

Spatial dependence and the representation of space in empirical models

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Abstract A well-formed spatial model should most likely not produce spatial autocorrelation at all. From this perspective spatial autocorrelation is not (pure) statistical nuisance but a sign of that a model lacks a representation of an important economic phenomenon. In a Knowledge Production Function (KPF) context, this paper shows that a representation of space reflecting the potential of physical interaction between localities by means of accessibility variables on the “right-hand-side”—a simple alternative to spatial lag and spatial error which can be estimated by OLS—captures substantive spatial dependence. Results are verified with Monte Carlo simulations based on Anselin’s (Int Reg Sci Rev 26(2):153–166, 2003) taxonomy of modelled and unmodelled effects. The analysis demonstrates that an accessibility representation of explanatory variables depict the network nature of spatial interaction, such that spatial dependence is actually modelled.

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1 Introduction

The so-called “1st law of geography” (Tobler 1970) states that everything in space is related but the relatedness of things decreases with distance. In any research that

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acknowledges such a law, spatial dependence among spatial units should be perceived as a generic occurrence that is subject to distance-related friction phenomena. Spatial dependence implies, e.g. that activities in one region have an effect on the activities in another region. Distance enters in the sense that the strength of such effects decreases with the distance between regions. “Interaction decreases with distance” (Beckman 2000) is an axiomatic statement in regional science.

Several theoretical approaches in regional and urban economics explicitly presume the existence of spatial dependence.¹ The theoretical underpinnings in the literature on geography and innovation, for instance, focus extensively on various forms of spatial externalities (Feldman 1999). Spatial externalities refer to externalities that are distance sensitive and whose spatial range is limited. Such externalities essentially imply that the characteristics of a location have effects on other locations which diminish with distance, and imply a form of spatial dependence.

It comes as no surprise then that the point in urban and regional economics is reached where it is nearly impossible to estimate an empirical model without receiving requests for spatial autocorrelation tests, since such correlation is caused by underlying spatial dependence among observations. The problem is that there are no truly well-formed spatial models. The two most common spatial models, the spatial lag and the spatial error model, acknowledge such dependence but do not indicate the mechanism by which it arises (Niebuhr 2001). Parameter estimates are however unbiased and consistent.

A well-formed model should most likely not produce spatial autocorrelation at all. From this perspective spatial autocorrelation is not (pure) statistical nuisance but a sign of that a model lacks representation of an important economic phenomenon. McMillen (2003) shows for example that spatial autocorrelation is often produced spuriously by model misspecification. Substantive (as opposed to nuisance) dependence refers precisely to dependence that arises from economic phenomena that incorporate spatial interaction.

The current paper is focused on how substantive spatial dependence among observations can be modelled directly in empirical spatial models. It shows that a model with spatially discounted explanatory variables, i.e. a form of spatial cross-regressive model, is a simple alternative to the spatial lag model. Such a model incorporates an explicit mechanism through which substantive spatial dependence is mediated and is a natural way to specify a model when theory suggests the presence of substantive spatial dependence.

The paper utilizes Swedish data on R&D and patent applications to the European Patent Office (EPO) to estimate a knowledge production function (KPF). It is demonstrated that the modelling of spatial relationships in the explanatory variables captures and explains the spatial dependence in the dependent variable. Accessibility variables are used to approximate the potential for interaction among localities (Weibull 1980)

¹ Spatial dependence has implicitly been emphasized for a long time in urban and regional economics. The classic work by Harris (1954), for instance, illustrated how the location of production is affected by the market potential. In Harris’ study the market potential of a location was defined as the location’s internal demand along with the distance-weighted sum of the demand at other locations. Such effects relate to the traditional central-place-system (CPS) framework, in which consumers travel to the central location to consume high-order goods, (cf. Dicken and Lloyd 1990).

and the overall interaction pattern among localities is recognized.² This is achieved by acknowledging that certain localities belong to the same functional region. Functional regions are delineated based on the frequency of observed interaction. Mobility and interaction naturally vary between different geographical scales, such as the local, the intra-regional and the inter-regional scale. The paper illustrates the importance of recognizing this generic structure among localities.

To validate the findings in the KPF context, the paper conducts Monte–Carlo simulations and presents tests of how the inclusion of spatially discounted variables on the “right-hand-side” (RHS) affect the test statistics of the most common tests for spatial dependence. Specifically, the paper examines: (1) if the inclusion of spatially discounted variables on the RHS in empirical spatial models removes (or reduces) spatial autocorrelation among residuals and (2) if significance of the estimated parameters of the spatially discounted variables can be interpreted as spatial dependence.

A distinct advantage of the model formulation advocated in the paper is that it is easy to implement and can in principle be estimated with OLS. The spatial lag and the spatial error model, for instance, require maximum-likelihood estimation. It can also readily be applied in more complicated situations than the OLS, such as the Poisson model. Moreover, it can account for both local and global spillovers.

The remainder of the paper is organised as follows: in Sect. 2 an overview of standard spatial dependence models are presented. Section 3 discusses the accessibility concept and shows how it can be used to incorporate spatially discounted variables on the RHS in empirical spatial models. Section 4 presents an empirical application of the method described in Sect. 3 by estimating a KPF across Swedish municipalities. Section 5 presents Monte Carlo simulations of the method applied in Sect. 4 to investigate to what extent the results in Sect. 4 can be generalized. Summary and conclusions are given in Sect. 6.

2 Spatial dependence in empirical models

Potential statistical problems associated with dependence among observations in cross-sectional data are extensively treated in spatial econometrics literature (e.g. [Anselin 1988a](#); [Anselin and Florax 1995](#)). [Anselin \(1988a\)](#) refers to two types of spatial dependence: substantive spatial dependence and nuisance dependence (see also [Anselin and Florax 1995](#); [Florax and van der Vlist 2003](#)). The first deals with the spatial interaction of the variable of interest, e.g. the dependent variable of the regression model. The second is about the spatial dependence between the ignored variables in the model, which reflects the error terms. While substantive spatial dependence necessitates the development of spatially explicit models, nuisance dependence involves adjustments of existing specifications, for example to express neighbourhood effects in the model ([Dubin 1992](#)).

The presence of any kind of spatial dependence can invalidate regression results. In the case of spatial error autocorrelation, OLS parameter estimates are inefficient and

² The accessibility measure used in the paper satisfies criteria of consistency and meaningfulness, as shown by [Weibull \(1976\)](#), and has a clear coupling to spatial interaction theory.

in the presence of spatial lag dependence parameters become biased and inconsistent (Anselin 1988a). The general expression for the spatial lag model is

$$y = \rho W y + x \beta + u \quad (1)$$

where y is the dependent variable, W is a spatial weight matrix, $W y$ is a vector of lagged dependent observations ρ is a spatial autoregressive parameter, x is a matrix of independent variables, β is a vector of regression parameters and u is a vector of independent disturbance terms, $u \sim N(0, \sigma^2)$.

The standard spatial model with autoregressive disturbances represents an alternative form of spatial dependence. Spatial error autocorrelation is modelled as follows:

$$y = x \beta + \varepsilon \quad (2a)$$

$$\varepsilon = \lambda W \varepsilon + u \quad (2b)$$

where ε is the spatially autoregressive error term, λ is the parameter of the spatially autoregressive errors $W \varepsilon$. The reduced form of (2) then becomes:

$$y = x \beta + (I - \lambda W)^{-1} u \quad (3)$$

The researcher's job is to determine which model (spatial lag or spatial error) best fits the data. Thus, the job is to determine whether $\rho = 0$ or $\lambda = 0$. It could be the case that both differ from zero and the question is then which model to choose. If $\rho = 0$ and $\lambda = 0$ then OLS is applicable (if other necessary conditions are met). Anselin and Rey (1991) showed in a Monte Carlo study that if tests for spatial lag and spatial error are both significant, the larger of the two statistics probably indicates the correct model.

The standard taxonomy of spatial lag and error models has been extended by Anselin (2003) (see also Anselin and Bera 1998; Anselin 2001). He distinguishes between a global and a local range of dependence, and analyzes how this distinction affects the specification of models with (1) spatially lagged dependent variables ($W y$), (2) spatially lagged explanatory variables ($W x$) and (3) spatially lagged error terms ($W u$). The taxonomy in Anselin (2003) has two dimensions. The primary dimension is whether the spatial correlation in the reduced form pertains only to unmodelled effects (error terms), to modelled effects (included explanatory variables), or to both. Specification tests and theoretical arguments should suggest the nature of the externalities and dictate the proper alternative. The second dimension in the taxonomy is the distinction between global and local spillovers. In the reduced form this comes down to the inclusion of a spatial multiplier effect of the form $(I - \lambda W)^{-1}$ versus a simple spatial lag term using spatial weights W .³ The taxonomy is presented in Table 1. Note that $W y$ only appears on the RHS for models that incorporate global spillovers.

³ The definition of geometric series gives $(I - \rho W)^{-1} = I + \rho W + \rho^2 W^2 + \dots$, and the interpretation is that every location is correlated with every other location in the system, but closer locations more so (since in most cases $|\rho| < 1$).

Table 1 Taxonomy of structural forms

| | Local externalities | Global externalities |
|-------------------------|---------------------------------------|--|
| Unmodelled effects, u | $y = x\beta + u + \gamma Wu$ | $y = \lambda Wy + x\beta - \lambda Wx\beta + u$ |
| Modelled effects, x | $y = x\beta + Wx\rho + u$ | $y = \rho Wy + x\beta + u - \rho Wu$ |
| Both u and x | $y = x\beta + Wx\rho + u + \gamma Wu$ | $y = (\rho + \lambda)Wy - \rho\lambda W^2y + x\beta - \lambda Wx\beta + u - \rho Wu$ |
| | $y = x\beta + Wx\rho + u + \rho Wu$ | $y = \rho Wy + x\beta + u$ |

Source: [Anselin \(2003\)](#)

If considering modelled effects and global spatial spillovers the specification is

$$y = (I - \rho W)^{-1}x\beta + u \tag{4}$$

and after multiplying with $(I - \rho W)$:

$$y = \rho Wy + x\beta + u - \rho Wu \tag{5}$$

This model contains both a spatially lagged dependent variable as well as a spatial moving average (SMA) error. When there are local spillovers in the explanatory variables the specification is instead

$$y = x\beta + Wx\rho + u \tag{6}$$

where ρ is not a scalar as in (4) but a column vector matching the column dimension of Wx . If W is a first order contiguity matrix (non-zero elements for locations with common boundaries), this model would be appropriate when the proper spatial range of the explanatory variables is the location and its immediate neighbours (and not neighbours' neighbours).

In Sect. 3 it is demonstrated how the inclusion of accessibilities on the RHS can account for global spillovers without estimating an equation like (5), which requires maximum likelihood (ML). The general idea is to use an expression similar to the one for local spillovers (6) but with a weight matrix, that incorporates all locations (not only the neighbours).

3 Accessibility, interaction patterns and the representation of space

3.1 Spatial discounting and the accessibility concept

A general presumption is that the extent of spatial dependence between localities depends on the frequency of various types of interaction between those localities.⁴

⁴ For instance, spatial externalities that are mediated by the labour market—i.e. pecuniary externalities—depend on the interaction on the labour market, which is a market in which mobility is highly limited by distance.

Spatial discounting procedures should thus relate to concepts from *spatial interaction theory*. Accessibility is precisely such a concept.

The accessibility concept has a long history in both regional science and transport economics. According to Martellato et al. (1998, p. 163), Hansen (1959) provided one of the first foundations for the use of accessibility “theory” and defined accessibility as *potential of opportunities for interaction*. Baradaran and Ramjerdi (2001) note that this way of defining accessibility is closely associated with gravity models based on the interaction of masses. In surveying the literature, Weibull (1980, p. 54) remarks that interpretations of accessibility usually relate to (see also Pirie 1979; Jones 1981)

1. Nearness
2. Proximity
3. Ease of spatial interaction
4. Potential of opportunities for interaction
5. Potentiality of contacts with activities and supplies

The most popular interpretation of accessibility relates to (3) and (4) above, which emphasize the link between accessibility and interaction. High accessibility between two locations translates into a high potential for interaction and a high potential for spatial externalities between the locations.⁵ In its most general form, the total accessibility of location i to an arbitrary opportunity x , A_i^X can be written as

$$A_i^X = x_1 f(c_{i1}) + \cdots + x_i f(c_{ii}) + \cdots + x_n f(c_{in}) = \sum_{j=1}^n x_j f(c_{ij}) \quad (7)$$

where $f(c)$ is a non-increasing function of distance. This function is often referred to as the distance-deterrence (or distance-decay) function. Note that (7) also includes location i 's internal accessibility to opportunity x .

There are several alternative ways in which numerical values of a location's accessibility can be calculated. By using an axiomatic approach to the measurement of accessibility Weibull (1976) narrowed down the measurement of accessibility to those measures that satisfy certain axioms. Accessibility measures satisfying the axioms fulfil requirements of consistency and meaningfulness (see Weibull 1976, pp. 359–362 for details). Weibull (1976) maintains that accessibility is related to choice contexts for spatial interaction. A choice context is represented by a configuration of opportunities for spatial interaction. Let d denotes the distance from a point of reference and let a denote the attractiveness (e.g. size of an opportunity) at a location in question. A configuration \bar{c} is then defined as an n -tuple of opportunities:

$$\bar{c} = \langle (d_1, a_1); (d_2, a_2); \dots; (d_n, a_n) \rangle = \langle (d_i, a_i) \rangle_{i=1}^n \quad (8)$$

where n is a finite positive integer, $n \in N = \{1, 2, \dots\}$. The author then defines an accessibility measure as a function that to any configuration \bar{c} attributes a finite and

⁵ In view of this, Karlsson and Manduchi (2001) have maintained that accessibility makes the general concept of geographical proximity, which is often emphasized in the literature on knowledge spillovers, operational.

non-negative real number $f(\bar{c})$, $f: C \rightarrow R_+$, where C is the class of all configurations \bar{c} .⁶

Accessibility measures with an exponential distance-decay function belong to the class of measures that satisfy the axioms stated in Weibull (1976).⁷ In this case (7) is formulated as:

$$A_i^X = \sum_{j=1}^n x_j \exp\{-\gamma t_{ij}\} \quad (9)$$

where t_{ij} is the time distance between location i and j , and γ is a time distance-friction (or sensitivity) parameter. Distance is often measured by the physical distance, but a better way to measure it is to use the time it takes to travel between different locations (Beckman 2000). Time distances are also crucial for the frequency of business meetings and the spatial borders of labour markets (see e.g. Johansson et al. 2002; Hugosson and Johansson 2001 for the Swedish case). For the interpretation of accessibility, it should be noted that the accessibility value in (9) may improve in two ways; either by an increase in the size of the opportunity, x_j , or by a reduction in the time distance between location i and j .

The accessibility measure in (9) satisfies criteria of meaningfulness and consistency, and can also be motivated theoretically by relating it to the preference structure in random choice theory. This procedure starts from a stochastic specification of the utility an individual (or firm/organization) in location i derives from accessing an opportunity x in location j , U_{ij} . A simple form of such a specification is shown below:

$$U_{ij} = V_{ij} + \varepsilon_{ij} \quad (10a)$$

$$V_{ij} = \ln x_j - \phi c_{ij} - \alpha t_{ij} \quad (10b)$$

where V_{ij} is the deterministically known utility and ε_{ij} denotes the random influence from non-observed factors. x_j is the opportunity in location j , c_{ij} denotes the cost of travelling from i to j , t_{ij} the time distance between the locations and ε_{ij} random influence from non-observed factors. Assuming that ε_{ij} is IID and extreme value (Gumbel) distributed, the probability that an individual in location i will choose to interact with location j (in the sense of accessing opportunity x in location j) is given by⁸

$$P_{ij} = \exp\{V_{ij}\} / \sum_{j \in M} \exp\{V_{ij}\} \quad (11)$$

where the set $M = \{1, \dots, i, \dots, j, \dots, n\}$ contains all the possible locations to access opportunity x . Equation (11) is the general expression for the choice probabilities in

⁶ In e.g. a country, the spatial distribution of an opportunity across locations and the infrastructure connecting these locations results in a given spatial configuration upon which accessibility calculations are made.

⁷ Furthermore, the negative exponential function emerges directly from an entropy maximizing framework with origin, destination and cost constraints (see e.g. Wilson 2000; Smith 1978).

⁸ This condition is derived in several texts, see inter alia Train (1986) and Maddala (1983).

the multinomial logit model. The numerator represents the preference value accessing opportunity x in location j whereas the denominator is the sum of all such preference values. This means that, *ceteris paribus*, the probability of interaction between location i and j with respect to the given individual increases with the size of the attraction factor x in location j and decreases with the cost of accessing x_j from location i .

Assuming that the cost of traveling between the locations, c_{ij} , is proportional to the time distance, such that $c_{ij} = \alpha c t_{ij}$ the denominator in (11) can be expressed as:

$$A_i^x = \sum_{j \in M} x_j \exp \{-\gamma t_{ij}\} \quad (12)$$

where $\gamma = (\phi\alpha_c + \alpha)$. The expression in (12) is exactly the one in (9). Equation (12) gives the sum of the preference values, conditional on a location in region i . Observe that the set of M alternatives is the same in any region in the set $M = \{1, \dots, n\}$. Thus, an individual located in region $s \in M$ faces the same set of alternatives, i.e. alternative regions, as an individual located in $i \in M$, but the sum of the preference values from that location is different, i.e. $t_{sj} \neq t_{ij}$. Hence, locations where the sum of such preference values is high can thus be interpreted as location with high attractiveness (cf. Ben-Akiva and Lerman 1979). Both the size of the attractor (x) and time distances in (12) are arguments in the preference function in (10).

3.2 Incorporating accessibility in spatial models

Having motivated the accessibility measure in (9) and (12) we consider an example of model specification. Suppose we are interested in the relationship between y (output) and x (input). Suppose also that the theory suggests spatial dependence which diminishes with distance. Then, a natural way of specifying the empirical model is

$$y_i = W_i x \beta + u_i \quad (13)$$

where $W_i x$ is A_i^x . This specification is highly related to the specification in (4). In fact (13) shows how the inclusion of accessibility on the RHS can account for global spillovers without estimating an equation like (5), which requires ML-estimation. Equation (13) can in principle be estimated by means of OLS.

The inclusion of a single variable that measures the total accessibility does not provide any information about the spatial range or the structure of (potential) dependencies. However, by recognizing different spatial levels and their associated interaction opportunities, their effect can be estimated. Suppose for instance that the locations under study are municipalities. A typical municipality belongs to a local labour market (LLM) region, which can be seen as an approximation of a functional unit region (Cheshire and Carbonaro 1995). An LLM-region consists of a number of municipalities that together constitute an integrated labour market. LLM-regions are connected to other LLM-regions by means of economic and infrastructure networks. The same prevails for the different municipalities within a LLM region. Moreover, each municipality can also be looked upon as a number of nodes connected by the same type of

networks. The borders between LLM-regions are in general characterized by a sharp decline in the intensity of economic interaction including commuting. With reference to such a structure, it is possible to define three different spatial levels with different characteristics in terms of mobility and interaction opportunities. Because of this, it is also possible to construct three different categories of accessibility (cf. [Johansson et al. 2002](#)). Specifically, the total accessibility of a location (municipality) to a specific opportunity can be decomposed into local, intra-regional and inter-regional:

$$A_i^X = A_{iL}^X + A_{iR}^X + A_{iOR}^X \tag{14}$$

where

$A_{iL}^X = x_i \exp \{-\gamma_L t_{ii}\}$, local accessibility to opportunity x for location i .

$A_{iR}^X = \sum_{r \in R, r \neq i} x_r \exp \{-\gamma_R t_{ir}\}$, intra-regional accessibility to opportunity x for location i .

$A_{iOR}^X = \sum_{k \notin R} x_k \exp \{-\gamma_{OR} t_{ik}\}$, inter-regional accessibility to opportunity x for location i .

In the equations above, r defines locations within the own region R , and k defines locations in other regions. It is also evident that the value of γ depends on whether the interaction is local (within location i), intra-regional (between locations in a region), or inter-regional (location i and j in different regions) (see [Johansson et al. 2003](#)). The time distances for the three types of accessibilities indicate that there may be a qualitative difference. With the location of Swedish municipalities as an example, the average commuting time distance within a municipality amounts to an average of 10 min. The corresponding average for commuting between municipalities in the same LMM region can be approximated by 25–30 min. The average commuting time from a municipality to municipalities outside the pertinent region is more than 45 min. [Johansson et al. 2003](#) showed that the commuters perceived the time friction differently for different time intervals. Hence, the commuters’ time sensitivity followed a non-linear form, according to the pattern $\gamma_L < \gamma_{OR} < \gamma_R$. That is, increased intra-regional commuting time will hamper the propensity to travel the most.

Rewriting (13) to include the decomposition in (14) yields

$$y_i = W_{i1}x\beta_1 + W_{i2}x\beta_2 + W_{i3}x\beta_3 + u_i \tag{15}$$

where $W_{i1}x = A_{iL}^X$, $W_{i2}x = A_{iR}^X$ and $W_{i3}x = A_{iOR}^X$. W_1 is a weight matrix with $w_{ii} \neq 0$ and $w_{ij} = 0$, W_2 is weight matrix with $w_{ii} = 0$ and $w_{ij} \neq 0$ if location i and j in the same region and W_3 a weight matrix with $w_{ii} = 0$ and $w_{ik} \neq 0$ if location i and k in different regions. Observe that (15) is still a specification with modelled effects and global spillovers. However, the inclusion of the three components separately means that the effect from each component can be revealed and compared with each other.⁹

⁹ Similar methods have been applied in a series of papers (see e.g. [Gråsjö 2005, 2006](#); [Andersson and Ejeremo 2004, 2005](#); [Andersson and Karlsson 2007](#)).

4 An empirical application

4.1 Presentation of model

This section employs the procedure described in Sect. 3 by estimating a knowledge production function across Swedish municipalities.¹⁰ The number of patent applications in municipality i is used as the dependent variable. Local, intra-regional and inter-regional accessibility to university and company R&D are explanatory variables. The time-sensitivity parameter value β_L is set to 0.02, β_R to 0.1 and β_{OR} to 0.05. Johansson et al. (2003) estimated these values by using data on commuting flows within and between Swedish municipalities in 1990 and 1998. The model to be estimated takes the following form:

$$\begin{aligned} Pat_i = & b_1 + b_2 A_{iL}^{uR \& D} + b_3 A_{iR}^{uR \& D} + b_4 A_{iOR}^{uR \& D} \\ & + b_5 A_{iL}^{cR \& D} + b_6 A_{iR}^{cR \& D} + b_7 A_{iOR}^{cR \& D} + u_i \end{aligned} \quad (16)$$

where uR&D denoted university R&D and cR&D denotes company R&D. Both types of R&D are measured in man-years. In order to check if the spatial weights defined in (14) perform better than simple one-zero weights, OLS estimations of (16) are compared with OLS estimations of

$$\begin{aligned} Pat_i = & b_1 + b_2 x_{iL}^{uR \& D} + b_3 x_{iR}^{uR \& D} + b_4 x_{iOR}^{uR \& D} \\ & + b_5 x_{iL}^{cR \& D} + b_6 x_{iR}^{cR \& D} + b_7 x_{iOR}^{cR \& D} + u_i \end{aligned} \quad (17)$$

In (17) x_{iL} , x_{iR} and x_{iOR} denotes R&D efforts locally, within own LLM region, and in other LLM regions. The error terms are investigated whether or not they are spatially autocorrelated. The estimations and the test results are presented in Sect. 4.3.

4.2 Weight matrices

Regressions on the patent data are conducted with two types of weight matrices; one with accessibility weights according to (9) and (15), which enters (16) and the other with binary weights, which enters (17). When checking for spatial dependence in the error structure and spatial autocorrelation in the variables (i.e. doing the tests) the results from using a row standardised binary matrix is compared with the results from an inverse time distance matrix. In the binary matrix, the weight w_{ij} is greater than zero if municipality i and j are in the same LLM-region, and zero otherwise. The inverse distance matrix has a weight greater than zero if i and j are within certain time distance bands from each other. The distance bands used are 30, 60, 90 and 120 min for the variable tests and additionally 180, 240 and 300 min for the regression tests.

¹⁰ We thank Olof Ejerno for providing patent data across Swedish municipalities.

Table 2 Moran's I values of variables for different weight matrices

| | Binary | $t < 30$ min | $t < 60$ min | $t < 90$ min | $t < 120$ min |
|---|----------------|---------------------|---------------------|---------------------|---------------|
| Number of patents, inventor | 0.082 (0.99) | 0.551 (3.07) | 0.290 (2.37) | 0.206 (2.02) | 0.157 (1.75) |
| Number of patents, proprietor/applicant | 0.021 (0.28) | 0.149 (0.84) | 0.077 (0.65) | 0.058 (0.59) | 0.043 (0.51) |
| Accessibility to university R&D, local | -0.015 (-0.13) | 0.014 (0.10) | 0.025 (0.23) | 0.012 (0.15) | 0.007 (0.11) |
| University R&D, local | -0.016 (-0.14) | 0.007 (0.06) | 0.020 (0.19) | 0.009 (0.12) | 0.004 (0.08) |
| Accessibility to company R&D, local | 0.012 (0.18) | 0.140 (0.79) | 0.067 (0.57) | 0.048 (0.50) | 0.035 (0.42) |
| Company R&D, local | 0.012 (0.18) | 0.142 (0.81) | 0.068 (0.58) | 0.049 (0.50) | 0.035 (0.42) |

$Z(I)$ values in parenthesis

Significant values in bold (95% confidence level)

4.3 Estimation and test results

4.3.1 Spatial autocorrelation in variables

Number of patents is a yearly average during the period 1994–1999 for the municipalities in Sweden. Two types of patent variables are used: (1) patents registered by inventor and (2) patents registered by proprietor/applicant. The last is often the company where the inventor is employed. If all companies are located in municipalities where the inventors live, then the two variables are identical, but this, however, is rarely the case. Accessibility to university and company R&D are computed using university R&D measured in man years (full-time equivalents) during the period 1993/1994–1999 for Swedish municipalities.

As can be seen from Table 2, the number of patents in a municipality registered by inventor is the only variable that is spatially autocorrelated. For all weight matrices, except the binary and the one with a time distance band less than 120 min, the Moran's I is statistically significant. The binary matrix has especially difficult to pick up potential spatial autocorrelation.

4.3.2 Spatial dependence in error terms

In order to establish if the inclusion of spatially weighted explanatory variables by means of the accessibility concept can reduce spatial dependence, the regressions are conducted with patents registered by inventor as the dependent variable. Then we have a spatially dependent variable on the LHS, whose spatial effects we are trying to model. Two questions are in focus.

1. Does the inclusion of accessibility have any affect on spatial dependence in the residuals?
2. Does the variable separation on different geographical levels (local, intra-regional and inter-regional) result in error terms that are spatially independent?

Table 3 Regression results and Moran's I , LM-err and LM-lag values for different weight matrices

| | R1 | R2 | R3 | R4 |
|---|-----------------------|-----------------------|---------------------|---------------------|
| University R&D, local | 0.029 (3.68) | | 0.029 (3.79) | |
| University R&D, intra-regional | | | 0.00005 (0.04) | |
| Company R&D, local | 0.376 (6.12) | | 0.367 (5.83) | |
| Company R&D, intra-regional | | | 0.013 (2.94) | |
| Access to university R&D, local | | 0.037 (3.70) | | 0.031 (3.62) |
| Access to university R&D, intra-regional | | | | -0.018 (-1.23) |
| Access to company R&D, local | | 0.421 (5.97) | | 0.440 (6.23) |
| Access to company R&D, intra-regional | | | | 0.200 (2.54) |
| Adjusted R^2 | 0.892 | 0.889 | 0.911 | 0.918 |
| LM-err, binary | 12.27 (0.0005) | 11.32 (0.0008) | 0.013 (0.908) | 0.036 (0.848) |
| LM-lag, binary | 0.413 (0.521) | 0.385 (0.535) | 0.0002 (0.988) | 0.020 (0.887) |
| Moran's I , binary | 0.176 (0.020) | 0.169 (0.024) | -0.010 (0.499) | -0.010 (0.486) |
| Moran's I , $t < 30$ min | 0.872 (6.0E-7) | 0.798 (4.6E-6) | 0.133 (0.415) | 0.115 (0.474) |
| Moran's I , $t < 60$ min | 0.417 (0.0003) | 0.379 (0.001) | -0.011 (0.499) | 0.019 (0.410) |
| Moran's I , $t < 90$ min | 0.304 (0.001) | 0.276 (0.003) | -0.001 (0.474) | 0.010 (0.433) |
| Moran's I , $t < 120$ min | 0.242 (0.004) | 0.220 (0.007) | 0.0004 (0.467) | 0.013 (0.418) |
| Moran's I , $t < 180$ min | 0.170 (0.014) | 0.155 (0.022) | -0.0006 (0.472) | 0.008 (0.434) |
| Moran's I , $t < 240$ min | 0.134 (0.028) | 0.121 (0.041) | -0.002 (0.0479) | 0.004 (0.447) |
| Moran's I , $t < 300$ min | 0.118 (0.038) | 0.107 (0.053) | -0.002 (0.478) | 0.003 (0.454) |

White's (1980) robust standard errors are used for t values. t values in parenthesis for parameter estimates
The intra-regional variables in R3 and R4 are collinear, but this does not harm the residuals

p values in parenthesis for Moran's I , LM-err and LM-lag

R1 and R3 are modifications of (17) and R2 and R4 are modifications of (16)

In binary matrix: weight $w_{ij} > 0$ if municipality i and j belong to the same LLM-region, 0 otherwise

Non-binary matrix: $w_{ij} = 1/(t_{ij})$

Significant values in bold (95% confidence level)

Test statistics for Moran's I and for LM-err and LM-lag is computed to assess the questions above.¹¹ The first tests are conducted on a simple specification, with only university and company R&D as explanatory variables. If the tests indicate spatial dependence the model must be changed in order to capture the spatial effects. Following the procedure discussed in Sect. 3, the strategy here is to add variables (first intra-regional and then if necessary inter-regional variables) on the RHS to account for the spatial effects. Regression and test results are presented in Table 3. In this table, all three tests, i.e. Moran's I , LM-err and LM-lag, are carried out based on a binary

¹¹ There are several test developed to detect spatial dependence. The most widely applied test statistic is Moran's I . Cliff and Ord (1972) and Hordijk (1974) applied the principle for spatial autocorrelation to the residuals of regression models for cross-sectional data. Tests for spatial error versus spatial lag can be conducted by using the Lagrange Multiplier (LM) principle (see e.g. Burridge 1980; Anselin 1988b; Anselin and Florax 1995).

W -matrix. However, the table also reports the Moran's I test statistic when using different cut-off values for the inverse time–distance matrix. As can be seen from the table, the spatial reach of spatial dependence among the residuals is reduced when we add accessibility variables on the RHS. A comparison of R1 and R2 in Table 3 answers the first question above. The result indicates that the accessibility concept has a minor advantage, since R2 has higher p values for the test statistics compared to R1. This is true for all weight matrices. But the error terms in both R1 and R2 are still spatially dependent (except with the inverse time distance matrix, $t < 300$ min in R2). In R3 and R4 the intra-regional variables are included and then the tests are not able to pick up any spatial dependence. Thus, specifications like R3 and R4 do model the spatial structure of patent registered by the inventor's home address (i.e. the municipality where he/she lives).

5 Monte Carlo simulations

By applying the taxonomy in Anselin (2003) in the Monte Carlo simulations, we test how the inclusion of spatially discounted variables on the RHS affects the extent of spatial autocorrelation. Data for the dependent variable and/or the error term are generated such that it is spatially dependent. Then a comparison is made between models with and without spatially discounted variables. Rejection frequencies of some common test for spatial autocorrelation (Moran's I , LM-lag and LM-err) are then presented for each model. Parameter accuracy is assessed by checking bias and root mean square error (RMSE) for the estimated parameters of each model. Moreover, to what extent significance of the estimated parameters of the spatially discounted explanatory variables can be interpreted as evidence of spatial dependence is also assessed.

The Monte Carlo simulations presented below follows the taxonomy in Anselin (2003) but cover only the case of global spillovers in order to save space. Results for the case of local spillovers can be obtained from the authors upon request. These results are qualitatively exactly the same as the results presented in this section.

5.1 Design of the simulations

Locations are normally generated randomly in Monte Carlo simulations. However, here the locations of the Swedish municipalities are used. There are several reasons for this

- (1) A uniform distribution of locations, which is often used, is not very realistic. It is more probable that the locations are clustered.
- (2) The municipalities in Sweden are divided into LLM-regions, which makes it possible to test intra-regional and inter-regional effects separately.
- (3) The distances between the municipalities are real travelling time, which is also more realistic than the often used Euclidean distance.

Regarding the data generating process, independent variables consist of an intercept and an x -variable drawn from a uniform distribution with range 0–5. This data remains

Table 4 Structural models in the data generating process

| | Global spillovers | True values, y | Error, e |
|---|----------------------|---------------------------|-------------------------|
| 1 | Unmodelled effects | $x\beta$ | $(I - \lambda W)^{-1}u$ |
| 2 | Modelled effects | $(I - \rho W)^{-1}x\beta$ | u |
| 3 | Both effects, global | $(I - \rho W)^{-1}x\beta$ | $(I - \lambda W)^{-1}u$ |

Note that ρ is a scalar in 2 and 3 and a column vector matching the column dimension of Wx in 5 and 6

the same for all repetitions. The “true” y values and the error structures are generated according to the structural equations of Anselin (2003) taxonomy (see Table 4).

The “true” values of the regression coefficients are $\beta_1 = 0.5$ and $\beta_2 = 1.0$. W is a weight matrix with inversed time distances. Two different weight matrices are used in the data generating process:

- W_{60} , with $w_{ij} \neq 0$ if $t_{ij} < 60$ min, zero otherwise
- W_{120} , with $w_{ij} \neq 0$ if $t_{ij} < 120$ min, zero otherwise

The reason for using different weight matrices is to test for the importance of inter-regional accessibility. The average number of joins for a location is then 7 for W_{60} and 24 for W_{120} . The error term u is drawn from a normal distribution with mean zero and variance one. The parameters ρ and λ determine the strength of the spatial dependence. Our aim is to generate data such that estimation of a model like $y_i = b_1 + b_2x_i + \varepsilon_i$ results in spatial autocorrelation. For this reason, the following parameter values have been used:

- $\rho = 0.4, \lambda = 0.75$ for W_{60}
- $\rho = 0.3, \lambda = 0.65$ for W_{120}

Each experiment is repeated 1,000 times and every repetition contains three different OLS regressions: (1) without accessibilities, $y_i(x_i)$, (2) with intra-regional accessibility, $y_i(x_i, A_{iR}^X)$ and (3) with intra- and inter-regional accessibility, $y_i(x_i, A_{iR}^X, A_{iOR}^X)$, with x matrices according to:¹²

$$\text{OLS 1 : } x = \begin{pmatrix} 1 & x_1 \\ 1 & x_2 \\ \cdot & \cdot \\ \cdot & \cdot \\ 1 & x_{288} \end{pmatrix}, \quad \text{OLS 2 : } x = \begin{pmatrix} 1 & x_1 & A_{1R}^X \\ 1 & x_2 & A_{2R}^X \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ 1 & x_{288} & A_{288R}^X \end{pmatrix},$$

¹² As can be seen from the notation, local accessibility is not used in the simulations. Hence, for a given location (observation) the local inputs, i.e. the inputs “inside” this location, are *not* spatially discounted whereas all inputs in other locations are spatially discounted.

Table 5 Factors varying in the different Monte Carlo experiments

| Factor | Symbol | Value (global spillovers) |
|--|-----------------|----------------------------|
| No. of exogenous variables (incl. intercept) | – | 2, 3, 4 |
| Spatial parameter for unmodelled effects | λ | 0.65, 0.75 |
| Spatial parameter for modelled effects | ρ | 0.3, 0.4 |
| Spatial parameters for both effects | $\lambda; \rho$ | (0.65; 0.3), (0.75; 0.4) |
| Weights in weight matrix for DGP | $w_{ij} = 1/t$ | > 0 if $t < 60, 120$ min |

$$OLS\ 3 : x = \begin{pmatrix} 1 & x_1 & A_{1R}^X & A_{1OR}^X \\ 1 & x_2 & A_{2R}^X & A_{2OR}^X \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ 1 & x_{288} & A_{288R}^X & A_{288OR}^X \end{pmatrix}$$

where A_{iR}^X and A_{iOR}^X are defined as in (3.9). Thus, following the procedure discussed in Sect. 3, the strategy is to add the accessibility variables one by one (first intra-regional and then inter-regional variables). Note that the values of β_3 and β_4 are set to zero in OLS 2 and 3.

For each experiment spatial dependence is tested for by using the Moran’s I statistic, and the Lagrange multiplier test statistics LM-err and LM-lag. We report how the rejection frequency of each of these tests is affected by the inclusion of accessibility variables on the RHS. A row standardised binary matrix, with weight $w_{ij} > 0$ if municipality i and j are in the same LLM-region, and zero otherwise, is used for all tests. Parameter accuracy is assessed by checking bias and RMSE

$$RMSE = \sqrt{\frac{\sum_{k=1}^{1,000} (\hat{\beta}_k - \beta_k)^2}{1,000}}, \quad k = \text{the number of repetitions}(1,000)$$

Details of factors varying and held constant in the Monte Carlo experiments are displayed in Tables 5 and 6.

5.2 Simulation results

The size of the tests shows the probability to reject the null hypothesis when the null is true. This is a measure that illustrates how the chosen tests behave when there is no spatial dependence. When the data generating process (DGP) in the simulations is no spatial effects, the rejection frequencies of the sizes are:

- Moran’s I : 0.030
- LM-err: 0.047
- LM-lag: 0.056

Table 6 Factors held constant in the Monte Carlo experiments

| Factor | Symbol | Value |
|------------------------------------|----------|---------------------------------------|
| No. of observations | N | 288 |
| Time distance between locations | t | * |
| Distribution of regressor | x | $U[0,5]$ |
| Distribution of errors | u | $N(0,1)$ |
| Weights in weight matrix for tests | w_{ij} | > 0 , if i and j in same region |
| Number of repetitions | – | 1,000 |

* Travelling distance in minutes between municipalities in Sweden

The rejection frequencies show the proportions of the repetitions that reject the null hypothesis, i.e. no spatial dependence at the 0.05 significance level. Thus, the sizes are at acceptable levels in order to move on to simulations with spatial effects as DGP's (see Table 7).

The following concluding results can be drawn from an examination of Table 7.

1. The power of the tests, i.e. the probability to reject the null (the rejection frequency) when the null is false is as expected when comparing the tests. LM-err has the highest power to detect “Unmodelled effects” and LM-lag has the highest power to detect “Both effects”.
2. A common result for all structural models is that the rejection frequencies are reduced when accessibility variables are included.
3. The largest decrease is not surprisingly when the data generating process (DGP) is “modelled effects”. The inter-regional accessibility is especially important when the data is generated with the use of W_{120} . Thus, intra-regional accessibility is then not alone capable to model and take care of the spatial dependence.
4. With “both unmodelled and modelled effects” as DGP the accessibility variables major contribution is to capture the spatial dependencies originated from “modelled effects”.

Table 8 shows the mean estimates of the regression coefficients, β_1 and β_2 . The table also present bias and RMSE calculations. An examination of Table 8 reveals the following conclusions.

1. When the DGP is “unmodelled effects”, the OLS regressions do not produce biased estimates, which are a well known fact (Anselin 1988a).
2. When the DGP is “modelled effects”, the constant term is heavily biased if the spatial dependencies are not modelled. The accessibility variables reduce the bias substantially. Note also the large impact of inter-regional accessibility, even in the cases when data was generated by W_{60} . There is also a large efficiency gain (reduced RMSE) when the accessibility variables are utilized in the OLS regressions.
3. When the DGP is “both unmodelled and modelled effects”, the underlying spatial structure is a spatially lagged dependent variable, which results in biased parameter estimates, in accordance with theory (Anselin 1988a). By including accessibility variables, i.e. spatially lagged explanatory variables, on the RHS, the problem

Table 7 Rejection frequencies, global spillovers, 1,000 iterations

| Unmodelled effects | W_{60} , time distance < 60 min ($\lambda = 0.75$) | | |
|--------------------|--|----------------------|---------------------------------|
| | $y_i(x_i)$ | $y_i(x_i, A_{iR}^X)$ | $y_i(x_i, A_{iR}^X, A_{iOR}^X)$ |
| Moran's I | 0.445 | 0.263 | 0.231 |
| LM-err | 0.653 | 0.470 | 0.437 |
| LM-lag | 0.372 | 0.274 | 0.239 |
| Unmodelled effects | W_{120} , time distance < 120 min ($\lambda = 0.65$) | | |
| | $y_i(x_i)$ | $y_i(x_i, A_{iR}^X)$ | $y_i(x_i, A_{iR}^X, A_{iOR}^X)$ |
| Moran's I | 0.397 | 0.285 | 0.250 |
| LM-err | 0.594 | 0.481 | 0.438 |
| LM-lag | 0.348 | 0.273 | 0.248 |
| Modelled effects | W_{60} , time distance < 60 min ($\rho = 0.4$) | | |
| | $y_i(x_i)$ | $y_i(x_i, A_{iR}^X)$ | $y_i(x_i, A_{iR}^X, A_{iOR}^X)$ |
| Moran's I | 0.995 | 0.022 | 0.005 |
| LM-err | 1.000 | 0.080 | 0.047 |
| LM-lag | 1.000 | 0.267 | 0.128 |
| Modelled effects | W_{120} , time distance < 120 min ($\rho = 0.3$) | | |
| | $y_i(x_i)$ | $y_i(x_i, A_{iR}^X)$ | $y_i(x_i, A_{iR}^X, A_{iOR}^X)$ |
| Moran's I | 0.906 | 0.307 | 0.019 |
| LM-err | 0.979 | 0.599 | 0.085 |
| LM-lag | 1.000 | 0.848 | 0.269 |
| Both effects | W_{60} , time distance < 60 min ($\rho = 0.4, \lambda = 0.75$) | | |
| | $y_i(x_i)$ | $y_i(x_i, A_{iR}^X)$ | $y_i(x_i, A_{iR}^X, A_{iOR}^X)$ |
| Moran's I | 0.952 | 0.363 | 0.280 |
| LM-err | 0.989 | 0.589 | 0.496 |
| LM-lag | 0.991 | 0.631 | 0.432 |
| Both effects | W_{120} , time distance < 120 min ($\rho = 0.3, \lambda = 0.65$) | | |
| | $y_i(x_i)$ | $y_i(x_i, A_{iR}^X)$ | $y_i(x_i, A_{iR}^X, A_{iOR}^X)$ |
| Moran's I | 0.942 | 0.761 | 0.372 |
| LM-err | 0.974 | 0.902 | 0.541 |
| LM-lag | 0.995 | 0.929 | 0.640 |

with biased parameter estimates is heavily reduced. Furthermore, the parameter estimates are much more efficient when the accessibility variables are included in the regressions. The results are, overall, very similar to the ones received when “modelled effects” was DGP.

Table 8 Global spillovers, true values: $\beta_1 = 0.5, \beta_2 = 1.0$

| Unmodelled effects | W_{60} , time distance < 60 min ($\lambda = 0.75$) | | | | | |
|--------------------|--|--------|----------------------|--------|---------------------------------|--------|
| | $y_i(x_i)$ | | $y_i(x_i, A_{iR}^X)$ | | $y_i(x_i, A_{iR}^X, A_{iOR}^X)$ | |
| | b_1 | b_2 | b_1 | b_2 | b_1 | b_2 |
| Mean | 0.5010 | 1.0003 | 0.5018 | 1.0004 | 0.4951 | 1.0007 |
| Bias | 0.20% | 0.03% | 0.36% | 0.04% | -0.99% | 0.07% |
| RMSE | 0.1540 | 0.0438 | 0.1558 | 0.0411 | 0.1695 | 0.0412 |
| Unmodelled effects | W_{120} , time distance < 120 min ($\lambda = 0.65$) | | | | | |
| | $y_i(x_i)$ | | $y_i(x_i, A_{iR}^X)$ | | $y_i(x_i, A_{iR}^X, A_{iOR}^X)$ | |
| | b_1 | b_2 | b_1 | b_2 | b_1 | b_2 |
| Mean | 0.5040 | 1.0006 | 0.5034 | 1.0005 | 0.4964 | 1.0008 |
| Bias | 0.80% | 0.06% | 0.68% | 0.05% | -0.72% | 0.08% |
| RMSE | 0.2045 | 0.0425 | 0.1735 | 0.0416 | 0.1713 | 0.0418 |
| Modelled effects | W_{60} , time distance < 60 min ($\rho = 0.4$) | | | | | |
| | $y_i(x_i)$ | | $y_i(x_i, A_{iR}^X)$ | | $y_i(x_i, A_{iR}^X, A_{iOR}^X)$ | |
| | b_1 | b_2 | b_1 | b_2 | b_1 | b_2 |
| Mean | 1.2305 | 1.0275 | 0.8607 | 0.9889 | 0.5409 | 0.9996 |
| Bias | 146% | 2.75% | 72.1% | -1.11% | 8.18% | -0.04% |
| RMSE | 0.7400 | 0.0494 | 0.3809 | 0.0427 | 0.1612 | 0.0412 |
| Modelled effects | W_{120} , time distance < 120 min ($\rho = 0.3$) | | | | | |
| | $y_i(x_i)$ | | $y_i(x_i, A_{iR}^X)$ | | $y_i(x_i, A_{iR}^X, A_{iOR}^X)$ | |
| | b_1 | b_2 | b_1 | b_2 | b_1 | b_2 |
| Mean | 1.5594 | 1.0189 | 1.3092 | 0.9928 | 0.7065 | 1.0129 |
| Bias | 211% | 1.89% | 162% | -0.72% | 41.3% | 1.29% |
| RMSE | 1.0660 | 0.0452 | 0.8184 | 0.0481 | 0.2588 | 0.0432 |
| Both effects | W_{60} , time distance < 60 min ($\rho = 0.4, \lambda = 0.75$) | | | | | |
| | $y_i(x_i)$ | | $y_i(x_i, A_{iR}^X)$ | | $y_i(x_i, A_{iR}^X, A_{iOR}^X)$ | |
| | b_1 | b_2 | b_1 | b_2 | b_1 | b_2 |
| Mean | 1.2311 | 1.0274 | 0.8620 | 0.9889 | 0.5401 | 0.9997 |
| Bias | 146% | 2.74% | 72.4% | -1.11% | 8.02% | -0.03% |
| RMSE | 0.7472 | 0.0517 | 0.3941 | 0.0425 | 0.1741 | 0.0412 |
| Both effects | W_{120} , time distance < 120 min ($\rho = 0.3, \lambda = 0.65$) | | | | | |
| | $y_i(x_i)$ | | $y_i(x_i, A_{iR}^X)$ | | $y_i(x_i, A_{iR}^X, A_{iOR}^X)$ | |
| | b_1 | b_2 | b_1 | b_2 | b_1 | b_2 |
| Mean | 1.5630 | 1.0191 | 1.3121 | 0.9929 | 0.7070 | 1.0131 |
| Bias | 213% | 1.91% | 162% | -0.71% | 41.4% | 1.31% |
| RMSE | 1.0825 | 0.0466 | 0.8304 | 0.0422 | 0.2687 | 0.0438 |

Table 9 Global spillovers, mean and *t* value, 1,000 iterations

| Unmodelled effects | W_{60} , time distance < 60 min ($\lambda = 0.75$) | | |
|--------------------|--|---------------------------------|--------|
| | $y_i(x_i, A_{iR}^X)$ | $y_i(x_i, A_{iR}^X, A_{iOR}^X)$ | |
| | b_3 | b_3 | b_4 |
| Mean | -0.0007 | -0.0005 | 0.0016 |
| <i>t</i> value | -0.025 | -0.017 | 0.068 |
| Unmodelled effects | W_{120} , time distance < 120 min ($\lambda = 0.65$) | | |
| | $y_i(x_i, A_{iR}^X)$ | $y_i(x_i, A_{iR}^X, A_{iOR}^X)$ | |
| | b_3 | b_3 | b_4 |
| Mean | 0.0005 | 0.0007 | 0.0017 |
| <i>t</i> value | 0.018 | 0.026 | 0.070 |
| Modelled effects | W_{60} , time distance < 60 min ($\rho = 0.4$) | | |
| | $y_i(x_i, A_{iR}^X)$ | $y_i(x_i, A_{iR}^X, A_{iOR}^X)$ | |
| | b_3 | b_3 | b_4 |
| Mean | 0.3066 | 0.3165 | 0.0762 |
| <i>t</i> value | 11.3 | 11.8 | 3.24 |
| Modelled effects | W_{120} , time distance < 120 min ($\rho = 0.3$) | | |
| | $y_i(x_i, A_{iR}^X)$ | $y_i(x_i, A_{iR}^X, A_{iOR}^X)$ | |
| | b_3 | b_3 | b_4 |
| Mean | 0.2074 | 0.2260 | 0.1437 |
| <i>t</i> value | 7.26 | 8.36 | 6.07 |
| Both effects | W_{60} , dist. < 60 min ($\rho = 0.4, \lambda = 0.75$) | | |
| | $y_i(x_i, A_{iR}^X)$ | $y_i(x_i, A_{iR}^X, A_{iOR}^X)$ | |
| | b_3 | b_3 | b_4 |
| Mean | 0.3061 | 0.3160 | 0.0767 |
| <i>t</i> value | 10.9 | 11.4 | 3.16 |
| Both effects | W_{120} , dist. < 120 min ($\rho = 0.3, \lambda = 0.65$) | | |
| | $y_i(x_i, A_{iR}^X)$ | $y_i(x_i, A_{iR}^X, A_{iOR}^X)$ | |
| | b_3 | b_3 | b_4 |
| Mean | 0.2080 | 0.2267 | 0.1442 |
| <i>t</i> value | 7.06 | 8.12 | 5.91 |

b_3 refers to the coefficient estimate of A_{iR}^X . b_4 refers to the coefficient estimate of A_{iOR}^X

Another way of confirming the importance of modelling the spatial effects is to analyse the statistical significance of the parameter estimates of the variables. In Table 9 these parameter estimates and corresponding *t* values are presented. Regarding “unmodelled

effects” the table shows that the coefficient estimates of both intra- and inter-regional accessibilities are insignificant. However, as shown in Table 7, even though the coefficient estimates are insignificant, the inclusion of the accessibility variables on the RHS reduces the rejection frequency of the tests drastically.

In the case of “modelled effects” it is evident that the coefficient estimates of the accessibility variables are significantly different from zero. From Table 7 we also know that the rejection frequency of the three tests is significantly reduced when these additional variables are included in the model. This is a clear indication that the inclusion of spatially weighted explanatory variables on the RHS in the case of modelled effects successfully captures a large part of the effects involved. In other words, the coefficient estimates of the accessibility variables have a bearing on the strength of spatial dependence.

The simulation results for “both unmodelled and modelled effects” suggest that the accessibility variables capture modelled effects but cannot fully account for unmodelled effects. This cannot be seen in Table 9 directly, so comparisons with Table 7 need to be made.

6 Summary and conclusions

This paper has shown a spatial empirical model with a coherent representation of space, which builds on the potential for interaction, accommodate spatial dependence and can be estimated with ordinary least squares (OLS). A model with accessibility variables on the RHS can account for and model substantive spatial dependence among observations. Spatial dependence is revealed via the parameter estimates of the RHS variables. This modelling strategy has a number of advantages. It is easy to implement and can in principle be estimated with OLS. The spatial lag and the spatial error model, for instance, require maximum-likelihood estimation. It can also readily be applied in more complicated situations than the OLS, such as the Poisson model. Furthermore, as has been shown in the paper, it can account for both local and global spillovers.

The results from Monte Carlo simulations which incorporate the taxonomy in [Anselin \(2003\)](#) can be summarized as follows:

- The coefficient estimates of the accessibility variables are significantly different from zero in the case of “modelled effects”. The rejection frequency of the three tests (Moran’s I, LM-lag and LM-err) is significantly reduced when these additional variables are included in the model.
- The bias of the parameter estimates is reduced when accessibility variables are incorporated into the model. Thus, by including spatially lagged explanatory variables on the RHS, the problem with biased parameter estimates is reduced even if the underlying spatial structure is spatially lagged dependent variables (“both unmodelled and modelled effects”).
- The coefficient estimates of the accessibility variables indicate the strength of substantive spatial dependence.

The results show that an accessibility representation of spatial explanatory variables can assimilate and reflect spatial dependence. A likely reason for this is that an

accessibility representation of explanatory variables in fact depicts the network nature of spatial interaction. In other words, spatial interdependence is actually modelled.

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