The Annals of Regional Science (2019) 62:211–246
https://doi.org/10.1007/s00168-018-0891-4

ORIGINAL PAPER

Cultural workers and the character of cities

Juan Carlos G. Lopez[1]

Received: 12 August 2017 / Accepted: 12 December 2018 / Published online: 2 January 2019

Abstract
This paper examines the location choice of cultural producers when cities differ in housing supply and income demographics. We develop a two-region spatial model containing three types of industries: a constant returns to scale traditional sector, a modern sector with external scale economies and a monopolistically competitive cultural sector. The model is initially analyzed when workers supply labor inelastically to their respective industry. Both integration and segregation are never a stable equilibrium, and the conditions for the stability of both concentrated and partially interior equilibrium are solved for. The model is extended to allow for cultural producers to divide their time between cultural production and moonlighting in the traditional sector, in order to smooth their income. Under this extension, there is an equilibrium where a share of cultural producers live isolated from a larger integrated market. We also identify an equilibrium where one region is able to sustain full-time cultural producers, while in the opposite region cultural producers must moonlight in the traditional sector. Under partially interior equilibria, the number of varieties of the cultural good is always larger in the region with the greater supply of housing.

JEL Classification R23 · R31 · R12 · Z1

1 Introduction

Cities often pride themselves on the scale and variety of their local cultural sector. Broadway in New York City, the West End in London or the Sunset Strip in Los Angeles are cultural emblems and indicators of urban vitality. As cities grow, it is important to understand the role that the cultural sector and its producers play in a city's expansion. The share of the population living in cities has been steadily increasing over the last century (United Nations 2015). However, growth has not been uniform across cities, with some metropolitan populations rising

✉ Juan Carlos G. Lopez
juancarlos.lopez@du.edu

1 Department of Economics, University of Denver, Sturm Hall, Room 246. 2000 East Asbury Ave., Denver, CO 80208, USA

while others have dwindled. Such regional shifts in the population have altered the economic fortunes of those cities involved. Growing cities can increasingly provide a greater variety of urban amenities which, in turn, attracts new residents. While declining cities are able to maintain fewer local amenities, reducing the local quality of life. However, when there are large income differentials and a limited housing supply within a growing metropolitan area, high-income households bid up rents beyond the reach of less affluent households. Therefore, thriving cities often struggle with providing affordable housing for lower-income workers. Many such cities are finding much of the urban core primarily occupied by high-income workers, while peripheral cities are increasingly being settled by migrating workers looking for cheaper housing (Florida 2017). This is particularly important with regard to cultural workers, many of whom are well educated but often receive lower wages than their equally well-educated counterparts employed in traditional professional industries (Alper et al. 1996). In fact, a recent survey of US musicians found that 64% of the sample had at least a bachelor's degree, yet the median income for the sample in 2018 was $35,000 (Krueger and Zhen 2018). In contrast, the average starting salary in 2016 for a US college graduate majoring in finance was $48,285 (CERI 2017). The contribution of this paper is in developing a framework to analyze cultural producer's location and labor supply decisions when cities differ in income demographics and available housing.

Cultural producers are often caught in the divide between high- and low-income residents in cities. One the one hand, locating in an affluent city where the demand for urban amenities is high provides a large income source for cultural producers. As casual empiricism suggests, high-income cities offer a greater variety of cultural goods and can accommodate more rarefied tastes, such as record stores devoted solely to jazz or classical music, specialty restaurants, experimental theater (Glaeser 2011). On the other hand, high-income cities tend to have more expensive housing. In contrast, peripheral cities may provide a lower level of aggregate demand for cultural goods, but offer more affordable housing options. A concern for cities where rents are rising rapidly is what will happen to the cultural makeup of the city as artists are unable to afford to live and work in those locations? The fear is that artists will leave to look for more affordable locations. Indeed, a number of smaller cities have proven to be a haven for the cultural class: Asheville, NC; Tucson, AZ; and Athens, GA, for instance. Interestingly, Detroit's recent financial distress has been a boon to both local and relocating artists who have taken advantage of the low cost of housing from the spate of foreclosures over the years (see Ewing and Grady 2012). To some extent, this is not surprising. Cities with a declining population retain a stock of both public and private infrastructures left behind by departing residents, which can be relatively quickly and inexpensively reconfigured for a new use. For example, popular cultural establishments in high-rent cities are often forced to close due to the high rental costs but resurface in lower-rent neighborhoods or cities nearby.

In this paper, we avoid normative statements regarding the quality of cultural work and make no direct comparison between, say, the value of fine art as compared to light entertainment (for a theoretical analysis of high and low art, see Cowen and

Tabarrok 2000). This allows for a particularly broad view of the ingredients of a specific city's cultural makeup and can include local restaurants and museums, music and theater venues, book and record stores. The primary focus is that households have some interaction in the city through the consumption of cultural goods beyond simply housing and employment needs.

Our model modifies the footloose entrepreneur model developed by Forslid and Ottaviano (2003) to explore the effect of agglomeration economies on the cultural makeup of cities. Additionally, this research extends the literature on land use in multiregion spatial models, pioneered by Helpman (1998) and extended by Tabuchi (1998), Murata and Thisse (2005), Pflüger and Süedekum (2007) and Pflüger and Tabuchi (2010). In our model, a variety of local cultural goods are produced requiring a fixed cost of the "talent" of a cultural worker. In our formulation, the New Economic Geography (NEG) framework is altered to incorporate a non-tradable cultural good with the intuition being that many urban amenities are perishable, an "experience," that cannot be equivalently replicated in another location (Glaeser et al. 2001). Therefore, two regions may have similar but not identical varieties of a good (e.g., a Broadway play in comparison with its nationally touring companion with a different cast). What we argue is that a major draw of a city is not the relative cost for all varieties in each city but rather the varieties available in one city which are simply not available in another.

Each city is endowed with a fixed supply of housing. There are three types of mobile workers: cultural producers, workers in a traditional industry and workers in a modern industry, which shows external economies of scale. Initially, we study worker's location decision when each type supplies labor inelastically to their respective industry. Under this specification, the concentration of all workers in a single region is a stable equilibrium when the "love of variety," represented by the elasticity of substitution between cultural goods, is strong, while full integration is never stable. Additionally, we derive the conditions for partially segregated cities, where modern and traditional workers live in opposite cities and cultural producers are divided between both regions, and partially integrated cities, where one of the two regions contains a share of all types of workers. In traditional NEG models, regions are endowed with a fixed number of immobile workers. In contrast, in our model when all workers are mobile there tends to be a degree of regional segregation between modern and traditional workers. Therefore, regional differences in average-income levels and available housing create two distinct labor markets for artists, and a different number of available varieties of cultural goods in each city.

Conceptually, the contrast we make between cultural producers and modern workers is akin to the divisions employed by Florida (2011) in defining the "creative class." Florida makes the distinction between the "super-creative core" which "...includes scientists and engineers, university professors, poets and novelists, artists, entertainers, actors, designers and architects, as well as thought leadership of modern society," and "creative professionals" consisting of a highly educated professional class including doctors, lawyers and businessmen. The "creative class" is set in contrast to a "working class" made up of industrial workers and a low human capital "service class." These latter two groups are consistent with our definition of a traditional sector. Florida (2011) provides data showing that the artistic class on average have incomes less than 50% above those of the "working class," whereas legal and managerial workers on average have incomes nearly triple those of the working class. This suggests that from an economic standpoint, cultural workers bear more resemblance to the working class than the professional class.

There has been considerable research on artists labor markets that debunks the vision of the starving artist (Throsby 1992; Alper and Wassal 2006). This is due to the fact that artists tend to be relatively well educated and have access to more lucrative outside options, allowing artists to smooth their income. Therefore, "moonlighting" in other industries provides cultural workers a more reliable income stream (See Alper and Wassall 2000). To account for this, our model is extended to allow for cultural producers to split their time between cultural production and production of the traditional good. As Throsby (1994) writes "... theories of labor supply in the arts will need to account for multiple jobholding by artists, and in particular for the differences in their motivations in supplying work to the arts and nonarts labor markets...artists as a group differ by virtue of the fact that their professional creative work alone is, in the majority of cases, unlikely to generate a living wage over a reasonable period of time, either because the hourly earnings are too low and/or because remunerative work opportunities are not available." Indeed, in one survey 61% of musicians indicate that they could not afford to live solely on music-related income (Krueger and Zhen 2018). Furthermore, the survey indicates that music-related income came from a number of different tasks including live performances, music lessons and merchandise sales, revealing the multiple jobholding of artists. In our modified model, some additional outcomes emerge. There is a stable equilibrium in which all traditional and modern workers concentrate in one region, while a portion of the cultural producers live isolated in the opposite region, creating what is described as an "artists' colony." Access to the traditional sector's labor market creates a safety net that is able to sustain a small community of cultural producers. However, we find that under this configuration artists in the larger market are able to devote more time to cultural production than those in the smaller market. Additionally, we construct an equilibrium where one region contains a number of artists that are able to devote all their time to artistic production, while in the opposite region cultural producers must spend a portion of their time moonlighting in the traditional sector. Finally, we show that under both a partially integrated and partially segregated equilibrium there are a greater number of varieties of cultural goods available in the region with the greater supply of housing.

The remainder of the paper is presented as follows: Section 2 develops the model. Section 3 analyzes a number of equilibrium configurations. Section 4 extends the model to allow for moonlighting by cultural producers. Section 5 provides a discussion of the results and a numerical analysis. Finally, Sect. 6 concludes.

2 The model

Consider two regions, indexed by $i = 1, 2$. Households supply labor inelastically to firms in one of three industries, modern, traditional, and cultural, indexed by $j = m, t, c$. Note that the terms *cities* and *regions*, as well as *cultural producers* and *artists*, are used interchangeably in the paper. Workers in industry j located in region i receive the wage w_{ji}. Each industry has an exogenous total supply of labor, λ_j, that is divided endogenously between both cities such that $\lambda_j = \lambda_{j1} + \lambda_{j2}$, where λ_{ji} is the endogenous share of a type j worker in region i. To reduce the number of parameters in the model, it is assumed that each industry has a unit measure of available workers so that the total population is equal to 3.[1] There is a fixed supply of homogeneous housing in each city, H_i, which carries the rental price r_i. All rents accrue to absentee landlords. The assumption that physical structures are solely used for residential purposes is common in the literature and similar to the assumption in the monocentric city model that while all production takes place at the CBD all land is devoted solely to housing (see Pflüger and Tabuchi 2010, which incorporates land use in production into the NEG framework). Workers have preferences over housing, H_{ji}, the numeraire traditional good, T_{ji} and the modern good, M_{ji}, which sells for the price p_m. Both the modern and traditional goods are assumed to be freely traded across regions. In addition, households have preferences over an aggregate of locally produced, horizontally differentiated cultural goods, C_{ji}, which carries the local price $p_{ci}(s)$ for each variety, s, of the good, $c_{ji}(s)$. Workers are mobile across regions; however, it is assumed that commuting between regions is prohibitively costly ensuring that all workers live and work in the same region. In addition, we ignore "temporary" access to varieties such as weekend trips into the city from a suburb or tourism. The utility function for a worker in industry j residing in region i is given by,

$$U_{ji} = T_{ji}^{\alpha} M_{ji}^{\beta} H_{ji}^{\gamma} C_{ji}^{\delta}, \quad \alpha + \beta + \gamma + \delta = 1, \tag{1}$$

$$C_{ji} = \left(\int_{s \in k_i} c_{ji}(s)^{(\sigma-1)/\sigma} \mathrm{d}s \right)^{\sigma/(\sigma-1)}, \tag{2}$$

where, α, β, γ and δ are parameters denoting the share of income devoted to consumption of the traditional good, modern good, housing and cultural goods, respectively. The total mass of varieties is K, with the number of varieties in each region denoted k_i. The parameter $\sigma > 1$ represents the elasticity of substitution between each variety of the cultural good, with a lower value of σ representing a stronger "love of variety."

[1] This assumption does not impact the qualitative results of the model. However in practice these shares likely vary. For example, using Florida (2011) the creative class comprised around one-third of the working population in 2000. In the model presented here, the share represented by the creative class would be divided between cultural producers and modern workers with the remaining share employed in the traditional sector.

Utility maximization yields the following demand functions

$$T_{ji} = \alpha w_{ji}, \quad M_{ji} = \beta \frac{w_{ji}}{p_m}, \quad H_{ji} = \gamma \frac{w_{ji}}{r_i}, \quad C_{ji} = \delta \frac{w_{ji}}{P_i}, \tag{3}$$

where

$$P_{ci} = \left(\int_0^{k_i} p_{ci}(s)^{1-\sigma} \, \mathrm{d}s \right)^{1/(1-\sigma)} \tag{4}$$

is the CES price index for cultural goods in region i. The demand for each variety is then

$$c_{ji}(s) = \delta \frac{w_{ji}}{p_{ci}(s)^{\sigma} P_{ci}^{1-\sigma}}. \tag{5}$$

The indirect utility function for a type j worker in region i is given by

$$V_i = \eta \frac{w_{ji}}{P_i}, \tag{6}$$

$$P_i \equiv p_m^{\beta} r_i^{\gamma} P_{ci}^{\delta}, \quad \eta \equiv \alpha^{\alpha} \beta^{\beta} \gamma^{\gamma} \delta^{\delta}, \tag{7}$$

where P_i represents the regional price index and η is a cluster of parameters.

2.1 Traditional and modern sectors

The traditional and modern production sectors are perfectly competitive with linear production functions. Unskilled workers in the traditional sector produce one unit of output per unit of labor. Given that the traditional good is the numeraire and is freely traded, we then have $p_t = w_{t1} = w_{t2} = 1$. The modern sector is comprised of a continuum of small firms that produce output using skilled labor with the perceived marginal product of labor A_{mi}, which firms take as given. However, there are Marshallian externalities generated from the share of modern workers in a city with $A_{mi} = A_m(1 + \lambda_{mi})^{\epsilon}$, where A_m denotes the marginal product of an isolated worker and ϵ is a measure of the degree of scale economies in the industry. The wage of modern workers is then the value of their average (rather than marginal) product, $w_{mi} = p_m A_{mi}$.

2.2 Cultural sector

Similar to the framework in Forslid and Ottaviano (2003) production of the cultural good requires both fixed and variable components. The fixed component represents the time and effort required by the artists to develop an idea. The idea is then combined with physical inputs to produce a physical output that can be consumed by local residents. Each cultural producer faces a marginal cost of B units of the numeraire good and a fixed cost of one unit of their labor to produce a distinct variety of the cultural good. Denoting $c_i(s)$ as the total demand for a variety s in region i and w_{ci} as the wage of a cultural producer in region i, total cost for producing a variety s in region i is then

$$TC_i(s) = Bc_i(s) + w_{ci}. \tag{8}$$

Equating the supply and demand for labor of cultural producers yields the number of varieties of cultural goods in each region

$$k_i = \lambda_{ci}. \tag{9}$$

The profit for a typical firm producing variety s in region i is given by,

$$\pi_i(s) = (p_{ci}(s) - B)c_i(s) - w_{ci}. \tag{10}$$

Under the Chamberlinian large group assumption, the producer of each variety acts as a monopolist, taking the prices of other varieties as given. The profit maximizing price is then a constant markup over marginal cost

$$p_{ci}(s) = B\frac{\sigma}{(\sigma - 1)}. \tag{11}$$

Given CES preferences there is no pro-competitive effect such that price of all varieties is the same both within and across cities (see Ottaviano et al. 2002 for model with preferences that generate pro-competitive effects in producer markups). We can then write $p_{ci}(s) = p_c$ and $c_{ji}(s) = c_{ji}$.

Free entry of firms drives profits to zero. Therefore, the quantity of output of each variety is chosen such that any net revenue covers the fixed labor costs for cultural producers.

$$c_{ji} = \frac{(\sigma - 1)}{B}w_{ci}. \tag{12}$$

Finally, from (4) the CES price index in each city can be written as

$$P_{ci} = B\frac{\sigma}{(\sigma - 1)}\lambda_{ci}^{1/(1-\sigma)}. \tag{13}$$

Note that the price index is declining in the number of cultural producers in each region.

2.3 Temporary equilibrium

It is assumed that in the short run the share of each type of worker in either region is fixed. The market clearing conditions for the modern good, cultural good and housing, respectively, are given by

$$p_m(\lambda_{m1}A_{m1} + \lambda_{m2}A_{m2}) = \beta\big(p_m(\lambda_{m1}A_{m1} + \lambda_{m2}A_{m2}) + 1 + \lambda_{c1}w_{c1} + \lambda_{c2}w_{c2}\big), \tag{14}$$

$$\sigma\lambda_{ci}w_{ci} = \delta\big(p_mA_{mi}\lambda_{mi} + \lambda_{ti} + \lambda_{ci}w_{ci}\big), \tag{15}$$

$$r_i H_i = \gamma \left(p_m A_{mi} \lambda_{mi} + \lambda_{ti} + \lambda_{ci} w_{ci} \right), \qquad i = 1, 2. \tag{16}$$

Solving for p_m, w_{ci} and r_i yields the short-run prices

$$p_m = \frac{\sigma\beta}{\sigma(1-\beta)-\delta}\left(\frac{1}{\lambda_{m1}A_{m1}+\lambda_{m2}A_{m2}}\right), \tag{17}$$

$$w_{ci} = \frac{\delta}{\sigma-\delta}\left(\frac{\sigma\beta}{\sigma(1-\beta)-\delta}\frac{\lambda_{mi}A_{mi}}{\lambda_{m1}A_{m1}+\lambda_{m2}A_{m2}} + \lambda_{ti}\right)\frac{1}{\lambda_{ci}}, \tag{18}$$

$$r_i = \frac{\gamma\sigma}{\sigma-\delta}\left(\frac{\sigma\beta}{\sigma(1-\beta)-\delta}\frac{\lambda_{mi}A_{mi}}{\lambda_{m1}A_{m1}+\lambda_{m2}A_{m2}} + \lambda_{ti}\right)\frac{1}{H_i}. \tag{19}$$

Note that the price of the modern good, p_m, peaks when modern workers are evenly divided, $\lambda_{m1} = 1/2$, and is at its trough when modern workers are concentrated in a single region, $\lambda_{m1} = 1$ or $\lambda_{m1} = 0$. The value of the modern workers wage, $w_{mi} = p_m A_{mi}$, is equivalent at $\lambda_{m1} = 1$, $\lambda_{m1} = 1/2$ and $\lambda_{m1} = 0$. Additionally, it can be shown that $\frac{\partial w_{mi}}{\lambda_{mi}}|_{\lambda_{mi}=1/2} > 0$ and $\frac{\partial w_{mi}}{\lambda_{mi}}|_{\lambda_{mi}=1} < 0$, indicating the trade-off between increasing productivity in the sector and the decreasing value of output as modern workers become more concentrated in a single region. For cultural workers, regional wages are decreasing in the local supply of artists and increasing in the total expenditure by modern and traditional workers. Rents in each region rise with total expenditure from modern and traditional workers and fall with the supply of housing. Note that the income of cultural workers is fully capitalized into rents, as λ_{ci} does not enter the short-run rent function.

2.4 Long-run equilibrium

In the long run, all workers locate in the region where they receive the higher utility. The adjustment process is governed by the utility gap between regions for each type of worker

$$\dot{\lambda}_j = V_{j1} - V_{j2}. \tag{20}$$

There are stable concentrated equilibria at $\dot{\lambda}_j \geq 0$ for $\lambda_{j1} = 1$ and $\dot{\lambda}_j \leq 0$ for $\lambda_{j1} = 0$. Interior equilibria occur at $\dot{\lambda}_j = 0$ for $\lambda_{j1} \in (0, 1)$. Interior equilibria are stable if the eigenvalues associated with the Jacobian of the adjustment process $\dot{V}_j$, $\forall j \in \{t, m, c\}$ are negative or have negative real parts (See Tabuchi and Zeng 2004). We employ the standard assumption in the literature regarding the timing of events, namely that prices adjust instantaneously while migration decisions follow as a reaction to the price changes.

3 Stable equilibrium configurations

Our aim is to analyze how regional differences in housing supply and scale economies in both the cultural and modern sectors generate relatively more or less integrated communities. We focus our analysis on the following equilibrium configurations:

Integration: Each city contains all three types of workers.
Concentration: All workers locate in a single city.
Segregation: A city has one type of worker and the other city has two types.
Partial integration: Each city has two types of workers and one city has three types.
Partial segregation: Each city has two types of workers at most.

We do not attempt to catalog all equilibria but instead focus only on a subset. First, many of the equilibria are qualitatively similar whether we focus on one region or the other. For example, if we consider a case of partial segregation where modern workers are located in region 1 and traditional workers in region 2, there is similar equilibrium where modern workers are in region 2 and traditional workers in region 1. Second, while some minor modifications do generate different equilibrium values, the salient qualitative properties remain unaffected. Therefore, we focus on the necessary relationship between the key parameters in the model that is required for each of the equilibrium configurations stated above to hold, and provide an examples in the text to showcase the results. We make the following assumption on the parameters:

Assumption 1 $\sigma > 1 + \delta$.

Assumption 2 $\frac{\sigma\beta}{\sigma(1-\beta)-\delta} > 1$.

Assumption 1 is the standard "no-black-hole condition," which ensures that all economic activities do not always collapse to a single region. Given that we are interested in wage inequality among workers, Assumption 2 indicates that the nominal wage for modern workers when either clustered in a single region or equally divided between regions is greater than that for traditional workers. The rationale for Assumption 2 is that one of the primary focuses of the paper is on spatial distribution of different income groups, given that all workers want access to as many cultural varieties as possible but must compete over a limited supply of housing. Assumption 2 implies that *ex ante* modern workers are more productive or have a set of specialized skills that traditional workers do not and thus receive a higher wage.

In addition, we make the implicit assumption that the supply of housing is sufficiently similar such that there is an adequately large number of cultural producers in each region. As will be shown in the proceeding sections, the population distribution hinges critically on the relative housing supply between regions. In the monopolistic competition framework presented by Dixit and Stiglitz (1977), the set of varieties is assumed to be "reasonably large" such that firms pricing decisions have a negligible effect on the overall price index over varieties of horizontally differentiated goods. Consumers in traditional NEG models generally have access to the whole set of available varieties, albeit at different prices due to interregional transport costs. However, in our framework consumers in either region only have access to those varieties that are produced within the region such that if the distribution of the population leaves one region with few cultural producers the Chamberlinian large group assumption may be violated. This assumption thus ensures that the market structure for differentiated products does not change in the model.

For future reference, the utility gap for each type is detailed below.

$$\begin{aligned}\dot{\lambda}_m = \kappa_m \Bigg(& \frac{(1+\lambda_{m1})^{\epsilon} H_1^{\gamma}}{(p_m \lambda_{m1} A_m (1+\lambda_{m1})^{\epsilon} + \lambda_{t1})^{\gamma} \lambda_{c1}^{\frac{\delta}{1-\sigma}}} \\ & - \frac{(1+(1-\lambda_{m1}))^{\epsilon} H_2^{\gamma}}{(p_m A_m (1-\lambda_{m1})(1+(1-\lambda_{m1}))^{\epsilon} + (1-\lambda_{t1}))^{\gamma} (1-\lambda_{c1})^{\frac{\delta}{1-\sigma}}} \Bigg),\end{aligned} \tag{21}$$

$$\begin{aligned}\dot{\lambda}_t = \kappa_t \Bigg(& \frac{H_1^{\gamma}}{(p_m \lambda_{m1} A_m (1+\lambda_{m1})^{\epsilon} + \lambda_{t1})^{\gamma} \lambda_{c1}^{\frac{\delta}{1-\sigma}}} \\ & - \frac{H_2^{\gamma}}{(p_m A_m (1-\lambda_{m1})(1+(1-\lambda_{m1}))^{\epsilon} + (1-\lambda_{t1}))^{\gamma} (1-\lambda_{c1})^{\frac{\delta}{1-\sigma}}} \Bigg),\end{aligned} \tag{22}$$

$$\begin{aligned}\dot{\lambda}_c = \kappa_c \Bigg(& \frac{\big(p_m A_m \lambda_{m1} (1+\lambda_{m1})^{\epsilon} + (1-\lambda_{t1})\big)^{1-\gamma} H_1^{\gamma}}{\lambda_{c1}^{\frac{1-\sigma+\delta}{1-\sigma}}} \\ & - \frac{\big(p_m A_m (1-\lambda_{m1})(1+(1-\lambda_{m1}))^{\epsilon} + (1-\lambda_{t1})\big)^{1-\gamma} H_2^{\gamma}}{\lambda_{c1}^{\frac{1-\sigma+\delta}{1-\sigma}}} \Bigg),\end{aligned} \tag{23}$$

where the terms

$$\kappa_m = \frac{\eta p_m^{1-\beta}}{p_c^{\delta}} \left(\frac{\sigma-\delta}{\gamma\sigma} \right)^{\gamma}, \quad \kappa_t = \frac{\eta}{p_m^{\beta} p_c^{\delta}} \left(\frac{\sigma-\delta}{\gamma\sigma} \right)^{\gamma}, \quad \kappa_c \equiv \frac{\eta}{p_m^{\beta} p_c^{\delta}} \left(\frac{\sigma-\delta}{\gamma\sigma} \right)^{\gamma} \left(\frac{\delta}{\sigma-\delta} \right),$$

are common terms which are factored out to improve clarity. For traditional workers there are two competing forces: the congestion effect that enters through higher housing rents as the local population increases and the agglomeration effect from access to a large share of cultural goods. Both of these competing forces enter into the location decision of modern workers, as well as the additional agglomeration force from the external scale economies, which alters the local wage. For cultural workers, these forces work in opposite directions. The wages of cultural workers, like rents, reflect the income level of the city; therefore, an increase in the number of traditional and modern workers raises regional income and thus the wage of cultural producers. Additional cultural producers act as a congestion force for other cultural producers via two channels. First, through increased congestion in the housing market which raises rents and thus the local cost of living. Second, by offering additional varieties new entrants reduce the demand for existing varieties, thus lowering the wage for cultural producers in the region.

3.1 Segregation and concentration

In this section, we show that segregation is not a stable equilibrium and derive the conditions for a stable concentrated equilibrium. The instability of a segregated equilibrium follows directly from the assumption of Cobb–Douglas preferences. Given that cultural goods must be consumed locally by workers, modern and traditional workers would receive no utility living in a region without cultural producers and thus always have an incentive to move toward the region with artists. Therefore, even if modern and traditional workers are segregated from one another each region would hold some measure of cultural producers.

Inspection of (21)–(23) indicates that when all workers are concentrated in a single region the utility level for each type is positive and finite in the populated region. For a concentrated equilibrium to be stable, the utility in the empty region must be no greater than in the populated region. Consider an initial population distribution given by $\lambda_{m1} = 0$ and $\lambda_{t1} \in (0, 1)$ and $\lambda_{c1} \in (0, 1)$. Integrating this distribution into (22) and (23), we can write the utility level in region 1 for traditional and cultural workers as

$$V_{t1} = \kappa_t H_1^{\gamma} \left(\frac{\lambda_{c1}^{\frac{\delta}{\sigma-1}}}{\lambda_{t1}^{\gamma}} \right), \tag{24}$$

$$V_{c1} = \kappa_c H_1^{\gamma} \left(\frac{\lambda_{t1}^{1-\gamma}}{\lambda_{c1}^{\frac{1-\sigma+\delta}{1-\sigma}}} \right). \tag{25}$$

Equations (24) and (25) highlight the competing forces present for each type of worker. Traditional workers benefit from a greater number of cultural producers and varieties of cultural goods, while a greater number of traditional workers increase competition for housing and raise local rents. For cultural producers, a greater number of traditional workers increase the size of the local market for cultural goods, while a greater number of cultural producers increase competition in the cultural goods market lowering the wage. The question is: as region 1 becomes depopulated will the utility level for traditional and modern workers, tend toward 0 or become infinitely high? Totally differentiating (24) and (25) with respect to λ_{t1} and λ_{c1} gives

$$dV_{t1} = \frac{\delta}{\sigma - 1}\frac{V_{t_1}}{\lambda_{c1}}d\lambda_{c1} - \gamma\frac{V_{t1}}{\lambda_{t1}}d\lambda_{t1}, \tag{26}$$

$$dV_{c1} = -\frac{1 - \sigma + \delta}{1 - \sigma}\frac{V_{c1}}{\lambda_{c1}}d\lambda_{c1} + (1 - \gamma)\frac{V_{t1}}{\lambda_{t1}}d\lambda_{t1}. \tag{27}$$

Define the notation $\hat{x} = \mathrm{d}x/x$ as the percentage change in a variable x. Now consider an initial distribution of cultural and traditional workers in region 1 with a lower total number but the same relative number of each. This implies an equal percentage reduction in both traditional and cultural workers so that $\hat{\lambda}_{t1} = \hat{\lambda}_{c1} = \hat{\lambda} < 0$. We can then rewrite (26) and (27) as

$$\hat{V}_{t1} = \left(\frac{\delta}{\sigma - 1} - \gamma\right)\hat{\lambda}, \tag{28}$$

$$\hat{V}_{c1} = \left(-\frac{1 - \sigma + \delta}{1 - \sigma} + (1 - \gamma)\right)\hat{\lambda}. \tag{29}$$

Utility declines with a reduction in the population if the terms in brackets in both (28) and (29) are positive, which holds when $\sigma < 1 + \delta/\gamma$. In which case V_{c1} and V_{t1} tend to 0 as λ_{c1} and λ_{t1} approach 0. In addition, modern workers would have no incentive to move away from region 2. To see this note that $V_{m1} = w_{m1}V_{t1}$, which clearly equals 0 when V_{t1} is 0. In words, when there is a strong "love of variety," for traditional workers the gain in utility from the reduction in housing rents is more than offset by the loss of available local varieties of the cultural good, while, for cultural producers the reduction in utility from decline in market size exceeds the benefits of reduced competition.

3.2 Integration, partial segregation and partial integration

3.2.1 Integration

This section focuses on integrated, partially integrated and partially segregated equilibrium. In the previous section, we found that when the "love of variety" is high (i.e., low σ) concentration was a stable equilibrium. We now focus on the case where this fails to hold, that is $\sigma > 1 + \delta/\gamma$. To begin we consider an equilibrium in which $\dot{V}_j = 0\ \forall j$, which implies that $P_1 = P_2$, $w_{c1} = w_{c2}$ and $w_{m1} = w_{m2}$. Solving (21)–(23), we yield the population distribution

$$\lambda_{c1}^{I} = \left(\frac{\frac{\beta\sigma}{\sigma(1-\beta)-\delta} + \left(\frac{H_2}{H_1}\right)^{\Psi}}{\frac{\beta\sigma}{\sigma(1-\beta)-\delta} + 1}\right)\frac{H_1^{\Psi}}{H_1^{\Psi} + H_2^{\Psi}}, \tag{30}$$

$$\lambda_{t1}^{I} = \frac{\frac{1}{2}\frac{\beta\sigma}{\sigma(1-\beta)-\delta}(H_1^{\psi} - H_2^{\psi}) + H_2^{\psi}}{H_1^{\psi} + H_2^{\psi}}, \tag{31}$$

$$\lambda_{m1}^{I} = \frac{1}{2}, \tag{32}$$

$$\psi \equiv \frac{\gamma(\sigma-1)}{\gamma(\sigma-1)-\delta}, \tag{33}$$

where I is a mnemonic for integration. Note the cluster of parameters denoted by ψ in (33) is positive when $\sigma > 1 + \delta/\gamma$ and that when $H_1 = H_2$, then $\lambda_{c1} = \lambda_{t1} = 1/2$. That is, the population of all types are evenly distributed across regions when the housing supply is equivalent in each region, as would be expected. However, we show in "Appendix" that an integrated equilibrium is unstable. The intuition is that when the cost of living is equivalent in each region, which is implied by the spatial equilibrium condition for traditional workers, modern workers have an incentive to pursue a higher wage by moving toward a more concentrated distribution. Consider a movement of modern workers toward region 1. The impact of this transition is to raise the average-income level and rents in the region with a greater number of modern workers. The higher-income level in the region raises the local wage for cultural producers; however, given the relatively weak "love of variety," the benefits are largely outweighed by the congestion effect in housing. Conversely, traditional workers see no change in their wage and migrate toward the region with a lower number of modern workers in search of more affordable housing. This outmigration of traditional workers from region 1 partially mitigates the congestion effect from an increase in modern workers which stabilizes the asymmetric equilibrium rather than pushing the population distribution back toward the integrated equilibrium. We next consider under what conditions on the parameters are partially segregated and partially integrated equilibrium stable.

3.2.2 Partial segregation

Under a partially segregated equilibrium, all modern workers live in one city, all traditional workers live in the opposite city and cultural workers are divided between both cities. Suppose that $\lambda_{m1} = 1$ and $\lambda_{t1} = 0$. The condition for the spatial equilibrium of cultural workers is given by

$$\dot{\lambda}_c \Big|_{\substack{\lambda_{m1}=1 \\ \lambda_{t1}=0}} = \kappa_c \left(\frac{\left(\frac{\sigma\beta}{\sigma(1-\beta)-\delta}\right)^{1-\gamma} H_1^{\gamma}}{\lambda_{c1}^{(\delta+1-\sigma)/(1-\sigma)}} - \frac{H_2^{\gamma}}{(1-\lambda_{c1})^{(\delta+1-\sigma)/(1-\sigma)}} \right) = 0. \tag{34}$$

Solving (34) for λ_{c1} we yield

$$\lambda_{c1}^{PS} = \frac{\left(\left(\frac{\sigma\beta}{\sigma(1-\beta)-\delta}\right)^{1-\gamma}\left(H_1\right)^{\gamma}\right)^{\Phi}}{\left(H_2^{\gamma}\right)^{\Phi} + \left(\left(\frac{\sigma\beta}{\sigma(1-\beta)-\delta}\right)^{1-\gamma}\left(H_1\right)^{\gamma}\right)^{\Phi}}, \tag{35}$$

$$\Phi \equiv \frac{\sigma - 1}{\sigma - 1 - \delta} > 0, \tag{36}$$

where *PS* is a mnemonic for partial segregation.

Equation (35) has a clear intuitive explanation that focuses on the trade-off cultural producers face between affordable housing and market demand for their products. The terms, $\left(\frac{\sigma\beta}{\sigma(1-\beta)-\delta}\right)^{1-\gamma}\left(H_1\right)^{\gamma}$, and, H_2^{γ}, are Cobb Douglas functions of the average income of non-cultural workers in each region (recalling that $w_t = 1$) and each region's housing stock weighted, respectively, by the expenditure share on non-housing consumption $1 - \gamma$, and housing, γ. When γ is large, the supply of housing is weighted more heavily as a larger stock of housing reduces rents. When γ is relatively small, the average regional income of non-cultural workers plays a greater role in the distribution of cultural workers. Additionally, we have the following comparative statics:

$$\frac{\partial \lambda_{c1}^{PS}}{\partial H_1} > 0, \quad \frac{\partial \lambda_{c1}^{PS}}{\partial H_2} < 0, \quad \frac{\partial \lambda_{c1}^{PS}}{\partial \sigma} < 0, \quad \frac{\partial \lambda_{c1}^{PS}}{\partial \delta} > 0, \quad \frac{\partial \lambda_{c1}^{PS}}{\partial \beta} > 0. \tag{37}$$

The signs in (37) show that the number of cultural producers in region 1 increases with H_1, which reduces local rents and decreases with H_2 as local rents become relatively more expensive. Note that the wage of modern workers when concentrated in a single region, $\frac{\sigma\beta}{\sigma(1-\beta)-\delta}$, falls with σ and rises with δ and β. Therefore, an increase in σ has two effects. The first effect is to reduce the markup for cultural goods, and thus, the wage of each cultural producer as goods become more substitutable. The second is to reduce the income of modern workers, cumulatively decreasing the number of cultural producers in region 1. An increase in δ has the opposite effect by increasing the share of income devoted to cultural goods, raising the wage of modern workers and thus shifting more cultural producers to region 1. While an increase in β raises the wage of modern workers and indirectly the wage of cultural producers.

From (34), it is straightforward to verify that the slope of $\dot{\lambda}_c$ is negative as it crosses the equilibrium point, provided that $\sigma > 1 + \delta/\gamma$. In order to ensure that (35) is a stable equilibrium value, traditional and modern workers should have no incentive to move. The condition is given by

$$\dot{\lambda}_m\Big|_{\lambda_{c1}^S} \geq 0, \quad \dot{\lambda}_t\Big|_{\lambda_{c1}^S} \leq 0. \tag{38}$$

The condition holds if

$$(2)^{\frac{\epsilon}{\gamma}\frac{\Psi}{\Phi}}\left(\frac{H_1}{H_2}\right)^{\Psi} \geq \left(\frac{\sigma\beta}{\sigma(1-\beta)-\delta}\right) \geq \left(\frac{H_1}{H_2}\right)^{\Psi}. \tag{39}$$

The inequality on the right ensures that the cost of living for traditional workers is no higher in region 2 than in region 1, while the inequality on the left ensures that the productivity, and thus the nominal wage, when all modern workers are concentrated in region 1 is sufficiently high to offset the higher cost of living in region 1. Given Assumption 2, if each region has an equivalent supply of housing so that $H_1 = H_2$ then the equilibrium will be stable if the external scale economies for modern workers are sufficiently strong. However, when one region contains significantly more housing than another and can accommodate a greater population with relatively lower rents, one type of worker will be tempted to move. In this case, we find a partially integrated equilibrium, which we will proceed to analyze in the next section.

3.2.3 Partial integration for traditional workers

Here we consider an equilibrium where modern workers are concentrated in a single city, while traditional and cultural workers are divided between cities. Again, assume $\lambda_{m1} = 1$. This configuration implies that $\dot{\lambda}_c = 0$, $\dot{\lambda}_t = 0$, $P_1 = P_2$ and $w_{c1} = w_{c2}$. Using (22) and (23), we can solve for the population shares for cultural and traditional workers,

$$\lambda_{c1}^{\mathrm{PI}} = \frac{H_1^{\Psi}}{H_1^{\Psi} + H_2^{\Psi}} \quad \lambda_{t1}^{\mathrm{PI}} = \frac{H_2^{\Psi}}{H_1^{\Psi} + H_2^{\Psi}}\left(\left(\frac{H_1}{H_2}\right)^{\Psi} - \left(\frac{\sigma\beta}{\sigma(1-\beta)-\delta}\right)\right) \tag{40}$$

where the superscript *PI* is a mnemonic for partial integration. Note that

$$\lambda_{t1}^{\mathrm{PI}} > 0 \iff \left(\frac{H_1}{H_2}\right)^{\Psi} > \left(\frac{\sigma\beta}{\sigma(1-\beta)-\delta}\right). \tag{41}$$

Furthermore, it can be shown that the number of cultural workers is greater than the number of traditional workers in the integrated city, i.e., $\lambda_{c1}^{\mathrm{PI}} > \lambda_{t1}^{\mathrm{PI}}$. The condition in (41) is the opposite of that for a stable segregated equilibrium set out in (39). In other words, for one region to have some degree of integration, that region must have a larger share of total available housing.

In order to verify whether the equilibrium is stable, the eigenvalues associated with the Jacobian matrix of $\dot{\lambda}_c$ and $\dot{\lambda}_t$, evaluated at the equilibrium values, $\lambda_{m1}^{\mathrm{PI}} = 1$, $\lambda_{c1}^{\mathrm{PI}}$ and $\lambda_{t1}^{\mathrm{PI}}$, must be negative or have negative real parts. If the trace of the Jacobian is negative and the determinant is positive when evaluated at the equilibrium values both eigenvalues will be negative or have negative real parts. Using (22) and (23) it is straightforward to verify that the trace is given by $\frac{\partial \dot{\lambda}_t}{\partial \lambda_{t1}} + \frac{\partial \dot{\lambda}_c}{\partial \lambda_{c1}} < 0$. Defining Δ^{PI} as the determinant and noting that $V_{ci} = w_{ci}V_{ti}$ and that in equilibrium $w_{c1} = w_{c2} = w_c$, $V_{t1} = V_{t2} = V_t$ and $V_{c1} = V_{c2} = V_c$, we have

$$
\begin{aligned}
\Delta^{\mathrm{PI}} &= \frac{\partial \dot{\lambda}_t}{\partial \lambda_{t1}}\frac{\partial \dot{\lambda}_c}{\partial \lambda_{c1}} - \frac{\partial \dot{\lambda}_t}{\partial \lambda_{c1}}\frac{\partial \dot{\lambda}_c}{\partial \lambda_{t1}} \\
&= \left(\frac{\partial V_{t1}}{\partial \lambda_{t1}} - \frac{\partial V_{t2}}{\partial \lambda_{t1}}\right)\left(\frac{\partial V_{c1}}{\partial \lambda_{c1}} - \frac{\partial V_{c2}}{\partial \lambda_{c1}}\right) - \left(\frac{\partial V_{t1}}{\partial \lambda_{c1}} - \frac{\partial V_{t2}}{\partial \lambda_{c1}}\right)\left(\frac{\partial V_{c1}}{\partial \lambda_{t1}} - \frac{\partial V_{c2}}{\partial \lambda_{t1}}\right) \\
&= \left(\gamma\left(\frac{1}{p_m A_{m1} + \lambda_{t1}^{\mathrm{PI}}} + \frac{1}{1-\lambda_{t1}^{\mathrm{PI}}}\right)V_t\right)\left(\frac{\sigma - 1 - \delta}{\sigma - 1}\left(\frac{1}{\lambda_{c1}^{\mathrm{PI}}} + \frac{1}{1-\lambda_{c1}^{\mathrm{PI}}}\right)V_c\right) \\
&\quad - \left(\frac{\delta}{\sigma - 1}\left(\frac{1}{\lambda_{c1}^{\mathrm{PI}}} + \frac{1}{1-\lambda_{c1}^{\mathrm{PI}}}\right)V_t\right)\left((1-\gamma)\left(\frac{1}{p_m A_{m1} + \lambda_{t1}^{\mathrm{PI}}} + \frac{1}{1-\lambda_{t1}^{\mathrm{PI}}}\right)V_c\right) \\
&= V_t V_c\left(\frac{1}{\lambda_{c1}^{\mathrm{PI}}} + \frac{1}{1-\lambda_{c1}^{\mathrm{PI}}}\right)\left(\frac{1}{p_m A_{m1} + \lambda_{t1}^{\mathrm{PI}}} + \frac{1}{1-\lambda_{t1}^{\mathrm{PI}}}\right)\left(\frac{\gamma(\sigma - 1 - \delta)}{\sigma - 1} - \frac{(1-\gamma)\delta}{\sigma - 1}\right).
\end{aligned}
\tag{42}
$$

The sign of the determinant hinges on the final term in brackets in (42), which is positive when $\sigma > 1 + \delta/\gamma$, indicating that the equilibrium is stable. Intuitively, when the housing supply in region 1 is large so that housing costs remain low even with a large population, some measure of traditional workers will choose to locate in region 1.

3.2.4 Partial integration for modern workers

In this section, we consider whether there is an equilibrium where all traditional workers are located in a single city while modern and cultural workers are divided between cities. Suppose that $\lambda_{t1} = 1$. The condition for a concentrated equilibrium for traditional workers is given by

$$
\dot{\lambda}_t \geq 0 \implies 1 \geq \frac{P_1}{P_2}, \tag{43}
$$

which says that the price index in city 2 must be at least as high as that of city 1. The condition for an interior equilibrium for modern workers is given by

$$
\dot{\lambda}_m = 0 \implies \frac{(1+\lambda_{m1})^{\epsilon}}{(1+(1-\lambda_{m1}))^{\epsilon}} = \frac{P_1}{P_2}. \tag{44}
$$

This says that the relative productivity of modern workers between city 1 and city 2 is equal to the relative price index between city 1 and city 2. Combining the two conditions in (43) and (44) implies that traditional workers will concentrate in city 1 if $\lambda_{m1} \leq 1/2$. The case that $\lambda_{m1} = 1/2$ can be ruled out as the system is overdetermined, with both $P_1 = P_2$ and $w_{c1} = w_{c2}$, providing two equations to solve for a single unknown, λ_{c1}. It must be the case then that $\lambda_{m1} < 1/2$.

While we cannot determine the equilibrium values analytically due to the nonlinearity of the modern worker's wage, we should expect that for a stable equilibrium two features should likely hold. First, given the results in Sect. 3.2.3, one region should have sufficiently more housing than the other. Second, external economies in

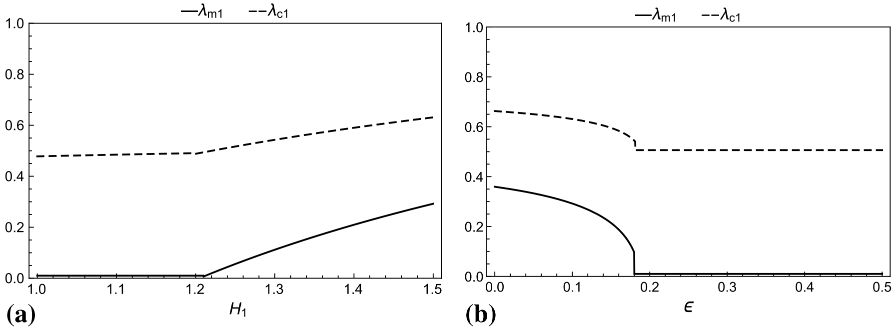

Fig. 1 Equilibrium population distributions of modern and cultural workers. Note: $\beta = .5$, $H_2 = 1$, $\gamma = .25$, $\delta = .1$, $A_m = 1$

the production of the modern good should be relatively low to ensure that the nominal wage gap between modern workers in each region is small. Here we undertake numerical simulations by solving $\dot{\lambda}_c = 0$ for λ_{c1} as a function of λ_{m1}. We then place this function into $\dot{\lambda}_m = 0$ to solve for λ_{m1}. That value is then used to simulate λ_{c1} and the eigenvalues of the Jacobian matrix from $\dot{\lambda}_c$ and $\dot{\lambda}_m$ for different values of the parameters.

Figure 1a, b shows the plots of the population distributions of modern and cultural producers for increasing values of housing supply in region 1, H_1, and the external scale parameter ϵ. In "Appendix", we provide simulations of the eigenvalues showing that the equilibrium values are indeed stable. In Fig. 1a, when housing supply is equivalent in each region, there is a partially segregated equilibrium. Once region 1 reaches a supply of housing of roughly 20% greater than that of region 2, a positive number of modern workers move to region 1 and cultural producers follow suit. In Fig. 1b, when scale economies are low, there is a relatively large share of modern workers in region 1, but those numbers steadily decline as scale economies increase.

The results from this section are summarized in the following proposition.

Proposition 1 *Neither integration nor segregation is a stable equilibrium. A concentrated equilibrium is stable when $\sigma < 1 + \delta/\gamma$. When $\sigma > 1 + \delta/\gamma$, if each region has a similar supply of housing and external scale economies in the modern sector are strong, a partially segregated equilibrium is stable. When one region has sufficiently more housing than the other region there is a stable, partially integrated equilibrium in which the region with the greater supply of housing has all three types of workers, while the other region contains solely traditional workers and cultural producers. When scale economies in the modern sector are weak and one region has sufficiently more housing than the other there is a stable, partially integrated equilibrium where the region with the greater supply of housing has all three types of workers, while the other region contains only modern workers and cultural producers.*

4 Extension: moonlighting by cultural producers in the traditional sector

In this section, we extend the model from the preceding section by considering the case where cultural producers supplement their income by moonlighting in the traditional sector. Define θ_i as the share of time that a cultural producer devotes to producing the cultural good in region i and receiving the wage w_{ci} with $1-\theta_i$ the share spent in the traditional sector and receiving $w_t = 1$. Therefore, a cultural producer's income is given by $\theta_i w_{ci} + (1-\theta_i)$. This is the only major change to the model. If the unit wage of cultural workers were above that of traditional workers, there would be no incentive for cultural producers to moonlight. Conversely, if the wage in the cultural sector were below that of the traditional sector, there would be no incentive to produce the cultural good. However, given the Cobb–Douglas preferences of consumers, all types demand a positive quantity of the cultural good and would thus bid up the price to ensure some positive level of production. Therefore, in an equilibrium where a cultural producer's time is divided between sectors, i.e., $\theta_i \in (0,1)$, the unit of wage of cultural workers must equalize with that of traditional workers. The remainder of this section focuses on the case where $w_{ci} = 1$. Given that the price of the traditional good, and thus the wage in the traditional sector, is the numeraire any wage effects from the increase in labor supply in the traditional sector are captured through the change in rents and the cultural goods price index as cultural workers adjust their labor supply to maintain the equality between wages in the cultural and traditional sectors.

When $\theta_i \in (0,1)$ and $w_{ci} = 1$, the number of varieties available in each region is given by $k_i = \theta_i \lambda_{ci}$ which is the product of the number of cultural producers in each region and the time each spends on cultural production. Plugging this wage into (14) and (16), the price for the modern good and regional rents can be rewritten as

$$p_m^M = \frac{2\beta}{1-\beta}\frac{1}{\lambda_{m1}A_{m1} + \lambda_{m2}A_{m2}}, \tag{45}$$

$$r_i^M = \gamma\left(\frac{2\beta}{1-\beta}\frac{\lambda_{m_i}A_{mi}}{\lambda_{m1}A_{m1} + \lambda_{m2}A_{m2}} + \lambda_{ti} + \lambda_{ci}\right)\frac{1}{H_i}, \tag{46}$$

where the M superscript is used as a mnemonic for "moonlighting" by cultural producers. The regional price index for cultural goods and the share of time devoted to cultural production can then be written as

$$P_{ci}^M = B\frac{\sigma}{\sigma-1}\left(\theta_i\lambda_{ci}\right)^{\delta/1-\sigma}, \tag{47}$$

$$\theta_i^M = \frac{\delta}{\sigma}\left(\frac{2\beta}{1-\beta}\frac{\lambda_{m_i}A_{mi}}{\lambda_{m1}A_{m1} + \lambda_{m2}A_{m2}} + \lambda_{ti} + \lambda_{ci}\right)\frac{1}{\lambda_{ci}}. \tag{48}$$

The effect of λ_{m1} and λ_{t1} on p_m and r_i is consistent with Sect. 2.3. However, notice that rents now include the size of the artists labor force. In addition, θ_i is decreasing in the supply of cultural producers. The greater the number of local artists, the more competitive the market for cultural goods; reducing the time artists can spend on cultural production. Additionally, we make the following assumption, consistent with Assumption 2, which ensures that when modern workers are concentrated in a single region or equally divided between regions they receive a higher wage than traditional workers.

Assumption 3 $\frac{2\beta}{1-\beta} > 1$.

For future reference, we again include the revised equation of motion for each type, noting that in equilibrium $V_{ti} = V_{ci}$,

$$\dot{\lambda}_m^M = \kappa_m^M \left(\frac{\left(1+\lambda_{m1}\right)^{\epsilon} H_1^{\gamma}}{\left(\lambda_{m1} p_m^M A_m (1+\lambda_{m1})^{\epsilon} + \lambda_{ti} + \lambda_{ci}\right)^{\frac{\delta+\gamma(1-\sigma)}{1-\sigma}}} - \frac{\left(1+(1-\lambda_{m1})\right)^{\epsilon} H_2^{\gamma}}{\left((1-\lambda_{m1}) p_m^M A_m (1+(1-\lambda_{m1}))^{\epsilon} + \lambda_{ti} + \lambda_{ci}\right)^{\frac{\delta+\gamma(1-\sigma)}{1-\sigma}}} \right), \tag{49}$$

$$\dot{\lambda}_l^M = \kappa_l^M \left(\frac{H_1^{\gamma}}{\left(\lambda_{m1} p_m^M A_m (1+\lambda_{m1})^{\epsilon} + \lambda_{ti} + \lambda_{ci}\right)^{\frac{\delta+\gamma(1-\sigma)}{1-\sigma}}} - \frac{H_2^{\gamma}}{\left((1-\lambda_{m1}) p_m^M A_m (1+(1-\lambda_{m1}))^{\epsilon} + \lambda_{ti} + \lambda_{ci}\right)^{\frac{\delta+\gamma(1-\sigma)}{1-\sigma}}} \right), \quad l = c, t \tag{50}$$

where

$$\kappa_m^M = \eta \frac{(p_m^C)^{1-\beta}}{(p_c^C)^{\delta}} \frac{\sigma^{\delta}}{\gamma^{\gamma}\delta^{\delta}}, \quad \kappa_l^M = \eta \frac{(p_m^C)^{-\beta}}{(p_c^C)^{\delta}} \frac{\sigma^{\delta}}{\gamma^{\gamma}\delta^{\delta}}, \quad l = c, t \tag{51}$$

are common terms which are factored out to ease exposition. The superscript M denotes moonlighting. Many of the qualitative results from the preceding section continue to hold under this extension; therefore, unless necessary, technical derivations of the stability of equilibria are provided in Appendices 7.3 and 7.4

4.1 Concentration or segregation with moonlighting

We now consider how moonlighting by cultural producers affects the stability of a concentrated equilibrium. Analysis of (49) and (50) reveals that regional utility is rising in the regional income level when $\sigma < 1 + \delta/\gamma$. In which case the concentrated equilibrium is stable, which is consistent with the results in Sect. 3.1. Setting $\lambda_{m1} = \lambda_{t1} = \lambda_{c1} = 1$ (or 0) gives the share of time that artists spend producing cultural goods

$$\theta_i^{CM} = \frac{\delta}{\sigma} \frac{2}{1-\beta}, \tag{52}$$

where CM denotes concentration with moonlighting. Time devoted to cultural production is increasing in the expenditure by households on cultural goods, δ, and modern goods, β, and is decreasing in the elasticity of substitution, σ, between varieties.

When $\sigma > 1 + \delta/\gamma$, an increase in the aggregate income level of a region acts as a congestion force, raising the local price level and reducing utility. Suppose that modern and traditional workers are all concentrated in region 1. In this case, we can show there is an interior equilibrium in which a share of the cultural producers are segregated in region 2. To avoid confusion with the case of partial segregation, this equilibrium is referred to as the "artists' colony." Setting $\dot{\lambda}_c^M = 0$ and solving for λ_{c1}, we yield

$$\lambda_{c1}^{\mathrm{AC}} = \frac{H_2^{\psi}}{H_1^{\psi} + H_2^{\psi}} \left(\left(\frac{H_1}{H_2} \right)^{\psi} - \frac{1+\beta}{1-\beta} \right), \tag{53}$$

$$\theta_1^{\mathrm{AC}} = \frac{\delta}{\sigma} \left(\frac{1+\beta}{1-\beta} \right) \left(\frac{\left(\frac{H_1}{H_2} \right)^{\psi}}{\left(\frac{H_1}{H_2} \right)^{\psi} - \left(\frac{1+\beta}{1-\beta} \right)} \right), \tag{54}$$

$$\theta_2^{\mathrm{AC}} = \frac{\delta}{\sigma}, \tag{55}$$

where the superscript AC is a mnemonic for artists' colony and ψ is defined in (33). For $\lambda_{c1}^{\mathrm{AC}}$ and θ_1^{AC} to be positive, we require

$$\left(\frac{H_1}{H_2} \right)^{\psi} > \left(\frac{1+\beta}{1-\beta} \right) \tag{56}$$

Given that $\dot{\lambda}_c^M = 0$ it follows directly that $\dot{\lambda}_t^M = 0$ and $\dot{\lambda}_m^M > 0$ and the slope of (50) is negative at the equilibrium point, when $\sigma > 1 + \delta/\gamma$, which imply that the equilibrium is stable. This result differs from the case where artists devoted all of their time to the production of cultural goods. Allowing cultural producers access to the labor market in the traditional sector ensures a minimum level of income for the artist's community, as artists adjust the time they devote to cultural goods production to maintain the wage equality between the traditional and cultural sectors. However, this equilibrium can only be sustained when region 1 has substantially more housing than region 2. Comparing the time that artists can devote to cultural production between regions, we have

$$\theta_1^{\mathrm{AC}} - \theta_2^{\mathrm{AC}} = \frac{\delta}{\sigma}\left(\left(\frac{1+\beta}{1-\beta}\right)\frac{\left(\frac{H_1}{H_2}\right)^{\Psi}}{\left(\frac{H_1}{H_2}\right)^{\Psi} - \left(\frac{1+\beta}{1-\beta}\right)} - 1\right) > 0. \tag{57}$$

Therefore, artists in the larger market are able to spend less time moonlighting than those in the “artists’ colony.” Furthermore, we can compare the number of varieties in each region. Recalling that $k_i = \theta_i \lambda_{ci}$, the difference is given by

$$k_1^{\mathrm{AC}} - k_2^{\mathrm{AC}} = \frac{\delta}{\sigma(1-\beta)}\frac{1}{H_1^{\Psi} + H_2^{\Psi}}\left(H_1^{\Psi}(1+\beta) - H_2^{\Psi}\right) > 0, \tag{58}$$

indicating that there are a greater number of varieties available in the larger market.

4.2 Partial segregation with moonlighting

Now consider a partially segregated equilibrium when cultural producers moonlight in the traditional sector. Suppose $\lambda_{m1} = 1$ and $\lambda_{t1} = 0$. Solving $\dot{\lambda}_c^M = 0$ for λ_{c1}, and using this to solve for θ_1 and θ_2 , we yield

$$\lambda_{c1}^{\mathrm{PSM}} = \frac{2H_2^{\Psi}}{H_1^{\Psi} + H_2^{\Psi}}\left(\left(\frac{H_1}{H_2}\right)^{\Psi} - \frac{\beta}{1-\beta}\right), \tag{59}$$

$$\theta_1^{\mathrm{PSM}} = \frac{\delta}{\sigma}\frac{H_1^{\Psi}}{(1-\beta)H_1^{\Psi} - \beta H_2^{\Psi}}, \tag{60}$$

$$\theta_2^{\mathrm{PSM}} = \frac{\delta}{\sigma}\frac{2H_2^{\Psi}}{(1+\beta)H_2^{\Psi} - (1-\beta)H_1^{\Psi}}, \tag{61}$$

where *PSM* denotes partial segregation with moonlighting. An interior solution will be stable when $\sigma > 1 + \delta/\gamma$. For $\lambda_{c1}^{\mathrm{PSM}} > 0$, $\theta_1^{\mathrm{PSM}} > 0$ and $\theta_2^{\mathrm{PSM}} > 0$ require

$$\frac{1+\beta}{1-\beta} > \left(\frac{H_1}{H_2}\right)^{\Psi} > \frac{\beta}{1-\beta}. \tag{62}$$

This constraint is consistent with the results for partial segregation in Sect. 3.2.2 where the stability of the equilibrium required that each region had a similar supply of housing. In this case $\dot{\lambda}_t^M = 0$ and $\dot{\lambda}_m^M > 0$, ensuring that the other two types of workers have no reason to deviate and that the equilibrium is stable.

The time spent on cultural production, θ_i^{PSM}, is declining in the local supply of housing and increasing in supply of housing in the other region, while the opposite effect holds for the number of cultural producers, $\lambda_{ci}^{\mathrm{PSM}}$. To understand this result consider an increase in the supply of housing in region 1. The initial effect is to reduce rents in region 1 which, in turn, induces migration toward the region adding to the number of local artists. Given the increase in the number of cultural producers, in order to maintain the equality between the wages in the traditional and cultural sectors, artists must reduce their share of time devoted to cultural production. Conversely, an increase in housing supply in the opposite region leads to outmigration reducing the supply of local artists. Those artists that remain in the region respond by increasing the amount of time they devote to cultural production. Comparing the time devoted to cultural production for artists in each region, we give

$$\theta_1^{\mathrm{PSM}} > \theta_2^{\mathrm{PSM}} \iff \beta > \frac{1 + \left(\frac{H_1}{H_2}\right)^{\Psi}}{3 + \left(\frac{H_1}{H_2}\right)^{\Psi} + 2\left(\frac{H_2}{H_1}\right)^{\Psi}}. \tag{63}$$

If each region has the same supply of housing, the right-hand side of (63) reduces to $\beta > 1/3$, which by Assumption 3 always implies artists in region 1 devote at least as much time to cultural production as those in region 2. However, the right-hand side of the second inequality in (63) is increasing in H_1, requiring a larger value of β in order for the condition to hold. The difference in the number of available varieties can be shown to be

$$k_1^{\mathrm{PSM}} - k_2^{\mathrm{PSM}} = \frac{2\delta}{\sigma(1-\beta)} \frac{1}{H_1^{\Psi} + H_2^{\Psi}} \left(H_1^{\Psi} - H_2^{\Psi}\right), \tag{64}$$

where the inequality follows from (56), indicating that the number of varieties is unambiguously larger in the region with a greater supply of housing. When both regions have an equal supply of housing the number of varieties is the same in each region, the number of cultural producers is greater in the region with traditional workers and cultural producers located in the region with modern workers spend less time moonlighting.

4.3 Partial integration with moonlighting

When labor choice is introduced for cultural workers, $V_c^M = V_t^M$ in equilibrium. This outcome reduces the system of equations by 1, with three equations to solve for 4 variables ($\lambda_{c1}, \lambda_{t1}, \theta_1$ and θ_2, respectively), leaving the system underdetermined. However, we can consider the case where artists in one region only work in cultural production, while in the other region artists moonlight in the traditional sector. Suppose that region 1 contains only artists that do not moonlight as well as all modern workers such that $\theta_1 = 1$, $\theta_2 \in (0,1)$ and $\lambda_{m1} = 1$. Using (48) and (50) to solve for the remaining variables, we yield

$$\lambda_{c1}^{\mathrm{PIM}} = \frac{2\delta}{\sigma(1-\beta)} \frac{H_1^{\psi}}{H_1^{\psi} + H_2^{\psi}}, \tag{65}$$

$$\lambda_{t1}^{\mathrm{PIM}} = 2\frac{(\sigma(1-\beta)-\delta)H_2^{\psi}}{\sigma(1-\beta)(H_1^{\psi}+H_2^{\psi})}\left(\left(\frac{H_1}{H_2}\right)^{\psi} - \frac{(\sigma\beta)}{(\sigma(1-\beta)-\delta)}\right), \tag{66}$$

$$\theta_2^{\mathrm{PIM}} = \frac{2\delta H_2^{\psi}}{\sigma(1-\beta)(H_1^{\psi}+H_2^{\psi}) - 2\delta H_1^{\psi}}, \tag{67}$$

where *PIM* denotes partial integration with moonlighting. There is a qualitatively similar equilibrium with $\theta_1 \in (0,1)$ and $\theta_2 = 1$. To ensure $\lambda_{t1} > 0$, we require

$$\left(\frac{H_1}{H_2}\right)^{\psi} > \frac{(\sigma\beta)}{(\sigma(1-\beta)-\delta)}. \tag{68}$$

This condition is equivalent to the case for partial integration with inelastic labor supply in (41). Furthermore, by taking the difference between (66) and (65) we have that when $H_1^{\psi}/H_2^{\psi} > \sigma\beta/(\sigma(1-\beta)-2\delta)$, the number of traditional workers is greater than the number of cultural workers in region 1. This is in contrast to the results without moonlighting, where the number of cultural workers exceeded that of traditional workers in the integrated city. Additionally, the number of cultural producers in region 1 is lower in the partially integrated equilibrium when workers can moonlight compared to that when they cannot, as specified in Sect. 3.2.3.[2] Intuitively, when artists in one city devote all of their time to cultural production, only a small number of artists can reside there to maintain the equality between the wages in the cultural and traditional sectors. This allows traditional workers in region 1 access to a large number of cultural goods while enjoying relatively low rents, as there are few local artists to compete with for housing. The difference in the number of varieties in each region is given by

$$\begin{aligned} k_1^{\mathrm{PIM}} - k_2^{\mathrm{PIM}} &= \lambda_{c1}^{\mathrm{PIM}} - \theta_2^{\mathrm{PIM}}(1-\lambda_{c1}^{\mathrm{PIM}}) \\ &= \frac{2\delta}{\sigma(1-\beta)} \frac{1}{H_1^{\psi}+H_2^{\psi}}\left(H_1^{\psi} - H_2^{\psi}\right), \end{aligned} \tag{69}$$

[2] To see this note that a stable equilibrium requires $\sigma > 1 + \delta/\gamma$. The difference between the distribution of cultural producers without moonlighting in (40) and with moonlighting is the first term on the right in (65), $2\delta/\sigma(1-\beta)$, which is decreasing in σ. Therefore, we only need to consider the smallest possible value for σ to verify whether the term is less than unity. Using $\sigma = 1 + \delta/\gamma$ and $1-\beta = \alpha+\gamma+\delta$, we then have

$$\sigma(1-\beta) = (1+\delta/\gamma)(\alpha+\delta+\gamma) > 2\delta$$

indicating that the multiple is less than 1.

which is the equivalent to the condition in (64), indicating that the number of varieties is greater in the region with the larger share of housing. In "Appendix" we verify the stability of the equilibrium for the case of partial integration of traditional workers and in "Appendix" we show that a partially integrated equilibrium with modern workers is unstable. The results of this section are summarized in the following proposition.

Proposition 2 *In the model with labor choice for cultural producers, when $\sigma < 1 + \delta/\gamma$ a concentrated equilibrium is stable. When $\sigma > 1 + \delta/\gamma$, if one region has sufficiently more housing than the other region, there is a segregated equilibrium where a share of the cultural producers are isolated in the smaller region, spend less time on cultural production and have access to fewer varieties of the cultural good than those in the larger region. When regions have a similar supply of housing there is a partially segregated equilibrium, where one region contains modern workers and the other traditional workers with cultural producers divided between the two regions. When one region has a sufficiently larger supply of housing there is a partially integrated equilibrium of traditional workers where cultural producers in one region work only as artists while those in the other region moonlight in the traditional sector. In both the partially segregated and partially integrated equilibrium, the number of available varieties is greater in the region with the larger supply of housing.*

5 Discussion of the results

There are three critical results from the preceding analysis: (1) whether a concentrated or (partially) interior equilibrium is stable depends on how strong the "love of variety" for cultural goods is, i.e., whether $\sigma \gtreqless 1 + \delta/\gamma$, (2) the relative supply of housing in each region is a determinant of both whether an interior equilibrium is partially integrated or partially segregated and the extent which cultural producers moonlight in the traditional sector, and (3) when scale economies in the modern sector are strong, i.e., high ϵ, modern workers will tend to cluster in a single region.

Glaeser et al. (2001) have argued that amenities are one of the primary drivers of urban growth, finding that cities with a greater variety of restaurants and theater performances have grown at a faster rate. Furthermore, Schiff (2015) has shown that larger, more concentrated cities offer a greater number of varieties of cuisine. The extent to which consumer's value the variety of cultural goods is a function of a number of factors including income, availability of leisure time, as well as idiosyncratic factors. However, casual empiricism suggests that while the number of available varieties in a city does tend to increase with both population and income levels, even relatively small communities often have a number of local cultural institution or restaurant options, suggesting that some degree of variety is valued. Therefore, under what conditions would we expect to see a concentrated equilibrium? A more appropriate way to answer this question may be to consider for what kind of consumers would we expect this condition to hold. Two candidates stand out in the data: high-income and highly educated consumers. The top 20% of the income distribution in the USA spent the largest share of their income on entertainment with 5.2% and spent 60% more on fees and admissions than the next highest quintile, 1.6% compared to 1%, respectively (Foster 2015). Additionally, the top income quintile spent 49% of their annual food expenditure share (11.2%) on food away from home implying a larger demand for local restaurant services (Bureau of Labor Statistics 2016a) . Finally, higher-income households devote a smaller share of their income to housing services, with the top third of the income distribution spending under 20% of their income if renting and an even lower share if they own and pay a mortgage (Trust The Pew Charitable 2016). Meanwhile, the share of expenditure on entertainment tends to rise with education levels, with those with master's, professional or doctoral degrees spending 5.1% on total entertainment and 1.8% on fees and admissions (Foster 2015). Suppose the expenditure share on cultural goods δ represents the sum of expenditure on entertainment and food away from home and consider the case where $\gamma = .15$ or 15% of income, consistent with the data presented for high-income households. This numbers indicate a value of $\delta/\gamma = (.112 \times .49 + .051)/.15 \cong .71$ so that concentration would require a value for $\sigma < 1.71$. Conversely, If we consider the bottom income quintile, expenditure on entertainment is 4.5% of income (Foster 2015), and expenditure on food away from home is 5.26% (Bureau of Labor Statistics 2016b), giving an expenditure share of 9.76% or $\delta = .0976$. However, for the lowest third of the income distribution the share of income spent on housing is substantially larger with the low-income renters spending nearly 50% of their income on housing (Trust The Pew Charitable 2016). Using a value of $\gamma = .5$, this gives a threshold of $\sigma < 1.195$. This suggests that higher-income and highly educated consumers would be more likely to concentrate in a single region than lower-income consumers. The intuition is that higher-income workers are able to devote more resources to enjoying cultural goods and thus receive a larger benefit from the number of varieties. Therefore, concentration can be sustained even for a weaker "love of variety."

An additional outcome of the preceding analysis is that for one region to contain all three types of workers that region must have a sufficiently larger supply of housing than the other, and that when this fails to hold there is income segregation between regions. Furthermore, the region with the greater supply of housing offers a larger number of varieties and artists in the region are able to devote more time to cultural production than those in the opposite region. The literature on migration and labor markets suggests that the growth from positive shocks to local demand that induces migration toward a region may be constrained if housing supply fails to keep up with population growth (Saks 2004, 2008; Glaeser et al. 2006). When housing supply elasticities are low, an increase in the population generates substantial increases in housing rental rates and the local cost of living, reducing the relative attractiveness of the location. Furthermore, amenities have been shown to be an important determinant in the household location decision (Glaeser et al. 2001). Our model suggests that attempts by cities to motivate amenity creation are firmly interlinked with local housing policy, as cities that are able to offer ample and affordable housing are better able to attract and retain cultural producers.

Table 1 The impact of σ and H_1 on population shares, moonlighting shares and regional rents under different equilibrium configurations. Note: $\beta = .4, \gamma = .25, \delta = .1, H_2 = 1$

	Initial	$\sigma = 1.3$	%Δ w.r.t σ		
Concentration ($H_1 = 1, \sigma = 1.2$)					
λ_{c1}^{CM}	1.000	1.000	0.00		
θ_1^{CM}	0.256	0.237	− 7.69		
λ_{t1}^{CM}	1.000	1.000	0.00		
λ_{m1}^{CM}	1.000	1.000	0.00		
r_1^{PSM}	0.769	0.769	0.00		
	Initial	$\sigma = 1.8$	%Δ w.r.t σ	$H_1 = 1$	%Δ w.r.t H_1
Partial segregation ($H_1 = 0.9, \sigma = 1.7$)					
$\lambda_{c1}^{\mathrm{PSM}}$	0.273	0.300	9.75	0.4615	68.83
θ_1^{PSM}	0.291	0.255	− 12.25	0.1961	− 32.51
θ_2^{PSM}	0.140	0.135	− 3.47	0.1681	20.24
$\lambda_{t1}^{\mathrm{PSM}}$	0.000	0.000	0.00	0.0000	0.00
$\lambda_{m1}^{\mathrm{PSM}}$	1.000	1.000	0.00	1.0000	0.00
r_1^{PSM}	0.338	0.344	1.97	0.3846	13.93
r_2^{PSM}	0.432	0.425	− 1.54	0.3846	− 10.90
	Initial	$\sigma = 1.8$	%Δ w.r.t σ	$H_1 = 1.3$	%Δ w.r.t H_1
Partial integration ($H_1 = 1.2, \sigma = 1.7$)					
$\lambda_{c1}^{\mathrm{PIM}}$	0.109	0.101	− 7.84	0.1174	7.22
θ_1^{PIM}	1.000	1.000	0.00	1.0000	0.00
θ_2^{PIM}	0.080	0.078	− 2.73	0.0721	− 10.25
$\lambda_{t1}^{\mathrm{PIM}}$	0.674	0.638	− 5.40	0.8009	18.75
$\lambda_{m1}^{\mathrm{PIM}}$	1.000	1.000	0.00	1.0000	0.00
r_1^{PIM}	0.388	0.378	− 2.42	0.3837	− 1.03
r_2^{PIM}	0.304	0.315	3.70	0.2704	− 11.05
	Initial	$\sigma = 1.8$	%Δ w.r.t σ	$H_1 = 2$	%Δ w.r.t H_1
Artist's colony ($H_1 = 1.9, \sigma = 1.7$)					
$\lambda_{c1}^{\mathrm{AC}}$	0.438	0.333	− 24.01	0.4905	12.10
θ_1^{AC}	0.228	0.272	19.09	0.2078	− 8.91
θ_2^{AC}	0.059	0.056	− 5.56	0.0588	0.00
$\lambda_{t1}^{\mathrm{AC}}$	1.000	1.000	0.00	1.0000	0.00
$\lambda_{m1}^{\mathrm{AC}}$	1.000	1.000	0.00	1.0000	0.00
r_1^{AC}	0.331	0.317	− 4.18	0.3209	− 3.00
r_2^{AC}	0.141	0.167	18.68	0.1274	− 9.41

"Initial" denotes the initial values of the variables evaluated using the parameter values in round brackets

To gauge the magnitude of the impacts of changes in the love of variety" and the supply of housing on each equilibrium configuration, we calibrate the model in Sect. 4. As motivated in introduction, the framework employed in Sect. 4 is the more empirically relevant as the majority of artists are reliant on outside employment opportunities to make ends meet. Parameters are chosen to be broadly realistic. We set the share of income devoted to cultural goods at $\delta = .1$ (Foster 2015; Bureau of Labor Statistics 2016b), consistent with the data presented above and the share of income devoted to housing, $\gamma = .25$ (Davis and Ortalo-Magne 2011). We choose $\beta = .4$ to ensure that Assumption 3 holds. Given that relative housing supply is the relevant parameter, we choose $H_2 = 1$ and vary H_1 to ensure that the restrictions in (56), (62) and (68) are met. Finally, there is little in the literature on the estimates of the elasticity of substitution for cultural goods. Therefore, we choose a value of $\sigma = 1.7$ which is the average of the estimates for varieties of shoes between the periods 1972 and 2001 from Broda and Weinstein (2006). The rationale for using estimates on varieties of shoes is that, similar to cultural goods, shoes are an item that consumers appear to have highly idiosyncratic preferences for and would thus benefit from having a number of available options. Additionally, we choose $\sigma = 1.2$ to analyze the concentrated equilibrium such that the restriction $\sigma < 1 + \delta/\gamma$ holds. The results are presented in Table 1.

The first set of values are the impact of σ on the equilibrium when the population is concentrated in a single region. It is clear that an increase in σ lowers the share of time artists devote to cultural production, consistent with a decline in the "love of variety." Because the change in σ does not alter the population distribution or wages, the remaining variables are left unaffected. In addition, when $\sigma > 1 + \delta/\gamma$ the choice of location for workers is independent of the supply of housing and thus has no impact on the equilibrium. In the second set of values are the simulation results for a partially segregated equilibrium which was shown to hold when each region has a similar supply of housing. An increase in σ leads to an increase in the number of artists and thus housing rent in the region with modern workers but there is a reduction in the time devoted to cultural production as demand for distinct varieties of cultural goods falls. While for artists in region 2 there is a decline in rents which leads to smaller decline in time to devoted to cultural production. However, in both scenarios artists spend more time on cultural production in the high-income region. An increase in the supply of housing in region 1 generates an increase in the number of cultural producers and housing rents; however, this increase in the population generates a reduction in θ_1^{PSM} due to the stiffer competition among artists. In contrast, the decline in the number of artists in region 2 reduces rents and competition. Furthermore, cultural producers in region 2 maintain a "captive audience," in that the number of traditional workers is unaffected by the change in the supply of housing. The cumulative effect is to increase θ_2^{PSM}.

In the third set of values are the simulation results for partial integration. Recall from Sect. 4.3 that under partial integration both θ_1 and θ_2 cannot be jointly determined. Here we consider the case where artists in region 1 devote all of their time to cultural production while those in region 2 are free to moonlight in the traditional sector. An increase in σ reduces the number of artists in region 1. This follows directly from the fact that θ_1^{PIM} is set at 1; thus, to accommodate the decline in demand for varieties some artists must move toward region 2. This leads to an increase in rents in region 2 which requires artists in the region to spend more time working in the traditional sector. Given that rents are higher in region 1, a decline in the “love of variety” motivates traditional workers to take advantage of lower rents in region 2. An increase in the supply of housing in region 1 increases the number of both artists and traditional workers. Interestingly, even with the increase in population there is still in decline in rents in the region. The loss of the population in region 2 lowers rents but reduces the aggregate demand for cultural goods leading to a decline in θ_2^{PIM}. This is in contrast to the case of partial segregation and points to the policy issue that changes in the supply of housing will have a different impact depending on whether the two cities are initially of similar size or initially one city is relatively larger than the other. The final set of values are the simulation results under the artist’s colony when region 1 has a significantly larger supply of housing. An increase in σ pushes cultural producers away from the larger region and toward the artist’s colony. The impact is to raise rents in region 2 and reduce the share of time that artists can devote to cultural production, while the opposite effect occurs in region 1. Finally, when region 1 sees an increase in the supply of housing under the artist’s colony equilibrium, the number of artists in region 1 rises which reduces θ_1^{AC} while housing rents decline in both regions.

As a final point, early empirical work on urban agglomeration economies tended to focus on city size as the determinant of economies of scale (See Combes and Gobillon 2015 for a survey). However, more recently there have been two important distinctions to be made. The first is that agglomeration economies may not be the same across industries (Abel et al. 2012). Cities that specialize in industries with stronger agglomeration economies will see greater economic gains from density and that such gains from density will not accrue evenly across the local population but instead depend on the industry makeup of a city. The second distinction is that the sorting among specific kinds of skilled workers plays an important role in knowledge spillover effects rather than city size per se (Combes et al. 2008). These results are consistent with the specification in our model where strong agglomeration economies in the modern sector lead to modern workers clustering in a single region, raising the average-income level in that city. The nominal wage gain for modern workers from clustering justifies the higher cost of living. However, there are no such wage gains for traditional workers who see their real wage decline and react by migrating to the opposite region. The model is thus able to replicate the displacement of lower-income workers, which in our model includes cultural producers, from cities that foster industries with strong agglomeration economies (Florida 2017).

6 Conclusion and future research

This paper has developed a spatial model with heterogeneous agents to explore the trade-offs between the market size in aggregate demand and market crowding effects in housing costs that cultural producers face in cities that foster industries with agglomeration economies. We first consider the case where all types of workers are interregionally mobile but intersectorally fixed. The conditions for the concentration, segregation and integration of workers were analyzed. We have shown that concentration is a stable equilibrium when the elasticity of substitution is sufficiently low. While if the elasticity of substitution is sufficiently high, partial segregation is stable when agglomeration economies in the modern sector are strong and each region has a similar supply of housing. Partially integrated equilibria are stable when one region has a significantly greater supply of housing. The model was extended to allow for cultural producers to moonlight in the traditional sector generating additional equilibrium outcomes. We construct an equilibrium in which a share of cultural producers are isolated in one region separate from a larger integrated market. Additionally we find an equilibrium where one region holds a share of artists that devote their time solely to cultural production, while the opposite region holds a share of artists that must moonlight in the traditional sector. Furthermore, under both a partially segregated and partially integrated equilibrium, we show that the region with the larger supply of housing contains a greater number of varieties of the cultural good.

We offer four suggestions for future research. The first is to allow cultural producers to "moonlight" in the modern sector. In practice, there are a number of industries that provide lucrative contract work for artists such as advertising, the film industry, and fashion. Additionally, such work is often more complementary to an artist's skill set. A second suggestion is to consider non-homothetic preferences over cultural goods which would allow for differences in consumption patterns for cultural goods across income groups as in Murata (2009) and may be applicable to less developed countries. The third suggestion is to endogenize housing supply. In the framework developed by Helpman (1998) and used extensively to provide a congestion effect in spatial models (see Redding and Rossi-Hansberg 2017), housing is an exogenous fixed factor. However, in practice we would expect that changes in the population distributions across regions would lead to new housing development to accommodate the change in demand. The fourth suggestion is to introduce a public sector and consider how the spatial organization of households leads to changes in the quality of the housing supply. In the model presented above, housing quality and quantity are fixed. However, the quality of housing and the level of public services in a community are determined by its income level. Therefore, by introducing a public sector that attempts to maximize the local utility level through the provision of public services, given the income constraint of the community, we could more fully explore issues of blight and gentrification associated with the migration patterns of cultural producers.

Acknowledgements I would like to thank an editor of this journal, Janet E. Kohlhase, and two anonymous referees for helpful comments that helped improve the quality of the manuscript. In addition, I would also like to thank John Carruthers for helpful suggestions at the 2017 WRSA meeting in Santa Fe and Richard Arnott for comments on an early draft of the paper. Finally, I would like to thank the Western Regional Science Association. This manuscript received the Springer Award for best paper by an early career scholar at the 56st Annual WRSA Meeting, Santa Fe, NM 2017.

Appendix

The instability of an integrated equilibrium

We show that when $\sigma > 1 + \delta/\gamma$ an integrated equilibrium is unstable. To begin, we consider an equilibrium in which $\dot{\lambda}_j = 0\ \forall j$. This implies that $P_1 = P_2$, $w_{c1} = w_{c2}$ and $A_{m1} = A_{m2}$ and the distribution is given by

$$\lambda_{c1}^{I} = \left(\frac{\frac{\beta\sigma}{\sigma(1-\beta)-\delta} + \left(\frac{H_2}{H_1}\right)^{\psi}}{\frac{\beta\sigma}{\sigma(1-\beta)-\delta} + 1} \right) \frac{H_1^{\psi}}{H_1^{\psi} + H_2^{\psi}}, \tag{70}$$

$$\lambda_{t1}^{I} = \frac{\frac{1}{2}\frac{\beta\sigma}{\sigma(1-\beta)-\delta}(H_1^{\psi} - H_2^{\psi}) + H_2^{\psi}}{H_1^{\psi} + H_2^{\psi}}, \tag{71}$$

$$\lambda_{m1}^{I} = \frac{1}{2}, \tag{72}$$

$$\psi \equiv \frac{\gamma(\sigma-1)}{\gamma(\sigma-1)-\delta}, \tag{73}$$

where I is a mnemonic for integration. The Jacobian matrix of first partial derivatives for Eqs. (21)–(23) and the corresponding signs are given by

$$J = \begin{bmatrix} \frac{\partial\dot{\lambda}_m}{\partial\lambda_{m1}} & \frac{\partial\dot{\lambda}_m}{\partial\lambda_{t1}} & \frac{\partial\dot{\lambda}_m}{\partial\lambda_{c1}} \\ \frac{\partial\dot{\lambda}_t}{\partial\lambda_{m1}} & \frac{\partial\dot{\lambda}_t}{\partial\lambda_{t1}} & \frac{\partial\dot{\lambda}_t}{\partial\lambda_{c1}} \\ \frac{\partial\dot{\lambda}_c}{\partial\lambda_{m1}} & \frac{\partial\dot{\lambda}_c}{\partial\lambda_{t1}} & \frac{\partial\dot{\lambda}_c}{\partial\lambda_{c1}} \end{bmatrix}, \quad \begin{bmatrix} ? & - & + \\ - & - & + \\ + & + & - \end{bmatrix} \tag{74}$$

The ambiguity of $\frac{\partial\dot{\lambda}_m}{\partial\lambda_{m1}}$ arises from the competing forces of the external economies which both raise the nominal wage and increase rents. Stability of the equilibrium requires all eigenvalues to be negative when evaluated at the equilibrium values. Given that the determinant of a matrix is equal to the product of its eigenvalues, a necessary (though not sufficient) condition for the stability of the Jacobian is that the determinant then be negative. We now show that the determinant is positive. It is useful to calculate the determinant using the bottom row of J and note that $V_{mi} = w_{mi}V_{ti}$ and that $w_{m1} = p_m A_{m1} = p_m A_{m2} = w_{m2}$ when $\lambda_{m1} = 1/2$. Furthermore $\frac{\partial w_{m1}}{\partial\lambda_{m1}}|_{\lambda_{m1}=1/2} = \frac{\partial w_{m2}}{\partial\lambda_{m1}}|_{\lambda_{m1}=1/2}$, and $V_{j1} = V_{j2} = V_j$, in equilibrium. Denoting the determinant of J by Δ yields

$$\Delta = \frac{\partial\dot{\lambda}_c}{\partial\lambda_{m1}}A - \frac{\partial\dot{\lambda}_c}{\partial\lambda_{t1}}B + \frac{\partial\dot{\lambda}_c}{\partial\lambda_{c1}}C, \tag{75}$$

where

$$\begin{aligned} A &= \frac{\partial \dot{\lambda}_m}{\partial \lambda_{t1}}\frac{\partial \dot{\lambda}_t}{\partial \lambda_{c1}} - \frac{\partial \dot{\lambda}_m}{\partial \lambda_{c1}}\frac{\partial \dot{\lambda}_t}{\partial \lambda_{t1}} \\ &= w_{m1}\frac{\partial \dot{\lambda}_t}{\partial \lambda_{t1}}\frac{\partial \dot{\lambda}_t}{\partial \lambda_{c1}} - w_{m1}\frac{\partial \dot{\lambda}_t}{\partial \lambda_{c1}}\frac{\partial \dot{\lambda}_t}{\partial \lambda_{t1}} \\ &= 0, \end{aligned} \tag{76}$$

$$\begin{aligned} B &= \frac{\partial \dot{\lambda}_m}{\partial \lambda_{m1}}\frac{\partial \dot{\lambda}_t}{\partial \lambda_{c1}} - \frac{\partial \dot{\lambda}_m}{\partial \lambda_{c1}}\frac{\partial \dot{\lambda}_t}{\partial \lambda_{m1}} \\ &= \left(2\frac{\partial w_{m1}}{\partial \lambda_{m1}}|_{\lambda_{m1}=1/2}V_t + w_{m1}\frac{\partial \dot{\lambda}_t}{\partial \lambda_{m1}}\right)\frac{\partial \dot{\lambda}_t}{\partial \lambda_{c1}} - w_{m1}\frac{\partial \dot{\lambda}_t}{\partial \lambda_{c1}}\frac{\partial \dot{\lambda}_t}{\partial \lambda_{m1}} \\ &= 2\frac{\partial w_{m1}}{\partial \lambda_{m1}}|_{\lambda_{m1}=1/2}V_t\frac{\partial \dot{\lambda}_t}{\partial \lambda_{c1}} \\ &= \frac{4}{3}V_m\frac{\partial \dot{\lambda}_t}{\partial \lambda_{c1}}, \end{aligned} \tag{77}$$

$$\begin{aligned} C &= \frac{\partial \dot{\lambda}_m}{\partial \lambda_{m1}}\frac{\partial \dot{\lambda}_t}{\partial \lambda_{t1}} - \frac{\partial \dot{\lambda}_m}{\partial \lambda_{t1}}\frac{\partial \dot{\lambda}_t}{\partial \lambda_{m1}} \\ &= \left(2\frac{\partial w_{m1}}{\partial \lambda_{m1}}|_{\lambda_{m1}=1/2}V_t + w_{m1}\frac{\partial \dot{\lambda}_t}{\partial \lambda_{m1}}\right)\frac{\partial \dot{\lambda}_t}{\partial \lambda_{t1}} - w_{m1}\frac{\partial \dot{\lambda}_t}{\partial \lambda_{t1}}\frac{\partial \dot{\lambda}_t}{\partial \lambda_{m1}} \\ &= \frac{4}{3}V_m\frac{\partial \dot{\lambda}_t}{\partial \lambda_{t1}}. \end{aligned} \tag{78}$$

We can then rewrite the determinant as

$$\begin{aligned} \Delta &= \frac{4}{3}V_m\left(-\frac{\partial \dot{\lambda}_c}{\partial \lambda_{t1}}\frac{\partial \dot{\lambda}_t}{\partial \lambda_{c1}} + \frac{\partial \dot{\lambda}_c}{\partial \lambda_{c1}}\frac{\partial \dot{\lambda}_t}{\partial \lambda_{t1}}\right) \\ &= \frac{4}{3}V_m\left(-(1-\gamma)\left(\frac{1}{\lambda_m 1 w_{m1} + \lambda_{t1}} + \frac{1}{(1-\lambda_{m1})w_{m2} + \lambda_{t1}}\right)V_c\right)\left(\frac{\delta}{\sigma-1}\left(\frac{1}{\lambda_{c1}} + \frac{1}{1-\lambda_{c1}}\right)V_t\right) \\ &\quad + \left(\frac{1+\delta-\sigma}{1-\sigma}\left(\frac{1}{\lambda_{c1}} + \frac{1}{1-\lambda_{c1}}\right)V_c\right)\left(\gamma\left(\frac{1}{\lambda_m 1 w_{m1} + \lambda_{t1}} + \frac{1}{(1-\lambda_{m1})w_{m2} + \lambda_{t1}}\right)V_t\right) \\ &= \frac{4}{3}V_m V_t V_c\left(\frac{1}{\lambda_{m1} w_{m1} + \lambda_{t1}} + \frac{1}{(1-\lambda_{m1})w_{m2} + \lambda_{t1}}\right)\left(\frac{1}{\lambda_{c1}} + \frac{1}{1-\lambda_{c1}}\right) \\ &\quad \times \left(\frac{\delta(1-\gamma)}{1-\sigma} + \frac{\gamma(1+\delta-\sigma)}{1-\sigma}\right) \end{aligned} \tag{79}$$

The sign of Δ is determined by the sign of the final term in brackets, which can readily shown to be positive when $\sigma > 1 + \delta/\gamma$.

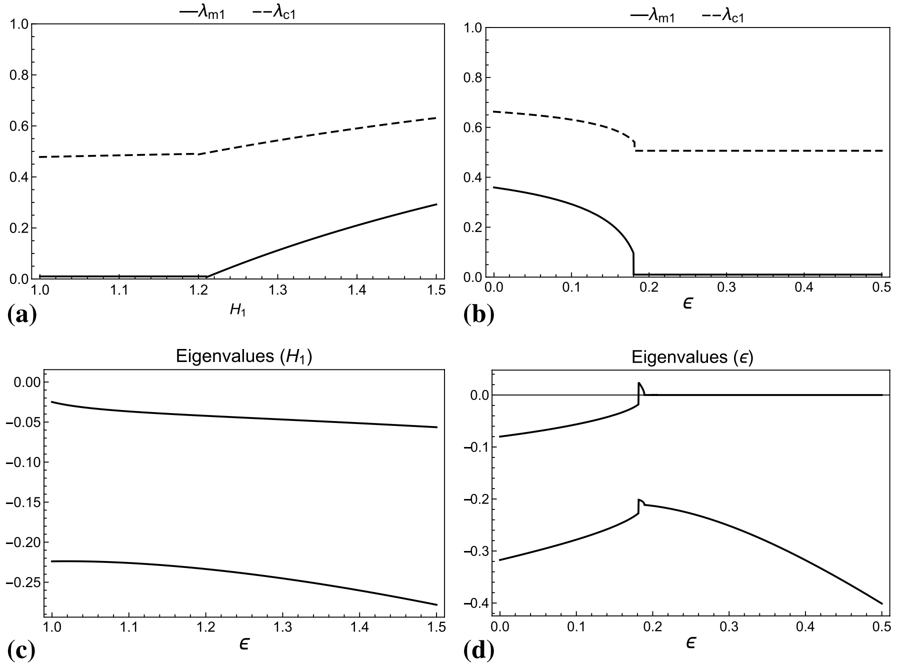

Fig. 2 Simulations of the equilibrium population distributions of modern and cultural workers and the associated eigenvalues for the Jacobian of the utility gaps. Note: $\beta = .5$, $H_2 = 1$, $\gamma = .25$, $\delta = .1$, $A_m = 1$

Simulations for partial integration of modern workers

This section provides simulations of the population distributions and the associated eigenvalues of the Jacobian matrix for the equations of motion under the partial integration of modern workers.

Figure 2a, b provides the population distributions. Figure 2c, d simulates the eigenvalues of the Jacobian matrix from (22) and (23) as the parameters H_1 and ϵ are increased. With regard to H_1, the eigenvalues are negative over the region where both $\lambda_{c1} > 0$ and $\lambda_{m1} > 0$. When $\lambda_{m1} = 0$ the stability of the equilibrium is laid out in the condition for partial segregation. We see that for low values of ϵ the eigenvalues are negative but as scale economies increase one eigenvalue becomes positive just as the number of modern workers in region 1 goes to 0.

Stability of a partially integrated equilibrium of traditional workers under moonlighting

Under this configuration, we will consider the case that $\theta_1 = 1, \theta_2 \in (0, 1)$ and $\lambda_{m1} = 1$. Therefore, the wage for cultural producers in region 1 and the price for the modern good are determined by the market clearing conditions in (14) and (16). Noting that $w_{c2} = 1$ we can then write the wage for modern workers and cultural producers as

$$p_m A_{m1} = \frac{\beta(\sigma-\delta)}{\sigma(1-\beta)-\delta}\left(1+(1-\lambda_{c1})+\frac{\delta}{\sigma-\delta}\lambda_{t1}\right), \tag{80}$$

$$w_{c1} = \frac{\delta}{\sigma(1-\beta)-\delta}\left(\frac{\beta(1+(1-\lambda_{c1}))+(1-\beta)\lambda_{t1}}{\lambda_{c1}}\right). \tag{81}$$

The equation of motion for traditional and cultural workers is then given by

$$\begin{aligned}\dot{\lambda}_c = & \frac{\eta}{p_m^\beta}\left(A_{c1}\left(\frac{\left(\beta(1+(1-\lambda_{c1}))+(1-\beta)\lambda_{t1}\right)^{1-\gamma}H_1^\gamma}{\lambda_{c1}^{\frac{\sigma-1-\delta}{\sigma-1}}}\right)\right. \\ & \left.-A_{c2}\left(\frac{H_2^\gamma}{\left((1-\lambda_{c1})+(1-\lambda_{t1})\right)^{\frac{\gamma(\sigma-1)-\delta}{\sigma-1}}}\right)\right),\end{aligned} \tag{82}$$

$$\begin{aligned}\dot{\lambda}_t = & \frac{\eta}{p_m^\beta}\left(A_{t1}\left(\frac{H_1^\gamma}{\left(\beta(1+(1-\lambda_{c1}))+(1-\beta)\lambda_{t1}\right)^{\gamma}\lambda_{c1}^{\frac{\delta}{1-\sigma}}}\right)\right. \\ & \left.-A_{t2}\left(\frac{H_2^\gamma}{\left((1-\lambda_{c1})+(1-\lambda_{t1})\right)^{\frac{\gamma(\sigma-1)-\delta}{\sigma-1}}}\right)\right),\end{aligned} \tag{83}$$

where A_{ij} are a bundle of constants that are independent of λ_{t1} and λ_{c1}. It is straightforward to verify that $\frac{\partial\dot{\lambda}_c}{\partial\lambda_{c1}} < 0$ and $\frac{\partial\dot{\lambda}_t}{\partial\lambda_{t1}} < 0$ when $\sigma > 1 + \delta/\gamma$, so that the trace of the Jacobian is negative. We now show that the determinant of the Jacobian is positive, implying that both eigenvalues are negative and that the equilibrium is stable. Denote the determinant of the Jacobian as Δ^{PITM} where PITM denotes partial integration of traditional workers with moonlighting. We then have

$$\begin{aligned}
\Delta^{\mathrm{PITM}} &= \frac{\partial \dot{\lambda}_c}{\partial \lambda_{c1}} \frac{\partial \dot{\lambda}_t}{\partial \lambda_{t1}} - \frac{\partial \dot{\lambda}_c}{\partial \lambda_{t1}} \frac{\partial \dot{\lambda}_t}{\partial \lambda_{c1}} \\
&= \left(\frac{\beta(1-\gamma)}{\beta(1+(1-\lambda_{c1}))+(1-\beta)\lambda_{t1}} + \frac{\sigma-1-\delta}{\sigma-1} \frac{1}{\lambda_{c1}} + \frac{\gamma(\sigma-1)-\delta}{\sigma-1} \frac{1}{(1-\lambda_{c1})+(1-\lambda_{t1})} \right) V_c \\
&\quad \times \left(\frac{\gamma(1-\beta)}{\beta(1+(1-\lambda_{c1}))+(1-\beta)\lambda_{t1}} + \frac{\gamma(\sigma-1)-\delta}{\sigma-1} \frac{1}{(1-\lambda_{c1})+(1-\lambda_{t1})} \right) V_t \\
&\quad - \left(\frac{(1-\beta)(1-\gamma)}{\beta(1+(1-\lambda_{c1}))+(1-\beta)\lambda_{t1}} - \frac{\gamma(\sigma-1)-\delta}{\sigma-1} \frac{1}{(1-\lambda_{c1})+(1-\lambda_{t1})} \right) V_c \\
&\quad \times \left(\frac{\delta}{\sigma-1} \frac{1}{\lambda_{c1}} + \frac{\beta\gamma}{\beta(1+(1-\lambda_{c1}))+(1-\beta)\lambda_{t1}} - \frac{\gamma(\sigma-1)-\delta}{\sigma-1} \frac{1}{(1-\lambda_{c1})+(1-\lambda_{t1})} \right) V_t \\
&= V_t V_c \Bigg(\frac{\gamma(\sigma-1)-\delta}{\sigma-1} \frac{1}{(1-\lambda_{c1})+(1-\lambda_{t1})} \bigg(\frac{\beta(1-\gamma)}{\beta(1+(1-\lambda_{c1}))+(1-\beta)\lambda_{t1}} + \frac{\sigma-1-\delta}{\sigma-1} \frac{1}{\lambda_{c1}} \\
&\quad + \frac{\delta}{\sigma-1} \frac{1}{\lambda_{c1}} + \frac{(1-\beta)\gamma}{\beta(1+(1-\lambda_{c1}))+(1-\beta)\lambda_{t1}} \bigg) + \frac{\gamma(\sigma-1)-\delta}{\sigma-1} \frac{1}{(1-\lambda_{c1})+(1-\lambda_{t1})} \\
&\quad \times \left(\frac{\gamma(1-\beta)+\beta(1-\gamma)}{\beta(1+(1-\lambda_{c1}))+(1-\beta)\lambda_{t1}} \right) \\
&\quad + \frac{(1-\beta)}{\sigma-1} \frac{1}{\lambda_{c1}} \frac{1}{\beta(1+(1-\lambda_{c1}))+(1-\beta)\lambda_{t1}} (\gamma(\sigma-1)-\delta) \Bigg).
\end{aligned} \tag{84}$$

If the final term in brackets is positive, the determinant is positive, which holds when $\sigma > 1 + \delta/\gamma$, indicating that the equilibrium is stable.

Instability of equilibrium for partial integration of modern workers with labor choice

In this section, we verify the instability of the equilibrium in which both modern and cultural workers are divided between regions, while traditional workers are concentrated in a single region. Suppose that $\lambda_{t1} = 1$. Given that when cultural workers split their time between industries they receive the same utility as traditional workers. Then in equilibrium it must be that case that $\dot{\lambda}_t = 0$ if $\dot{\lambda}_c = 0$, which implies that the price index is equivalent in each region. This further implies modern workers are equally divided between regions in an interior equilibrium so that $\lambda_{mi} = 1/2$. Define regional income as

$$I_i \equiv \lambda_{mi} p_m A_{mi} + \lambda_{ci} + \lambda_{ti}. \tag{85}$$

We can then write the equations of motion for cultural and modern workers as

$$\dot{\lambda}_m = \kappa_m^C \left(\frac{(1+\lambda_{m1})^{\epsilon}}{I_1^{\frac{1-\sigma+\delta}{1-\sigma}}} - \frac{(1+(1-\lambda_{m1}))^{\epsilon}}{I_2^{\frac{1-\sigma+\delta}{1-\sigma}}} \right), \tag{86}$$

$$\dot{\lambda}_c = \kappa_c^C \left(\frac{1}{I_1^{\frac{1-\sigma+\delta}{1-\sigma}}} - \frac{1}{I_2^{\frac{1-\sigma+\delta}{1-\sigma}}} \right). \tag{87}$$

Denote the determinant of the Jacobian Δ^{PIMM} where *PIMM* denotes partial integration of modern workers under moonlighting. In equilibrium $V_{m1} = V_{m2} = V_m$ and $V_{c1} = V_{c2} = V_c$. Furthermore denote,

$$\frac{d\lambda_{mi} p_m A_{mi}}{d\lambda_{m1}} = p_m A_{m1} \frac{\partial \lambda_{mi}}{\partial \lambda_{m1}} + \lambda_{mi} A_{m1} \frac{\partial p_m}{\partial \lambda_{m1}} + \lambda_{mi} p_m \frac{\partial A_{m1}}{\partial \lambda_{m1}},$$

where $\frac{\partial p_m}{\partial \lambda_{m1}}|_{\lambda_{m1}=1/2} = 0$. We then have

$$\begin{aligned}
\Delta^{PIMM} &= \frac{\partial \dot{V}_m}{\partial \lambda_{m1}} \frac{\partial \dot{V}_c}{\partial \lambda_{c1}} - \frac{\partial \dot{V}_m}{\partial \lambda_{c1}} \frac{\partial \dot{V}_c}{\partial \lambda_{m1}} \\
&= -\left(\frac{4\epsilon}{3} - \frac{1-\sigma+\delta}{1-\sigma} \left(\frac{\frac{d\lambda_{m1} p_m Am1}{d\lambda_{m1}}}{I_1} + \frac{\frac{d\lambda_{m2} p_m A_{m2}}{d\lambda_{m1}}}{I_2} \right) \right) V_m \times \left(\frac{1-\sigma+\delta}{1-\sigma} \left(\frac{1}{I_1} + \frac{1}{I_2} \right) \right) V_c \\
&\quad - \left(\frac{1-\sigma+\delta}{1-\sigma} \left(\frac{1}{I_1} + \frac{1}{I_2} \right) \right) V_m \times \left(\frac{1-\sigma+\delta}{1-\sigma} \left(\frac{\frac{d\lambda_{m1} p_m Am1}{d\lambda_{m1}}}{I_1} + \frac{\frac{d\lambda_{m2} p_m A_{m2}}{d\lambda_{m1}}}{I_2} \right) \right) V_c \\
&= -\frac{4\epsilon}{3} \left(\frac{1-\sigma+\delta}{1-\sigma} \left(\frac{1}{I_1} + \frac{1}{I_2} \right) \right) V_m V_c < 0.
\end{aligned} \tag{88}$$

The result implies that one eigenvalue must be positive; therefore, any such equilibrium is unstable.

References

Abel JR, Dey I, Gabe TM (2012) Productivity and the density of human capital. J Reg Sci 52:562–586

Alper NO, Wassall GH (2000) More than once in a blue moon: multiple jobholdings by American artists, national endowment for the arts. Seven Locks Press, Santa Ana

Alper NO, Wassal GH (2006) Artists' careers and their labor markets. In: Reprinted in Ginsburgh and Throsby (eds) Handbook of the economics of arts and culture. Elsevier, New York, pp 813–864

Alper NO, Wassall GH, Jeffri J, Greenblat R, Kay AO, Butcher SGW, Chartrand HH (1996) Artists in the work force: employment and earnings 1970 to 1990. NEA Research Division Report No. 37. National Endowment for the Arts, Washington

Broda C, Weinstein DE (2006) Globalization and the gains from variety. Q J Econ 121:541–585

Bureau of Labor Statistics, U.S. Department of Labor (2015) Consumer spending on entertainment by household income in 2013. The Economics Daily. Retrieved from https://www.bls.gov/opub/ted/2015/consumer-spending-on-entertainment-by-household-income-in-2013.htm. Visited May 11, 2017

Bureau of Labor Statistics, U.S. Department of Labor (2016a) High-income households spent half of their food budget on food away from home in 2015. The Economics Daily. Retrieved from https://www.bls.gov/opub/ted/2016/high-income-households-spent-half-of-their-food-budget-on-food-away-from-home-in-2015.htm. Visited June 03, 2018

Bureau of Labor Statistics, US Department of Labor (2016b) Consumer expenditures-2015. Retrieved from https://www.bls.gov/news.release/pdf/cesan.pdf. Visited June 3, 2018
Collegiate Employment Research Institute (CERI) (2017) Recruiting trends 2016–2017, 46th edition. Brief 3: starting salaries. Retrieved from http://www.ceri.msu.edu/wp-content/uploads/2016/10/Recruiting-Trends-2016-17-Brief-3-Starting-Salaries-10-2-16.pdf. Visited June 20, 2018
Combes PP, Gobillon L (2015) The empirics of agglomeration economies. In: Duranton G, Henderson V, Strange W (eds) Handbook of Regional and Urban Economics, vol 5A. Elsevier, Amsterdam, pp 247–348
Cowen T, Tabarrok A (2000) An economic theory of avant-garde and popular art, or high and low culture. South Econ J 67:232–253
Combes PP, Duranton G, Gobillon L (2008) Spatial wage disparities: sorting matters!. J Urban Econ 63:723–742
Davis MA, Ortalo-Magne F (2011) Household expenditures, wages and rents. Rev Econ Dyn 14:248–261
Dixit AK, Stiglitz JE (1977) Monopolistic competition and optimum product diversity. Am Econ Rev 67:297–308
Ewing H, Grady R, directors (2012) Detropia. Loki Films
Florida R (2011) The rise of the creative class revisited. Basic Books, New York
Florida R (2017) The new urban crisis. Basic Books, New York
Forslid R, Ottaviano GIP (2003) An analytically solvable core-periphery model. J Econ Geogr 3:229–240
Foster AC (2015) Movies, music, and sports: U.S. entertainment spending, 2008–2013. Beyond the numbers: prices and Spending 6. Retrieved from https://www.bls.gov/opub/btn/volume-4/movies-music-sports-entertainment-spending.htm
Glaeser E (2011) Triumph of the city. Penguin Books, New York
Glaeser EL, Kolko J, Saiz A (2001) Consumer city. J Econ Geogr 1:27–50
Glaeser EL, Gyourko J, Saks RE (2006) Urban growth and housing supply. J Econ Geogr 6:71–89
Helpman E (1998) The size of regions. In: Reprinted in Pines, Sadka, Zilcha (eds) Topics in public economics. Cambridge University Press, Cambridge, pp 33–54
Krueger AB, Zhen Y (2018) Inaugural music industry research association (MIRA) survey of Musicians. Retrieved from Music Industry Research Association Website https://img1.wsimg.com/blobby/go/53aaa2d4-793a-4400-b6c9-95d6618809f9/downloads/1cgjrbs3b_761615.pdf. Visited June 25th, 2018
Murata Y, Thisse J-F (2005) A simple model of economic geography à la Helpman–Tabuchi. J Urban Econ 58:137–155
Murata Y (2009) On the number and composition of varieties. Econ J 119:1065–1087
Ottaviano G, Tabuchi T, Thisse JF (2002) Agglomeration and trade revisited. Int Econ Rev 43:409–435
Pflüger M, Süedekum J (2007) Integration, agglomeration and welfare. J Urban Econ 63:544–566
Pflüger M, Tabuchi T (2010) The size of regions with land use for production. Reg Sci Urban Econ 40:481–489
Redding SJ, Rossi-Hansberg E (2017) Quantitative spatial economics. Annu Rev Econ 9:21–58
Saks RE (2004) Housing supply restrictions across the United States. Wharton Real Estate Review, Fall
Saks RE (2008) Job creation and housing construction: constraints on metropolitan area employment growth. J Urban Econ 64:178–195
Schiff N (2015) Cities and product variety: evidence from restaurants. J Econ Geogr 15:1085–1123
Tabuchi T (1998) Urban agglomeration and dispersion: a synthesis of Alonso and Krugman. J Urban Econ 44:333–351
Tabuchi T, Zeng D (2004) Stability of spatial equilibrium. J Reg Sci 44:641–660
Throsby D (1992) Artists as workers. In: Reprinted in Towse and Khakee (eds) Cultural economics. Springer, New York, pp 201–208
Throsby D (1994) The production and consumption of the arts: a view of cultural economics. J Econ Lit 32:1–29
Trust The Pew Charitable (2016) Household expenditures and income. Pew Research Center, Washington
United Nations, Department of Economic and Social Affairs, Population Division (2015) World urbanization prospects: the 2014 revision (ST/ESA/SER.A/366)

Publisher's Note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.