

# Interurban population distribution and commute modes

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**Abstract** This paper explores the interplay between trade costs and urban costs within a new economic geography model in which workers are mobile. As in former research, we show that workers tend at the same time to agglomerate in order to limit trade costs of manufactured goods and to scatter in order to alleviate the burden of urban costs due to large urban areas. In this paper, special attention is paid to the role of congestion, which acts as a dispersion force and hampers workers from agglomerating in the same urban area. We show that the development of public transport, or the construction of road infrastructure, modifies the spatial organization of the economy and fosters agglomeration, as it reduces congestion.

**JEL Classification** R120 Size and Spatial Distributions of Regional Economic Activity · R410 Transportation: Demand, Supply, and Congestion; Travel Time; Safety and Accidents; Transportation Noise · R300 Real Estate Markets, Spatial Production Analysis, and Firm Location: General

## 1 Introduction

Spatial organizations of industrial activities across the world emerge from the interplay between dispersion and agglomeration forces. Explaining these forces is the main focus of the new economic geography. The canonical model of this field, developed by [Krugman \(1991\)](#) and known as the Core–Periphery model, relies on a dispersion

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force which is rooted in the agricultural sector whose share in employment and expenditure has sharply decreased in most industrialized countries, and thus does not fit well contemporary space economies. Nowadays, the main dispersion force seems to lie in the existence of urban costs borne by workers living in large urban agglomerations (Murata and Thisse 2005). Excess concentration in large cities brings negative externalities, notably due to longer commuting costs and scarce land for housing.

Housing and commuting costs are interrelated and shape the spatial structure of urban agglomerations. Workers seek to reduce their commuting costs by choosing a living place in the vicinity of their working place. However, because of the scarcity of land, everybody cannot live close to its job. Competition for land among workers gives rise to a land rent that varies inversely with the distance to the city center, thereby compensating workers that live far from their workplace and have to commute between the workplace and their living place. In other words, there is a trade-off between commuting and housing costs: The former increases with distance, whereas the latter decreases (Fujita 1989).

Numerous economic geography models have been built in order to address urban costs. Helpman (1998) first dealt with urban costs and introduced a housing market into an economic geography model in which all workers are mobile. However, commuting costs were not considered because of the absence of an explicit spatial extension of cities. Independently, Tabuchi (1998) developed a model in which both housing and commuting costs were described. In Tabuchi's analysis, commuting costs combine time and pecuniary costs. Unfortunately, his model had a significant shortcoming: Analytical results are available only for two extreme cases. Few years later, Murata and Thisse (2005) overcame this difficulty and introduced iceberg commuting costs affecting labor supplied by a worker, and thus its income. When agglomeration occurs, the city spread, commuting costs increase and labor supply shrinks. The idea is that if workers save time on commuting, more labor is available for production. These three models, and all the following others, showed the impacts of urban costs on the economic spatial organization. However, as far as we know, none of them deals explicitly with transport congestion.

Congestion is an important component of commuting costs, as it increases fuel consumption and commuting times. In 2011, the Texas Transportation Institute estimates that congestion caused urban Americans to purchase an extra 2.9 billion gallons of fuel and to travel 5.5 billion hours more to drive the same commuting distances without congestion, for a congestion cost of \$121 billion, namely up to 25% of car running costs (Schrang et al. 2012). It also has major public health impacts. Sitting in traffic leads to higher tailpipe emissions which everyone is exposed to. Currie and Walker (2009) show that congestion enhances prematurity and low birth weight among mothers living close to congested roads. Road traffic congestion poses thus a challenge for all large and growing urban areas, especially as it has increased substantially over the last decades (OECD 2007). It is a major issue for urban decision makers, and it needs to be further studied in order to highlight transport policies.

The aim of this paper is to propose a model of economic geography integrating transport congestion. As it is standard, the agglomeration force finds its origin in the need to reduce trade costs of manufactured goods between the two regions, and the main dispersion forces stem from land consumption and the resulting need for

workers to commute. We retain the general equilibrium framework of monopolistic competition *la Dixit–Stiglitz* with the standard iceberg trade costs *la Samuelson*. But, instead of iceberg commuting costs, as in [Murata and Thisse \(2005\)](#), or generalized commuting costs, as in [Tabuchi \(1998\)](#), we introduce a time constraint. [Zahavi and Talvitie \(1980\)](#) considered first the existence of a transport-specific time constraint as a fundamental travel behavior. The idea is that households need to spend time commuting to do most of their activities (work, leisure, shopping) and they invest a nearly constant travel-time budget per day for transportation, on average, despite widely differing transportation infrastructures, geographies, cultures and per capita income levels ([Schafer and Victor 2000](#)). This travel-time budget captures the well-known idea that a rise of commuting speed leads to increase in distance travelled and not less commuting time ([Levinson and Kumar 1994](#)). The assumption of the time constraint allows us to introduce the speed of transport modes within a city. Congestion reduces road transport speeds depending on the capacity of the urban road network, which is captured by a macroscopic fundamental diagram. A reproducible macroscopic fundamental diagram relates the number of circulating vehicles to the average commuting speed on large urban areas ([Daganzo et al. 2011](#)). The idea of such a diagram is quite old ([Godfrey 1970](#)), but the verification of its existence is recent ([Geroliminis and Sun 2011](#)).

This paper explores the interplay between trade costs of manufactured goods between the two region and workers' urban costs. The results suggest that housing and commuting costs act as a dispersion force and modifies the spatial organization of the economy. Workers tend at the same time to agglomerate in order to limit trade costs of manufactured costs and to scatter in order to alleviate the burden of urban costs due to large urban areas. Like [Murata and Thisse \(2005\)](#), agglomeration appears to be a stable equilibrium when trade costs are large but dispersion prevails when they are low. Indeed, the decline of trade costs reduces the need of being located in the largest market, but it has no direct impact on workers' commuting and housing costs. However, unlike [Murata and Thisse \(2005\)](#), partial agglomeration in one region can be a stable equilibrium when trade costs are large. One of the most striking features of many new economic geography models—probably because it is so unexpected—is that, beyond a given value of trade costs, catastrophic agglomeration occurs. The only stable outcome is then the full agglomeration in one region. This unsatisfactory result has been overcome, in particular with the works of [Murata \(2003\)](#) and [Tabuchi and Thisse \(2002\)](#), which assume individual taste heterogeneities in perceptions of regional differences. Our model allows also for partial agglomeration to be a stable equilibrium through the integration of transport congestion and housing demand. The results thus highlight the role of congestion which enhances dispersion forces in the spatial organization of the economy. The development of alternative transport mode less subject to congestion, or the construction of road infrastructure, may foster agglomeration, as it reduces congestion.

The model is introduced in Sect. 2. The properties of the spatial equilibrium with one transport mode are derived in Sect. 3 and with two mode in Sect. 4, whereas Sect. 5 concludes.

## 2 The model

### 2.1 The spatial economy

Consider an economy involving two regions (labeled  $r = 1, 2$  or  $j = 1, 2$ ) and one industrial sector producing numerous varieties,  $n_r$ , of a horizontally differentiated good. Any variety of this good, produced under monopolistic competition and increasing returns to scale, can be shipped from one region to the other according to iceberg transportation costs *à la Samuelson*:  $\tau > 1$  units of the variety must be sent from the origin for one unit to arrive at destination.

As in [Murata and Thisse \(2005\)](#), each region is formed by a city spread along a one-dimensional space  $X = [0, X_r]$ . All firms located in region  $r$  are set up at the central business district (in short CBD) situated at the origin  $x = 0$ . The economy is endowed with a unit mass of identical and mobile workers, which settle around the CBD and commute to the CBD for work or leisure. There is no interregional commuting. Each worker provides one unit of labor. Let  $\lambda$  denote the fraction of workers residing in region 1 so that the mass of workers in region 1 and 2 is, respectively, given by  $L_1 = \lambda$  and  $L_2 = 1 - \lambda$ .

### 2.2 Consumption

Within each region, workers choose their location facing a trade-off problem ([Alonso et al. 1964](#)) between accessibility to the CBD and surface of their housing. As accessibility decreases with distance to the CBD, incentives to move closer to the CBD are important. Nearby the CBD, competition for land is thus fiercer and housing prices increase. Workers bid against one another, paying higher rent for proximity to the center of business based on respective accessibility. In those places, workers afford thus a smaller housing surface. The equilibrium is reached within the city until workers have no more incentives to move, namely when their utilities are constant throughout the region. If workers settle far away from the CBD, they suffer poor accessibility, but they enjoy bigger dwellings, thanks to a low housing rent, whereas close to the CBD, they enjoy good accessibility, but suffer from high housing prices.

Accessibility includes both pecuniary and time costs associated with getting to and from work, visiting relatives and friends, shopping and other such activities. The accessibility is defined here as a number of journeys to the CBD a worker can take under a time constraint  $T$ . The existence of a transport-specific time constraint  $T$  was first suggested by [Zahavi and Talvitie \(1980\)](#) and was found to be nearly constant in numerous different metropolitan areas which differ from transportation infrastructure endowments, geographies, cultures and per capita income levels ([Schafer and Victor 2000](#)). For sake of simplicity, we neglect pecuniary costs associated with commuting. As we focus on congestion costs, which enhance mostly travel-time costs, the effect of pecuniary costs on workers' behavior does not change qualitatively our results. Indeed, extra pecuniary costs due to congestion are much lower than extra travel-time costs. According to the Texas Transportation Institute, in 2011, travel-time costs due

to congestion account for more than 90% of total congestion costs (Schrank et al. 2012).

In a location  $x$ , workers' utility depends on their consumption of differentiated goods  $C_r(x)$ , their consumption of housing surface  $h_r(x)$  and their accessibility to the CBD  $a_r(x)$ . In our model, the accessibility appears in the workers' utility function in order to take directly into account the time constraint,  $T$ . If workers use travel time saved through higher speeds or shorter commuting distances to visit more destinations, they increase their utility. In region  $r$ , workers located at the distance  $x$  from the CBD maximize the following utility function:

$$U_r(x) = C_r(x)^{(1-\delta^a-\delta^H)} h_r(x)^{\delta^H} a_r(x)^{\delta^a} \tag{1}$$

where  $\delta^a$  and  $\delta^H$  are the elasticities of utility with respect to accessibility and housing surfaces, respectively.

We assume that the distance from location  $x$  to the CBD is  $x$  as the city is spread along a one-dimensional space. Let be  $v_r(x)$  the average speed to reach the CBD from the location  $x$  in region  $r$ , the number of journeys to the CBD a worker can take under the time constraint  $T$  is<sup>1</sup>:

$$a_r(x) = \frac{v_r(x)T}{x}$$

Besides, the consumption of differentiated goods  $C_r(x)$  is made up of a number of differentiated varieties. Let  $\epsilon > 1$  be the substitution elasticity between two varieties and  $c_{jr}(x)$  the amount of a variety produced in region  $j$  consumed by an household located at distance  $x$  from the CBD of region  $r$ . Thus,

$$C_r(x) = \left( \sum_{j=1}^2 n_j (c_{jr}(x)^{\epsilon-1/\epsilon}) \right)^{\epsilon/\epsilon-1}$$

Utility maximization under time and budget constraints,  $T$  and  $Y_r$ :

$$\begin{cases} Y_r = P_r C_r(x) + R_r(x) h_r(x) \\ T = \frac{a_r(x)x}{v_r(x)} \end{cases}$$

<sup>1</sup> The city border is defined as the location  $X_r$  where a worker can exactly make a round trip within the time constraint  $T$ , namely  $a_r(X_r) = 2$ .

gives:

$$\begin{cases} c_{jr}(x) = \frac{1 - \delta^a - \delta^H}{1 - \delta^a} (\tau_{jr} p_j)^{-\epsilon} \frac{Y_r}{P_r^{1-\epsilon}} \\ h_r(x) = \frac{\delta^H}{1 - \delta^a} \frac{Y_r}{R_r(x)} \\ a_r(x) = \frac{v_r(x)T}{x} \end{cases} \quad (2)$$

where  $P_r$  is the price index of the differentiated good and  $R_r(x)$  the rent per surface unit. The price index is the same within a region and:

$$P_r^{1-\epsilon} = \sum_{j=1}^2 n_j (\tau_{jr} p_j)^{1-\epsilon} \quad (3)$$

where  $\tau_{jr} = \begin{cases} \tau & \text{if } j \neq r \\ 1 & \text{if } j = r \end{cases}$  and  $p_r$  the production cost of varieties in region  $r$ .

Each region is characterized by an housing supply at distance  $x$  from the CBD,  $H_r(x)$ . The housing market entails:

$$H_r(x) = h_r(x)L_r(x) \quad (4)$$

This equation sets the number of workers living at distance  $r$  from the CBD. As seen above, workers split up the housing supply in order their utility to be constant wherever they settle. We assume the housing supply to be constant within each region, as in [Helpman \(1998\)](#) and [Tabuchi \(1998\)](#).

## 2.3 Production

Each firm produces a single variety under monopolistic competition using labor as its sole input, as in [Murata and Thisse \(2005\)](#). The total number of varieties is thus fixed through the number of firms  $n_r$ . The production of a variety requires a fixed and a variable amount of labor. To produce  $q_r$  units of output,  $l^v q_r + l^f$  units of labor are required. The profit is as follows:

$$\pi_r = p_r q_r - w_r (l^v q_r + l^f)$$

where  $w_r$  is the wage. The price  $p_r$  that maximizes profits is determined by markup pricing over marginal costs, which is a standard rule in monopolistic competition:

$$p_r = \frac{\epsilon}{\epsilon - 1} l^v w_r \quad (5)$$

As in [Krugman \(1991\)](#), entry and exit of firms on the market are free so that profits are zero in equilibrium. The equilibrium output per firm in region  $r$  is then obtained

from the zero-profit condition and depends only on the local labor productivity and the substitution elasticity between varieties.

$$q_r = \frac{l^f (\epsilon - 1)}{l^v} \tag{6}$$

**2.4 Income and revenue allocations**

We consider each region as an independent jurisdiction that owns the land of its region. As a result, housing expenditures are shared between every region’s residents and added to the wage that each worker earns for providing one unit of labor (Helpman 1998). Accordingly, each worker receives an income equals to:

$$Y_r = w_r + \frac{\delta^H}{1 - \delta^a} Y_r \tag{7}$$

**2.5 Market clearing conditions**

The short-term equilibrium is characterized by equilibria on the differentiated good market and the labor market. On the differentiated good market, the production of every firm is consumed either in region of production or in the other region. The differentiated good market clearing condition is:

$$q_r = \sum_{j=1}^2 \int_X \tau_{rj} c_{rj}(x) L_j(x) dx \tag{8}$$

The labor market clearing condition in region *r* is given by the equality between active workers in the region and the labor needed to produce  $n_r q_r$  outputs.

$$L_r = n_r (l^v q_r + l^f)$$

This condition implies that any change in the population of workers in a given region must be accompanied by a corresponding change in the number of varieties. From (5) and (6), we obtain:

$$L_r = \epsilon l^f n_r \tag{9}$$

The labor market clearing condition and the zero-profit condition have major implications for our analysis in the long term: It is sufficient to describe the migration of workers because the supply of entrepreneurs in each region is supposed to be large enough for the zero-profit condition to be satisfied regardless of the number of workers. Workers are attracted by the region that yields the higher utility level. Workers who live in a region with a lower utility level gradually migrate to the region with the highest

utility level. The quantitative dynamic in the workers' migration is described by:

$$\frac{dL_r}{dt} = \frac{U_r - U_j}{\omega^U} \Big|_{j \neq r}$$

where  $\omega^U$  represents the time lag in the workers' migration. At the long-term equilibrium, their location choices end in:

$$\forall x, U_1(x) = U_2(x)$$

It is worth noting that firms' interests are seamlessly related to workers'. This derives from the fact that labor is the only productive factor.

### 3 The spatial equilibrium with one mode

#### 3.1 Assumptions and preliminary results

For the sake of simplicity, we assume that the two regions have exactly the same features, and differ only by their number of workers. Therefore, the symmetric configuration, where  $L_1 = L_2$ , appears always to be a spatial equilibrium. In this section, the only means of commuting from home to the CBD is supposed to be private cars. Recent experimental work has shown that the average speed and average users density within certain urban road networks are related by a unique, reproducible curve known as the macroscopic fundamental diagram (Geroliminis and Daganzo 2008). The empiric estimates of the macroscopic fundamental diagram show a smoothly declining curve (Geroliminis and Sun 2011). The average speed on the network decreases with the number of users due to congestion. We assume that the average speed of commuting at the distance  $x$  from the CBD has the following expression, plotted in Fig. 1.

$$v_r(x) = \frac{v_0}{1 + \left(\frac{L_r}{\kappa_r}\right)^\alpha} \tag{10}$$

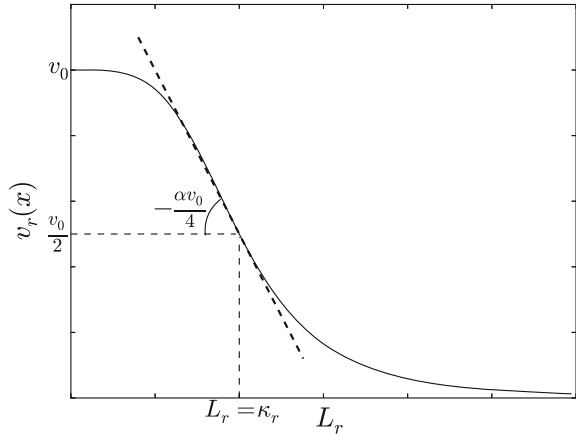
where  $v_0$  is the average speed without congestion,  $\alpha$  a parameter that captures the decline of the average speed with respect to the number of users of the road network and  $\kappa_r$  the infrastructure capacity of the urban road network of the region  $r$ . We assume that workers commute all at the same time, so the number of users of the road network is also the number of workers. Besides, as workers commute always for the same time  $T$ , the number of users of the road network does not depend on the number of journeys they make. The average commuting speed does not vary much when  $L_r \ll \kappa_r$ . But it drops dramatically when congestion occurs, i.e., when  $L_r \sim \kappa_r$ . An increase of the population in one region decreases the average commuting speed and the accessibility of workers. Congestion thus acts as a dispersion force. We assume that the average speed without congestion,  $v_0$ , is constant within each region.<sup>2</sup>

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<sup>2</sup> The city border is then equal to  $X_r = \frac{v_0 T}{2 \left(1 + \left(\frac{L_r}{\kappa_r}\right)^\alpha\right)}$ .



**Fig. 1** Macroscopic fundamental diagram



### 3.2 Symmetric configuration $\lambda = 1/2$

To start with, we focus on the symmetric configuration  $\lambda = 1/2$ . As seen above, this configuration is always a spatial equilibrium. To study its stability, we derive the elasticity of the utility in one region with respect to the number of workers in that region. From Eqs. (1), (2) and (4), the utility of a worker living at distance  $x$  from the CBD is:

$$U_r(x) = \left( \frac{1 - \delta^a - \delta^H}{1 - \delta^a} \frac{Y_r}{P_r} \right)^{1 - \delta^a - \delta^H} \left( \frac{H_r(x)}{L_r(x)} \right)^{\delta^H} \left( \frac{v_r(x)T}{x} \right)^{\delta^a}$$

The spatial equilibrium condition states that workers' utility is constant throughout the city. We then have:

$$\forall x, \left( \frac{H_r(x)}{L_r(x)} \right) \frac{1}{x^{\frac{\delta^a}{\delta^H}}} = \text{cte} \Leftrightarrow \frac{H_r(x)}{x^{\frac{\delta^a}{\delta^H}}} = \text{cte } L_r(x)$$

Integrating both sides, we obtain the constant term:

$$\frac{\int_X \frac{H_r(x)}{x^{\frac{\delta^a}{\delta^H}}} dx}{L_r} = \text{cte}$$

From Eq. (7), we eventually have:

$$U_r(x) = \left( \frac{w_r}{P_r} \right)^{1 - \delta^a - \delta^H} \left( \frac{\int_X \frac{H_r(x)}{x^{\frac{\delta^a}{\delta^H}}} dx}{L_r} \right)^{\delta^H} \left( \frac{v_0 T}{1 + \left( \frac{L_r}{\kappa_r} \right)^\alpha} \right)^{\delta^a}$$

The utility increases with housing supply, but the repartition of dwellings plays a role. The further a dwelling is built from the CBD, the less contribution to the utility it has. Indeed, a worker located close to the CBD would commute less than someone settled further. The weight of a given dwelling in the utility depends thus on the relative share of income workers allocate to housing  $\delta^H$  and to commuting  $\delta^a$ .

Totally differentiating  $U_r(x)$  and evaluating the resulting expression at  $\lambda = 1/2$ , we obtain

$$\frac{dU_r}{U_r} = (1 - \delta^a - \delta^H) \frac{dw_r}{w_r} - (1 - \delta^a - \delta^H) \frac{dP_r}{P_r} - \left( \delta^H + \delta^a \frac{\alpha(2\kappa_r)^{-\alpha}}{1 + (2\kappa_r)^{-\alpha}} \right) \frac{dL_r}{L_r} \tag{11}$$

As usually (Fujita et al. 2001), let

$$Z = \frac{1 - \tau^{1-\epsilon}}{1 + \tau^{1-\epsilon}}$$

$Z$  is an index of trade cost, with values between 0 and 1. If trade is perfectly costless,  $\tau = 1$ ,  $Z$  takes the value 0; if trade is impossible, it takes value 1. Then, Eqs. (5) and (8) imply

$$\left( \frac{\epsilon}{Z} - 1 \right) \frac{dw_r}{w_r} = (\epsilon - 1) \frac{dP_r}{P_r} + \frac{dL_r}{L_r} \tag{12}$$

Similarly, totally differentiating the price indexes (3) yields

$$\left( \frac{1 - \epsilon}{Z} \right) \frac{dP_r}{P_r} = (1 - \epsilon) \frac{dw_r}{w_r} + \frac{dL_r}{L_r} \tag{13}$$

Solving (12) and (13), we get

$$\frac{dw_r}{w_r} = \frac{Z}{\epsilon(Z + 1) - Z} \frac{dL_r}{L_r} \tag{14}$$

$$\frac{dP_r}{P_r} = - \frac{\epsilon Z}{(\epsilon - 1)(\epsilon(Z + 1) - Z)} \frac{dL_r}{L_r} \tag{15}$$

Inserting (14) and (15) into (11), we have the elasticity of the utility at  $\lambda = 1/2$ :

$$\frac{dU_r}{U_r} = (1 - \delta^a - \delta^H) \left( \frac{(2\epsilon - 1)Z}{(\epsilon - 1)(\epsilon(Z + 1) - Z)} - \frac{\delta^H + \delta^a \frac{\alpha(2\kappa_r)^{-\alpha}}{1 + (2\kappa_r)^{-\alpha}}}{1 - \delta^a - \delta^H} \right) \frac{dL_r}{L_r}$$

It is convenient to define the parameter  $\mu$  as follows:

$$\mu = \frac{1 - \delta^a - \delta^H}{\delta^H + \delta^a \frac{\alpha(2\kappa_r)^{-\alpha}}{1 + (2\kappa_r)^{-\alpha}}}$$

$\mu$  captures the burden of urban costs faced by workers in a given region, with values between zero and infinity. When housing surface and accessibility matter to workers, namely when  $\delta^H$  and  $\delta^a$  are large,  $\mu$  is small. In that case, an increase of urban costs alters significantly workers' utility. On the contrary, when housing surface and accessibility do not matter to workers, namely when  $\delta^H$  and  $\delta^a$  are small,  $\mu$  is large. An increase in urban costs has few impact on workers' utility.  $\mu$  depends also on the features of the urban road network. Indeed, commuting costs do not vary linearly with the population, unlike housing costs. The relation between commuting costs and the population relies on the macroscopic fundamental diagram. The burden of urban costs depends then on the elasticity of the average commuting speed with respect to the population,  $\alpha \frac{(2\kappa_r)^{-\alpha}}{1+(2\kappa_r)^{-\alpha}}$ . A strong dependence of the commuting speed to the population combined with a large  $\delta^a$  lessens  $\mu$ .

We can rewrite the elasticity of the utility at  $\lambda = 1/2$  as:

$$\frac{dU_r}{U_r} = \frac{(1 - \delta^a - \delta^H)}{\mu} \left( \mu \frac{(2\epsilon - 1)Z}{(\epsilon - 1)(\epsilon(Z + 1) - Z)} - 1 \right) \frac{dL_r}{L_r} \tag{16}$$

Equation (16) allows us to study the stability of the symmetric equilibrium.

**Proposition 1** *If  $\mu > \epsilon - 1$ , then there exists an unique  $\tau$ -break point given by,*

$$Z^* = \frac{\epsilon(\epsilon - 1)}{(2\epsilon - 1)\mu - (\epsilon - 1)^2}$$

*and the symmetric configuration is a stable equilibrium if and only if  $Z < Z^*$ . However, there exists no  $\tau$ -break point and the symmetric equilibrium is always stable regardless of trade costs if and only if  $\mu < \epsilon - 1$ .*

*Proof* See the ‘‘Appendix.’’

The stability of the symmetric configuration depends on the interplay of agglomeration and dispersion forces. Workers tend to agglomerate because they look for a higher real wage: A larger market access leads to higher nominal wages (*home market effect*) as firms' revenues are less hampered by trade costs, and a higher number of firms, linked to a bigger population, increase the number of variety locally produced and lowers the price index (*price index effect*). But at the same time, housing and commuting act as dispersion forces, because workers' agglomeration reduces housing sizes and commuting speeds. Therefore, a slight increase of population in one region will intensify both forces.

The magnitude of the agglomeration and dispersion forces depends, though, on different parameters. On the one hand, dispersion forces, which rely on urban costs, are more important when workers value housing surface and accessibility and when the average commuting speed is strongly reliant on the population, namely when  $\mu$  is relatively small. On the other hand, agglomeration forces are weaker when varieties are close substitutes, i.e., when  $\epsilon$  is large, and when trade costs are moderate, i.e., when  $Z$  is small. Indeed, if varieties are close substitutes, the benefit of a better access to more varieties is small and workers receive few gain from being agglomerated.

Likewise, if trade costs are low, firms of each region access pretty much the same market wherever they settle and workers consume indifferently products from the two regions, as their price indexes are quite the same.

Proposition 1 shows up two cases, depending on the relative values of  $\mu$  and  $\epsilon$ . If  $\mu < \epsilon - 1$ , dispersion forces overpass agglomeration forces, and the symmetric configuration is always a stable equilibrium. On the contrary, if  $\mu > \epsilon - 1$ , there is a trade cost,  $Z^*$ , above which agglomeration forces are bigger than dispersion forces in the symmetric configuration. In that case, the net benefit of having most of varieties locally produced is sufficiently large to outweigh the higher urban costs that workers must bear by being agglomerated. Therefore, only a combination of strong agglomeration forces and weak dispersion forces can lead to regions that are unequal in size.

### 3.3 Full agglomeration $\lambda = 1$

We now focus on the case of total agglomeration ( $\lambda = 1$ ). From (8) and (6), we have in region 2:

$$\begin{cases} p_2 = \tau^{(\epsilon-1)/\epsilon} p_1 \\ w_2 = \frac{\epsilon - 1}{\epsilon l_2^v} \tau^{(\epsilon-1)/\epsilon} p_1 \end{cases}$$

Thus,

$$\frac{U_1}{U_2} \underset{\lambda \rightarrow 1}{\sim} \left( \tau^{\frac{2\epsilon-1}{\epsilon}} \right)^{1-\delta_a-\delta_H} \left( \frac{1}{1 + \left(\frac{1}{\kappa}\right)^\alpha} \right)^{\delta_a} (1 - \lambda)^{\delta_H} \quad (17)$$

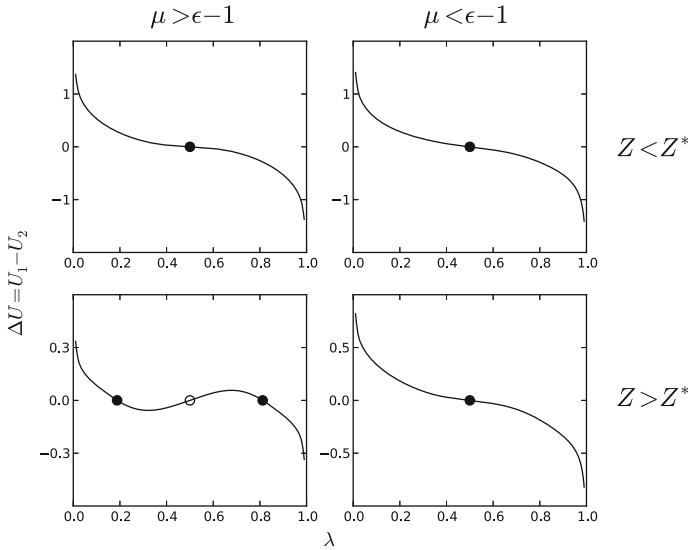
**Proposition 2** *Full agglomeration is never a spatial equilibrium.*

*Proof* From (17), we have

$$\lim_{\lambda \rightarrow 1} \frac{U_1}{U_2} = 0 < 1$$

If workers agglomerate entirely in region 1, utility in region 2 becomes much larger than utility in region 1, and the incentives to move from region 1 to region 2 are important.

The dispersion force mainly responsible of the instability of the full agglomeration configuration results from the housing market in region 2. The migration of the population from region 2 to region 1 decreases housing prices in region 2, as the dwelling stock remains the same. When population is close to zero, utility in region 2 raises sharply and offsets the agglomeration benefits. Albeit to a lesser extent, the average speed of commuting also plays a role. Depending on the transport infrastructure in each region, it reduces the utility in the more crowded area. Therefore, dispersion forces always prevent full agglomerations to occur. In that case, dispersion forces outweigh considerably agglomeration forces.



**Fig. 2** Utility difference between the two regions in different cases

### 3.4 Bifurcation diagrams

The previous analysis has dealt with two extreme cases so far, the symmetric configuration and the full agglomeration configuration. We now focus on the intermediate configurations.

In Fig. 2, the utility difference between the two regions is plotted versus population in different cases and for two values of trade costs. The stable equilibria are displayed with a black mark, whereas unstable ones are displayed with blank marks. When  $\mu < \epsilon - 1$ , as well as when  $\mu > \epsilon - 1$  and  $Z < Z^*$ , the utility differential is positive if  $\lambda$  is less than  $1/2$ , negative if  $\lambda$  is more than  $1/2$ . This means that if a region has more than half the manufacturing labor force, it is less attractive to workers than the other region. Urban costs always outweigh benefits from agglomeration. Clearly, in these cases, the economy converges to a long-run symmetric equilibrium in which manufacturing is equally divided between the two regions. When  $\mu > \epsilon - 1$  and  $Z > Z^*$ , by contrast, the wage differential slopes strictly upward in  $\lambda$  around the symmetric configuration. This upward slope results from the agglomerations forces discussed in the previous section. The *price index effect* and the *home market effect* exceed urban costs. The important point here is that although an equal division of manufacturing between the two regions is still an equilibrium, it is now unstable. The region with a slightly larger industrial sector will attract more and more workers until urban costs will dramatically increase due to scarcity of land and road congestion and will avoid full agglomeration. Two stable equilibria will then arise, as in [Murata and Thisse \(2005\)](#).

We now focus on the two cases  $\mu > \epsilon - 1$  and  $\mu < \epsilon - 1$  for all values of trade costs. As for the two values of trade studied above, the symmetric configuration is always a stable equilibrium when  $\mu < \epsilon - 1$ , namely when varieties are close substitutes

and utility is very sensitive to urban costs, and there exists no other equilibrium, whatever the capacity of the urban road network. Dispersion forces are then always greater than agglomeration forces. When  $\mu > \epsilon - 1$ , at sufficiently low trade costs, there is a unique stable equilibrium in which population is evenly divided between the two regions. Agglomeration forces do not offset dispersion forces. Being settled in a larger region has little benefits as trade costs do not hamper market access. However, when  $Z > Z^*$ , agglomeration forces can rival with dispersion forces. Trade costs are then sufficiently large to make location matter. The symmetric agglomeration is then unstable and two equilibria arise, which are characterized by partial agglomeration in one region.

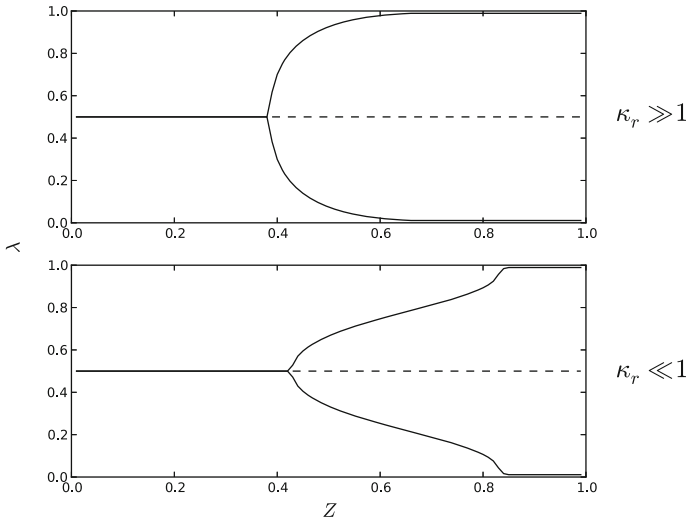
Figure 3 shows how the types of equilibria vary with trade costs in the case where  $\mu > \epsilon - 1$ . Solid lines indicate stable equilibria and broken lines unstable. Two cases are distinguished, depending on the urban road network:  $\kappa_r \gg 1$  on the top and  $\kappa_r \ll 1$  on the bottom. When  $\kappa_r \gg 1$ , the number of commuters is always smaller than the capacity of the road network and congestion never occurs. Only housing plays a dispersion role. On the contrary, when  $\kappa_r \ll 1$ , congestion can happen if population in one region is sufficiently large. Commuting and housing act then both as dispersion forces. Agglomeration happens for greater values of trade costs ( $Z^*$  increases with  $\kappa_r$ ), and when it happens, agglomeration is more partial. The form of the diagram bifurcation depends thus on the capacity of the urban network. When trade costs increase from  $Z^*$ , agglomeration forces can first be confronted to the reduction of the average speed of commuting, if the urban network cannot support a thin rise of population (as shown on the bottom). The utility difference due to an increasing competition on the housing market in one region and a decreasing dwelling price in the other region is thus slight because the difference of population is relatively small. When trade costs increase, the agglomeration forces overpass the reduction of mobility, but then are hampered by the scarcity of land. On the top, commuting does not play its dispersion role, because the urban network is sufficiently developed, and only the housing market acts as a dispersion force.

Congestion plays only a role in the spatial geography if the infrastructure cannot deal with influx of new commuters. The case exhibited on the bottom of Fig. 3 seems to be consistent with what is observed in reality. Urban networks are often undersized for new comers. In the following section, we will study the implication of a new transport mode, whose speed is not dependent on the number of users.

## 4 The spatial equilibrium with two modes

### 4.1 Assumptions and preliminary results

We now add in each region a public transport network to the road network described in the previous section. The features of the public mode, labeled PT and the private mode, labeled PC, differ from each other. We assume that the average speed of the private mode,  $v_r^{\text{PC}}$ , depends on the number of workers commuting on the urban road network, whereas the number of users of the public transport does not modify its speed,  $v_r^{\text{PT}}$ . Through this assumption, we consider that congestion in public transport does not alter



**Fig. 3** Bifurcation diagrams for different urban networks

the speed. Public transport is considered as a public goods.<sup>3</sup> However, public transport speeds depend on the location within each region. Some areas are more accessible by public transportation than others. Following equation (10), we have:

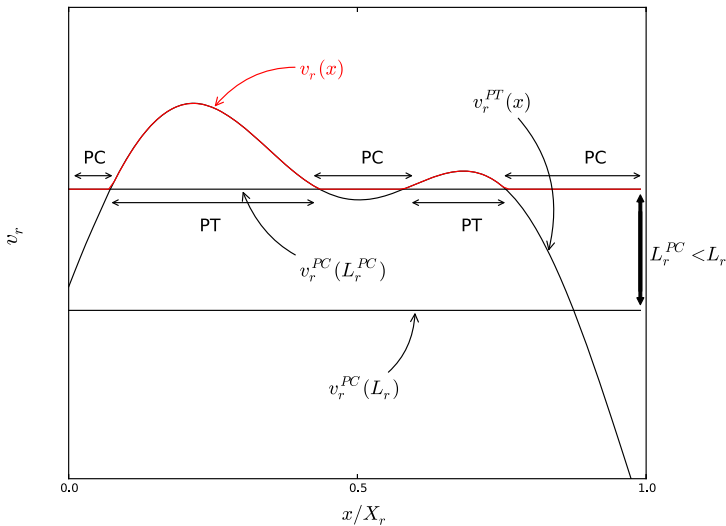
$$\begin{cases} v_r^{PC}(L_r^{PC}) = \frac{v_0^{PC}}{1 + \left(\frac{L_r^{PC}}{\kappa_r}\right)^\alpha} \\ v_r^{PT}(x) \end{cases}$$

where  $L_r^{PC}$  is the population living in region  $r$  who chooses the private mode to commute. Workers make their choice of travel mode in order to maximize their utility. Following Wardrop’s first principle (Wardrop 1952), a worker located in  $x$  chooses public transport mode if, and only if:

$$U_r^{PT}(x) > U_r^{PC}(x) \Leftrightarrow v_r^{PT}(x) > v_r^{PC}(L_r^{PC}) \tag{18}$$

Following expression (18), if the speed of public transport is always smaller than the speed by private cars ( $\forall x, v_r^{PT}(x) < v_r^{PC}(L_r)$ ), whatever choice of mode workers make, workers will use the private mode, as it maximizes their utilities. On the contrary, if there exists locations ( $\exists x, v_r^{PT}(x) > v_r^{PC}(L_r)$ ) in the urban area where public transport is faster than private cars, workers will use the public mode in these locations. The urban area population will then divide up between public transport and

<sup>3</sup> These assumption is straightforward and may seem far from the facts. Indeed, the speed of public modes, such as buses, may be subject to congestion (this is less true for subways, trams and bus rapid transits). However, the assumption made allows us to represent the main features of a public transport network and to study its implication on spatial organization.



**Fig. 4** Travel speeds for the two modes in region  $r$

private cars. The quantitative dynamic in the number of workers living at distance  $x$  from the CBD and choosing the private mode to commute,  $L_r^{PC}(x)$ , is described by:

$$\frac{dL_r^{PC}(x)}{dt} = \frac{v_r^{PC}(L_r^{PC}) - v_r^{PT}(x)}{\omega^v}$$

where  $\omega^v$  represents the time lag in workers' modal choice. At the equilibrium,<sup>4</sup> workers have no incentive to switch from a mode to another.

As an example, we consider an urban area in which public transport speed  $v_r^{PT}(x)$  varies as plotted in Fig. 4. If the entire population drive to commute ( $L_r^{PC} = L_r$ ), public transport is faster almost everywhere (except on the fringe of the urban area). Workers will then switch from the private mode to the public mode, and as congestion will be reduced, the speed of private cars will increase everywhere in the urban area. Private cars will then be faster than public transport in some locations where public transport was faster when the entire population drove to commute. In these locations, workers will still use the private mode. When workers have no incentive to switch from a mode to another, the speed of private cars is higher than when the entire population drive to commute, as shown in Fig. 4. Congestion is then lowered thanks to the public transport network, as some workers now commute using public transport.

<sup>4</sup> This equilibrium is reached at the short term, long before the equilibrium of workers' migrations between the two regions is reached (namely  $\omega^v \ll \omega^U$ ).



### 4.2 Bifurcation diagrams

A public transport network may limit congestion. Workers shift from one mode to another considering their relative speeds. This modal shift weakens the dispersion force relative to commuting. We consider a urban network such that  $\kappa_r < 1$ . As seen above, the urban road network cannot then bear the influx of many new commuters and, in the case where there is no public transport, the average speed of commuting drops when agglomeration occurs due to congestion. We assume now that another mode exists: A public transport mode.

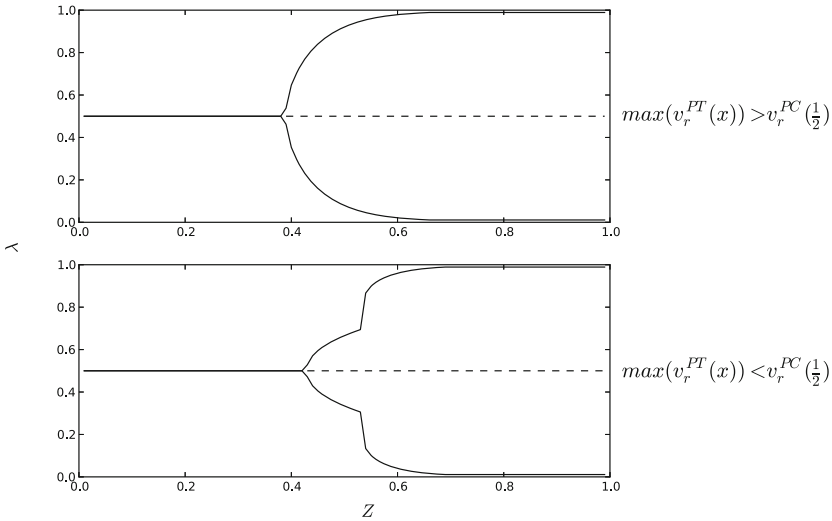
Two cases can be distinguished depending on the relative speed of the public mode and the private mode when the population is split evenly between the two regions. When  $\forall x, v_r^{PT}(x) < v_r^{PC}(\frac{1}{2})$ , workers in region  $r$  do not use public transportation until the population grows enough to reduce the private mode speed due to congestion, i.e.,  $\exists x, v_r^{PC}(L_r) < v_r^{PT}(x)$ . Public transport start then being used when population in region  $r$  gets over  $L_r^*$ :

$$L_r^* = \kappa_r \left[ \frac{v_0^{PC}}{\max(v_r^{PT}(x))} - 1 \right]^{1/\alpha}$$

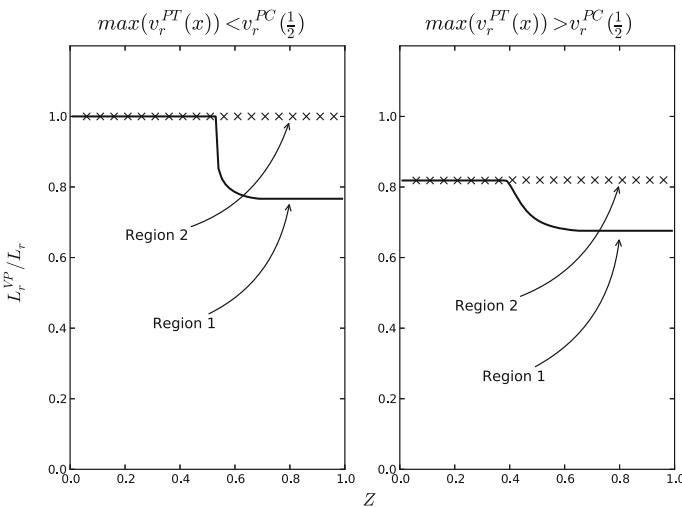
When  $\exists x, v_r^{PC}(\frac{1}{2}) < v_r^{PT}(x)$ , a share of the population in region  $r$  always chooses public transportation to commute. When the population grows, the average speed remains unchanged as more and more workers choose the public mode to commute. The number of users of the urban road network stays the same and private mode speed does not decrease with the arrival of new workers.

Figure 5 shows how the new equilibria vary with trade costs in those two different cases. At the top of Fig. 5, the public mode allows the speed of the private mode to remain the same, even if population grows. Commuting does not act as a dispersion force, although the urban road network cannot deal with the influx of new commuters. Only housing plays a dispersion role, as at the top of Fig. 3. At the bottom of Fig. 5, the same phenomenon occurs. However, the public transport system is not as efficient as in the previous case. When agglomeration happens, workers still prefer to use the private car, even if the commuting speed decreases. Commuting then acts as a dispersion force. This goes on until the speed of the private mode gets smaller than the speed of the public mode. Commuting does not play a dispersion role anymore, and agglomeration is fostered. The equilibria on the bottom of Fig. 5 are then the same than those on the top.

Modal shift from one mode to another mitigates the dispersion force due to commuting and encourages agglomeration. Figure 6 exhibits the workers' mode choice in region 2 (lines) and in region 1 (cross marks). When agglomeration occurs for high trade costs, workers report from private transport mode subject to congestion to public transport mode whose speed does not change with population. On the left of Fig. 6, the case where  $\max(v_r^{PT}(x)) < v_r^{PC}(\frac{1}{2})$  is drawn. Workers choose public transportation when  $L_r > L_r^*$ . That happens only in region 1 at high trade costs. On the right of Fig. 6, when  $v_r^{PC}(\frac{1}{2}) < \max(v_r^{PT}(x))$ , public transport is always used. But the use increases, when agglomeration in region 2 grows. In the two cases, the modal shares



**Fig. 5** Bifurcation diagrams for different public transport speeds



**Fig. 6** Modal share when agglomeration occurs in region 1

in region 1 stabilize when trade costs (and agglomeration) rise. Indeed, the average speed of the private mode does not evolve much with the population when  $L_r > \kappa_r$ , which is the case. The speed is almost constant, and there is no longer substantial shift from a mode to another.

### 5 Concluding remarks

This paper has developed a model inspired by the New Economic Geography that includes an explicit description of congestion and its interaction with the cost of trad-

ing goods. This description has two important features: Several different modes can be considered and the role of the urban transport network capacity can be assessed. It appears that commuting costs, and especially time costs, prevent population from agglomerating. As in Murata and Thisse (2005), the role of the two spatial costs is reversed: Low commuting time costs instead of high trade costs foster agglomeration, and vice versa. By being agglomerated, workers save on trade costs of differentiated product, but bear higher housing and commuting costs. Indeed, an increase of the population jams transport network and lowers the speed of travel within the city. However, the magnitude of the dispersion force due to commuting costs depends on the capacity of the urban road network and the availability of alternative modes. Investment in public transport or in road capacity will foster agglomeration. Furthermore, unlike Murata and Thisse (2005), partial agglomeration in one region can be a stable equilibrium. This more realistic feature is obtained through an explicit description of transport congestion and housing demand.

Therefore, what really matters for the structure of the space economy is not just the level of economic integration, but the interplay between trade costs and urban costs. Our model can be used as a building block to study the impacts of different transportation policies carried out in urban areas, or whether to elaborate more complex models calibrated on real urban structures or that endogenize infrastructure investments.

### Appendix

*Proof 1* The symmetric configuration is a stable equilibrium if, and only if,  $\frac{dU_r}{U_r}$  has not the same sign than  $\frac{dL_r}{L_r}$ . From Eq. (16), we have:

$$\begin{aligned} \frac{dU_r}{U_r} \frac{dL_r}{L_r} < 0 &\Leftrightarrow \mu \frac{(2\epsilon - 1)Z}{(\epsilon - 1)(\epsilon(Z + 1) - Z)} < 1 \\ &\Leftrightarrow [(2\epsilon - 1)\mu - (\epsilon - 1)^2] Z < \epsilon(\epsilon - 1) \\ &\Leftrightarrow \begin{cases} Z < \frac{\epsilon(\epsilon - 1)}{(2\epsilon - 1)\mu - (\epsilon - 1)^2} & \text{if } (2\epsilon - 1)\mu - (\epsilon - 1)^2 > 0 \\ Z > \frac{\epsilon(\epsilon - 1)}{(2\epsilon - 1)\mu - (\epsilon - 1)^2} & \text{if } (2\epsilon - 1)\mu - (\epsilon - 1)^2 < 0 \end{cases} \end{aligned}$$

Note that  $Z \in [0, 1]$ . We obtain:

$$\begin{cases} \frac{\epsilon(\epsilon - 1)}{(2\epsilon - 1)\mu - (\epsilon - 1)^2} < 0 \Leftrightarrow \mu < \frac{(\epsilon - 1)^2}{2\epsilon - 1} \\ \frac{\epsilon(\epsilon - 1)}{(2\epsilon - 1)\mu - (\epsilon - 1)^2} > 1 \Leftrightarrow \mu < \epsilon - 1 \end{cases}$$

Thus, when  $\mu < \frac{(\epsilon - 1)^2}{2\epsilon - 1}$  or  $\mu < \epsilon - 1$ , we have  $\forall Z, \frac{dU_r}{U_r} \frac{dL_r}{L_r} < 0$ . And, the symmetric configuration is always a stable equilibrium. However,

$$\forall \epsilon > 1, \quad \epsilon - 1 > \frac{(\epsilon - 1)^2}{2\epsilon - 1}$$

Let

$$Z^* = \frac{\epsilon(\epsilon - 1)}{(2\epsilon - 1)\mu - (\epsilon - 1)^2}$$

Eventually, when  $\mu < \epsilon - 1$ , the symmetric configuration is always a stable equilibrium. Otherwise, the symmetric configuration is stable if, and only if,  $Z < Z^*$ .

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