varphi) + 1 + \alpha\varphi)K^R \\ &- \alpha((1-\varphi)(1-\beta) + \varphi(1-\alpha))K^D.\end{aligned} \tag{34}$$

Using (30), (33), and (34), we find

$$\operatorname{sgn}(F(\varphi, \beta)) = \operatorname{sgn}\left(\frac{\mathrm{d}Y^R}{\mathrm{d}\sigma}\right) \quad \text{for } \sigma = s_r, s_w.$$

Because $\mathrm{d}Y^R/\mathrm{d}\sigma < 0$ (for $\sigma = s_r, s_w$) if $\beta < \alpha$ (i.e., $k_1 > k_2$), as shown in Proposition 1, we focus on the case where $\beta > \alpha$. In this case, the value of φ that satisfies $F(\varphi, \beta) = 0$ in (34) and is located in (0, 1) is

$$\varphi_1 = 1 + \frac{(1+\alpha)K^R + \alpha K^D - \sqrt{\left((1+\alpha)K^R + \alpha K^D\right)^2 + 4K^R K^D \alpha(1-\alpha)}}{2K^R(\beta-\alpha)}, \tag{35}$$

which increases as β is greater. This represents the boundary curve for $\mathrm{d}Y^R/\mathrm{d}\sigma = 0$ (for $\sigma = s_r, s_w$) in Fig. 3. From (32) and (35), we derive

$$(1 >)\varphi_1 > \varphi^c \quad \text{if } \beta \approx 1.$$

Thus, the boundary curve is located as illustrated in Fig. 3.

As for the boundary curve in Fig. 4, applying (26) to Φ_2 in (30) gives

$$\Phi_2 = \frac{k_1^{\alpha+1}[-((\alpha-\beta)(2-\alpha)-\alpha(1-\alpha))+\varphi(\alpha-\beta)(2-\alpha))]}{\alpha(1-\beta)(1-\varphi)}.$$

From (30), the signs of $\mathrm{d}Y^R/\mathrm{d}z$ and $\mathrm{d}Y^R/\mathrm{d}\epsilon$ are the same as the sign of Φ_2. The value of φ that results in $\Phi_2 = 0$ is

$$\varphi = 1 - \frac{\alpha(1-\alpha)}{(\alpha-\beta)(2-\alpha)},$$

which is the boundary curve of $\mathrm{d}Y^R/\mathrm{d}\sigma = 0$ (for $\sigma = z, \epsilon$) in Fig. 4.

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