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Measuring concentration: Lorenz curves and their decompositions

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Abstract This paper first reveals the basic properties behind the relative concentration measurement when using employment Lorenz curves. This involves axioms adapted not only from the literature on income distribution, but also from that on occupational segregation. Second, additive decompositions of this curve by subsectors and by groups of locations are proposed, since, as far as we know, no decompositions of these curves have yet been suggested in the field. Finally, this approach is used to analyze the concentration of the Spanish manufacturing industry. In particular, we study whether the technological intensity of an industry affects the extent of its relative concentration level.

JEL Classification R12 · D63

1 Introduction

The literature of regional science offers a variety of measures to quantify the concentration of economic activity. Some are formally derived from location models, as those proposed by Ellison and Glaeser (1997), Maurel and Sédillot (1999), and Guimarães et al. (2007). Others are borrowed from the literature on spatial statistics, as is the case of the distance-based measures put forward by Marcon and Puech (2003) and Duranton and Overman (2005). However, some of the most widely used concentration measures are derived from the literature on income inequality. Thus, the Gini index has been adapted to analyze spatial location patterns of manufacturing industries (Krugman 1991; Amiti 1999; Brülhart 2001; Suedekum 2006 among many others), and some

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of the indexes included in the generalized entropy family have been adapted as well (Brülhart and Traeger 2005; Mori et al. 2005; Brakman et al. 2005; Pérez-Ximénez and Sanz-Gracia 2007; Cutrini 2009).

In the literature on income distribution, these inequality indexes have been proven to be consistent with non-crossing Lorenz curves; i.e., if the Lorenz curve of an income distribution lies at no point below the other and at some point above, the Gini index and the generalized entropy family will lead to a lower inequality level at the former distribution, which makes the measurement of inequality by using Lorenz curves a rather robust procedure. In addition, the Lorenz curve has been characterized in that field in terms of basic axioms. However, a modified version of this curve has been proposed to measure the concentration of employment without characterizing it axiomatically. This implies that the properties which regional economics implicitly assume when using employment Lorenz curves to compare the concentration of two different sectors have been not identified yet.

To fill this gap, this paper first reveals the basic properties behind the concentration measurement when using employment Lorenz curves. These properties are borrowed not only from the literature on income distribution, but also from that of occupational segregation (Hutchens 1991, 2004; Alonso-Villar and Del Río 2007). Second, additive decompositions of this curve by subsectors and by groups of locations are proposed, since even though additive decompositions of the generalized entropy family of indexes have been used to measure industrial concentration (Brülhart and Traeger 2005), as far as we know, no decompositions of the employment Lorenz curves have yet been suggested. One of the decompositions parallels that proposed by Bishop et al. (2003) in a context of income distribution. Third, the employment Lorenz curves and their decompositions are finally used to analyze the concentration of manufacturing industries in Spain in 2008. In particular, we study whether the technological intensity of an industry affects the extent of its concentration level. An advantage of using these curves is that it allows comparisons among spatial distributions (the Lorenz curves of which do not cross) by using the lesser number of value judgments, which makes this measurement rather appealing and robust.

The paper is structured as follows. Section 2 presents some basic properties and characterizes the employment Lorenz curves in terms of them. Section 3 offers two decompositions of the employment Lorenz curve, one by groups of sectors, and another by groups of locations. This approach is later used in Sect. 4 to analyze the concentration of the Spanish manufacturing industry. Thus, on one hand, manufacturing industries are grouped by technological intensity to determine whether this characteristic affects the extent of concentration of an industry. On the other hand, the location units are grouped by their human capital level to go deeper into the analysis. Finally, Sect. 5 presents the main conclusions.

2 Measuring concentration with Lorenz curves

Consider an economy with $L > 1$ location units (counties, regions, countries, etc.). In measuring the concentration of a sector, this paper follows a relative notion, so that the distribution of this sector is compared with that of reference. Let $t \equiv (t_1, t_2, \ldots, t_L)$

denote this benchmark, and $T = \sum_l t_l$. If concerned, for example, with the concentration of manufacturing industries, t could represent the distribution of total manufacturing employment among regions (as in Amiti 1999 and Brülhart 2001). If concerned with a broader perspective, t could represent instead the distribution of overall employment, services included (as in Brülhart and Traeger 2005). Let us denote by $x \equiv (x_1, x_2, \ldots, x_L)$ the employment distribution of the sector in which we are interested, and by X its employment level ($X = \sum_l x_l$). In this paper, an index of relative concentration is a function $I_c : D \rightarrow \mathbb{R}$ such that $I_c(x; t)$ represents the concentration level of the sector having distribution x when comparing it with the distribution of reference t, where $D = \bigcup_{L>1} \{(x; t) \in \mathbb{R}^L_+ \times \mathbb{R}^L_{++} : x_l \leq t_l \ \forall l\}$.

In this section, we characterize the employment Lorenz curve of a sector in terms of basic properties borrowed from the literature on income distribution and occupational segregation.

2.1 Basic properties

Property 1 *Symmetry in locations*. If $(\Pi(1), \ldots, \Pi(L))$ represents a permutation of locations, then $I_c(x\Pi; t\Pi) = I_c(x; t)$, where $x\Pi = (x_{\Pi(1)}, \ldots, x_{\Pi(L)})$ and $t\Pi = (t_{\Pi(1)}, \ldots, t_{\Pi(L)})$.

This property means that if locations are enumerated in a different order, the concentration index should remain unchanged.[1]

Property 2 *Movement between locations*. If $(x'; t) \in D$ is obtained from $(x; t) \in D$ in such a way that:

(i) Location i loses employment in the sector of study, while the opposite happens to location h, i.e., $x'_i = x_i - d$, $x'_h = x_h + d$ $(0 < d \leq x_i)$, where i and h are two locations satisfying that $t_i = t_h$, and $x_i < x_h$;

(ii) The employment level of the sector of study does not change in the remaining locations, i.e., $x'_l = x_l \ \forall l \neq i, h$; then $I_c(x'; t) > I_c(x; t)$.

In other words, if location i has initially the same manufacturing employment level as location h, but a lower employment level in chemicals, then a movement of employment in chemicals from location i to location h would be considered a disequalizing movement fostering the concentration of the sector.[2]

Property 3 *Scale Invariance*. If the distribution of the sector of study, x, is multiplied by a positive scalar, a, and the distribution of reference, t, is multiplied by another positive scalar, b, in such a way that $ax_l \leq bt_l$, then the concentration level of the sector does not change; i.e., $I_c(ax; bt) = I_c(x; t)$.

[1] In the income distribution literature, this axiom is called "symmetry" or "anonymity," and it requires that the inequality index does not change when individuals' incomes swap. In the literature on occupational segregation, it is named "symmetry in groups" (see Hutchens 1991).

[2] This axiom corresponds to the Pigou–Dalton principle of transfers, which requires a transfer of income from a poorer to a richer person to increase inequality. It has also been adapted to measure occupational segregation, where it is called "movement between groups" (Hutchens 2004).

This property means that the value of the concentration index should not change when the employment level of the distribution of reference and/or that of the sector under consideration varies, so long as the weight that each location represents in distributions t and x remains unaltered. In other words, this property means that in measuring concentration it is only employment shares that matter, not employment levels.[3]

The next property we present, insensitivity to proportional subdivisions of locations, is not borrowed from the literature on income distribution, but from that on occupational segregation.[4] This property requires that subdividing a location into several units of equal size, both in terms of aggregate employment and in terms of employment in the sector of study, does not affect the concentration level of the sector. Without loss of generality, the subdivision in the next property is written for the last location to make notation easier.

Property 4 *Insensitivity to proportional subdivisions of locations.* If $(x'; t') \in D$ is obtained from $(x; t) \in D$ in such a way that:

(i) All locations except the last one remain unaltered both in terms of aggregate employment and employment in the sector of study, i.e., $t'_l = t_l$ and $x'_l = x_l$ for any $l = 1, \ldots, L-1$;
(ii) The last location is subdivided in M location units without introducing any difference among them in terms of employment shares, i.e., $x'_l = x_L/M, t'_l = t_L/M$ for any $j = L, \ldots, L+M-1$,

then, $I_c(x'; t') = I_c(x; t)$.

2.2 Characterizing employment Lorenz curves

In this subsection, we show that symmetry in locations, movement between locations, scale invariance, and insensitivity to proportional subdivisions of locations (i.e., Properties 1–4) are the basic properties of the concentration measurement behind employment Lorenz curves, since any concentration measure satisfying them is consistent with non-crossing employment Lorenz curves.

The Lorenz curve of the employment distribution of a sector is usually constructed as follows. First, locations are lined up in ascending order of the ratio of the Hoover–Balassa index $\frac{x_l/X}{t_l/T}$, which is equivalent to ranking according to $\frac{x_l}{t_l}$.[5] Next, the cumulative proportion of aggregate employment, $\sum_{i\le l} \frac{t_i}{T}$, is plotted on the horizontal axis and the cumulative proportion of employment in the sector of study, $\sum_{i\le l} \frac{x_i}{X}$, is plotted on the vertical axis. Therefore, if we denote by $\tau_l \equiv \sum_{i\le l} \frac{t_i}{T}$ the proportion of

[3] In the literature on income distribution, an index is said to satisfy this property if and only if inequality remains unaltered when multiplying all incomes by the same positive scalar. In a context of occupational segregation, a similar axiom has been proposed by Alonso-Villar and Del Río (2007).

[4] The corresponding axiom is named "insensitivity to proportional divisions" (see Hutchens 2004).

[5] See, for example, Brülhart (2001). Alternatively, Krugman (1991) and Amiti (1999) rank locations in descending order, but it is the ascending order that is consistent with income distribution literature.

cumulative aggregate employment represented by the first l locations ranked according to the above criterion, the employment Lorenz curve can be written as follows:

$$L_{(x;t)}(\tau_l) = \frac{\sum_{i \leq l} x_i}{X}.$$

The first decile of the distribution represents 10% of aggregate employment, and it includes those locations where the sector of study has the lowest relative presence. The second cumulative decile represents 20% of aggregate employment, and it also includes locations where the sector has the lowest relative presence, and so on. Each point of the employment Lorenz curve indicates the proportion of employment in the sector corresponding to each cumulative decile of aggregate employment. In other words, the curve shows the under-representation of the sector with respect to aggregate employment, decile by decile. In the case in which a sector of study was distributed across locations in the same manner as the distribution of reference, the employment Lorenz curve would be equal to the 45°-line and no concentration would exist.[6]

As with standard Lorenz curves, we can say that distribution $(x; t) \in D$ *dominates in concentration* distribution $(x'; t') \in D$ if the employment Lorenz curve of the former lies at no point below the latter and at some point above. In addition, a concentration index I_c can be defined as Lorenz consistent if for all pairs $(x; t), (x'; t') \in D$:

(a) When $L_{(x;t)}$ dominates $L_{(x';t')}$, then $I_{(x;t)} < I_{(x';t')}$.
(b) When both employment Lorenz curves coincide, $I_{(x;t)} = I_{(x';t')}$.

Proposition 1 *A concentration index I_c is consistent with the employment Lorenz criterion if and only if it satisfies symmetry in locations, movement between locations, scale invariance, and insensitivity to proportional subdivisions of locations (i.e., Properties 1–4).*

Proof First implication

Assume that I_c satisfies Properties 1–4 and consider distributions $(x; t), (x'; t') \in D$. In what follows, we first transform any employment distribution $(x; t)$ into a hypothetical "income" distribution, the Lorenz curve of which is equal to $L_{(x;t)}$, which allows us to use some well-known results from the literature on income distribution. Next, by following steps analogous to those followed by Foster (1985) in a context of income distribution, we multiply employment distributions $(x; t)$ and $(x'; t')$ by positive scalars in such a way that their corresponding "income" distributions share the same dimension and mean, while keeping the concentration unaltered.[7]

To build the fictitious "income" distribution, consider an economy with $l = 1, \ldots, L$ locations, where total "income" in location l is equal to x_l, and there are

[6] As happens in the case of other concentration measures used in the literature (such as the locational Gini index, the generalized entropy family, and the indexes proposed by Ellison and Glaeser 1997, and Maurel and Sédillot 1999), the concentration measurement captured by the employment Lorenz curve does not take into account distances among location units.

[7] The Lemmas included in Foster (1985) and mentioned in this proof are given in Appendix.

t_l "individuals" living in that location and receiving an equal amount of "income" $\frac{x_l}{t_l}$. Therefore, total "income" in this economy is $X = \sum_{l=1}^{L} t_l \frac{x_l}{t_l} = \sum_{l=1}^{L} x_l$, and the total number of "individuals" is $T = \sum_{l=1}^{L} t_l$. It is easy to verify that the employment Lorenz curve corresponding to distribution $(x; t)$, $L_{(x;t)}$, is equal to the standard Lorenz curve corresponding to fictitious "income" distribution $\left(\underbrace{\frac{x_1}{t_1}, \ldots, \frac{x_1}{t_1}}_{t_1}, \ldots, \underbrace{\frac{x_L}{t_L}, \ldots, \frac{x_L}{t_L}}_{t_L}\right)$. The same relationship can be established between employment distribution $(Tx'; Tt')$ and fictitious "income" distribution $y \equiv \left(\underbrace{\frac{Tx_1'}{Tt_1'}, \ldots, \frac{Tx_1'}{Tt_1'}}_{Tt_1'}, \ldots, \underbrace{\frac{Tx_L'}{Tt_L'}, \ldots, \frac{Tx_L'}{Tt_L'}}_{Tt_L'}\right)$, and between $\left(T' \frac{\frac{X'}{T'}}{\frac{X}{T}} x; T't\right)$ and

$$z \equiv \left(\underbrace{T'\frac{X'}{X}\frac{T}{T'}\frac{x_1}{T't_1}, \ldots, T'\frac{X'}{X}\frac{T}{T'}\frac{x_1}{T't_1}}_{T't_1}, \ldots, \underbrace{T'\frac{X'}{X}\frac{T}{T'}\frac{x_L}{T't_L}, \ldots, T'\frac{X'}{X}\frac{T}{T'}\frac{x_L}{T't_L}}_{T't_L}\right).$$

Note that y and z have the same number of "individuals" (TT') and "income" mean $(\frac{X'}{T'})$. Without loss of generality in what follows, we assume that $\frac{X}{T} > \frac{X'}{T'}$.

By using Lemma 2 proposed in Foster (1985), the Lorenz curves of the "income" distributions corresponding to $(x; t)$ and $(T' \frac{\frac{X'}{T'}}{\frac{X}{T}} x; T't)$ coincide, since the latter is a (T' times) replication of the former multiplied by a positive scalar $(\frac{X'}{X}\frac{T}{T'})$. The same applies to distributions $(x'; t')$ and $(Tx'; Tt')$. Consequently, $L_{(x;t)} = L_{(T'\frac{X'}{X}\frac{T}{T'}x;T't)}$ and $L_{(x';t')} = L_{(Tx';Tt')}$.

Two cases can be distinguished:

(a) If $L_{(x;t)}$ coincides with $L_{(x';t')}$, the employment Lorenz curve of distribution $(T' \frac{\frac{X'}{T'}}{\frac{X}{T}} x; T't)$ coincides with that of $(Tx'; Tt')$. By using Lemma 1 proposed in the aforementioned paper, it follows that the ordered distribution (from low to high values) corresponding to y, labeled $\hat{y}$, majorizes that of z, labeled $\hat{z}$, and vice versa.[8] In other words, distributions $\hat{y}$ and $\hat{z}$ are identical, which implies that $I_c(\hat{y}; 1^{TT'}) = I_c(\hat{z}; 1^{TT'})$, where $1^{TT'} \equiv \left(\underbrace{1, \ldots, 1}_{TT'}\right)$. Note that, on one hand, I_c satisfies the properties of insensitivity to proportional subdivisions of locations and scale invariance, which implies that $I_c(y; 1^{TT'}) = I_c(Tx'; Tt')$

[8] Given two income distributions with the same dimension and ranked in ascending order, one is said to majorize the other if and only if both distributions have the same total income, and the cumulative income level of the former, up to next to last individual, is lower than that of the latter.

and $I_c(z; 1^{TT'}) = I_c(T' \frac{\frac{X'}{T'}}{\frac{X}{T}} x; T't)$. On the other hand, by using the scale invariance property, $I_c(Tx'; Tt') = I_c(x'; t')$ and $I_c(T' \frac{\frac{X'}{T'}}{\frac{X}{T}} x; T't) = I_c(x; t)$ (since $\frac{X}{T} > \frac{X'}{T'}$). Consequently, $I_c(x; t) = I_c(x'; t')$.

(b) By following analogous steps, if $L_{(x;t)}$ dominates in concentration $L_{(x';t')}$, the employment Lorenz curve of distribution $(T' \frac{\frac{X'}{T'}}{\frac{X}{T}} x; T't)$ also dominates that of $(Tx'; Tt')$, which implies, by Lemma 3 in the aforementioned paper, that $\hat{y}$ is obtained from $\hat{z}$ by a finite sequence of regressive transfers. Therefore, since I_c satisfies the property of symmetry and that of movement between locations, $I_c(y; 1^{TT'}) > I_c(z; 1^{TT'})$. In addition, the properties of insensitivity to proportional subdivisions of locations and scale invariance mean that $I_c(y; 1^{TT'}) = I_c(Tx'; Tt') = I_c(x'; t')$ and $I_c(z; 1^{TT'}) = I_c(T' \frac{\frac{X'}{T'}}{\frac{X}{T}} x; T't) = I_c(x; t)$. Therefore, $I_c(x'; t') > I_c(x; t)$.

Second implication

Assume now that I_c is consistent with the employment Lorenz criterion. As mentioned above, the employment Lorenz curve corresponding to distribution $(x; t)$ coincides with the standard Lorenz curve of the corresponding hypothetical "income" distribution $\left(\underbrace{\frac{x_1}{t_1}, \ldots, \frac{x_1}{t_1}}_{t_1}, \ldots, \underbrace{\frac{x_L}{t_L}, \ldots, \frac{x_L}{t_L}}_{t_L}\right)$. Therefore, when comparing two employment distributions, there is consistency between the conclusions reached by using the employment Lorenz curves and those attained with the standard Lorenz curves of the fictitious "income" distributions.

(a) I_c satisfies scale invariance, since the Lorenz curve of the hypothetical "income" distribution $\left(\underbrace{\frac{\alpha x_1}{\beta t_1}, \ldots, \frac{\alpha x_1}{\beta t_1}}_{\beta t_1}, \ldots, \underbrace{\frac{\alpha x_L}{\beta t_L}, \ldots, \frac{\alpha x_L}{\beta t_L}}_{\beta t_L}\right)$ coincides with that of $\left(\underbrace{\frac{x_1}{t_1}, \ldots, \frac{x_1}{t_1}}_{t_1}, \ldots, \underbrace{\frac{x_L}{t_L}, \ldots, \frac{x_L}{t_L}}_{t_L}\right)$ and I_c is consistent with the Lorenz criterion (in terms of both employment Lorenz curves and standard Lorenz curves of the corresponding "income" distributions).

(b) I_c satisfies symmetry, since "individuals" play symmetric roles in standard Lorenz curves and I_c is consistent with the Lorenz criterion.

(c) I_c satisfies the movement between locations property, since any disequalizing movement from location i to h of the type mentioned in Property 2 leads to a sequence of regressive transfers in the fictitious “income” distribution world, which results in an increase in inequality according to the Lorenz criterion. As a consequence, the concentration index I_c also increases.

(d) I_c satisfies insensitivity to proportional subdivisions of locations because when a location l is subdivided into two locations (l' and l'') such that $x_{l'} = x_{l''} = \frac{x_l}{2}$ and $t_{l'} = t_{l''} = \frac{t_l}{2}$, the Lorenz curve of the corresponding “income” distribution does not change. □

The Lorenz criterion, when conclusive, has the advantage of allowing comparisons among spatial distributions by using the lesser number of value judgments, which makes this measurement rather appealing. However, when employment Lorenz curves cross, it is necessary to make use of concentration indexes. We should be aware that in these cases, even when using concentration indexes satisfying Properties 1– 4, such as the locational Gini coefficient and the generalized entropy family, the results do not necessary coincide, since each focuses on a different aspect of the employment distribution.

3 Decomposing employment Lorenz curves

While additive decompositions of the generalized entropy family of indexes have been proposed in the literature of industrial concentration (Brülhart and Traeger 2005), as far as we know, no decompositions of the employment Lorenz curves have yet been suggested. Following the decomposition of the standard Lorenz curve proposed by Bishop et al. (2003) in an inequality context, in what follows, we offer two forms of decomposition of these curves: one by groups of locations, and the other by subsectors.

Proposition 2 *Assume that locations can be classified into K mutually exclusive groups so that the distributions x and t can be expressed as* $(x;t) = (x^1, \ldots, x^K; t^1, \ldots, t^K)$*, where* x^k *denotes the employment distribution of the sector across locations in group k, and* t^k *is that of aggregate employment* $(k = 1, \ldots, K)$*. Then, the employment Lorenz curve,* $L_{(x;t)}$*, can be decomposed as follows:*

$$L_{(x;t)}(\tau_l) = \sum_k \frac{X_k}{X} \tilde{L}_{(\tilde{x}^k;t)}(\tau_l),$$

where X_k *is the employment level of the sector in group k,* $\tilde{L}_{(\tilde{x}^k;t)}(\tau_l)$ *is like the employment Lorenz curve of distribution* $(\tilde{x}^k;t)$ *except that locations are ranked according to ratios* $\frac{x_l}{t_l}$*, and* $\tilde{x}^k$ *is an L-dimensional vector resulting from enlarging vector* x^k *with zero-values for those locations that are not included in group k.*

Proof Following the decomposition of the standard Lorenz curve by population subgroups proposed by Bishop et al. (2003), we define indicator G_l^k so that $G_l^k = 1$ if location (region) l belongs to group (country) k and $G_l^k = 0$ otherwise ($l = 1, \ldots, L$, and $k = 1, \ldots, K$). By using vector x^k, we can build $\tilde{x}^k$ as follows: $\tilde{x}^k = (x_1 G_1^k, \ldots, x_L G_L^k)$. In other words, $\tilde{x}^k$ is a fictitious employment distribution having the same dimension as the original distribution x so that it can be compared to the distribution of total employment t. By keeping locations ranked in ascending order of the ratios $\frac{x_l}{t_l}(l = 1, \ldots, L)$, and maintaining $\tau_l \equiv \sum_{i \le l} \frac{t_i}{T}$, we could define $\tilde{L}_{(\tilde{x}^k;t)}(\tau_l)$ as the proportion of employment in the sector corresponding to the locations ranked before l that are included in group k. In other words, $\tilde{L}_{(\tilde{x}^k;t)}(\tau_l) = \frac{\sum_{i \le l} x_i G_i^k}{X_k}$. Then, the employment Lorenz curve corresponding to distribution $(x; t)$ can be decomposed as $L_{(x;t)}(\tau_l) = \frac{\sum_{i \le l} x_i}{X} = \sum_k \frac{X_k}{X} \frac{\sum_{i \le l} x_i G_i^k}{X_k} = \sum_k \frac{X_k}{X} \tilde{L}_{(\tilde{x}^k;t)}(\tau_l)$, which completes the proof. □

Consequently, expression:

$$\frac{X_k}{X} \frac{\tilde{L}_{(\tilde{x}^k;t)}(\tau_l)}{L_{(x;t)}(\tau_l)} \tag{1}$$

measures the contribution of group k to the value of the employment Lorenz curve in the corresponding percentile. Assume, for example, that our location units are the European regions and that we are interested in grouping them by country. Consider again that we focus on the chemical sector. As mentioned above, the first decile of the employment distribution includes those regions where the chemical industry has the lowest relative presence, and it accounts for 10% of manufacturing employment in Europe. By using the above decomposition for the first decile, one could determine whether the regions with the lowest employment in chemicals belong to Spain, France, Italy, etc.[9] Note that both cumulative and non-cumulative percentiles can be decomposed according to a given location classification.[10] If we want to decompose non-cumulative deciles, for example, we have to use instead the following expression:

$$\frac{X_k}{X} \frac{\tilde{L}_{(\tilde{x}^k;t)}(\tau_l + 0.1) - \tilde{L}_{(\tilde{x}^k;t)}(\tau_l)}{L_{(x;t)}(\tau_l + 0.1) - L_{(x;t)}(\tau_l)}. \tag{2}$$

Note, on the other hand, that function $\tilde{L}_{(\tilde{x}^k;t)}$ also enables one to determine how the sector of study in country k is distributed among non-cumulative deciles by using expression

$$\tilde{L}_{(\tilde{x}^k;t)}(\tau_l + 0.1) - \tilde{L}_{(\tilde{x}^k;t)}(\tau_l), \tag{3}$$

which indicates the proportion of employment in the sector in country k in each (non-cumulative) decile. This analysis would permit one, for example, to find out whether the distribution of chemicals in France across the deciles of manufacturing employment in Europe differs from that of Germany.[11] Note that this decomposition allows going

[9] In the empirical section, Sect 4, we only have data for one country, Spain, so that another classification of locations is considered. In particular, three groups of locations are built according to their human capital level.

[10] For the sake of simplicity, in Sect. 4, we have chosen only to decompose non-cumulative deciles.

[11] In the empirical section, we analyze whether the distribution of provinces with high human capital level across ventiles differs from the distribution of the remaining groups.

deeper into the analysis within each percentile of the total employment distribution, but it does not allow one to conclude which group of sectors is more agglomerated by comparing their corresponding curves $\tilde{L}_{(\tilde{x}^k;t)}(\tau_l)$, since these curves are obtained by ranking locations according to a common criterion: $\frac{x_l}{t_l}$ ratios.[12] If one were interested in that analysis, the Lorenz curve for each group of locations (countries, for example) would have to be calculated.

Next, without loss of generality, let employment in the sector be classified into two mutually exclusive subsectors, A and B, so that $(x_1, \ldots, x_L) = (x_1^A + x_1^B, \ldots, x_L^A + x_L^B)$. Denote by X^A (respectively X^B) the employment level of subsector A (respectively B).

Proposition 3 *If the sector can be partitioned into two mutually exclusive subsectors A and B so that $(x;t) = (x^A + x^B; t)$, then the employment Lorenz curve, $L_{(x;t)}$, can be decomposed as follows:*

$$L_{(x;t)}(\tau_l) = \frac{X^A}{X}\tilde{L}_{(x^A;t)}(\tau_l) + \frac{X^B}{X}\tilde{L}_{(x^B;t)}(\tau_l),$$

where $\tilde{L}_{(x^A;t)}(\tau_l)$ is like the employment Lorenz curve corresponding to $(x^A;t)$, and $\tilde{L}_{(x^B;t)}(\tau_l)$ is like the employment Lorenz curve corresponding to $(x^B;t)$, except that locations have been ranked according to ratios $\frac{x_l}{t_l}$.

Proof Let us define $\tilde{L}_{(x^A;t)}(\tau_l) = \frac{\sum_{i\le l} x_i^A}{X^A}$ and $\tilde{L}_{(x^B;t)}(\tau_l) = \frac{\sum_{i\le l} x_i^B}{X^B}$. The proof is immediate by noting that the employment Lorenz curve corresponding to distribution $(x;t)$ can be decomposed as $L_{(x;t)}(\tau_l) = \frac{X^A}{X}\frac{\sum_{i\le l} x_i^A}{X^A} + \frac{X^B}{X}\frac{\sum_{i\le l} x_i^B}{X^B} = \frac{X^A}{X}\tilde{L}_{(x^A;t)}(\tau_l) + \frac{X^B}{X}\tilde{L}_{(x^B;t)}(\tau_l)$. □

This decomposition can also be easily generalized to more than two subsectors so that expression:

$$\frac{X^A}{X}\frac{\tilde{L}_{(x^A;t)}(\tau_l)}{L_{(x;t)}(\tau_l)} \tag{4}$$

measures the contribution of subsector A to the employment Lorenz curve of the sector in each cumulative decile. This analysis would permit one, for example, to determine whether, in the first decile, chemical employment corresponds mainly to pharmaceutical products, manufacture of pesticides and other agrochemical products, or manufacture of man-made and synthetic fibers, and so on. If we want to decompose each non-cumulative decile, rather than each cumulative decile, we have to use, instead, this expression:

$$\frac{X^A}{X}\frac{\tilde{L}_{(x^A;t)}(\tau_l + 0.1) - \tilde{L}_{(x^A;t)}(\tau_l)}{L_{(x;t)}(\tau_l + 0.1) - L_{(x;t)}(\tau_l)}. \tag{5}$$

12 Note that the decomposition suggested in Proposition 2 depends crucially on the linearity properties of the cumulative x-shares, holding the t-shares fixed.

Furthermore, expression:

$$\tilde{L}_{(x^A;t)}(\tau_l + 0.1) - \tilde{L}_{(x^A;t)}(\tau_l) \quad (6)$$

enables one to determine how subsector A is distributed among (non-cumulative) deciles of the whole employment distribution.[13] In particular, this would allow one to determine whether a given subsector of the chemical industry is located mainly in regions where the chemical industry has a high presence or, on the contrary, it follows a different location pattern than the sector as a whole. It also allows, for example, studying whether the distribution of pharmaceutical products among European regions differs from that of manufacturers of man-made and synthetic fibers.[14]

4 An empirical illustration

The data used in this paper comes from the Labor Force Survey (EPA) conducted by the Spanish Institute of Statistics (INE) by following EUROSTAT's guidelines. Our data correspond to the second quarter of the year 2008. Manufacturing industries are considered at a two- and three-digit level of the National Classification of Economic Activities (CNAE-1993 Rev_1), and the territorial scale is that of provinces (nuts III).

When studying the concentration of manufacturing industries at the two or more digit level, scholars usually find evidence of remarkable concentration levels both in sectors that can be classified as high-tech and in others that are included in the low-tech group (see, for example, Maurel and Sédillot 1999, in France; Bertinelli and Decrop 2005, in Belgium; and Guimarães et al. 2007, in Portugal). In this section, we are interested in analyzing whether the set of high-tech industries is concentrated to a higher extent than that of industries with lower technological intensity.[15] By following the OECD and *INE* classification of manufacturing industries by technological intensity, four groups of sectors have been considered (see Table 1).[16]

Figure 1 shows the distribution of the ratios $\frac{x_l}{t_l}$ across provinces for each technological group. The overall pattern suggests that the high-tech industry is more concentrated in fewer locations than the low-tech industry. In other words, the employment of the low-tech group tends to be more ubiquitous (as compared with total manufacturing employment) than that of the high-tech group.

[13] This decomposition is used in the empirical section when decomposing each technological group by subindustries.

[14] Note again that this decomposition allows deeper analysis within each percentile of the total employment distribution, but if one were interested in comparing the spatial concentration of two subsectors, the employment Lorenz curve for each subsector would have to be calculated. As in Proposition 1, this decomposition depends crucially on properties of the cumulative x-shares, holding the t-shares fixed.

[15] In the European context, Cutrini (2009) also finds that innovative industries tend to agglomerate at a higher extent than traditional industries by using the decomposition of the entropy index.

[16] The *INE* includes *building and repairing of ships and boats* in the medium-high group, while the OECD classifies this industry in the medium-low-tech group. We chose to follow the *INE* classification.

Table 1 Classification of manufacturing industries by technological intensity

High-technology group	Aircraft and spacecraft (**353**)
	Pharmaceuticals (**244**)
	Office, accounting and computing machinery (**30**)
	Radio, TV and communications equipments (**32**)
	Medical, precision and optical instruments (**33**)
Medium-high-technology group	Motor vehicles, trailers and semi-trailers (**34**)
	Building/repairing of ships and boats, railroad equipment and transport equipment n.e.c. (**35–353**)
	Electrical machinery and apparatus n.e.c. (**31**)
	Machinery and equipment n.e.c. (**29**)
	Chemicals excluding pharmaceuticals (**24–244**)
Medium-low-technology group	Rubber and plastic products (**25**)
	Metallurgy (**27**)
	Fabricated metal products (machinery and equipment excluded) (**28**)
	Coke, refined petroleum products and nuclear fuel (**23**)
	Other non-metallic mineral products (**26**)
Low-technology group	Paper industry (**21**)
	Publishing, graphic arts, and reproduction of recorded supports (**22**)
	Food products and beverages (**15**)
	Tobacco (**16**)
	Wood and cork industry, except furniture (**20**)
	Textile industry (**17**)
	Clothing and fur industry (**18**)
	Leather and footwear (**19**)
	Manufacture of furniture and other manufacturing industries n.e.c. (**36**)
	Recycling (**37**)

To go further in the analysis, Fig. 2 shows the employment Lorenz curves of the four groups.[17] We can see that high-tech industries are the most concentrated group since the corresponding Lorenz curve is clearly dominated by those of the remaining groups. This means that according to any concentration index consistent with the Lorenz criterion such as the locational Gini coefficient and any of the members of the generalized entropy family of concentration indexes, the high-tech group is the most concentrated industry, which suggests that this is a rather robust result. Note also that the employment Lorenz curve of the medium-high technology group is dominated by that of the low technology group.

However, since the employment Lorenz curve of medium-low-tech industries crosses those of low- and medium-high-tech industries, we cannot rank these industries according to their concentration level in a robust way. We have to make, instead, use of concentration indexes to know which one is more concentrated. For this purpose, seven different inequality-based concentration indexes have been calculated: the

[17] In this paper, to quantify the concentration of a sector, we follow a widespread approach that does not take into account distances between locations (as in Ellison and Glaeser 1997; Maurel and Sédillot 1999; Brülhart and Traeger 2005 and Cutrini 2009). However, an alternative approach could be followed. One could use, for example, the $G(d)$ statistic proposed by Getis and Ord (1992) to measure the spatial association between location units. In this case, one would find that the null hypothesis of spatial independence for each technological sector would not be rejected, which suggests that there is no evidence of clustering at the provincial level (the finest territorial scale for which there are available data). The values of this index and the corresponding z-values for each technological group are given in Appendix (see Table 3).

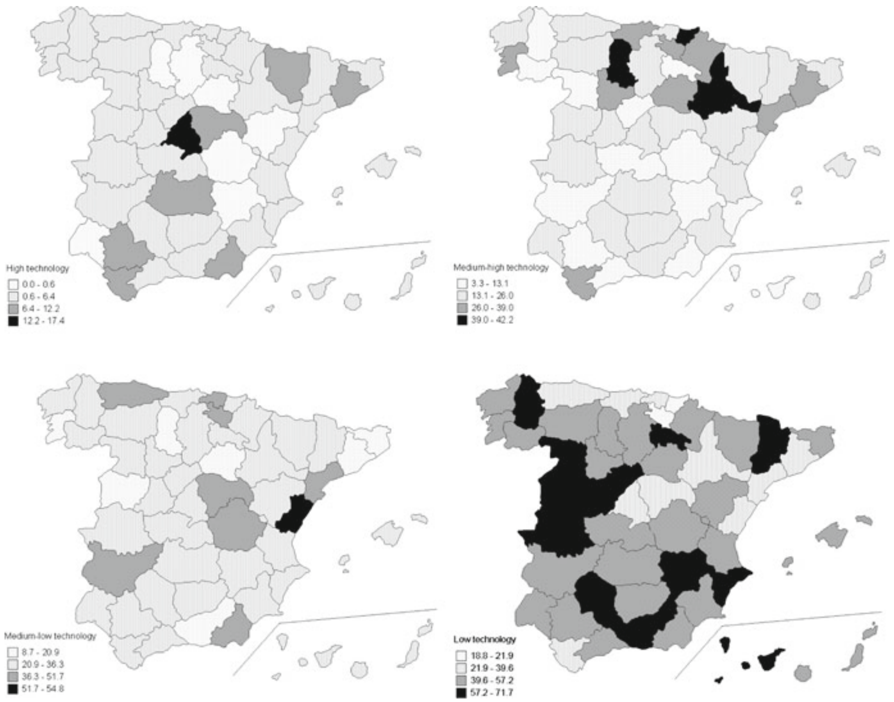

Fig. 1 Maps of the distribution of ratios $\frac{x_l}{t_l}$ across provinces for each technological group

locational Gini index and five members of the generalized entropy family (GE henceforth) ($\Psi_\alpha, \alpha = -1, 0, 0.1, 0.5, 1$, and 2).[18] As Table 2 shows, except for the case Ψ_{-1}, which pays more attention to the lower tail of the employment distribution (i.e., where the sector is more underrepresented), we find that the higher the technological intensity of the group, the higher its concentration level.

To check the robustness of this finding, we use bootstrapping for the above Ψ_α indexes, since there are arguments in favor of using this method in the case of entropy measures (Brülhart and Traeger 2005; Cutrini 2009). The bootstrap inference, based on 1,000 replications, confirms the previous conclusion for $\alpha = 0.1, 0.5, 1$, and 2 at the 95% level of significance (see Table 4 in Appendix).

When analyzing high-tech sectors in more detail, we find that there are important differences among them in terms of concentration. Thus, Fig. 3 shows that both the Lorenz curve of *aircraft and spacecraft* (**353**) and that of *office, accounting and computing machinery* (30) are dominated by those of the remaining high-tech sectors, which suggests that the concentration of these two sectors is especially intense.

In fact, as Fig. 4 shows, when using the decomposition of the non-cumulative deciles by subindustries, as explained in expression (5), we find that sectors **353** and

[18] The expressions of these concentration indexes are given in Appendix. Some members of the generalized entropy family (GE) cannot be calculated for those sectors that do not have employment in a location. The GE family of concentration indexes has been used, among others, by Brülhart and Traeger (2005).

Fig. 2 Employment Lorenz curves for four large sectors

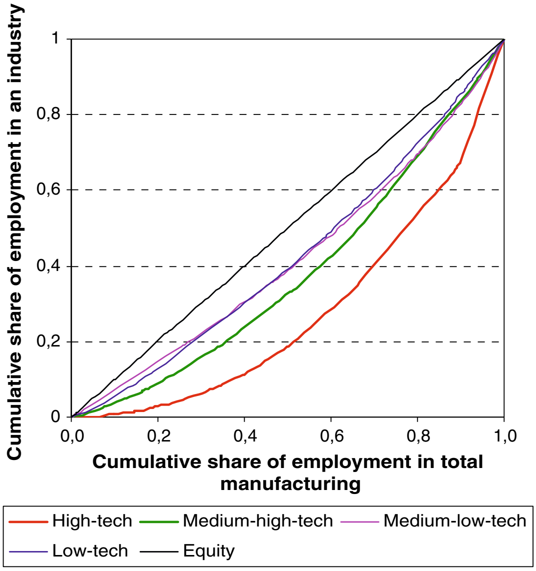

Table 2 Concentration indexes

	Ψ_{-1}	Ψ_0	$\Psi_{0.1}$	$\Psi_{0.5}$	Ψ_1	Ψ_2	Gini
High-tech	*	*	0.767	0.389	0.341	0.372	0.441
Medium-high-tech	*	*	0.098	0.091	0.086	0.082	0.231
Medium-low-tech	0.040	0.041	0.041	0.042	0.044	0.048	0.159
Low-tech	0.043	0.039	0.039	0.038	0.037	0.036	0.151

30 tend to appear from the fifth decile onwards. In other words, these two sectors tend to show up in provinces with a high presence of high-tech industries. Moreover, the decomposition of the high-technology industry by subindustries (using expression (6) for ventiles; i.e., $\tilde{L}_{(x^A;t)}(\tau_l + 0.2) - \tilde{L}_{(x^A;t)}(\tau_l)$) shows that almost 57% of the employment of sector **353** is in the fifth ventile of the manufacturing employment distribution (i.e., it is located in the most high-tech provinces of the country); in the case of sector **30**, over 68% of its employment is in the fifth ventile (see Fig. 5). However, for those groups with lower technology intensity, the agglomeration of their industries is not so intense since they are distributed across ventiles in a more homogeneous way (see Figs. 14, 15, and 16 in Appendix).

Within the class of medium-high technology industries, we find that the *manufacture of other transport material*, aircraft excluded, (**35–353**), is by far the most concentrated sector (see Fig. 6), while in the medium-low tech industry it is the *manufacture of coke, refinement of petroleum and treatment of nuclear fuels* (**23**) the one experiencing the highest concentration level (Fig. 7). Finally, note that differences among low-technology industries are not as strong as in the previous groups; see Fig. 8.

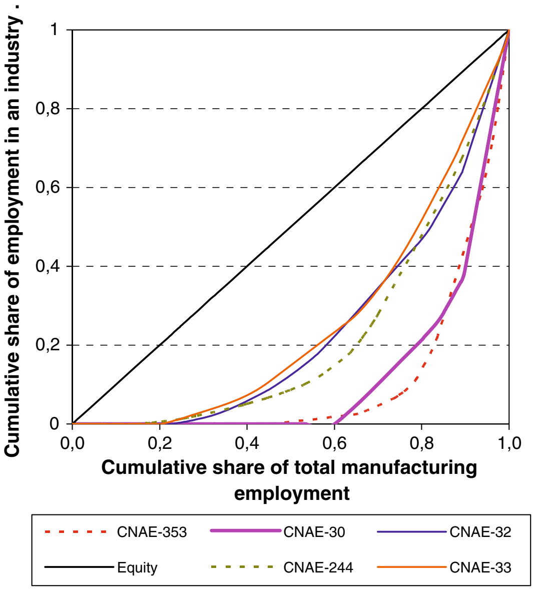

Fig. 3 Employment Lorenz curves of high-tech industries

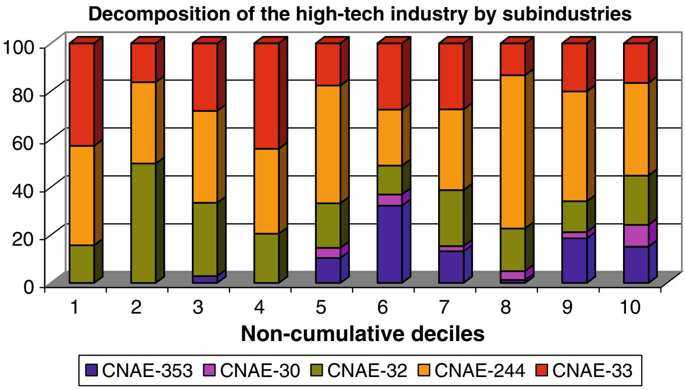

Fig. 4 The high-tech-industry: decomposition by subindustries (expression (5))

By using data for the U.S. economy, Audretsch and Feldman (1996) found that industries in which knowledge spillovers are more widespread tend to cluster at a higher extent than industries in which these externalities are less important. In line with this finding, in what follows, we are interested in determining whether human capital affects the location patterns of high-tech industries in Spain, and also if this variable is more relevant for this industry than for those with lower technological intensity.[19] For this purpose, the 52 Spanish provinces are classified into three groups

[19] An analysis of the sources of agglomeration economies is beyond the scope of this paper. For a general discussion on this matter, see Rosenthal and Strange (2004), in which a wide range of explanations are given (input sharing, labor market pooling, knowledge spillovers, home market effects, economies in consumption, and rent-seeking). For recent econometric analyses of the determinants of agglomeration in

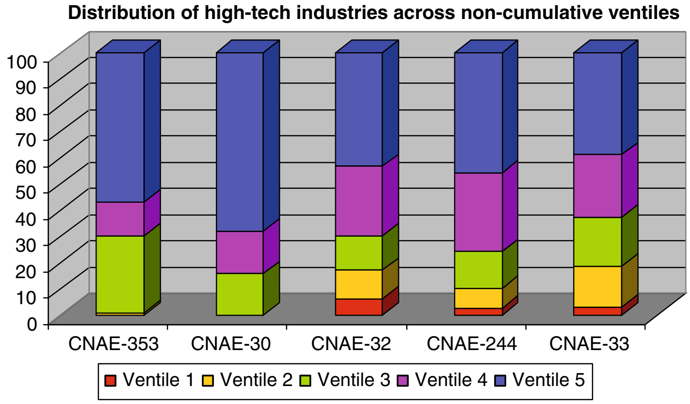

Fig. 5 Decomposition by subindustries (expression (6))

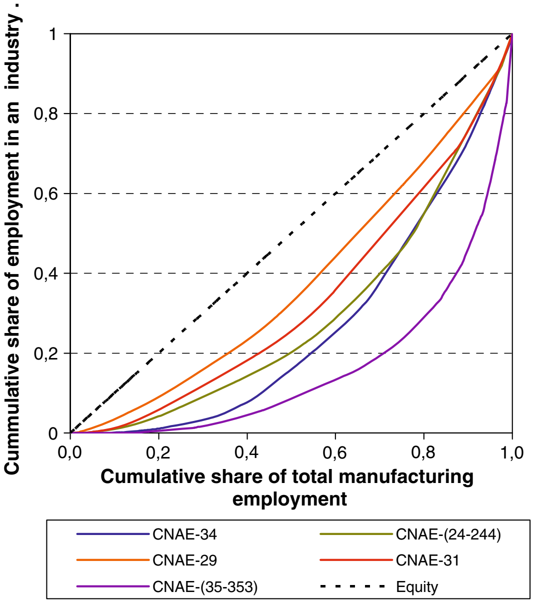

Fig. 6 Employment Lorenz curves of medium-high-tech industries

according to their human capital level.[20] By using the decomposition by groups of locations (see expression (2)), it is possible to determine whether each industry tends to concentrate in those locations with higher human capital. Figure 9 shows that the

Footnote19 continued
Portugal, Belgium, Ireland, and Germany, see Barrios et al. (2009) and Alecke et al. (2006). Specific factors favoring the concentration of the high-tech industry can be found in García Muñiz et al. (2009), in which it is shown that input-output linkages work as a vehicle of knowledge transmission and innovation.

[20] The level of human capital of a province has been determined by the proportion of individuals aged 30–65 having a university degree or a degree in "formación professional superior" (vocational training, secondary technical college).

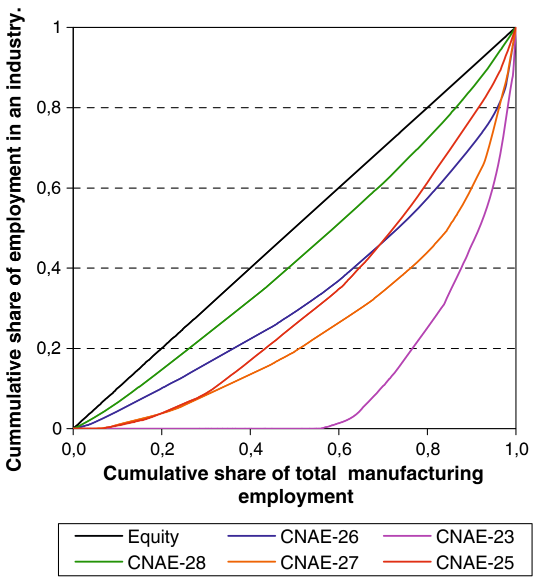

Fig. 7 Employment Lorenz curves of medium-low-tech industries

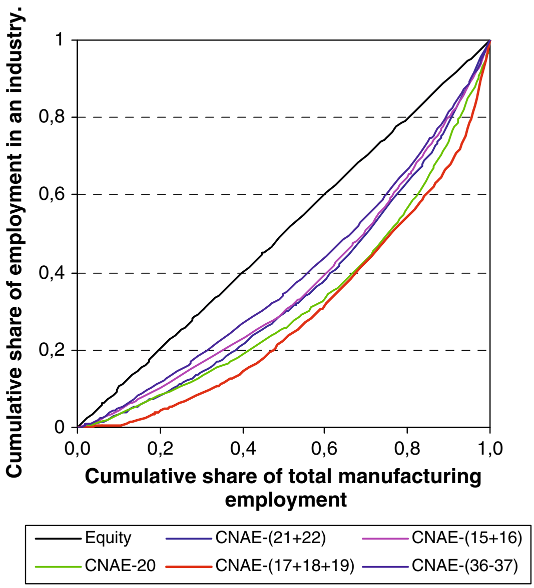

Fig. 8 Employment Lorenz curves of low-tech industries

high-tech employment included in the first decile of the distribution (which represents 10% of the total manufacturing employment and includes those provinces with the lowest presence of high-tech industries) corresponds to locations with an intermediate human capital level, while the high-tech employment included in the top (non-cumulative) decile (which includes provinces with the highest presence of the sector) is located in the provinces with the highest education level. Moreover, even though

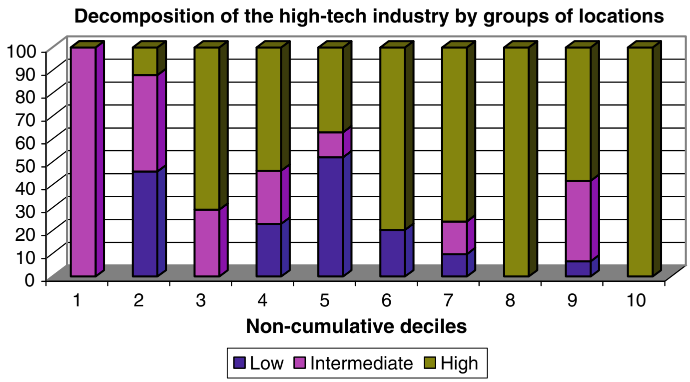

Fig. 9 The high-tech industry: decomposition (2) by groups of provinces (education)

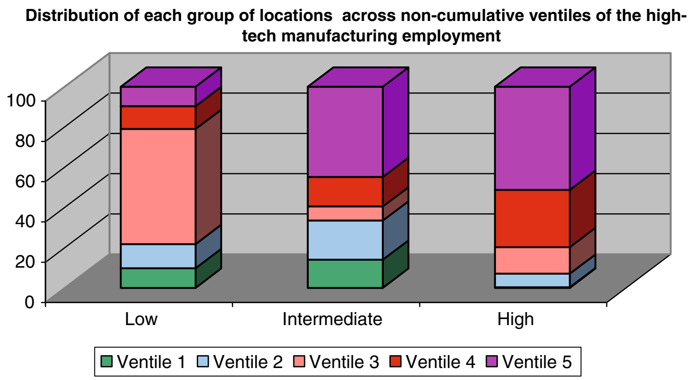

Fig. 10 The high-tech industry: decomposition (3) by groups of provinces (education)

the relationship between the presence of high-tech industry and education level is not monotonic, we find that the proportion of provinces with high human capital level is larger in the last deciles than in the first ones. In other words, it seems that high-tech industries tend to concentrate in locations with the highest human capital levels of the country.

On the other hand, as shown in Fig. 10 (using expression (3) for ventiles; i.e., $\tilde{L}_{(\tilde{x}^k;t)}(\tau_l + 0.2) - \tilde{L}_{(\tilde{x}^k;t)}(\tau_l)$), while over 51% (44%, respectively) of the high-tech employment located in provinces with high (intermediate, respectively) education are concentrated in the fifth ventile of the high-tech industry, less than 10% of the high-tech employees located in provinces with low human capital are in the fifth ventile. In other words, the high-tech industry located in provinces with high and intermediate level of education tends to agglomerate at a higher extent.[21]

[21] When using decomposition (3) for the remaining industries, we find that, in general, the concentration intensity tends to be lower in the group of provinces with lower human capital (see Figs. 17, 18 and 19 in Appendix).

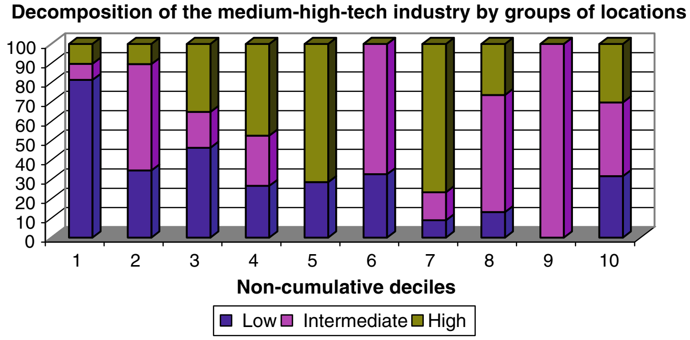

Fig. 11 The medium-high-tech industry: decomposition (2) by groups of provinces (education)

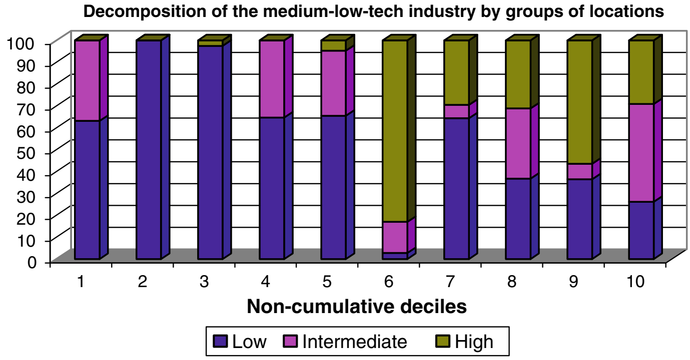

Fig. 12 Medium-low-tech industry: decomposition (2) by groups of locations (education level)

Regarding the medium-high-tech industry, we find that the relationship between localization and human capital is weaker, even though this sector tends to concentrate in provinces with intermediate and high human capital level (Fig. 11).[22] With respect to the medium-low-tech industry, we find that in this industry the weight of provinces with low human capital is much higher than in the remaining industries (the low-tech included), even though this weight decreases in the top deciles (Fig. 12). Finally, we see that the low-tech industry tends to concentrate in provinces with low and intermediate levels of human capital (Fig. 13).

5 Conclusions

This paper has characterized the Lorenz curves used to measure the relative concentration of economic activity in terms of basic properties. This analysis has allowed us to unveil the properties that the literature on regional economics is implicitly assuming when using these curves. This paper has also offered two decompositions of these curves since, as far as we know, no decompositions have yet been suggested in the

[22] In Fig. 10, locations are ranked according the corresponding ratio for the medium-high-tech group.

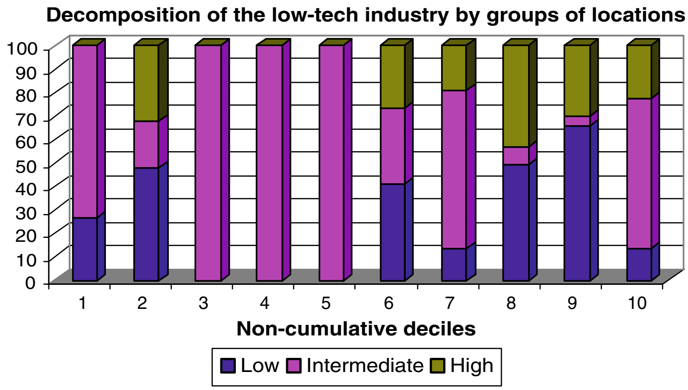

Fig. 13 Decomposition of the low-tech industry by groups of locations

field. One is obtained when locations are partitioned into different groups, while the other is obtained by classifying the sector into several subsectors.

These curves and their decompositions have been used to analyze the concentration of manufacturing industries in Spain in 2008. By grouping industries according to their technological intensity, we find evidence that in Spain the high-tech industry tends to concentrate at a higher extent than the remaining industries, especially in the case of aircraft and spacecraft and office, accounting, and computing machinery.

This result is in line with that previously obtained by Alonso-Villar et al. (2004) by using other concentration measures, which suggests that our finding for Spain seems rather robust. This pattern departs, however, from what was suggested by other scholars when studying the concentration of manufacturing industries in other countries at a two-or-more digit level, since those studies find either evidence of remarkable concentration levels both in high- and low-tech sectors (as Maurel and Sédillot 1999, in France; Bertinelli and Decrop 2005, in Belgium; and Guimarães et al. 2007, in Portugal), or did not find evidence of higher concentration for high-tech industries (as pointed out by Devereux et al. 2004, in the UK).

Finally, even though the relationship between technological intensity and education does not seem monotonic, we have shown that high-tech industries in Spain tend to concentrate in locations with the highest human capital level in the country, while the low-tech industry tends to do so in provinces with low and intermediate levels. This finding is line with those obtained by García Muñiz et al. (2009) and Baptista and Mendonça (2009), since the former find that in Spain high and medium technological sectors are more efficient than the remaining sectors and foster knowledge diffusion in the system, while the latter give evidence of the importance of local access to knowledge and human capital to explain the entry of knowledge-based firms. All the above suggests that the agglomeration of high technology industries is a detectable phenomenon in Spain, and also that knowledge spillovers may be working as an important source of agglomeration in this kind of sector.

Acknowledgments Financial support from the *Xunta de Galicia* (INCITE08PXIB300005PR) and from FEDER is gratefully acknowledged.

Table 3 The $G(d)$ statistic proposed by Getis and Ord (1992)

	$G(d)$	z-value
High-tech	0.084	−0.265
Medium-high-tech	0.104	0.044
Medium-low-tech	0.103	0.046
Low-tech	0.107	0.360

Table 4 Intervals of confidence for the concentration indexes at the 95% of significance (bootstrap, 1,000 replications)

	Ψ_{-1}	Ψ_0	$\Psi_{0.1}$	$\Psi_{0.5}$	Ψ_1	Ψ_2
High-tech	*	*	(0.730, 0.802)	(0.378, 0.400)	(0.331, 0.350)	(0.360, 0.384)
Medium-high-tech	*	*	(0.094, 0.101)	(0.088, 0.093)	(0.084, 0.088)	(0.079, 0.084)
Medium-low-tech	(0.038, 0.040)	(0.039, 0.041)	(0.0399, 0.041)	(0.040, 0.042)	(0.042, 0.044)	(0.046, 0.049)
Low-tech	(0.041, 0.044)	(0.037, 0.040)	(0.037, 0.0397)	(0.036, 0.038)	(0.035, 0.037)	(0.034, 0.036)

Appendix

A: Lemmas

Lemma 1 *Given two ordered income distributions with the same dimension and mean, one distribution dominates the other in the Lorenz sense if and only if the latter majorizes the former.*

Proof See Foster (1985).

Lemma 2 *The Lorenz curve satisfies symmetry, homogeneity, population principle, and Pigou–Dalton transfer principle.*

Proof See Foster (1985).

Lemma 3 *Given two order income distributions with the same dimension and mean, one distribution dominates the other in the Lorenz sense if and only if the latter can be obtained from the former by a finite sequence of regression transfers.*

Proof See Hardy et al. (1934).

B: Expressions of the locational Gini index and the GE family

$$G = \frac{\sum_{l,l'} \frac{t_l}{T}\frac{t_{l'}}{T}\left|\frac{x_l}{t_l} - \frac{x_{l'}}{t_{l'}}\right|}{2\frac{X}{T}}$$

$$\Psi_\alpha(x;t) = \begin{cases} \frac{1}{\alpha(\alpha-1)}\sum_l \frac{t_l}{T}\left[\left(\frac{x_l/X}{t_l/T}\right)^\alpha - 1\right] & \text{if } \alpha \neq 0, 1 \\ \sum_l \frac{x_l}{X}\ln\left(\frac{x_l/X}{t_l/T}\right) & \text{if } \alpha = 1 \end{cases}$$

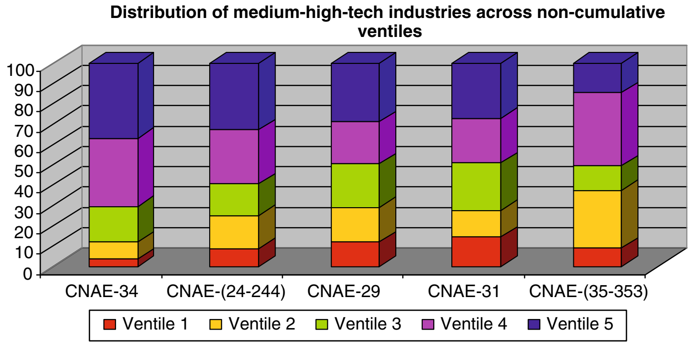

Fig. 14 The medium-high-tech industry: decomposition by subindustries (expression (6))

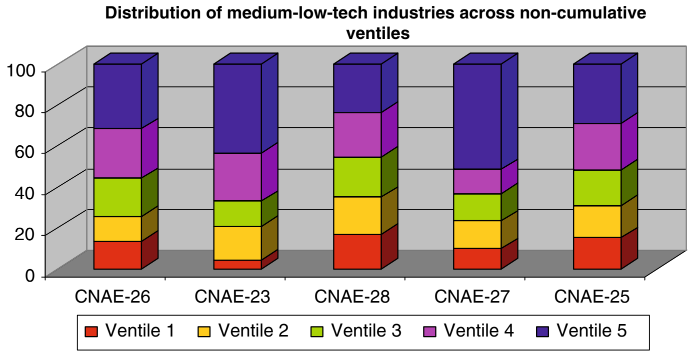

Fig. 15 The medium-low-tech industry: decomposition by subindustries (expression (6))

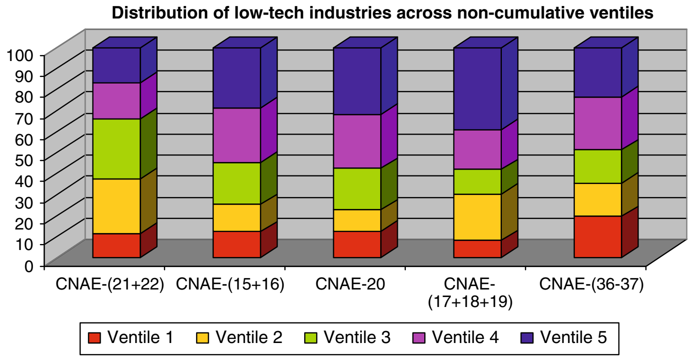

Fig. 16 The low-tech industry: decomposition by subindustries (expression (6))

C: The $G(d)$ statistic

The $G(d)$statistic proposed by Getis and Ord (1992) has been calculated when the variable under analysis is the location quotient:

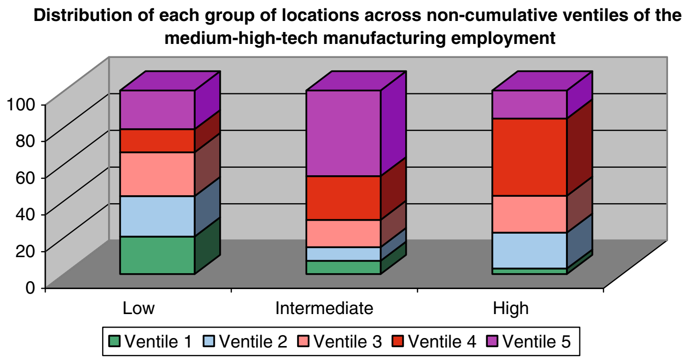

Fig. 17 The medium-tech industry: decomposition (3) by groups of provinces (education)

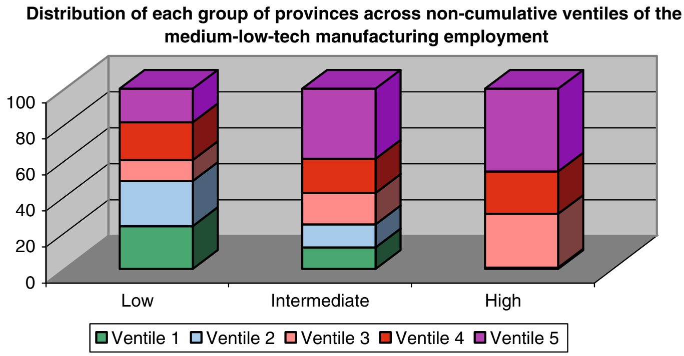

Fig. 18 The medium-low-tech industry: decomposition (3) by groups of provinces (education)

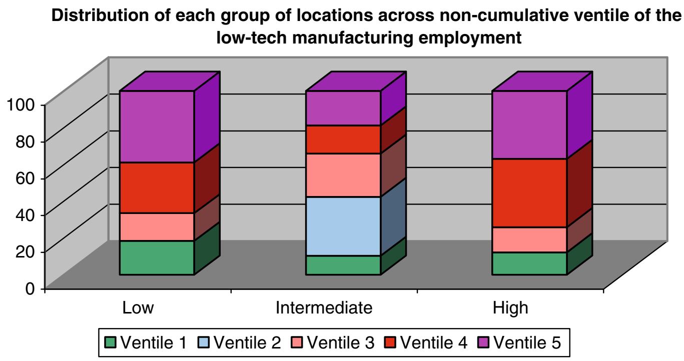

Fig. 19 The low-tech industry: decomposition (3) by groups of provinces (education)

$$G(d) = \frac{\sum_{l=1}^{L} \sum_{l'=1}^{L} w_{ll'}(d) \frac{x_l/X}{t_l/T} \frac{x_{l'}/X}{t_{l'}/T}}{\sum_{l=1}^{L} \sum_{l'=1}^{L} \frac{x_l/X}{t_l/T} \frac{x_{l'}/X}{t_{l'}/T}}, \quad l \text{ not equal to } l'.$$

In this paper, the most popular weighing scheme has been followed, so that $w_{ll'}(d) = 1$ if locations l and l'share a border, and $w_{ll'}(d) = 0$ otherwise (Table 3).

D: Intervals of confidence for Ψ_α

See Table 4.

E: Decomposition by subsectors

See Figs. 14, 15 and 16.

F: Decomposition by locations

See Figs. 17, 18 and 19.

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