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Structure and Matrix Models for Tolerance Analysis from Configuration to Detail Design

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Abstract. *A substantial amount of all quality problems that arise during assembly can be referred back to the geometrical design, and especially the geometrical concept of the product, i.e. the way in which parts are designed and located with each other. Special emphasis should thus be put on geometry design, especially during the early design phases, to try to find robust concepts and avoid solutions that may cause down-stream production problems.*

This paper presents a generic set of evaluation tools for robust geometry design encountering (i) potential tolerance chain detection in configuration design, (ii) assembly robustness evaluation in concept design, and (iii) tolerance sensitivity analysis in detail design. Special attention is given to the development of a new matrix-based evaluation tool for the configuration design part. The tool presented is based on a new way of representing geometry variation constraints in an enhanced function-means tree structure model. Different parts of the function-means tree that are of interest for analysis purposes are then extracted and converted to matrix representation. The reason for doing this is that the structure model is most suitable for modeling, but becomes unsuitable for analysis as the model complexity increases. For this latter purpose, the matrix representation is far better. The use of the different tools is demonstrated in the design of a new vehicle front system for which the geometry a priori *is unknown.*

Keywords: Conceptual design; Robust Design; Tolerance analysis

1. Introduction

Product design is a complex activity in which a product specification in the end must be fulfilled by a set of sub-systems, components and manufacturing processes. Very often, geometry-related quality problems are discovered during the assembly process when different parts from different sources are assembled using some kind of assembly strategy. In fact, quite a substantial amount of all quality problems that arise during assembly can be referred back to the geometrical concept of the product, i.e. the way in which parts are designed and located with each other. Economically, a late design change is very costly compared with an early change. Because of this, very strong focus should be put on the early design phases to try to find robust concepts and avoid solutions that may cause production problems later on.

Early avoidance of tolerance problems brings product modeling and tolerance design closer together. Pimmler and Eppinger (1994) describe a method for specifying interactions between sub-systems on the same hierarchical level in a product hierarchy. Characterization of functional couplings was discussed in Johannesson (1996). Identification of potential tolerance chains during configuration and concept design is covered by Söderberg and Johannesson (1998, 1999). In Mantripragada and Whitney (1998), a 'datum flow chain' is used to relate the datum logic explicitly to the product's key characteristics (KCs). The use of KCs and KC hierarchies to systematically describe all parameters that significantly affect a product's performance, function and form is discussed by Lee and Thornton (1996) and Thornton (1997). Thornton (1999) describes a mathematical framework for the key characteristic process. Tolerance design from an axiomatic perspective has been dealt with by El-Haik and Yang (1999).

Today, a number of commercial CAT (Computer Aided Tolerancing) tools are available, that can assist in trying to foresee and avoid geometrical problems that are related to geometrical variation (see Chase (1991), Salomonson (1997), Söderberg (1998, 1999) and Parkinson (1999)). These tools are most often used too late when CAD models are developed, and when real manufacturing data is available, i.e. the

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concept is almost ready and the processes are known. A design change at this stage is often quite costly. In this paper, we will present some new matrix-based CAT functionality that can assist robust design during both concept design and detail design.

1.1. Scope of the Paper

This paper presents a generic evaluation procedure that supports geometrical robustness and tolerance analysis through the different design phases, from a high level of abstraction to detail design. The work presented here is based on earlier work by the authors that has been further developed towards a more consistent analysis strategy that can be used in all design phases. The first part of this work, described in Section 2, has a purpose similar to that of the *datum flow chain* presented in Mantripragada and Whitney (1998), i.e. to track tolerance chains by studying the locating schemes of the assembly. In Section 2 in this paper, we will use an 'enhanced function-means modeling technique' as a basis for modeling and analyzing relations between objects during configuration design. In our object-oriented working procedure, the designer is forced to start to think about the locations of parts already in the configuration phase. Location schemes are treated as objects of the model, and are already used to detect potential tolerance chains in the configuration phase. By doing this, tolerance chains can be avoided or at least highlighted further on in concept and detail design. New in this work is also the idea of extracting the different parts of the function-means tree, and the use of a matrix representation of each part for the purposes of analysis. The main reason for doing this is that the structure model, which is a suitable base for modeling, soon becomes very messy for a complex product, and thereby unsuitable for analysis. The matrix representation is much better for this latter purpose.

Matrix-based sensitivity analysis in design has been performed by a number of authors; see, for example, Suh (1990), Thornton (1999) and Söderberg and Carlson (1999). Different types of sensitivity analysis are also available in some of the commercial CAT (Computer Aided Tolerancing) tools on the market. In Gao et al., sensitivity matrices are generated from a vector loop model based on kinematic joints. The method is a closed form method, and is therefore very well suited for mechanisms. Section 3 in this paper presents a CAD-based evaluation tool that allows assemblies to be evaluated with respect to geometrical sensitivity. The matrix-based sensitivity analysis used here (*stability analysis*) differs from the work presented by others in the way that it focuses on the part location schemes, which is in line with the first part of this paper. It combines the benefits of sensitivity analysis, matrix representations, CAD-based assembly definition and axiomatic design to evaluate the degree of geometrical coupling in an assembly, what parts and locators that are important, and in what order to adjust it.

Section 4 presents another new idea. Here the locating scheme stability analysis is extended to show the influence of defined tolerances in comparison with the unit disturbance, so as to be able to judge weather selected tolerances increase or decrease the locating scheme sensitivity in the assembly. Also, here the focus is on the locating schemes of the individual parts in the assembly.

In Section 5, the different analysis procedures are applied to an engine sub-frame in a case study.

The set of analysis and evaluation tools presented supports the design part of the geometry assurance process, and they can all be included in or attached to a PDM (Product Data Management) system using information extracted from a geometry and process database.

In this paper, *configuration design* refers to the early design stages, where an overall design task is hierarchically decomposed into sub-solutions and new requirements by zigzagging between the functional and the physical domains. It should especially be noted that no detailed geometry is available during this phase. It is nevertheless very important to be able to predict potential tolerance problems, even at this early stage. Since the focus here is on geometry, *concept design* refers to early embodiment design, where the overall geometry concept is evaluated with respect to geometrical robustness. *Detail design* refers in this work to the late embodiment design phase, during which the final geometry is set and tolerances are selected with respect to product constraints, geometrical sensitivity and process variation.

2. Tolerance Analysis in Configuration Design

During configuration design, the product specification, describing requirements and constraints for the product, is generally decomposed into sub-requirements and sub-constraints, i.e. geometrical constraints on individual sub-solutions. During this phase, the final geometry of individual parts has generally not been decided (even though ideas do exist), and the modeling is carried out on a more abstract level by mapping functional requirements to physical solutions.

2.1. Function-means Modeling and Constraints Decomposition

A function-means tree is an object oriented graphical hierarchical representation of the different items (assemblies, sub-assemblies, parts and features) and their governing criteria that constitute a complex product. Both items and criteria are modeled as objects. Different relations are used to model how different objects are linked to each other.

The objects used in the model are *Functional Requirements* (FRs), *Design Parameters = Means* (DPs) and *Constraints* (Cs). An FR is here defined as an independent criterion that completely characterizes the functional need of the design. DPs are the key items or variables that characterize the physical entity, i.e. the means, created by the design process to fulfil an FR (Suh, 1990). Cs are criteria that bound the solution space, i.e. put restrictions on the DPs. Thus, a DP represents the design solution, an FR is a solution driver and a *C* is a solution restrainer. In the graphical models, objects are drawn as boxes (FRs and DPs) or ellipses (*C*s).

Six different relations, *isb, icb, rf, ipmb, iw* and *iw*, are used to relate the FR-, DP- and C-objects to each other. The relations are, when they are shown, represented in the graphical models by different lines and arrows between objects. The six relations are defined as follows:

- An FR is_solved_by (isb) a DP;
- A DP is_constrained_by (icb) a C;
- A DP requires_functions (rf) FRs on the next lower hierarchical level (note that this relation is not always shown as a line or arrow in the graphical representations. Instead FRs and DPs can be drawn close together).
- A *C is_partly_met_by (ipmb)* DPs on the next lower hierarchical level;
- Parallel solution DPs with the same ‘parent’ may *interact_with (iw)* each other;
- Occasionally, the fulfilment of an FR *is_influenced_by (iib)* the choice of a parallel solution (DP).

While the *rf*- and *isb*-relations are the keys for decomposition of FRs and DPs, the *icb*- and *ipmb*-relations are the keys for decomposition of *C*s when creating the function-means tree in a top-down manner. In Söderberg and Johannesson (1998 and 1999), this function-means modeling technique has been used to cover spatial constraint decomposition and geometrical coupling analysis.

Figure 1 shows how a top-level geometrical constraint, *C*, is decomposed into geometrical

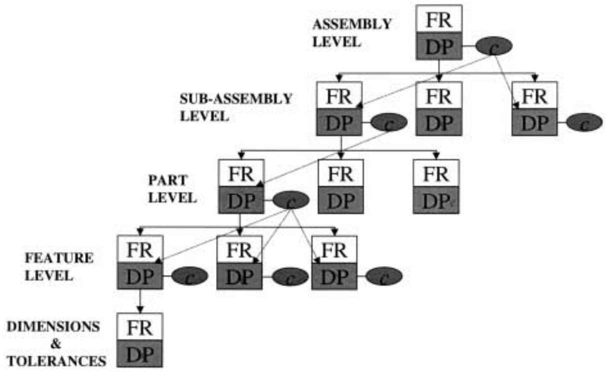


Fig. 1. Multi-level constraint decomposition.

constraints on sub-assemblies, parts and, finally, on individual geometrical features such as planes, holes and surfaces. On the last hierarchical FR/DP level, feature level constraints are transformed into functional requirements concerning the geometry of the feature, solved by design parameters as nominal dimensions and tolerances. For example, on a vehicle, top level constraints may be flush and gap dimensions and tolerances, whereas bottom-level DP:s may be nominal dimensions and tolerance call-outs for all individual part features and fixtures involved.

Top-down constraint decomposition is performed by studying each object on every hierarchical level, asking: *Will this physical solution allow small spatial deviation without violating an overall spatial constraint?* If the answer is *yes*, the sub-solution will not participate in meeting the higher level constraint. If the answer is *no*, the sub-solution will probably participate in meeting this constraint, and a new sub-constraint must then be introduced.

2.2. Spatial Constraints Couplings – Tolerance Chains

A *geometrical* (*spatial*) *constraint coupling* between parallel design solutions means that two or more interacting parallel design solutions share the same geometrical constraint, and must be adjusted to each other in order to meet that constraint. The solutions must thus be developed simultaneously and in close cooperation (see Fig. 2). Parallel design solutions are defined as design solutions at the same hierarchical level that have the same ‘parent’ on the nearest higher level.

Tolerance allocation is typically a situation in which a number of dimensions share an overall geometrical constraint (a critical assembly dimension) and therefore become coupled, i.e. tolerance chains

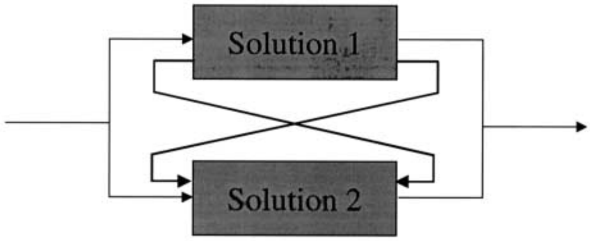


Fig. 2. Coupled solutions.

arise. Tolerances on one dimension must be selected both with respect to the overall constraint and with respect to the other dimensions in the chain.

Söderberg and Johannesson (1999) describes a method for potential tolerance chain detection during early constraint decomposition. The basic evaluation criteria is that: *if an overall geometrical constraint affects interacting parallel subsolutions on a lower hierarchical level, and if the solutions therefore become geometrically coupled, a tolerance chain may arise.*

2.3. Structure and Matrix Representations of Tolerance Chains

Two different kinds of relation loops should be looked for to identify tolerance chains in a function-means hierarchy. The first kind of loop starts with a constraint object (a *C*). Then follows an *ipmb* (is partly met by) relation to a solution object (a DP) on the next lower hierarchical level. This continues via an *iw* (interacts with) relation to a new parallel solution object (a DP) on the same level and then back to the starting point via an *ipmb* relation. More *iw* connected parallel DP:s may be passed before the last step back. This first kind of relation loop, *type 1*, indicating a potential constraint caused tolerance chain is shown in Fig. 3.

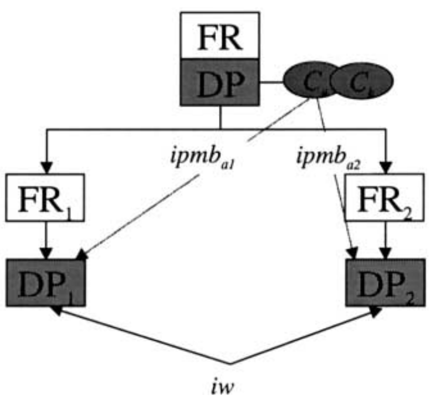


Fig. 3. Potential tolerance chain type 1.

Example 1
Consider designing a car, and that, referring to Fig. 3, DP = 'Exterior shell' with the geometrical constraint C_a = 'Nice fits and gaps'. Two, of many, functional requirements on 'Exterior shell' are FR_1 = 'Cover front wheel' and FR_2 = 'Allow entrance/exit of car'. FR_1 *is_solved_by* DP_1 = 'Fender' and FR_2 *is_solved_by* DP_2 = 'Door'. The design parameters 'Fender' and 'Door' *interact_with* each other, and the geometrical variations connected to each one of them influence the fulfillment of the constraint C_a = 'Nice fits and gaps' on the 'Exterior shell'. Thus C_a *is_partly_met_by* both the 'Fender' and the 'Door', and there is a potential tolerance chain of *type 1* between 'Fender' and 'Door' as they are sharing the constraint 'Nice fits and gaps'.

The *type 1* loop identified in the structure model can also be identified by studying matrix representations of the structure model, which is beneficial when details in the structure model becomes difficult to see due to high complexity.

To detect a *type 1* loop, two matrices are needed. First, a *constraint-solution matrix* is constructed by taking the *C*s constraining the current DP as matrix rows and the 'children' of the current DP, here DP_1 and DP_2, as matrix columns. Then the relations between the *C*s and the 'children' DPs (i.e. the *ipmb*-relations) will appear in the matrix rows and columns intersections. Secondly, *a solution-solution matrix* is generated by taking the 'children' DPs both as matrix rows and matrix columns. In this matrix, the relations between the 'Children' DPs (i.e. the *iw*-relations) will appear in the row-column intersections. The diagonal will of course be empty in the *solution-solution matrix*. Such matrices as *constraint-solution and solution-solution matrices* can automatically be generated by using information modeling software such as Metis Software (Metis 1997) for structure modeling. (In concept and detail design, discussed in Sections 3 and 4, similar matrices are generated in our software RD&T.)

A *type 1* loop can be identified by studying a *constraint-solution matrix* and a *solution-solution matrix* together (see Fig. 4).

From the *constraint-solution matrix* in Fig. 4, it can be seen that constraint C_a is partly met by both solution DP_1 and solution DP_2. The *solution-solution matrix* further shows that solutions DP_1 and DP_2 interact with each other. Thus, there is a *type 1* loop between C_a, DP_1 and DP_2. This observation can be generalized and expressed as follows:

C \ DP	DP_1	DP_2
C_a	$ipmb_{a1}$	$ipmb_{a2}$
C_b	0	0

DP \ DP	DP_1	DP_2
DP_1		iw_{12}
DP_2	iw_{21}	

$iw_{12}=iw_{21}=iw$

Fig. 4. Constraint-solution and solution-solution matrices.

> **If** $iw = iw_{ij} = iw_{ji} \neq 0$ **and** $ipmb_{xi} \neq 0$ *and* $ipmb_{xj} \neq 0$, **then** there is a *type 1* loop and a potential tolerance chain caused by constraint C_x.

The second kind of loop to look for starts with a Functional Requirement (FR) caused by a spatial constraint. This follows an *iib* (is influenced by) relation to a parallel solution object (a DP), continues via an *iw* (interacts with) relation to the next parallel DP, and returns to the starting point following an *iib* relation. Before the last step back, it may pass more *iw* connected parallel DPs. This second kind of relation loop, *type 2*, indicating a potential constraint caused tolerance chain, is shown in Fig. 5.

In our extended function-means model, the designer is forced to start thinking about how parts and sub-assemblies are going to be located and assembled, even before the final geometry is available. We therefore include the part locating schemes as DPs governed by positioning FRs in the model.

Example 2

Let DP in Fig. 5 be the sub-assembly 'Door' in the previous example. On this sub-assembly, we put new more detailed requirements, and among these FR_1 = 'Position door in car body', FR_2 = 'Carry door parts' and FR_3 = 'Allow locking of door'. Now FR_1 *is_solved_by* DP_1 = 'Sub-assembly locating scheme', FR_2 *is_solved_by* DP_2 = 'Door frame', and FR_3 *is_solved_by* DP_3 = 'Lock'. The 'Lock' and the 'Door frame' *interact_with* each other as the 'Lock' is to be mounted on the frame. If we now assume that the physical location of the locating points of DP_1 will be selected on the parts ('Door frame' and 'Lock') of the sub-assembly 'Door', the fulfillment of FR_1 = 'Position door in car body' *is_influenced_by* the variations in the 'Door frame' and the 'Lock'. As these two interacting design solutions are both influencing the fulfillment of the positioning requirement FR_1, there is a potential type 2 tolerance chain between the 'Door frame' and the 'Lock'.

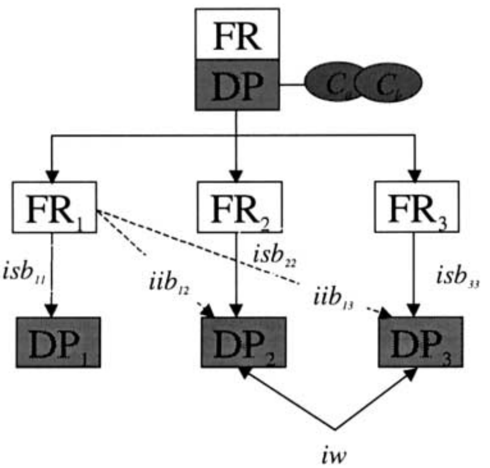


Fig. 5. Potential tolerance chain type 2.

The matrices of interest for representation of a *type 2* loop are *a requirement-solution matrix* and a *solution-solution matrix*. See Fig. 6, which shows the matrix representation of the structure in Fig. 5.

The *solution-solution matrix* is the same as above. New is the requirement-solution matrix, which is constructed by taking the governing FRs of the 'children' DPs (i.e. FR_1, FR_2 and FR_3 in Fig. 5), as matrix rows, and the 'children DPs DP_1, DP_2 and DP_3 as matrix columns. The relations between FRs and DPs that will appear in the row-column intersections are the *isb*s and the *iib*s. The *isb*s will be located in the matrix dagonal. Note that the *requirement-solution matrix* is equivalent to the axiomatic design matrix (see Suh 1990).

In the *requirement-solution matrix*, it can be seen that FR_1 is influenced by both DP_2 and DP_3, and the *solution-solution matrix* shows that DP_2 and DP_3 interact with each other. Thus, there is a *type 2* loop between FR_1, DP_2 and DP_3. This observation can be generalized and expressed as follows:

> **If** FR_i is a functional requirement caused by a constraint **and** $iw = iw_{jk} = iw_{kj} \neq 0$ **when** $iib_{ij} \neq 0$ **and** $iib_{ik} \neq 0$, **then** there is a *type 2* loop and a potential tolerance chain caused by the constraint-caused functional requirement FR_i.

The combined use of structure modeling and matrix modeling techniques is proposed for reasons of their different strengths in different phases of the configuration design process. During the modeling phase, when product structures are created, it is very easy to

FR \ DP	DP_1	DP_2	DP_3
FR_1	isb_{11}	iib_{12}	iib_{13}
FR_2	0	isb_{22}	0
FR_3	0	0	isb_{33}

DP \ DP	DP_1	DP_2	DP_2
DP_1		0	0
DP_2	0		iw_{23}
DP_3	0	iw_{32}	

$iw_{23}=iw_{32}=iw$

Fig. 6. Requirement-solution and solution-solution matrices.

use an object-oriented approach based on objects and relations between objects modeled in a function-means tree structure. However, if the product is complex and contains many objects and relations, the structure model soon becomes very messy, and it becomes difficult to see the details in the model. This is necessary during the analysis phase, and it is then very favorable to convert the parts of the structure model that are to be analyzed to matrix representation, where all details are easily seen.

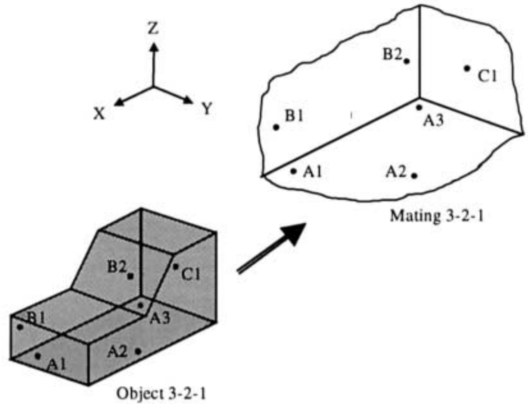


Fig. 7. The 3-2-1 locating scheme.

3. Tolerance Analysis in Concept Design

The way in which parts are located to each other is critical for how geometrical variation will propagate through the assembly and cause variation in critical product or assembly dimensions (see Söderberg and Carlsson 1999).

In concept design (early embodiment design), when part geometry suggestions are available, qualitative sensitivity evaluations may be performed. In this section, we will describe how matrix evaluations, similar to the solution-solution matrices described in the previous section, may be performed on the basis of the way in which parts are located to each other.

3.1. Part Location Schemes

Figure 7 shows the six-point locating scheme, often referred to as the 3-2-1 locating scheme. Here, six theoretical locating points are used to lock six degrees of freedom for a part (see Söderberg and Lindkvist 1999a). The locating points are represented in reality by physical locators such as planes, holes and slots. In this paper, the notation *P-frame* (positioning frame) will be used for a 3-2-1 locating scheme.

Generally, each part may have one *↑P-frame* (its 'master location system') that positions the part in the assembly, and a number of *↓P-frames* positioning other parts to it. In other words, one object (part or subassembly) is located via its *↑P-frame* to a mating *↓P-frame* on another part or another subassembly. Figure 8 describes the positioning system for a part as a function-means model. Generally, each part also has a local reference, a *D-frame* (datum frame), that may be separate from the *↑P-frame*. In this work, the *↑P-frame* is used both as local reference (*D-frame*) and master location system. Therefore, the features constituting the *↑P-frame* do not have tolerances. Features constituting *↓P-frames* are constrained by tolerances however, as shown in Fig. 8. Figure 8 also illustrates how overall spatial constraints are decomposed into constraints on individual *↓P-frame* features, and how these constraints are transformed into variation requirements fulfilled by tolerances, specified according to the standard used. A tolerance constraint on a physical *↓P-frame* feature, a position tolerance for example, may refer to physical features of the *↑P-frame*, for example A, B and C datums.

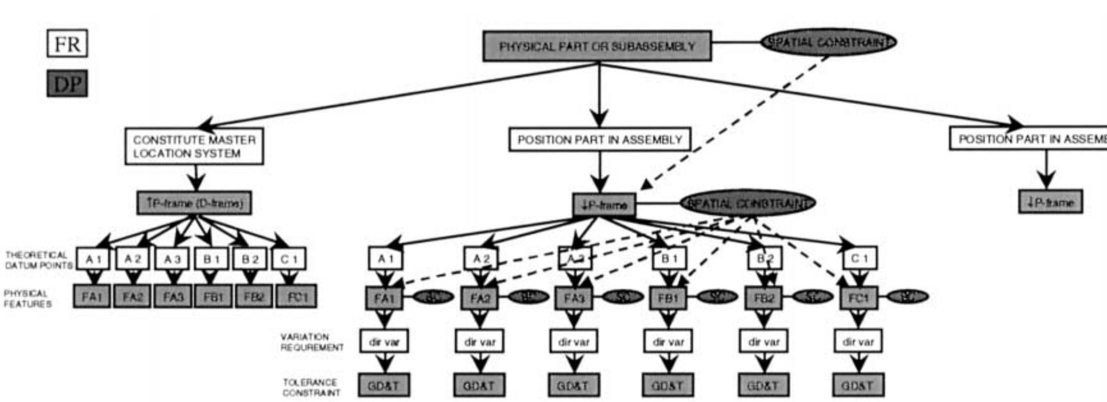


Fig. 8. Object positioning system.

3.2. Stability Analysis

Figure 9 shows two types of assemblies and their *stability matrices*. In the parallel assembly, the position of each part is controlled by its own *P-frame* only, which represents an *uncoupled design*, which is easy to adjust and tune. The serial assembly represents a coupled design, which is more difficult and time-consuming to adjust and tune. A triangular stability matrix may however be adjusted if done in the correct order, starting with A, then B, and so on. Real assemblies are often a mix of the parallel and the serial case, involving assembly fixtures as well.

By varying each locating point in each *P-frame* of an assembly with a small increment, *Δinput*, one at a time, *Δoutput*/*Δinput* may be determined in the X, Y and Z directions separately for a number of output points, n, representing each part geometry. The *RMS* values for all points, corresponding to variation in each of the six locating points, i, is then determined. The total *RSS* influence of all six locating points (absolute or in each direction) is then calculated, and shows how well a certain locating scheme controls the position stability of a certain part. In an assembly consisting of a number of parts with individual locating schemes, the individual *RSS* values are presented in the *stability matrix*. The following example shows how the stability analysis is used to analyze a door concept.

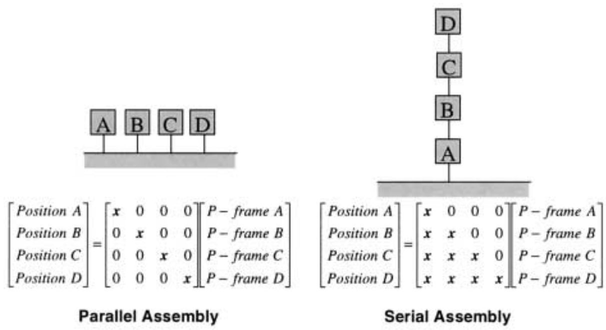


Fig. 9. Uncoupled and coupled assembly.

Figure 10 shows the *P-frame* stability analysis performed on a door concept including an assembly fixture (see Söderberg and Lindkvist 1999a). The matrix shows the locating scheme sensitivities of the assembly and can be seen as a solution-solution matrix, as described in Section 2, focused on the location schemes for the parts in the assembly. The *RSS* sensitivity values for the locating schemes can be expanded to show the sensitivity coefficient for each locating point. The analysis is performed in our CAD based analysis system RD&T.

By studying the stability matrix, two positioning aspects may be judged: the *degree of coupling* and the *robustness*. Here, a fully uncoupled design is represented by a diagonal matrix (see Söderberg and Lindkvist (1999a) and Suh (1990)). The robustness is judged by studying the values of the matrix elements. A value higher than one means that variation is amplified by the *P-frame*, whereas a value below one means that the input variation (variation in the contact points) is suppressed by the *P-frame*. A value equal to zero indicates no coupling at all between input and output. By using the two goodness measures, R (reangularity) and S (semangularity) (see Suh 1990), different concepts can be compared. R and S values equal to one represent a fully uncoupled solution, whereas values below one indicate couplings. Performing this type of sensitivity analysis in early concept design makes it possible to elaborate with the locating schemes on individual parts, as well as with the use of assembly fixtures. This means that robustness can be built in from the beginning without detailed process knowledge. The analysis combines

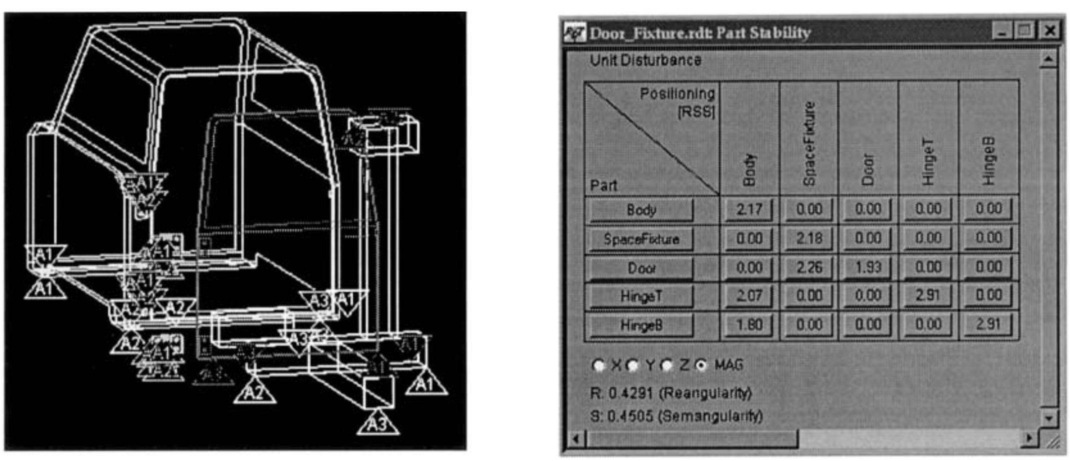


Fig. 10. P-frame stability analysis in concept design.

the benefits of sensitivity analysis, matrix representations, CAD-based assembly definition and axiomatic design to evaluate the degree of geometrical coupling in an assembly, what part locating schemes and locators that are important, and in what order to adjust it.

4. Tolerance Analysis in Detail Design

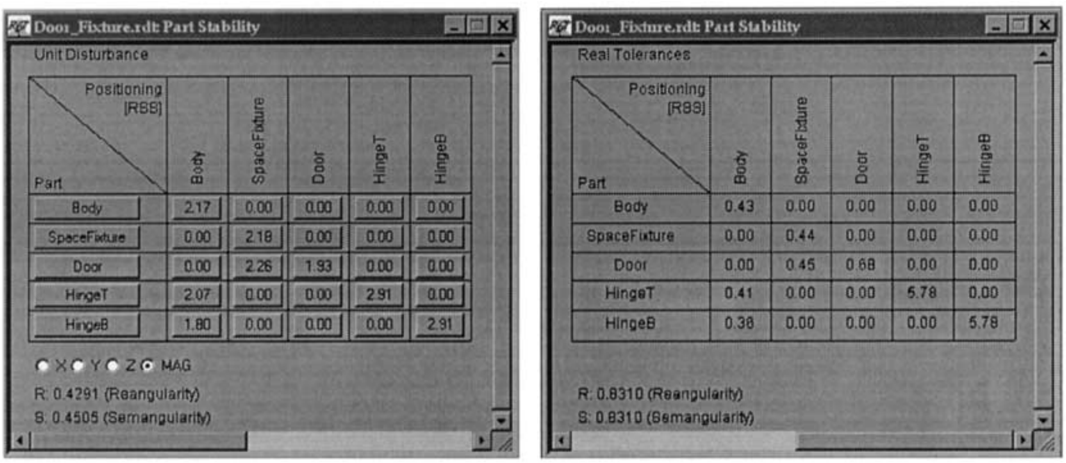
During *detail design* (late embodiment design), the design is adjusted to the manufacturing and assembly process. Geometrical concept sensitivity must then be met by manufacturing and assembly precision, and the final choice of suppliers of the individual parts must be made. The situation is often characterized by the facts that: (i) geometry and CAD models are more highly developed; (ii) product sensitivity is known (from concept design); and (iii) manufacturing and assembly distribution is available. At this stage, tolerances may be allocated with respect to: product constraints, assembly sensitivity and manufacturing and assembly variation and cost.

Traditional CAT (Computer Aided Tolerancing) technology found in systems such as VSA, 3DCS, CE/Tol and Valisys provides two types of analyses that can be used during detail design to optimize the selection of tolerances: *variation* analysis and *contribution* analysis. In variation analysis, statistical data such as standard deviation, mean value, tolerance range and acceptance rate for the total population are calculated for the specified critical assembly dimensions. In contribution analysis, the 3D influence of variation in each geometrical feature, according to specified tolerance and distribution, is ranked for specified critical dimensions, including effects of feature *position, direction* and variation *magnitude*. A review of commercial CAT tools is presented in Salomonsen et al. (1997).

Typical for the available CAT systems is that they provide help for analyzing *one* critical dimension at a time for a problem that is usually quite coupled. Sensitivity analysis and tolerance allocation of an assembly with a number of parts and critical dimensions is a multi-objective, multivariate optimization problem. Often, a trade-off between a number of different design and manufacturing interests must be made. Today, trade-off decisions are often based on experience and discussions in cross-disciplinary groups and with very little analytical data as input. To support these kinds of decisions and to be able to judge the overall degree of coupling and locating scheme robustness for an assembly, the matrix representation is also quite useful in this phase.

The solution-solution matrix described in Section 2, represented by the stability matrix in Section 3.2, captures the relations between all assembly inputs (disturbance in locating schemes) and outputs (part position or critical assembly dimension) in a way that makes it possible to judge the overall robustness of the design. During concept design, a unit disturbance is applied to all locating points in their locating direction to analyze geometrical sensitivity with respect to locator position and variation direction. The same matrix also can be used also in detail design, then including the effect of the variation range (tolerance value). Figure 11 shows the two matrices for the door assembly.

In the first matrix, a unit disturbance is used for all contact points. Each matrix elements then show the RSS *sensitivity* of the six locating points in each locating scheme (*P-frame*) with respect to position variation in a particular part. In the second matrix,


Door_Fixture.rdt: Part Stability

Unit Disturbance

Part \ Positioning [RSS]	Body	SpaceFixture	Door	HingeT	HingeB
Body	2.17	0.00	0.00	0.00	0.00
SpaceFixture	0.00	2.18	0.00	0.00	0.00
Door	0.00	2.26	1.93	0.00	0.00
HingeT	2.07	0.00	0.00	2.91	0.00
HingeB	1.80	0.00	0.00	0.00	2.91

X Y Z MAG

R: 0.4291 (Reangularity)

S: 0.4505 (Semangularity)

Door_Fixture.rdt: Part Stability

Real Tolerances

Part \ Positioning [RSS]	Body	SpaceFixture	Door	HingeT	HingeB
Body	0.43	0.00	0.00	0.00	0.00
SpaceFixture	0.00	0.44	0.00	0.00	0.00
Door	0.00	0.45	0.68	0.00	0.00
HingeT	0.41	0.00	0.00	5.78	0.00
HingeB	0.38	0.00	0.00	0.00	5.78

R: 0.8310 (Reangularity)

S: 0.8310 (Semangularity)

Fig. 11. Unit disturbance (sensitivity) and real tolerance (contribution) matrix.

real tolerance values are used. Each matrix element then shows the RSS *contribution* (tolerance × sensitivity) of the six locating points in each locating scheme (*P-frame*). Since the two matrices have different units, the relation between individual elements can be evaluated only within the matrix. As can be seen by comparing the two matrices, the R and S values of the contribution matrix indicate a less coupled design than the sensitivity matrix because, with real tolerances applied, the fixture variation is only about a tenth of the part variation. The real tolerance matrix shows that tolerances have been applied to parts and fixtures in a way that makes the concept *less coupled* as compared with the unit variation (real sensitivity) case.

The same type of analysis can also be performed with respect to stability in individual critical assembly dimensions, as we will show in the example in the next section. In this case, the matrix is not always square, which is why the R and S values cannot be used for evaluation. However, studying the matrix elements provides information about assembly sensitivity and main contributors.

5. Example

In this section, we will use an example of a vehicle 'front system' to demonstrate and discuss how geometrical constraints can be handled and broken down into tolerances on individual part features. The example will cover the decomposition from early concept and configuration design to tolerance allocation in late embodiment design.

5.1. Case Presentation

A conceptual solution for an engine sub-frame and suspension arrangement for a vehicle is to be modeled. At this early stage, no detailed geometric models are available, only principle sketches that will evolve in parallel with the top-down modeling of the function-means tree. Figure 12 shows a simple principle sketch reflecting the first ideas on how to provide the functionality of the required sub-assembly. The engine and the left and right arms are mounted to the subframe, which is mounted to the body of the vehicle. The left and right suspensions are then mounted to the arms and the inner fenders of the vehicle.

In this case, we will study the two camber and caster angles, which are critical for the behavior of the vehicle. The camber angles are controlled by the relation in the Y-direction between the assembly points for the suspension on the fender and the arm on each side, respectively. The caster angle is controlled by the same two points in the X-direction. The permitted camber and caster angle deviations are specified as four constraints acting on the vehicle.

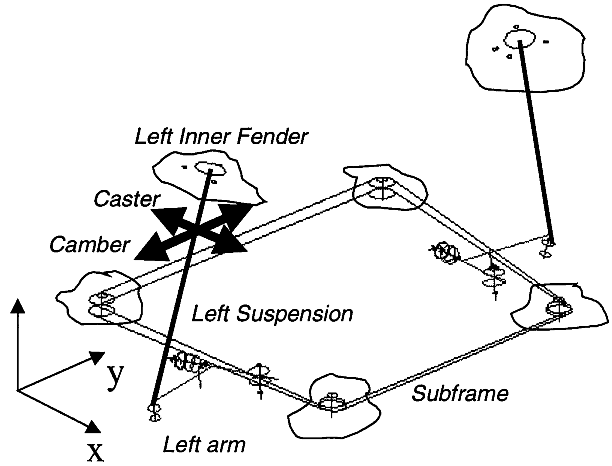


Fig. 12. Front system.

5.2. Function-Means Model

Figure 13 shows the function-means model for the vehicle and the front system. In the model a white box represents a Functional Requirement (FR), and a gray box represents a Design Parameter (DP). Note that there are functional requirements in the model that have not been formulated. These functional requirements are just represented with 'FR'. Design constraints are modeled as ellipses attached to the constrained design items. The relations between different objects shown in the function-means tree are modeled with different lines and arrows. In this model, the *is_solved_by* relation is not shown since the FRs and the DPs are modelled as linked objects.

The development of the model starts with the formulation of the overall functional requirement, solved by the DP 'vehicle'. On 'vehicle', four new functional requirements are formulated. Constraints on 'vehicle' are left out at this point, and will be treated in the next section. The new requirements are modeled on the next lower hierarchical level and linked to 'vehicle' with *requires_function* relations. The solutions to these requirements are solved by the DPs' 'right suspension', 'left suspension', 'body' and 'front system'.

The vehicle also requires the following positioning functions linked to 'vehicle' with *requires_function* relations: *provide means for the two suspensions and the body to be positioned in the vehicle*. The solutions

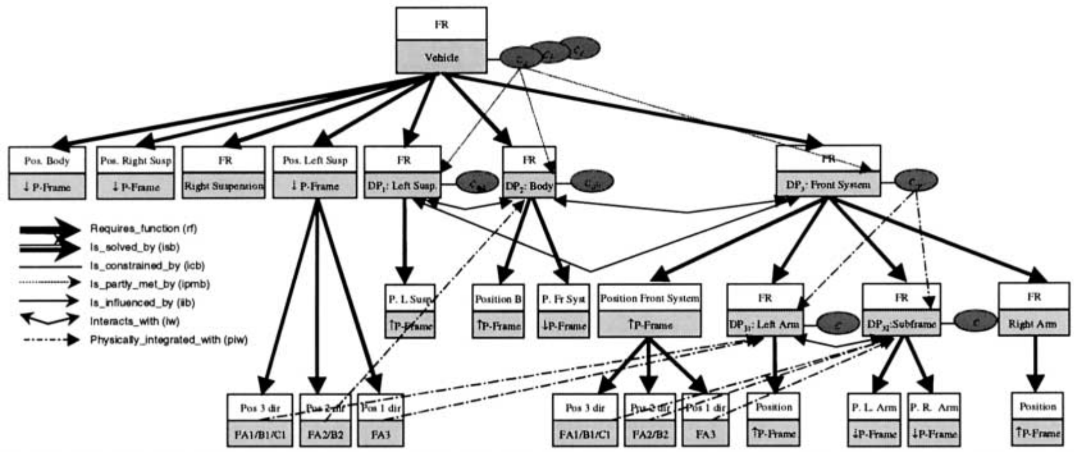


Fig. 13. Function-means hierarchy for front system.

to these position requirements are '↓*P-frame*' DPs. As the two suspensions are equivalent from a modeling point of view, only the 'left suspension' will be treated in the following.

The 'left suspension' is not decomposed any further but, as we are focusing on geometry and assembly requirements, the FR 'position left suspension' (in the 'vehicle') is formulated on the next hierarchical level. It is linked to the DP 'left suspension' with a *requires_function* relation. The solution to this FR is the DP '↑*P-frame*'.

The 'body' is treated in the same way as the 'left suspension', i.e. only position requirements and their assigned design solutions are modeled on the next hierarchical level. The 'body' requires the following positioning functions: *provide means for the front system to be positioned on the body, and position the body itself in the vehicle*. The last requirement is solved by a '↑*P-frame*' DP and the first by an '↓*P-frame*'-DP. All requirements are linked to 'body' with *requires_function* relations and to their assigned solutions with (not shown) *is_solved_by* relations.

Besides a positional requirement, three functional requirements, solved by the DPs 'left arm', 'sub-frame' and 'right arm', are required by the 'front system'. The positional requirement is: position the 'front system' in the 'body'. This position requirement is solved by a '↑*P-frame*' DP. All requirements are linked to 'front system' by *requires_function* relations. As the two arms are equivalent from a modeling point of view, only the 'left arm' is treated in the following discussion.

The 'left arm' has a positioning requirement linked to 'left arm' by a *requires_function* relation: *position arm on subframe*, which is solved by a '↑*P-frame*' DP. Accordingly, there are two positioning requirements on the 'sub-frame', linked to the 'subframe' by *requires_function* relations: *provide means to position 'left /right arm' on 'subframe'*. These requirements are solved by '↓*P-frame*' DPs. The positioning of the 'subframe' itself is handled by the DP that positions the complete 'front system' on the 'body'.

Of special interest here is the '↓*P-frame*' DP which provides the means to position the 'left suspension' in the vehicle. Referring to Fig. 13, the requirements on this DP are: *position in 3 directions*, solved by features FA1/B1/C1, *position in 2 directions*, solved by features FA2/B2 and *position in 1 direction*, solved by feature FA3. It should be noted especially that these feature DPs are found partly on the 'body' and partly on the 'left arm', as it is a component of the 'front system'. In other words, here we have a '↓*P-frame*' realized by physical features located in different subsystems and parts in the design.

This is a compound *P-frame* of another kind than the compound *P-frame* that was identified and related to a subassembly in Söderberg and Johannesson (1999). *That first 'upwards compound P-frame' positioned a subassembly itself upwards in a higher level subsystem. This new 'downwards compound P-frame' provides a means for another part or subsystem to be positioned*. As can be seen, this second kind of compound *P-frame* will lead to a closed interaction loop between the 'body', the 'front system' and the 'left suspension' which in turn indicates a potential closed tolerance chain. This is in contrast to the *P-frame* positioning the 'front system' in the 'body'. This *P-frame* is realized by geometrical features which are all located on the 'subframe', i.e. within the same part, so that this *P-frame* does not suffer from any part interaction problems.

5.3. Constraint Decomposition and Tolerance Chain Identification

The final step in the function-means modeling procedure is to analyze the model and to predict potential interactions between the different parts and subsystems. It is obvious here that the 'suspensions' interacts both with the 'body' and the 'front system' as the 'suspensions' are physically connected to both. Likewise, the 'body' and the 'front system' interact as they are also physically connected to each other. Within the 'front system', each 'arm' is physically connected to the 'subframe', thus resulting in interactions. All these interactions are modeled with *interacts_with* relations in the model.

Referring to Fig. 13, the overall constraints C_a and C_b on 'vehicle' that are of special interest here are the restricted variations of the caster and camber angles on left and right sides, respectively. For the left side of the 'vehicle', these variations are caused by variations in the 'left suspension' (DP_1), the ''body' (DP_2) and the 'front system' (DP_3). In other words, C_a is partly met by the 'left suspension', the 'body' and the 'front system' as indicated by the *is_partly_-met_by* relations in the structure model. Thus, to meet constraint C_a on 'vehicle', decomposed constraints C_{aa}, C_{ab} and C_{ac} must be imposed on the 'left suspension', the 'body' and the 'front system', respectively, on the next hierarchical level.

C_{ac} which constrains the left side of the 'front system' is partly met on the next lower level by the 'left arm' (DP_{31}) and the 'sub-frame' (DP_{32}), which is indicated by *is_partly_met_by* relations in the structure model. These two parts also interact as they are physically connected, as indicated by the *interacts_with* relation.

For clarity, the parts of the model in Fig. 13 that are of interest for the focused tolerance chain analysis are zoomed up in Figs 14 and 15.

By identifying *type 1* relation loops, as discussed in Section 2.3, in the structure model it can be seen that we have potential constraint-caused tolerance chains between 'left suspension' and 'body', between 'body' and 'front system' and between 'front system'' and 'left suspension', as these three DPs interact with each other and share the overall constraint C_a. Furthermore, it can be seen that tolerance chain DP_1-DP_2-DP_3 is closed by *interacts_with* relations.

In a similar manner, constraint C_{ac} on the 'front system' is shared by the 'left arm' and the 'subframe'. As they interact, there is a potential tolerance chain in the 'front system' between the 'left arm' and the 'subframe'.

The corresponding analyses of the consequences caused by C_a and C_{ac} using the matrix representation put forward in Section 2.3 appears as follows. The

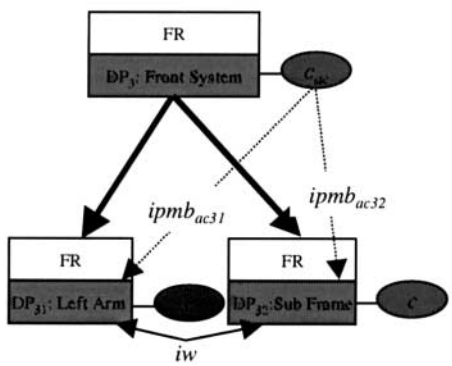


Fig. 15. C_{ac} relation loop (type 1).

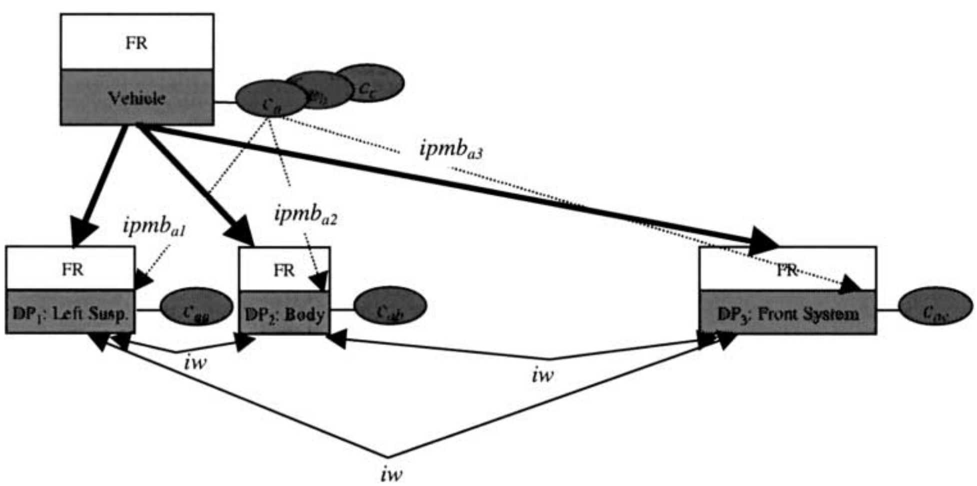


Fig. 14. C_a relation loop (type 1).

C \ DP	DP_1	DP_2	DP_3
C_a	$ipmb_{a1}$	$ipmb_{a2}$	$ipmb_{a3}$
C_b	--	--	--
C_c	--	--	--

DP \ DP	DP_1	DP_2	DP_2
DP_1		iw_{12}	iw_{13}
DP_2	iw_{21}		iw_{23}
DP_3	iw_{31}	iw_{32}	

Fig. 16. C_a constraint-solution and solution-solution matrices.

C \ DP	DP_{31}	DP_{32}
C_{ac}	$ipmb_{ac31}$	$ipmb_{ac32}$
C_{bc}	--	--

DP \ DP	DP_{31}	DP_{32}
DP_{31}		iw_{3132}
DP_{32}	iw_{3231}	

Fig. 17. C_{ac} constraint-solution and solution-solution matrices.

constraint-solution and solution-solution matrices related to C_a are shown in Fig. 16 and the corresponding matrices related to C_{ac} in Fig. 17.

From the matrices in Fig. 16, the following conclusions can be drawn:

A: *As* $iw_{12} = iw_{21} \neq 0$ *and* $ipmb_{a1} \neq 0$ *and* $ipmb_{a2} \neq 0$, *there is a potential tolerance chain between* DP_1 *and* DP_2

B: *As* $iw_{23} = iw_{32} \neq 0$ *and* $ipmb_{a2} \neq 0$ *and* $ipmb_{a3} \neq 0$, *there is a potential tolerance chain between* DP_2 *and* DP_3

C: *As A and B are TRUE, there is a potential tolerance chain between* DP_1, DP_2 *and* DP_3

D: *As C is TRUE and* $iw_{13} = iw_{31} \neq 0$, *the potential tolerance chain* DP_1-DP_2-DP_3 *is closed*

A *closed physical interaction chain* like the one between the 'suspension', the 'body' and the 'front system' describes an *over-constrained* design. Since there is no gap (or constrained critical dimension) that can vary with the component variations, this situation must be avoided, since it puts very high demands on tight tolerance. In this particular situation, the final closed physical interaction chain will probably contain flexible sleeves that allow for some component variation. The arms should also be released from their fixturing device during assembly, which provides one rotational degree of freedom.

From the matrices in Fig. 17, the following conclusion can be drawn:

A: *As* $iw_{3132} = iw_{3231} \neq 0$ *and* $ipmb_{ac31} \neq 0$ *and* $ipmb_{ac32} \neq 0$, *there is a potential tolerance chain between* DP_{31} *and* DP_{32}

In addition to the conclusion above, the analysis also leads to the following general conclusions:

- Compound *⇂P-frames* must be avoided, since they contain internal tolerance chains between location features.
- Closed loops of physical interactions between parallel subsolutions sharing the same overall constraint must be avoided, since they represent physically over-constrained design solutions.

5.4. Sensitivity and Tolerance Analysis

The previous section showed how tolerance chains and geometrical couplings can be detected at a very early design stage, before any real geometrical models are available, mainly by studying the function-means model and the relations between objects. The next development step will probably be to roughly decide upon the main dimensions of the parts in the assembly. Here, overall spatial constraints for the vehicle must be considered and shared among the individual parts. At this stage, with knowledge of main dimensions and possible locating schemes for all parts, sensitivity analysis, as described in Sections 3 and 4, can be performed. Figure 18 shows the locating scheme *stability analysis* for the geometrical

Unit Disturbance

Measure \ P-Frame	Subf	Arm_L	Arm_R
Camber_L	0.02	0.02	0.00
Camber_R	0.02	0.00	0.02
Caster_L	0.03	0.05	0.00
Caster_R	0.06	0.00	0.05

Real Tolerances

Measure \ P-Frame	Subf	Arm_L	Arm_R
Camber_L	0.03	0.02	0.00
Camber_R	0.03	0.00	0.02
Caster_L	0.05	0.05	0.00
Caster_R	0.10	0.00	0.05

Fig. 18. P-frame stability analysis.

concept described roughly in Fig. 12. The analysis is performed in RD&T using rough CAD sketches and information about locating schemes and mating conditions. The mean influence of each *P-frame* on each critical dimension (overall constraint) is presented in the stability matrix. Each matrix element may be expanded to show the relative influence of each individual *P-frame* point on a critical dimension.

As shown by the two matrices, the *P-frame* locating the subframe to the body results in an asymmetrical behavior of the two caster angles. As can be seen, the right caster angle is twice as sensitive to variation in the locating scheme of the subframe as is the left caster angle. The same result is found both with unit disturbance in all locating points and with real tolerances (expected distributions). All other sensitivities are symmetrical. With this information, it is now possible either to change the locating scheme definition for the sub-frame to try to reach a symmetric behavior, or to try to compensate with extra tight tolerances on the right side of the vehicle. The first alternative is of course preferable.

With knowledge of the final concept sensitivity, which is given by the unit disturbance matrix for the final solution, tolerances may now be allocated. Generally, tight tolerances are required for sensitive areas and wider tolerances can be accepted in areas with lower geometrical sensitivity. Manufacturing cost must of course also be considered. To support final tolerance analysis, *variation* and *contribution* analysis, provided by a number of commercial software systems, is used. Figure 19 shows the Monte Carlo-based variation analysis used to predict the final variation in critical dimensions. Here, the distribution, the expected range and the mean and the standard deviations for critical specified critical assembly dimensions (in this case, the camber and

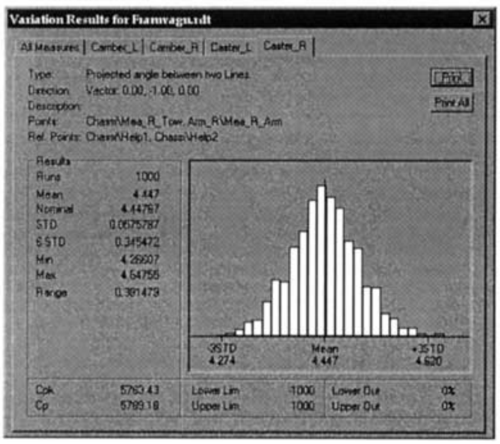


Fig. 19. Variation analysis.

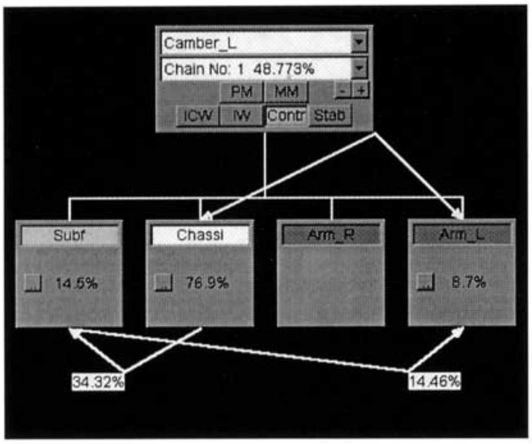


Fig. 20. Contribution analysis.

caster angles) are presented. Figure 20 shows the RD&T contribution analysis result for the assembly. Here, the total contribution of each component and each physical contact is presented for a selected critical assembly dimension (an overall constraint). Figure 20 describes the tolerance chain and the contributions for the camber angle on the left side.

The contribution analysis is used to visualize tolerance chains and to identify main variation sources. The analyses presented in this paper are carried out in the RD&T software (see also Söderberg and Lindkvist (1999a) and Lindkvist and Söderberg (1999)).

6. Summary

A substantial number of all the quality problems that arise during assembly can be referred to the geometrical design, and especially to the geometrical concept of the product, i.e. the way in which parts are designed and located to each other.

This work presents a new structure model and matrix-based geometry design procedure that can assist robust geometry design during *configuration, concept* and *detail* design. Special attention is given to the configuration design part, which presents a new way of representing geometry variation constraints in an enhanced function-means tree structure model. Different parts of the function-means tree that are of interest for analysis purposes are then extracted and converted to matrix representation. The reason for using one representation during modeling, and another during analysis is that the structure model is suitable for modeling but becomes unsuitable for analysis when the complexity of the model increases. For this latter purpose, the matrix representation is far better.

During concept design (early embodiment design), early geometry solutions are available for all parts in an assembly. The robustness of the concept is then evaluated using *stability matrices*, showing the degree of geometrical coupling and sensitivity in an assembly with respect to the way parts are located to each other. In this stage, assembly fixtures are included in the analysis. During detail design (late embodiment design), tolerances are allocated to parts in the assembly with respect to geometrical sensitivity, process variation and cost. This step is supported by traditional Monte Carlo analysis. The concept and detail design support tools are implemented in the CAT (Computer Aided Tolerancing) environment RD&T, and allow part geometry from different CAD systems to be imported and used for assembly evaluation.

The use of the different parts of the procedure is demonstrated on the design of a vehicle front system where the geometry is *a priori* unknown. In addition to showing the procedure and the different analysis tools, the example also gives the following general conclusions:

- Matrix representations are very useful for tolerance analysis in all stages of the design process.
- Compound ↓*P-frames* must be avoided, since they contain internal tolerance chains between location features.
- Closed loops of physical interactions between parallel subsolutions sharing the same overall constraint must be avoided, since they represent physically over-constrained design solutions.

The procedure and the different analysis tools presented in the paper provide a structured base for geometrical variation control throughout the different stages of product development and production preparation.

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