Ann Reg Sci (2012) 48:33–44
DOI 10.1007/s00168-010-0399-z

ORIGINAL PAPER

Spatial Cournot competition and transportation costs in a circular city

Toshihiro Matsumura · Noriaki Matsushima

Received: 27 August 2009 / Accepted: 15 July 2010 / Published online: 30 July 2010

Abstract We reconsider a Cournot spatial competition in a circular city. We discuss an oligopoly model. We find that two equilibria exist if the transport cost function is nonlinear in distance, while a continuum of equilibria exists if it is linear. Thus, the result of the real indeterminacy of equilibria in the linear transport cost case is knife edge.

JEL Classification R32 · L13

1 Introduction

Hamilton et al. (1989) and Anderson and Neven (1991) carried out pioneering works on spatial Cournot competition.[1] In their models, each firm chooses its location in a linear city and delivers its product for each point in the city. They showed that all firms agglomerate at the central point. Pal (1998) showed that their result depends on the assumption of the linear city. He investigated a circular city duopoly model and found that an equidistant location pattern appears in equilibrium; that is, locational

[1] Spatial Cournot models are also widely adopted for analyzing competition and public policies. See, Chen and Lai (2008), Matsumura (2003), Matsushima (2001b), Matsushima and Matsumura (2003, 2006), Nikae and Ikeda (2006), and Sun (2009).

T. Matsumura
Institute of Social Science, University of Tokyo, 7-3-1 Hongo,
Bunkyo, Tokyo 113-0033, Japan
e-mail: matsumur@iss.u-tokyo.ac.jp

N. Matsushima (✉)
Institute of Social and Economic Research, Osaka University,
Mihogaoka 6-1, Ibaraki, Osaka 567-0047, Japan
e-mail: nmatsush@iser.osaka-u.ac.jp

dispersion appears. He also suggested that equidistant location pattern constitutes an equilibrium in oligopoly. Matsushima (2001a) extended Pal's model to an n-firm oligopoly and showed another equilibrium where half of the firms locate at one point and the other half locate at the opposite point (partial agglomeration). Gupta (2004) showed that a continuum of equilibria exists if n is even and larger than 3. Thus, we face the real indeterminacy problem of equilibria in the circular city model.

Although the above three works on the circular city models use the linear transport cost function in distance, this assumption might not be innocuous. We use a linear-quadratic transport cost function to allow both convex and concave transport costs. For analytical simplicity, we investigate 4-firm oligopoly because a continuum of equilibria exists if the number of firms is even and larger than 3 under the linear transport cost function (Gupta 2004). We find that only two location patterns are equilibrium outcomes when the transport cost is either strictly convex or concave; one is an equidistance type pointed out by Pal (1998) and the other is a partial agglomeration type shown by Matsushima (2001a). Although multiple equilibria still exist, the real indeterminacy problem found in Gupta et al. (2004) disappear when we use a nonlinear transport costs.

We especially emphasize that the (strictly) concave transport cost is quite important in shipping models (delivered pricing models). The transport cost functions for firms are usually concave with respect to the distance. Air, ship, and railway freight costs are usually concave with respect to the distance. For instance, in the transport by air, the transportation cost is far from proportional to the distance (Brander and Zhang 1990). Large costs are incurred at take off and landing while the actual time in the air requires low costs, which yield concave transportation cost functions. Another example is the following situation. When the distance is small (large), the firm uses a truck (air cargo). Marginal cost for truck transport is larger than that for air transport, thus the transportation cost becomes concave (Dorta-González et al. 2005).

Regarding non-linear transport costs in circular city Cournot spatial models, Matsumura and Shimizu (2006, 2008) also adopted nonlinear transport cost in a duopoly model but did not discuss oligopoly cases. Matsumura et al. (2005) discussed nonlinear transport cost in a four-market model with four firms. In their model, possible locations are only four, so it excluded the possibility of a continuum of equilibria. Our work is closely related to Gupta et al. (2006). They investigated a Cournot spatial model with two complementary goods and showed that linear and convex transport cost yields different equilibrium locations and the set of equilibrium outcomes in the latter case is smaller than that in the former case. However, they did not examine the case of concave transport cost.

The remainder of the paper is organized as follows. Section 2 presents the basic model. Section 3 provides the result. Section 4 concludes.

2 The model

We present a two-stage location–quantity game. Let x_i ($i \in \{1, 2, \ldots, n\}$) be the locations of firm i. x_i is the point on the circle located at a distance from 0 (measured clockwise). In the first stage, each firm i ($i \in \{1, 2, \ldots, n\}$) simultaneously chooses its location x_i. Let $q_i(x)$ denote the firm i's output offered at each point $x \in [0, 1]$.

x is the point on the circle located at a distance from 0 (measured clockwise). In the second stage, each firm i ($i \in \{1, 2, \ldots, n\}$) observes its competitors' locations and simultaneously chooses $q_i(x) \in [0, \infty)$ for $x \in [0, 1]$. Let $p(x)$ denote the price of the product at x and $q(x) \equiv \sum_{i=1}^{n} q_i(x)$ denote the total quantity supplied at x. We assume that the demand function at each point x is linear and is given by: $p(x) = 1 - q(x)$.

Let $d(x, x_i)$ denote the distance between x and x_i. This signifies the shorter distance of the two possible ways to transfer the goods along the perimeter. To ship a unit of the product from its own location to a consumer at point x, each firm i ($i \in \{1, 2, \ldots, n\}$) pays a transport cost $T(x, x_i) \equiv td(x, x_i) + \tau d(x, x_i)^2$, where t and τ are constant values. We assume that $t > 0$ and $\tau > -t$ so as to ensure that T is increasing in $d(x, x_i)$. Firms are able to discriminate among consumers since they control transportation. Consumer arbitrage is assumed to be prohibitively costly. Each of (n) firms has identical technology and constant marginal cost of production, which is normalized to zero. These assumptions are standard and also made in many other location–quantity models.[2]

3 Equilibrium

In this section, we discuss the equilibrium in the model formulated above. We use subgame perfection as the equilibrium concept. The game is solved by backward induction. First, we discuss the equilibrium outcomes in the second-stage subgames given the location of each firm.

3.1 Quantity choice

We follow the Cournot assumption that firms compete in quantities at each point in the market. Since marginal production costs are constant, quantities set at different points by the same firm are strategically independent. Cournot equilibria can be characterized by a set of independent Cournot equilibria, one for each point x. Let $\pi_i(x)$ denote firm i's ($i \in \{1, 2, \ldots, n\}$) profit at x, given the locations of all firms;

$$\pi_i(x) = (1 - q(x) - T(x, x_i)))\, q_i(x). \tag{1}$$

The first-order condition of firm i ($i \in \{1, 2, \ldots, n\}$) is given by

$$1 - T(x, x_i) - q_i(x) - \sum_{i=1}^{n} q_i(x) = 0. \tag{2}$$

[2] Gupta (2004) discussed linear but non-identical transport costs. Shimizu (2002) and Yu and Lai (2003) discussed product differentiation in spatial Cournot models but they use liner transport costs. So as to focus on the effect of nonlinear transport costs, we do not incorporate these effects.

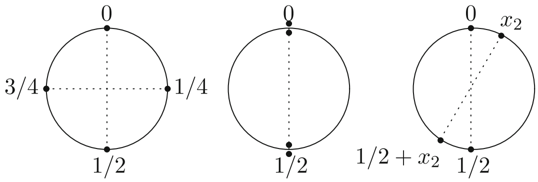

Fig. 1 The equilibrium location patterns

In this paper, we assume that the whole market will always be served by the firms. This assumption is satisfied if $4 > n(2t + \tau)$.[3]

The output quantity and profit of firm i supplying for market x are given by

$$q_i(x, x_i) = \frac{1 + \sum_{i=1}^{n} T(x, x_i) - (n+1)T(x, x_i)}{n+1}, \quad \pi_i(x, x_i) = q_i(x, x_i)^2. \tag{3}$$

The total quantity supplied and the price are:

$$q(x) = \frac{n - \sum_{i=1}^{n} T(x, x_i)}{n+1}, \quad p(x) = \frac{1 + \sum_{i=1}^{n} T(x, x_i)}{n+1}. \tag{4}$$

3.2 Location choice

In this subsection, the equilibrium locations are discussed. We consider a four firm case. $n = 4$ is the minimum number of firms yielding a continuum of equilibria.[4]

Without loss of generality, we set that the equilibrium location of firm 1 is 0. As shown by Gupta et al. (2004), when the transportation cost is linear in distance ($\tau = 0$), a continuum of equilibria exists.

Result 1 (Gupta et al. 2004) *Suppose that $n = 4$. If the transportation cost function is linear in distance (i.e., $\tau = 0$), $(x_1, x_2, x_3, x_4) = (0, x_a, 1/2, 1/2 + x_a)$ constitutes an equilibrium for any $x_a \in [0, 1/2)$.*

The first case of equilibrium location in Fig. 1, equidistant location pattern, was discovered by Pal (1998). The second case of equilibrium location in Fig. 1, partial agglomeration, was pointed out by Matsushima (2001a). Gupta et al. (2004) showed that many other equilibrium location patterns exist (third case in Fig. 1). The

[3] A similar assumption (small t and τ) is also made in many studies of quantity-setting spatial models. See, among others, Anderson and Neven (1991) and Pal (1998).

[4] If $n = 2$, the unique equilibrium is equidistance type for any increasing transport cost functions. See Matsumura and Shimizu (2006). If $n = 3$ and $\tau = 0$, Gupta et al. (2004) showed that two equilibria exist, one is equidistance type and the other is partial agglomeration type, where one firm locates at 0 and two firms locate at 1/2. We can show that the latter is equilibrium only if $\tau = 0$, while the former is always an equilibrium regardless of τ. More generally, if n (≥ 3) is odd, the location patterns where $(n-1)/2$ firms locate at 0 and $(n+1)/2$ firms locate at $1/2$ constitutes an equilibrium if and only if $\tau = 0$. If $n(\geq 2)$ is even, the location patterns where $n/2$ firms locate at 0 and $n/2$ firms locate at $1/2$ constitutes an equilibrium for all τ. For any $n \geq 1$, firm i locates at $(i-1)/n$ constitutes an equilibrium for all τ. The proofs are available upon request for authors.

following proposition indicates that the result drastically changes if the transport cost is nonlinear (i.e., $\tau \neq 0$). The first and the second cases are equilibria but others are not.

Proposition 1 *Suppose that* $n = 4$. *Suppose that* $\tau \neq 0$ (*i.e., the transport cost function is nonlinear*). $(x_1, x_2, x_3, x_4) = (0, x_a, 1/2, 1/2+x_a)$ *constitutes an equilibrium if and only if* $x_a = 0$ *or* $x_a = 1/4$.

Proof See Appendix. □

Remarks We now mention two remarks. As mentioned earlier, Matsumura et al. (2005) discussed general transport cost functions in a spatial Cournot competition model with four firms (a four-market setting), and showed that Pal's results (equidistance) are more robust than Matsushima's conclusions (partial agglomeration). Proposition 1 does not show such a property because the linear-quadratic transport cost function cannot capture extremely concave functions. We can make an example in which only Pal's location pattern appears in equilibrium. Consider the following transportation cost function:

$$T(x, x_i) = t\left(\frac{1}{16} - \left(\frac{1}{2} - d(x, x_i)\right)^4\right).$$

T is a concave function with respect to d. Under the cost function, the location pattern (0, 0, 1/2, 1/2) does not appear in equilibrium, while (0, 1/4, 1/2, 3/4) constitutes an equilibrium (see Appendix).

In our paper, we assume the transport cost function nonlinear in distance but linear in quantity. However, air, ship, and railway freight costs are also usually concave with respect to the quantity. Even if we replace the transport cost $TT(q(x), d(x)) = T(d(x))q(x)$ with $TT(q(x), d(x)) = T(d(x) + a)q(x) + bq(x)^2$, our results hold true as long as marginal cost is positive and the full coverage condition (the whole market will always be served by the firms) is satisfied.

We briefly discuss what happens if $n > 4$. As Gupta et al. (2004) showed, the set of equilibrium locations is quite complicated if the transport cost is linear. It is also true with non-linear transport cost. Proposition 1 states that the number of equilibrium location pattern is two when $n = 4$ but it is not true for $n > 4$. For example, if $n = 8$, we can show that the following three location patterns constitutes equilibria; $(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8) = (0, 1/8, 1/4, 3/8, 1/2, 5/8, 3/4, 7/8)$ (equidistance), (0, 0, 0, 0, 1/2, 1/2, 1/2, 1/2) (two point agglomeration), (0, 0, 1/4, 1/4, 1/2, 1/2, 3/4, 3/4) (hybrid 1), (0, 0, 0, 1/4, 1/2, 1/2, 1/2, 3/4) (hybrid 2). We now focus on the second equilibrium location pattern.

Result 2 (Gupta et al. 2004) *Suppose that n is even. Suppose that the transport cost function is linear in distance. Suppose that* $x_1 = x_2 = \cdots = x_{n/2-1} = 0$ *and* $x_{n/2-1} = \cdots = x_{n-2} = 1/2$. $x_{n-1} = x_b$ *and* $x_n = x_b + 1/2$ *constitutes an equilibrium for any* $x_b \in [0, 1/2)$.

A continuum of equilibria exists when the transport cost function is linear. However, if costs are strictly concave or convex, most strategies again fail to qualify as equilibria. The same principle can apply to other three patterns.

Proposition 2 *Suppose that n is even. Suppose that $\tau \neq 0$ (i.e., the transport cost function is nonlinear). Suppose that $x_1 = x_2 = \cdots = x_{n/2-1} = 0$ and $x_{n/2} = \cdots = x_{n-2} = 1/2$. $x_{n-1} = x_b$ and $x_n = x_b + 1/2$ constitutes an equilibrium if and only if $x_b = 0$ or $x_b = 1/4$.*

We omit the proof because the way to prove it is similar to that of Proposition 1. We can also show that the location pattern (0, 0, 0, 1/3, 1/3, 1/2, 2/3, 2/3) does not appear in equilibrium if the transport cost function is nonlinear, whereas this pattern appears as an equilibrium outcome in Gupta et al. (2004). Given the location patter, a firm locating at 1/3 or 2/3 has an incentive to move to point 0. For $n \leq 9$, we can derive equilibrium location patters as summarized in Fig. 2. Basically, asymmetric location patterns in Gupta et al. (2004) disappear in our model.

We guess that a sufficiently large number of equilibrium location pattern exists if $n \to \infty$. However, we believe that it is at most countable under nonlinear transport costs. We can also show that equidistance location pattern pointed out by Pal (1998) always constitutes an equilibrium and that two point agglomeration pointed out by Matsushima (2001a) also constitutes an equilibrium as long as n is even. Although many other hybrid type equilibria exist, we think that it is quite natural to focus on these two equilibria as two polar cases.

4 Concluding remarks

This study investigates a location-quantity model of oligopoly markets. The equilibrium location pattern crucially depends on whether or not the transport cost is linear. Our result indicates that linearity of the transport cost yields quite specific results. This makes sharp contrast to the discussion in the linear city model. As Anderson and Neven (1991) showed, the equilibrium location patterns is agglomeration at the center whether transport cost is linear and nonlinear. Our result indicates that in circular city model, we should care about the robustness of the equilibrium pattern when we use linear transport costs.

In the main part, we show an example in which Pal (1998) type location pattern appears as an equilibrium outcome whereas Matsushima (2001a,b) type does not appear. We have a conjecture that the Pal type is more robust than the Matsushima type. Investigating the conjecture is a considerable future research.

Appendix

Proof of Proposition 1 (only if part) $(x_1, x_2, x_3, x_4) = (0, k, 1/2, k + 1/2)$ constitutes an equilibrium only if $x_2 = k$ is firm 2's best reply given $(x_1, x_3, x_4) = (0, 1/2, k + 1/2)$. Without loss of generality we assume that $k \leq 1/4$.

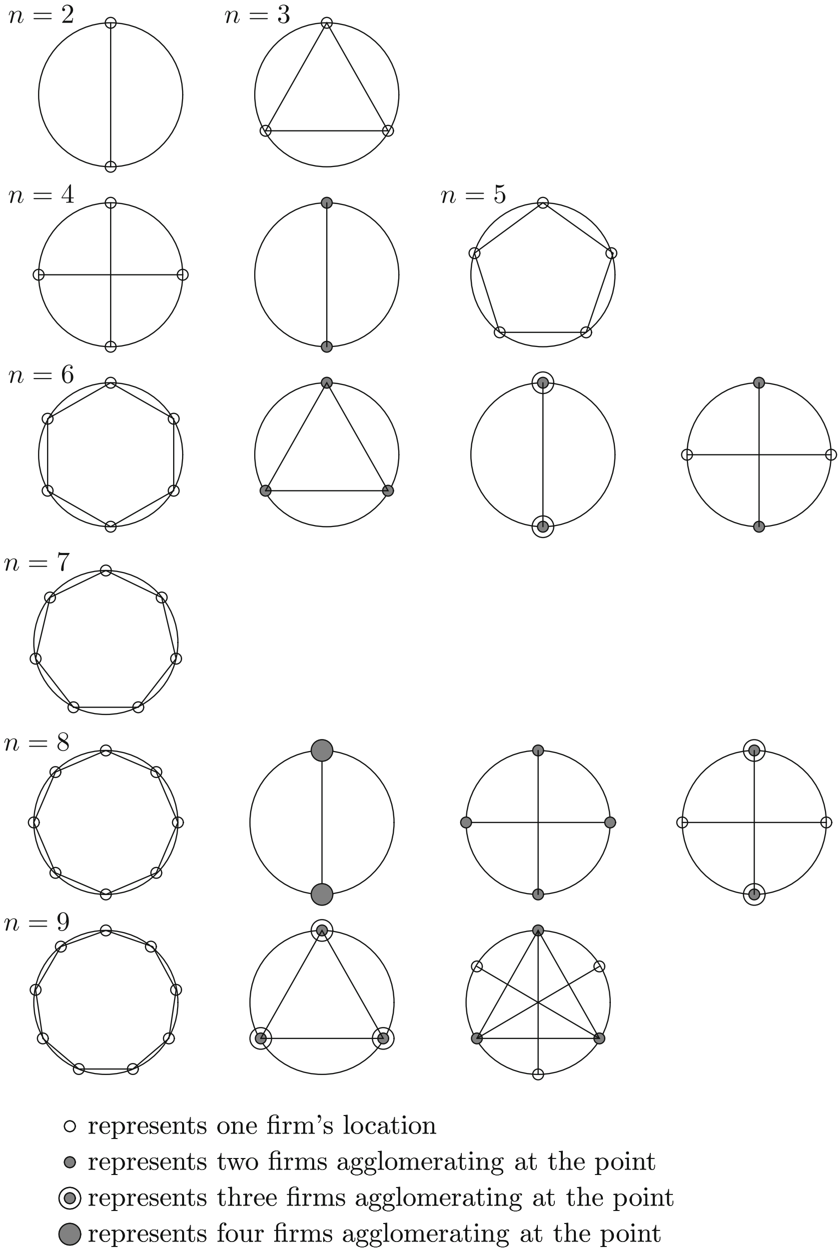

Fig. 2 Equilibrium location patterns for $n = 2, 3, \ldots, 9$

When $x_2 \le k$, the profit function of firm 2 is

$$\pi_2(x_2) = \int_0^1 \left(\frac{1 + \sum_{i=1}^4 T(x, x_i) - 5T(x, x_2)}{5} \right)^2 \mathrm{d}m$$

$$= \frac{20(6(2-t) + (7 - 48k^2 + 64k^2)t^2) - 5(8 - (27 - 192k^2 + 256k^3)t)\tau}{6000}$$

$$+ \frac{(33 - 200k^2 + 160k^3 + 160k^4)\tau^2 + 320k(1-2k)(6t(t+\tau) + (1+2k)\tau^2)x_2}{6000}$$

$$+ \frac{-960((1-4k)t(t+\tau) - 2k^2\tau^2)x_2^2 - 640(2t(t+\tau) + (1+2k)\tau^2)x_2^3 + 960\tau^2 x_2^4}{6000}.$$

Thus, we have

$$\frac{\partial \pi_2(x_2)}{\partial x_2} = \frac{4k(1-2k)(6t(t+\tau) + (1+2k)\tau^2) - 24((1-4k)t(t+\tau) - 2k^2\tau^2)x_2}{75}$$
$$+ \frac{-24(2t(t+\tau) + (1+2k)\tau^2)x_2^2 + 48\tau^2 x_2^3}{75}, \quad (5)$$

$$\frac{\partial^2 \pi_2(x_2)}{\partial x_2^2} = -\frac{8[((1-4k)t(t+\tau) - 2k^2\tau^2) + 2(2t(t+\tau) + (1+2k)\tau^2)x - 6\tau^2 x^2]}{25}. \quad (6)$$

Substituting $x_2 = k$ into (5) we have

$$\left. \frac{\partial \pi_2(x_2)}{\partial x_2} \right|_{x_2 = k} = \frac{4k(1-2k)(1-4k)\tau^2}{75} \ge 0,$$

and the equality is satisfied only if $k = 0$ or $k = 1/4$. Substituting $x_2 = k$ into (6) we have

$$\left. \frac{\partial^2 \pi_2(x_2)}{\partial x_2^2} \right|_{x_2 = k} = -\frac{8(t + \tau - 2k\tau)(t + 2k\tau)}{25}.$$

This is always negative for $k = 0$ and $k = 1/4$ because we have assumed $t > 0$ and $\tau > -t$.

When $k \le x_2 \le 1/2$, the profit function of firm 2 and the derivative of it are

$$\pi_2(x_2) = \int_0^1 \left(\frac{1 + \sum_{i=1}^4 T(x, x_i) - 5T(x, x_2)}{5} \right)^2 \mathrm{d}m$$

$$= \frac{20(6(2-t) + (7 - 48k^2 - 64k^2)t^2) - 5(8 - (27 - 192k^2 - 256k^3)t)\tau}{6000}$$

$$+\frac{(33-200k^2+160k^3+160k^4)\tau^2+320k(1-2k)(6t(t+\tau)+(1-2k)\tau^2)x_2}{6000}$$

$$+\frac{-960((1+4k)t(t+\tau)-2k^2\tau^2)x_2^2+640(2t(t+\tau)-(1+2k)\tau^2)x_2^3+960\tau^2x_2^4}{6000}.$$

$$\frac{\partial\pi_2(x_2)}{\partial x_2}=\frac{4k(1+2k)(6t(t+\tau)+(1-2k)\tau^2)-24((1+4k)t(t+\tau)-2k^2\tau^2)x_2}{75}$$

$$+\frac{24(2t(t+\tau)-(1+2k)\tau^2)x_2^2+48\tau^2x_2^3}{75}, \tag{7}$$

$$\frac{\partial^2\pi_2(x_2)}{\partial x_2^2}=-\frac{8[((1+4k)t(t+\tau)-2k^2\tau^2)-2(2t(t+\tau)-(1+2k)\tau^2)x-6\tau^2x^2]}{25}. \tag{8}$$

Substituting $x_2 = k$ into (5) we have

$$\left.\frac{\partial\pi_2(x_2)}{\partial x_2}\right|_{x_2=k}=\frac{4k(1-2k)(1-4k)\tau^2}{75}\geq 0,$$

and the equality is satisfied only if $k = 0$ or $k = 1/4$. Substituting $x_2 = k$ into (8) we have

$$\left.\frac{\partial^2\pi_2(x_2)}{\partial x_2^2}\right|_{x_2=k}=-\frac{8(t+\tau-2k\tau)(t+2k\tau)}{25}.$$

This is always negative for $k = 0$ and $k = 1/4$ because we have assumed $t > 0$ and $\tau > -t$.

Thus, if $k \neq 0$ and $k \neq 1/4$, $\pi_2(x_2)$ is strictly increasing when $x_2 = k$ (the first-order condition is never satisfied). Therefore, $x_2 = k$ can be its best reply only if $k = 0$ or $k = 1/4$. □

Proof of Proposition 1 (if part) We show that $x_2 = k$ is firm 2's best reply given $(x_1, x_3, x_4) = (0, 1/2, k + 1/2)$ when $k = 0$ and $k = 1/4$. In the proof of only if part, we have shown that the first-order condition is satisfied when $x_2 = k$ if $k = 0$ or $k = 1/4$. Thus, we now show that $x_2 = k$ is globally optimum if $k = 0$ or $k = 1/4$.

First, we consider the case in which $k = 0$. Substituting $k = 0$ into (7) we have

$$\frac{\partial\pi_2(x_2)}{\partial x_2}=-\frac{8x_2(1-2x_2)(t(t+\tau)+\tau^2x_2)}{25}\leq 0.$$

$\pi_2(x_2)$ is non-increasing in x_2 for $x_2 \in [0, 1/2]$ and strictly decreasing in x_2 for $x_2 \in (0, 1/2)$. These imply that $x_2 = 0$ is best reply of firm 2.

Second, we show that $x_2 = k$ is globally optimum when $k = 1/4$. We can show that given $(x_1, x_3, x_4) = (0, 1/2, 3/4)$, $\pi_2(y) > \pi_2(1/2 + y)$ for $y \in (0, 1/2)$.[5] Thus, the best reply of firm 2 lies on $[0, 1/2]$.

By symmetry, $\pi_2(z) = \pi_2(1/2 - z)$ for $z \in [0, 1/4]$. Thus, we can restrict our discussion on the case where $x_2 \le 1/4$. Substituting $k = 1/4$ into (5), we have

$$\frac{\partial \pi_2(x_2)}{\partial x_2} = \frac{(1 - 4x_2)((2t + \tau)^2 + 16t(t + \tau)x_2 + 8\tau^2(1 - 2x_2)x_2)}{100},$$

$\pi_2(x_2)$ is non-decreasing in x_2 for $x_2 \in [0, 1/4]$ and strictly increasing in x_2 for $x_2 \in [0, 1/4)$. This implies that $x_2 = 1/4$ is the globally optimum point. □

An example that only Pal-type location pattern appears As mentioned in the main part, we consider the following transport cost function:

$$T(x, x_i) = t\left(\frac{1}{16} - \left(\frac{1}{2} - d(x, x_i)\right)^4\right).$$

First, we show that $x_2 = 0$ is not firm 2's best reply given $(x_1, x_3, x_4) = (0, 1/2, 1/2)$. It implies that $(x_1, x_2, x_3, x_4) = (0, 0, 1/2, 1/2)$ does not constitutes an equilibrium.

When $x_2 \le 1/2$, the profit function of firm 2 is

$$\begin{aligned}\pi_2(x_2) &= \int_0^1 \left(\frac{1 + \sum_{i=1}^4 T(x, x_i) - 5T(x, x_2)}{5}\right)^2 \mathrm{d}m \\ &= \frac{40320 - 4032\tau + 319\tau^2}{1008000} \\ &\quad + \frac{x_2^2\left(5184 - 26880x_2 + 48384x_2^2 - 64512x_2^3 + 64512x_2^4 - 18432x_2^5 + 13824x_2^6\right)\tau^2}{1008000}.\end{aligned}$$

Thus, we have

$$\frac{\partial \pi_2(x_2)}{\partial x_2} = \frac{x_2(1 - 2x_2)(9 - 52x_2 + 64x_2^2 - 152x_2^3 + 32x_2^4 - 48x_2^5)\tau^2}{875}.$$

We can easily find that this is zero if $x_2 = 0$ and positive if $0 < x_2 \le 1/10$ (this inequality is a sufficient condition that the partial derivative is positive). Therefore,

[5] When $0 < y \le 1/4$, we have

$$\pi_2(y) - \pi_2(y + 1/2) = \frac{y\left(4t(t + \tau)(3 - 16y^2) + 3(1 - 16y^2)\tau^2\right)}{150} > 0.$$

By symmetry, $\pi_2(y) = \pi_2(1/2 - y)$. We also have $\pi_2(y) > \pi_2(1/2 + y)$ for $y \in [1/4, 1/2)$. Therefore, $\pi_2(y) > \pi_2(1/2 + y)$ for $y \in (0, 1/2)$.

$x_2 = 0$ is not optimal for firm 2. Therefore, $(x_1, x_2, x_3, x_4) = (0, 0, 1/2, 1/2)$ does not constitutes an equilibrium.

Next, we show that $x_1 = 0$ is firm 1's best reply given $(x_2, x_3, x_4) = (1/4, 1/2, 3/4)$. When $x_1 \leq 1/4$, the profit function of firm 1 is

$$\pi_1(x_1) = \int_0^1 \left(\frac{1 + \sum_{i=1}^4 T(x, x_i) - 5T(x, x_1)}{5} \right)^2 \mathrm{d}m$$
$$= \frac{10321920 - 1032192\tau + 96235\tau^2}{258048000}$$
$$- \frac{x_1^2 \left(656640 - 2709504x_1^2 - 12386304x_1^4 - 3538944x_1^6\right) \tau^2}{258048000}.$$

Thus, we have

$$\frac{\partial \pi_1(x_1)}{\partial x_1} = -\frac{3x_1 \left(95 - 784x_1^2 - 5376x_1^4 - 2048x_1^6\right) \tau^2}{56000}.$$

We can easily show that this is zero if $x_1 = 0$ and negative if $0 < x_1 \leq 1/4$. Therefore, $x_2 = 0$ is local optimal.

When $1/4 \leq x_1 \leq 1/2$, the profit function of firm 1 is

$$\pi_1(x_1) = \int_0^1 \left(\frac{1 + \sum_{i=1}^4 T(x, x_i) - 5T(x, x_1)}{5} \right)^2 \mathrm{d}m$$
$$= \frac{10321920 - 1032192\tau + 344107\tau^2}{258048000}$$
$$- \frac{x_1 \left(3241728 - 15019776x_1 + 35696640x_1^2 - 49158144x_1^3 + 45416448x_1^4\right) \tau^2}{258048000}$$
$$+ \frac{x_1 \left(28901376x_1^5 - 9437184x_1^6 + 3538944x_1^7\right) \tau^2}{258048000}.$$

Thus, we have

$$\frac{\partial \pi_1(x_1)}{\partial x_1} = -\frac{(1 - 2x_1)\tau^2}{112000}$$
$$\times \left[1407 - 10224x_1 + 26032x_1^2 - 33280x_1^3 + 32000x_1^4 - 11264x_1^5 + 6644x_1^6)\right].$$

We can easily show that this is zero if $x_1 = 1/2$ and negative if $1/4 \leq x_1 < 1/2$.

From the discussion, we find that $x_1 = 0$ is the optimal location of firm 1. By symmetry, this logic applies to the other firms' location choices. Therefore, $(x_1, x_2, x_3, x_4) = (0, 1/4, 1/2, 3/4)$ constitutes an equilibrium.

Acknowledgments We would like to thank anonymous referees and the editor for their helpful comments and suggestions. The authors gratefully acknowledge financial support from a Grant-in-Aid for Encouragement of Young Scientists and for Basic Research from the Japanese Ministry of Education, Science and Culture. Needless to say, we are responsible for any remaining errors.

References

Anderson SP, Neven DJ (1991) Cournot competition yields spatial agglomeration. Int Econ Rev 32(4): 793–808
Brander JA, Zhang A (1990) Market conduct in the airline industry: an empirical investigation. Rand J Econ 21(4):567–583
Chen C-S, Lai F-C (2008) Location choice and optimal zoning under Cournot competition. Reg Sci Urban Econ 38(2):119–126
Dorta-González P, Santos-Peñate DR, Suárez-Vega R (2005) Spatial competition in networks under delivered pricing. Pap Reg Sci 84(2):271–280
Gupta B (2004) Spatial Cournot competition in a circular city with transport cost differentials. Econ Bull 4(15):1–6
Gupta B, Lai F-C, Pal D, Sarkar J, Yu C-M (2004) Where to locate in a circular city? Int J Ind Organ 22(6):759–782
Gupta B, Pal D, Sarkar J (2006) Product differentiation and location choice in a circular city. J Reg Sci 46(2):313–331
Hamilton JH, Thisse J-F, Weskamp A (1989) Spatial discrimination: Bertrand vs. Cournot in a model of location choice. Reg Sci Urban Econ 19(1):87–102
Hotelling H (1929) Stability in competition. Econ J 39:41–57
Matsumura T (2003) Consumer-benefiting exclusive territories. Can J Econ 36(4):1007–1025
Matsumura T, Ohkawa T, Shimizu D (2005) Partial agglomeration or dispersion in spatial Cournot competition. South Econ J 72(1):224–235
Matsumura T, Shimizu D (2006) Cournot and Bertrand in shipping models with circular markets. Pap Reg Sci 85(4):585–598
Matsumura T, Shimizu D (2008) A noncooperative shipping Cournot duopoly with linear-quadratic transport costs and circular space. Jpn Econ Rev 59(4):498–518
Matsushima N (2001a) Cournot competition and spatial agglomeration revisited. Econ Lett 73(2):175–177
Matsushima N (2001b) Horizontal mergers and merger waves in a location model. Aust Econ Pap 40(3):263–286
Matsushima N, Matsumura T (2003) Mixed oligopoly and spatial agglomeration. Can J Econ 36(1):62–87
Matsushima N, Matsumura T (2006) Mixed oligopoly, foreign firms, and location choice. Reg Sci Urban Econ 36(6):753–772
Nikae D, Ikeda T (2006) Exclusive territories in the presence of upstream competition. Econ Bull 4(26):1–6
Pal D (1998) Does Cournot competition yield spatial agglomeration? Econ Lett 60(1):49–53
Shimizu D (2002) Product differentiation in spatial Cournot markets. Econ Lett 76(3):317–322
Sun C-H (2009) Spatial Cournot competition in a circular city with directional delivery constraints. Annals of Regional Science. doi:10.1007/s00168-009-0294-7
Yu C-M, Lai F-C (2003) Cournot competition in spatial markets: Some further results. Pap Reg Sci 82(4):569–580