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Jet Noise: Since 1952[1]

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Abstract. Jet noise research was initiated by Sir James Lighthill in 1952. Since that time, the development of jet noise theory has followed a very tortuous path. This is, perhaps, not surprising for the understanding of jet noise is inherently tied to the understanding of turbulence in jet flows. Even now, our understanding of turbulence is still tenuous. In the fifties, turbulence was regarded as consisting of a random assortment of small eddies. As a result, the primary focus of jet noise research was to quantify the noise from fine-scale turbulence. This line of work persisted into the eighties. The discovery of large turbulence structures in free shear flows in the early seventies led some investigators to begin questioning the validity of the then established theories. Some went further to suggest that, for high-speed jets, it was the large turbulence structures/instability waves of the flow that were responsible for the dominant part of jet mixing noise. Development of a quantitative theory of noise from large turbulence structures/instability waves took place during the next 15 years. Precision instrumentation and facilities for jet noise measurements became available in the mid-eighties. This permitted a large bank of high-quality narrow band jet noise data to be gathered over the subsequent years. Recent analysis of these data has provided irrefutable evidence that jet noise, in fact, is made up of two basic components; one from the large turbulence structures/instability waves, the other from the fine-scale turbulence. This is true even for subsonic jets. In this paper, some of the crucial research results of the past 44 years, that form the basis of our present understanding of jet noise generation and propagation, are discussed.

1. Introduction

The year 1952 was a very special year for jet noise research, for it was this year when Sir James Lighthill published the first of his two-part paper (Lighthill, 1952, 1954) on aerodynamic sound in the *Proceedings of the Royal Society of London*. This paper has since been regarded as marking not only the beginning of jet noise research but also the birth of the research area “Aeroacoustics.” Forty-four years have now elapsed. During that time, the development of jet noise theory has followed a very tortuous path. This is, perhaps, not surprising, for the understanding of jet noise is inherently tied to the understanding of turbulence in jet flows. Even today, our understanding of turbulence is still tenuous.

The aim of this paper is to highlight the important developments of jet noise theory since Lighthill’s original work. Special emphasis is given to the more recent findings that appear to offer a new perspective on jet noise characteristics and generation mechanisms.

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2. The Fifties to the Seventies

2.1. The Acoustic Analogy Theory

To develop a jet noise theory, intuitively, it seem that the first thing to do is to identify the sources of noise. In his 1952 and 1954 papers, Lighthill tackled this problem by establishing the renowned Acoustic Analogy Theory. The basic idea is to cast the compressible equations of motion into a form representing the propagation of acoustic waves. Whatever terms that are left are then moved to the right side of the equation. The result is an inhomogeneous wave equation of the form

$$\frac{\partial^2 \rho}{\partial t^2} - a_\infty^2 \nabla^2 \rho = \frac{\partial^2 T_{ij}}{\partial x_i \, \partial x_j} \tag{1}$$

where ρ is the density, a_∞ is the ambient sound speed, and $T_{ij} = \rho v_i v_j + (p - \rho a_\infty^2)\delta_{ij} - \tau_{ij}$ is known as the Lighthill stress tensor, v_i, p, and τ_{ij} are the velocity, pressure, and viscous stresses. δ_{ij} is the Kronecker delta. Within the framework of the Acoustic Analogy Theory, the following reasoning is advanced. By design, the left side of (1) represents acoustic wave propagation. It follows, therefore, that the right side of the equation must be the sources that generate the noise field. The source terms involve second spatial derivatives. They are referred to as quadrupoles.

During the fifties, the prevailing view of turbulence was that it consisted of a random assortment of small eddies. Thus, although no formal relationship between quadrupoles and small turbulent eddies was ever established, the implication was that the quadrupoles were related in some way to the small eddies. In present day terminology, we call the small eddies the fine-scale motion of the turbulent flows.

One important result that can be derived from the Acoustic Analogy Theory is the noise scaling law. By using Green's function of the wave equation, Lighthill obtained a formal solution of (1). On applying dimensional analysis to the formal solution, he established that the acoustic power radiated by a jet should vary as the eighth power of the jet velocity V_j. This became the celebrated V_j^8 Law.

2.2. The Source Convection Effect

In jets, the quadrupoles are convected downstream by the mean flow at a relatively high speed. That is, these are moving sources. It is easy to verify that moving sources tend to radiate more noise in the direction of motion. This is somewhat analogous to synchrotron radiation. Lighthill recognized immediately the significance of the source convection effect on the directivity of jet noise. At higher jet speeds, this effect is more pronounced. This was investigated by Ffowcs-Williams (1963). By extending Lighthill's dimensional argument including the effect of source convection, Ffowcs-Williams found that for very high-speed jets, the power of the radiated noise should vary as the third power of the jet velocity. That is $P \sim V_\mathrm{j}^3$ where P is the acoustic power radiated. This and the Lighthill V_j^8 Law are the two most important results of the classical Acoustic Analogy Theory.

2.3. Effect of Refraction

Once sound is generated by the quadrupoles, the acoustic waves have to propagate through the jet flow to reach the far field. The mean flow of the jet is highly nonuniform. Thus, the radiated sound undergoes refraction in its passage through the jet.

Figure 1 illustrates the refraction of a ray of sound emitted by a point source *S* located in the mixing layer of a jet. To see why the ray bends outward, one needs only to consider the propagation of the wave front *AB*. The point *A* moves at a speed equal to the local sound speed plus the local flow velocity of the jet. So does the point *B*. If the jet is nearly isothermal, the speed of sound is the same at *A* and *B*. However, the flow velocity at *B* is higher. As a result, as the wave front propagates, it becomes tilted as *A′B′*. Obviously, this effect of refraction is even more severe for hot jets. In this case, the sound speed at *B* is higher than that at *A*. One of the important consequences of mean flow refraction is that less sound can be radiated in the direction of jet flow. This creates a relatively quiet region around the jet axis commonly known as the "cone of silence." Experimentally, the presence of a cone of silence in which the noise intensity drops by more than 20 dB has been demonstrated by Atvars *et al.* (1965).

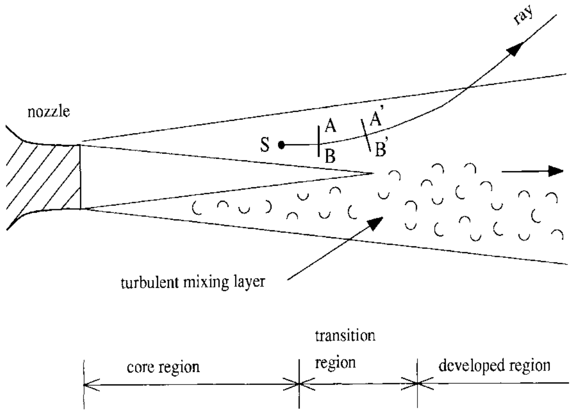

Figure 1. Schematic diagram showing a localized noise source at S in the mixing layer of a jet. Also shown are the core, transition, and developed region of the jet.

2.4. Variants of the Acoustic Analogy Theory

In the years after the work of Lighthill, there have been many attempts to modify or improve the Acoustic Analogy Theory. Many of these efforts involved modifying the wave propagation operator on the left side of (1). One immediate result is that each new theory produces a slightly different set of noise source terms. Just as in the original theory, the noise source terms of all the modified theories are themselves unknowns. In other words, these theories are not self-contained. A good deal of turbulence information has to be input into the theory before a prediction can be made.

One good reason to modify the wave propagation operator is to account for the mean flow refraction effect. Lilley (1974) suggested that the correct wave propagation operator was the linearized Euler equations. In his paper, the linearization was performed over the mean flow of the jet. In this way, a version of the Acoustic Analogy Theory designed to include mean flow refraction effect was developed. Although it is not clear why the full Euler equations, which, in principle, not only account for the mean flow refraction effect, but also the nonlinear propagation effects, are not the more appropriate wave propagation operators. This must have been rejected because it would leave almost no noise source terms. Lilley's proposal received immediate acceptance. It has been followed by numerous subsequent investigators. This approach formed the main direction of jet noise research throughout the seventies and early eighties. Further discussions and references to other jet noise theories built around the Acoustic Analogy concept of Lighthill may be found in a review article by Lilley (1991).

3. The Seventies and the Eighties

Turbulence research took an abrupt turn in the early seventies when it was discovered independently by Crow and Champagne (1971), Brown and Roshko (1974), and Winant and Broward (1974) that turbulence in jets and free shear layers is made up of large turbulence structures as well as fine-scale turbulence. The large turbulence structures are somewhat more deterministic than the fine-scale turbulence motion. They dominate the dynamics and overall mixing processes of jets and free shear layers.

Soon after the discovery of the large turbulence structures, Crow and Champagne, who studied subsonic jets, and others began to propose that they were important jet noise sources. It took a few years of investigation before it became clear that the large turbulence structures are definitely important direct noise sources of supersonic jets. However, they are not as important for subsonic jets. For imperfectly expanded supersonic jets, a shock cell structure automatically develops in the jet plume. In the presence of the shock cells, the jets emits two additional components of noise. They are referred to as screech tones and broadband shock associated noise. In this paper, we do not consider these shock-related noise components. Our attention is confined to turbulent mixing noise alone.

3.1. Large Turbulence Structures Model

To predict the noise radiated by the large turbulence structures in jets, a mathematical description of these entities is necessary. The familiar turbulence modeling approach (see, e.g., Speziale, 1991) is inappropriate. It has no large structures in its formulation.

The first statistical description of the large turbulence structures in free shear flows in the form of a stochastic instability wave model was proposed by Tam and Chen (1979). This stochastic model approach has since been used and extended by Plaschko (1981), Morris *et al.* (1990), Viswanathan and Morris (1992), Tam and Chen (1994), and others in their mixing layers, jet flows, and jet noise studies. The crucial observation that forms the basis of the model is that the turbulent jet flow spreads out very slowly. This means that the flow variables as well as the turbulence statistics change only very slowly in the downstream direction; i.e., the turbulence statistics are nearly constants locally. If, indeed, all the turbulence statistics are true constants, the flow is statistically stationary in time and in the flow direction. This implies that dynamically the turbulence fluctuations are locally in quasi-equilibrium. For a system in dynamical equilibrium, statistical mechanics theory suggests that the large scale fluctuations of the system can be represented mathematically by a superposition of its normal modes. In the case of high-speed jets, the large-scale fluctuations are the large turbulence structures while the dominant normal modes are the instability wave modes of the mean flow. The turbulence statistics and mean flow of jets and mixing layers are well predicted by the stochastic instability wave model. More detailed descriptions and references of large turbulence structure models can be found in the two reviews by Tam (1991, 1995).

3.2. Mach Wave Radiation

It may seem natural and reasonable to represent large turbulence structures by the instability wave of the mean flow. However it is well known that instability wave solutions decay to zero exponentially away from a jet or mixing layer. In other words, there is no acoustic radiation associated with instability waves. Tam and Morris (1980) recognized this difficulty. They correctly pointed our that the difficulty arose because of the locally parallel flow assumption. This assumption is routinely used in classical hydrodynamic stability analysis. They showed that to determine sound radiation, a global solution of the entire instability wave propagation phenomenon along the jet is necessary. Earlier, Saric and Nayfeh (1975), Crighton and Gaster (1976), and others had succeeded in accounting for the slow divergence of the mean flow by using a multiple-scales expansion method. Tam and Morris, however, demonstrated that the multiple-scales expansion instability wave solution is not uniformly valid outside the jet or the mixing layer. Thus, it is not surprising that the multiple-scales type solution still produces no acoustic radiation.

To construct a uniformly valid instability wave solution inside and outside the jet, Tam and Burton (1984) employed the method of matched asymptotic expansions. The inner solution is the multiple-scales instability wave solution. This solution is valid inside and in the neighborhood immediately outside the jet. The outer solution is essentially a solution of the acoustic wave equation taking into consideration the growth and decay of the instability wave amplitude in the flow direction. The outer solution is valid in the near field outside the jet all the way to the far field.

The matched asymptotic expansions solution reveals the basic mechanism by which sound in the form of Mach wave radiation is generated by the large turbulence structures/instability waves of the flow. The inner solution, which is the instability wave solution, is valid out to the near field immediately outside the jet. This means that the influence of the instability waves extends beyond the mixing layer of the jet flow. The change-over from instability wave solution to acoustic wave solution takes place near the edge of the jet and is contained in the outer solution. The sound field of the Mach wave radiation may, therefore, be regarded to be generated near the edge of the jet flow. Physically an instability wave behaves like a wavy wall moving at a high speed in the downstream direction. When the wave speed is supersonic relative to the ambient sound speed, Mach waves are generated (see Figure 2). This Mach wave radiation is highly directional. Since this sound field is generated near the edge of the jet, it is, therefore, not subjected to the mean flow reaction effect. In other words, there is no cone of silence for larger turbulence structures noise. The near field sound pressure level contours predicted by the matched asymptotic expansions solution of Tam and Burton for a Mach 2.1 moderate Reynolds number jet agree well with the measurements of Troutt and McLaughlin (1982). Recent work by Hixson *et al.* (1995) using direct numerical simulation clearly shows strong Mach

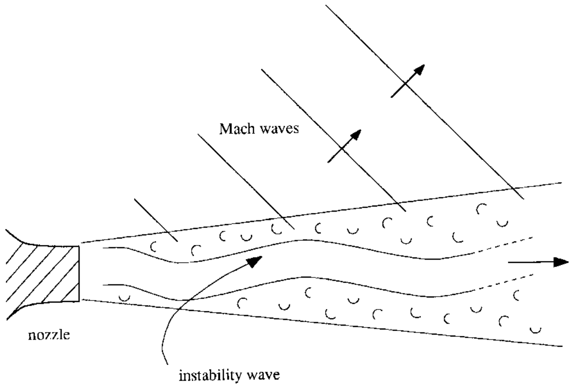

Figure 2. Schematic diagram showing Mach wave radiation generated by the large turbulence structures/instability waves of a high-speed jet.

wave radiation associated with the instability waves of the jet. Their computed sound pressure level contours are in good agreement with the analytical results of Tam and Burton and the experimental measurements of Troutt and McLaughlin.

Tam and Burton (1984) pointed out that the wavy wall analogy must be modified to account for the growth and decay of the instability wave as it propagates downstream. The growth and decay of the wave amplitude are important to the noise radiation process. For a fixed frequency wave of constant amplitude, the wave spectrum is discrete. With a single wave number there is only a single wave speed, so the Mach waves are radiated in a single direction. The growth and decay of the instability wave amplitude lead to a broadband wave number spectrum. This results in Mach wave radiation over large angular directions. Furthermore, a single frequency subsonic wave of constant amplitude would not radiate sound according to the Mach wave radiation mechanism. However, with growth and decay of the wave amplitude, a part of the broadband wave number spectrum could have supersonic phase velocity. These supersonic phase disturbances will lead to noise radiation. The single instability wave matched asymptotic expansions solution has recently been extended by Tam and Chen (1994) to include a broad frequency spectrum in a stochastic model theory of supersonic jet noise from the large turbulence structures. Their calculated noise directivities for Mach 2 jets at different temperatures are in good agreement with the measurements of Seiner *et al.* (1992).

The Mach wave radiation mechanism discussed above relies on the existence of supersonic phase components (relative to ambient sound speed). For highly supersonic jets, especially at high temperature, this is an extremely efficient noise generation process. However, if the jet speed is subsonic, the efficiency is greatly reduced. Thus, for subsonic jets the fine-scale turbulence is probably the more dominant noise source, except in the cone of silence.

Mach wave radiation by the large turbulence structures/instability waves of high-speed jets discussed above is not to be confused with eddy Mach wave radiation considered by a number of investigators in the sixties (e.g., Phillips, 1960; Ffowcs-Williams and Maidanik, 1965). The eddy Mach waves proposal is that eddies moving supersonically relative to ambient sound speed would generate strong Mach wave radiation. However, a closer examination reveals that this is physically an untenable process. Eddies are, by definition, localized entities with a limited range of influence spatially. While an eddy may be moving supersonically relative to the ambient gas outside the jet, it is definitely moving subsonically relative to the fluid in its immediate surroundings (eddies are convected downstream by the mean flow). That being the case, no Mach waves can be produced.

4. The Nineties

Until the late eighties, jet noise was, invariably, measured in $\frac{1}{3}$-octave bands. The experimental facilities available were generally incapable of producing high-speed jets at a high jet temperature. A $\frac{1}{3}$-octave band spectrum artificially enhances the importance of the high frequency noise component. This inevitably complicates the physical interpretation of the data. With improvements in instrumentation and experimental facilities beginning near the end of the eighties, jet noise data, for research purposes, has since been routinely

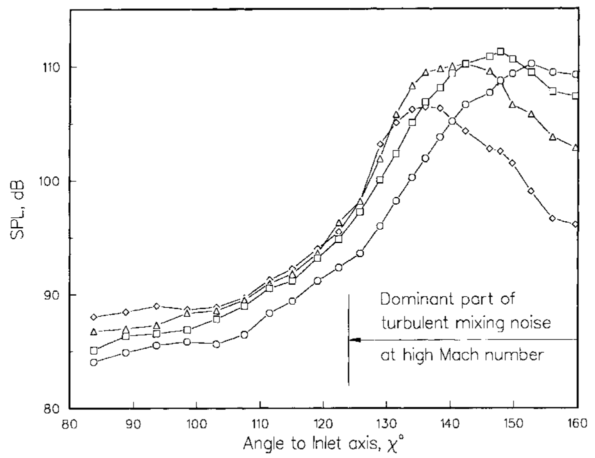

Figure 3. Measured noise directivities at selected Strouhal numbers of a Mach 2 jet at a total temperature of 500 K ($St = fD/U_{\mathrm{j}}$): ○ St = 0.067; □ St = 0.12; △ St = 0.20; ◇ St = 0.40.

processed in narrow bands. Also, very high temperature data, up to a temperature ratio of five, have been measured at the Jet Noise Laboratory of the NASA Langley Research Center. This new data offers the aeroacoustics community an unprecedented opportunity not only to study the characteristics of the noise of high-speed jets but also to test the validity of jet noise theories developed since Lighthill's original work.

With the coming of the nineties, a number of investigators (e.g., Tam and Chen, 1994; Seiner and Krejsa, 1989; Tam, 1995; and others) suggested that turbulent mixing noise from supersonic jets actually consists of two distinct components. One component is produced by the large turbulence structures/instability waves of the jet flow in the form of Mach wave radiation. The other component is generated by the fine-scale turbulence of the jet. Figure 3 shows the noise directivities measured by Seiner *et al.* (1992) at selected Strouhal numbers of a Mach 2 perfectly expanded jet. It is clear in this figure that the dominant part of jet noise is radiated in the downstream direction in the sector with an inlet angle, χ, larger than 125°. Tam and Chen (1994) showed that this highly directional noise component was generated by the large turbulence structures of the jet flow. They also observed that for an inlet angle, χ, less than 110° (see Figure 3) the jet noise radiation was almost uniform without a strongly preferred direction. They suggested that this low-level, almost uniform, background noise was generated by the fine-scale turbulence of the jet flow. In other words, the fine-scale turbulence noise is dominant over inlet angles smaller than 110° for the experiment of Figure 3. By implication, in the intervening angular directions $110° < \chi < 125°$, both noise components are important.

4.1. The Similarity Spectra

In the mixing layer of a turbulent jet, there is no inherent geometrical length scale. Also, it is well known that at high Reynolds number, viscosity is not a relevant parameter. Based on these observations, it is easy to see that there are no intrinsic length and time scales in the mixing layer in the core and the transition regions of a jet flow (see Figure 1). As a result, the mean flow as well as all the turbulence statistics of the flow must exhibit self-similarity. Over the years, that the mean flow and turbulence statistics of a high-speed turbulent jet possesses a similarity profile has been well verified experimentally. Since noise is generated by the turbulence of the jet, the above facts and reasonings strongly suggest that the noise spectra of the two independent turbulent mixing noise components should also exhibit similarity. In the absence of an intrinsic time or frequency scale, the frequency f must be scaled by f_{L}, the frequency at the peak of the large turbulence structures/instability waves noise spectrum, or f_{F}, the frequency at the peak of the fine-scale turbulence noise spectrum.

The noise of a high-speed jet naturally depends on the jet operating parameters V_{j} (the fully expanded jet velocity), T_{r} (the reservoir temperature), D_{j} (the fully expanded jet diameter), T_{∞} (the ambient temperature), χ (the direction of radiation), and r (the distance of the measurement point from the nozzle exit). On accounting for the contributions of the two independent noise components, the jet noise spectrum, S, may,

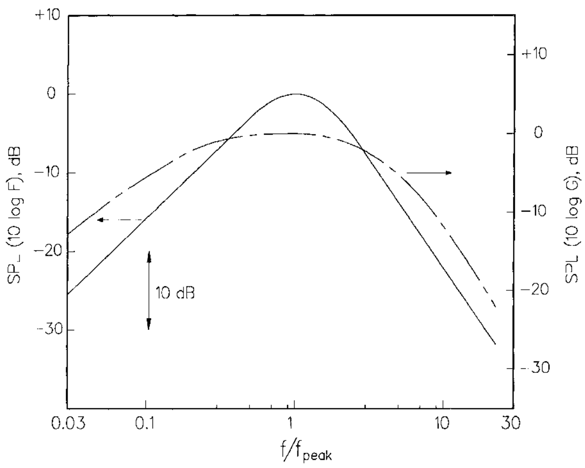

Figure 4. Similarity spectra for the two components of turbulent mixing noise. ——, large turbulence structures/instability waves noise; — - —, fine-scale turbulence noise.

therefore, be expressed in the following similarity form:

$$S = \left[AF\left(\frac{f}{f_L}\right) + BG\left(\frac{f}{f_F}\right)\right]\left(\frac{D_j}{r}\right)^2, \tag{2}$$

where $F(f/f_L)$ and $G(f/f_F)$ are the similarity spectra of the large turbulence structures noise and the fine-scale turbulence noise. These spectrum functions are normalized such that $F(1) = G(1) = 1$. In (2), A and B are the amplitudes of the independent spectra; they have the same dimensions as S. The amplitudes A and B and the peak frequencies f_L and f_F are functions of the jet operating parameters, V_j/a_∞, T_r/T_∞, and inlet angle χ.

To provide concrete experimental evidence that turbulent mixing noise from supersonic jets is, indeed, made up of two distinct components, Tam *et al.* (1996) performed a careful analysis of all the jet noise spectral data (1900 spectra in all) measured in the Jet Noise Laboratory of the NASA Langley Research Center. By means of this set of data, they were able to identify the similarity spectrum functions $F(f/f_L)$ and $G(f/f_F)$. Figure 4 shows the shapes of the empirically determined spectrum functions in dB scale versus $\log(f/f_{peak})$, where $f_{peak} = f_L$ for the large turbulence structures noise and $f_{peak} = f_F$ for the fine-scale turbulence noise. (Note: In a $\log(f/f_L)$ plot, the graph of $10 \log F$ should fit the entire measured spectrum, if it is dominated by the large turbulence structures noise, when the peak of the graph is aligned with the peak of the measured spectrum. The same is true for the fine-scale turbulence noise). The two spectrum shapes are distinctly different. The $10 \log F$ function has a relatively sharp peak and drops off linearly as shown. The $10 \log G$ function, on the other hand, consists of a very broad peak and rolls off extremely gradually.

Figure 5 shows typically how well the spectrum function $10 \log F$ fits the measured data. In these examples, the jet Mach numbers, M_j, are 1.5 and 2.0. The jet to ambient temperature ratio increases from 1.11 to 4.89. The direction of radiation, χ, varies from 138° to 160°. As can be seen, there is good agreement in all the cases. Figure 6 illustrates typical comparisons between the spectrum function $10 \log G$ and the measured data for perfectly expanded supersonic jets at $M_j = 1.5$ and 2.0 in directions for which the noise from fine-scale turbulence dominates. The jet to ambient temperature ratio covers the range of 0.98 to 4.89. The inlet angle, χ, varies from 83.3° to 120° Clearly, there is good agreement over the entire measured frequency range.

For angular directions neither too far upstream nor downstream both mixing noise components are important. Figure 7 shows examples of how the two noise spectra can be added together to reproduce the measured spectra. To obtain a good fit to the data, the amplitude function A and B as well as the peak frequencies f_L and f_F are adjusted in each case. The separate contributions from each of the two noise components are shown in the figure.

The similarity spectra have been checked by comparing the entire data band (1900 spectra). They fit all the measured spectra over the entire range of Mach number and temperature ratio of the NASA Langley data.

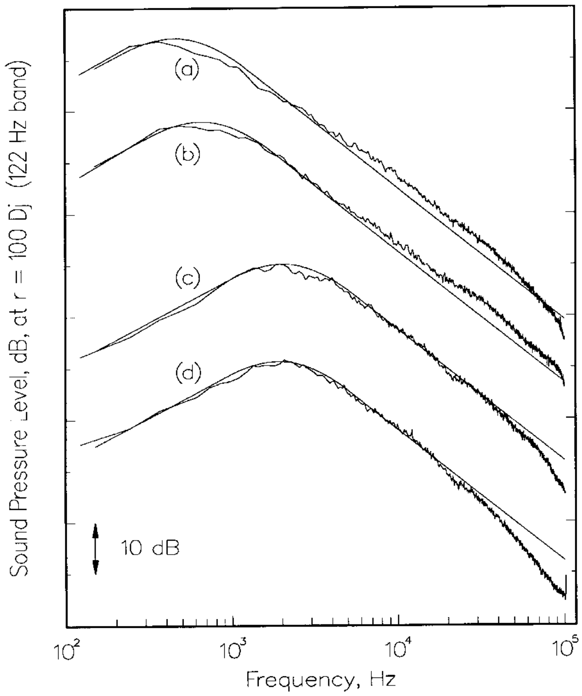

Figure 5. Comparison of the similarity spectrum of large turbulence structures/instability waves noise and measurements. (a) M_j = 2.0, T_r/T_∞ = 4.89, χ = 160.1°, SPL_max = 124.7 dB, (b) M_j = 2.0, T_r/T_∞ = 1.12, χ = 160.1°, SPL_max = 121.6 dB, (c) M_j = 1.96, T_r/T_∞ = 1.78, χ = 138.6°, SPL_max = 121.0 dB, (d) M_j = 1.49, T_r/T_∞ = 1.11, χ = 138.6°, SPL_max = 106.5 dB.

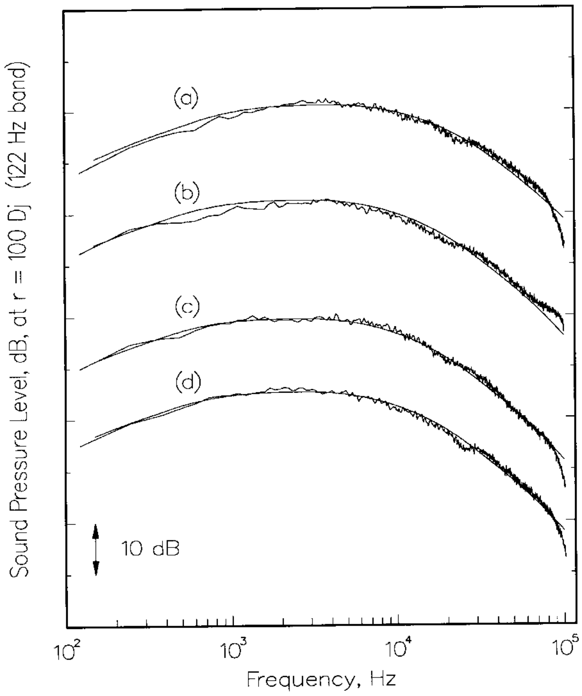

Figure 6. Comparison of the similarity spectrum of fine-scale turbulence noise and measurements. (a) M_j = 1.49, T_r/T_∞ = 2.35, χ = 92.9°, SPL_max = 96 dB, (b) M_j = 2.0, T_r/T_∞ = 4.89, χ = 83.8°, SPL_max = 107 dB, (c) M_j = 1.96, T_r/T_∞ = 0.99, χ = 83.3°, SPL_max = 95 dB, (d) M_j = 1.96, T_r/T_∞ = 0.98, χ = 120.2°, SPL_max = 100 dB.

4.2. Turbulent Mixing Noise from Subsonic Jets

Tam *et al.* (1996), in their data analysis effort, found that M_j, the fully expanded jet Mach number, is not a useful parameter for characterizing turbulent mixing noise. However if M_j is unimportant, then the finding that turbulent mixing noise consists of two distinct components should be true regardless of M_j. In other words, it must be valid for supersonic jets ($M_\mathrm{j} > 1$) as well as subsonic jets ($M_\mathrm{j} < 1$).

It has been discussed before that the mean flow refraction effect creates a cone of silence for the fine-scale turbulence noise around the direction of the jet flow. On the other hand, as pointed out above, this is the principal direction of Mach wave radiation by the large turbulence structures. Thus, the noise spectrum and characteristics inside the cone of silence of the fine-scale turbulence noise of a subsonic jet should be those associated with Mach wave radiation. They would, therefore, be distinctly different from noise radiated at angles outside the cone of silence. The experimental measurements of Lush (1971) and Ahuja (1973) prove that this is, indeed, the case. Inside the cone of silence, their measured noise intensity is not only not small but is the highest. Moreover, the spectrum shape is distinctly different from those in directions at larger exhaust angles.

To provide concrete experimental evidence that turbulence mixing noise from subsonic jets, just as their supersonic counterparts, consists of two distinct components, comparisons between the measured data of Ahuja (1973) and the two similarity noise spectra have been carried out. Figure 8 shows the noise spectrum of a Mach 0.98 jet measured by Ahuja at $T_\mathrm{r}/T_\infty = 1.0$ and χ = 160° (inside the cone of silence). The smooth curve in this figure is the similarity spectrum $10 \log F$. It is evident that the curve is an excellent fit to the data providing irrefutable evidence that this is, in fact, the noise from the large turbulence structures of the subsonic jet. Figure 9 shows the corresponding measured noise spectrum at χ = 90°. In this direction, the noise is from the fine-scale turbulence. Here the smooth curve is the similarity noise spectrum of $10 \log G$. The agreement between the data and the similarity noise spectrum is very good, giving strong support to

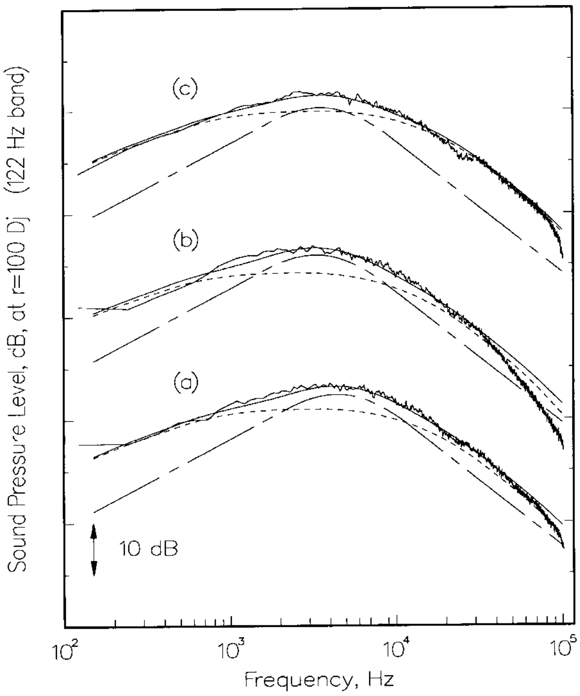

Figure 7. Comparison of the sum of the similarity spectra of both large and fine-scale turbulence noise and measurements at the intermediate direction of radiation. —-—, large turbulence structures noise; – – – –, fine scale turbulence noise; ——, total. (a) M_j = 1.49, T_r/T_∞ = 2.25, χ = 112.6°, SPL_{max} = 101.5 dB, (b) M_j = 1.49, T_r/T_∞ = 1.33, χ = 126.9°, SPL_{max} = 100.5 dB, (c) M_j = 1.96, T_r/T_∞ = 1.79, χ = 120.2°, SPL_{max} = 107.0 dB.

the contention that the second component of turbulent mixing noise from subsonic jets is, as in the case of supersonic jets, fine-scale turbulence noise.

4.3. Noise Intensity Scaling Formulas

In decibel scale, the directional dependence of the large turbulence structures noise amplitude turns out to be quite simple. A typical case is given in Figure 10. Here SPL is effectively $10\log(A/p_{\text{ref}}^2) - 40\,\text{dB}$

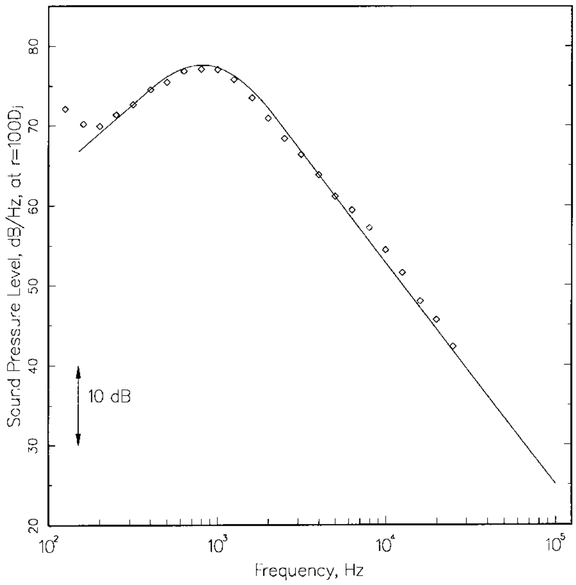

Figure 8. Comparison of the similarity spectrum of large turbulence structures/instability waves noise and subsonic jet noise measurement of Ahuja. M_j = 0.98, χ = 160°, T_r/T_∞ = 1.0.

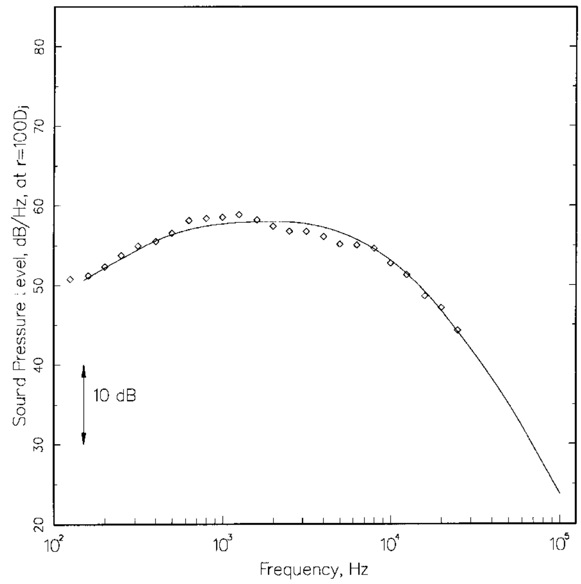

Figure 9. Comparison of the similarity spectrum of fine-scale turbulence noise and subsonic jet noise measurement of Ahuja. M_j = 0.98, χ = 90°, T_r/T_∞ = 1.0.

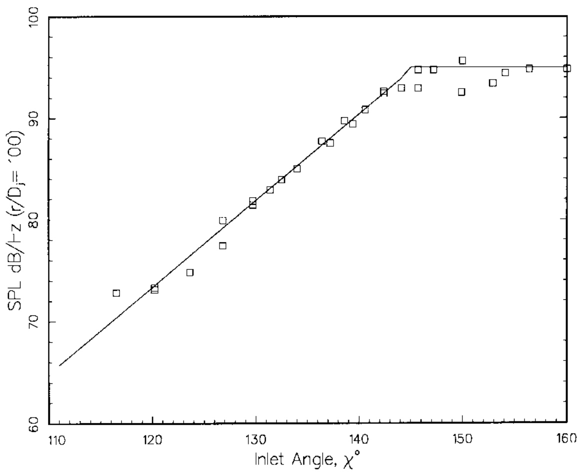

Figure 10. Variation of the peak amplitude of the noise spectrum of large turbulence/instability waves with direction of radiation. M_j = 1.8, T_r/T_∞ = 1.0.

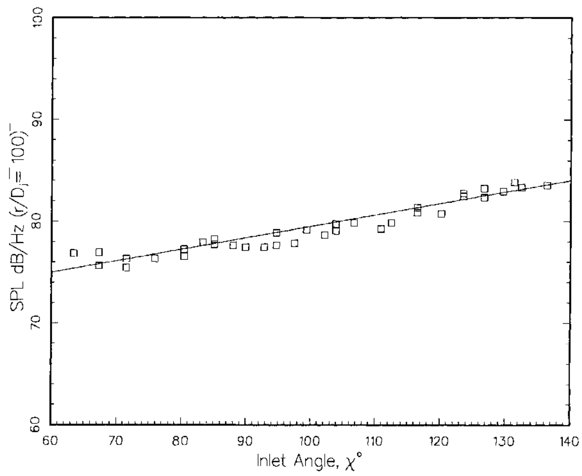

Figure 11. Variation of the peak amplitude of the noise spectrum of fine-scale turbulence with direction of radiation. M_j = 2.24, T_r/T_∞ = 1.0.

(p_ref = 2×10^{-5} N/m^2 is the reference pressure for the decibel scale). This quantity increases linearly with χ until a plateau is reached where the noise amplitude is practically constant. In the plateau region, the noise intensity is maximum. This maximum intensity is a function of the jet operating parameters, V_j/a_∞ and T_r/T_∞. Figure 11 shows a typical dependence of the amplitude of the fine-scale turbulence noise, $10\log(B/p^2_\mathrm{ref}) - 40$ dB, on directivity. Again, there is a linear increase with χ. The slope of the straightline relationship is a function of the jet operating parameters.

To obtain an idea of how the intensity of the large turbulence structures noise varies with the jet velocity and temperature, Tam *et al.* concentrated their attention on χ = 160°. Figure 12 is a plot of $10\log S$ versus

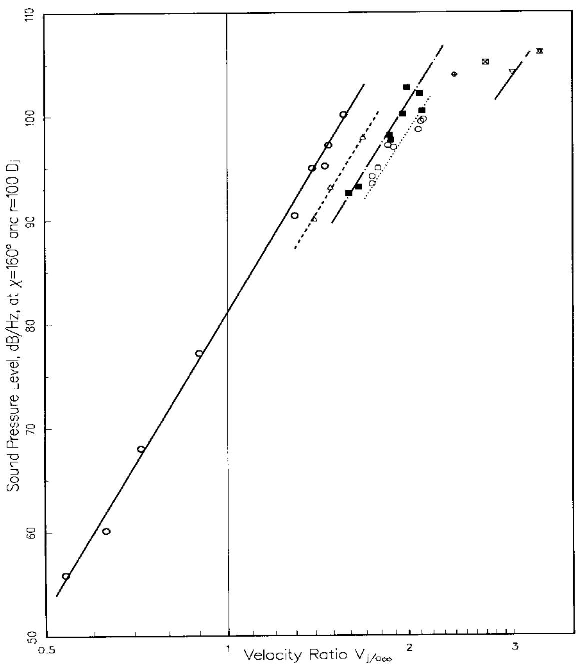

Figure 12. Variation of the peak amplitude of the noise spectrum of large turbulence structures/instability waves at χ = 160° with jet velocity and temperature ratios.

T_r/T_∞	1.0	1.4	1.8	2.2	2.7	3.3	4.1	4.9
Measurements	○	△	■	◯	⊕	⊗	▽	⧖
Equation (3)	——	– – –	—·—	......			—-—	

$\log(V_j/a_\infty)$ with T_r/T_∞ as a parameter at χ = 160° and r/D_j = 100. One obvious feature of this figure is that data corresponding to the same jet to ambient temperature ratio align themselves along nearly parallel straight lines. A good fit to the entire set of data is (in dB per 1 Hz band)

$$10\log\left(\frac{A}{p_{\mathrm{ref}}^2}\right) = 75 + \frac{46}{(T_r/T_\infty)^{0.3}} + 10\log\left(\frac{V_j}{a_\infty}\right)^n, \tag{3}$$

where

$$n = 10.06 - 0.495\,\frac{T_r}{T_\infty}. \tag{4}$$

For cold jets, the velocity exponent n is approximately equal to 9.5. That is significantly larger than 8 or 3, the velocity exponents predicted by the Acoustic Analogy Theory. (Note: Although the velocity exponent is large, the noise power radiated by a jet expanding to its maximum velocity, i.e., into a vacuum, is still only a small fraction of its mechanical power).

To assess the correct scaling formula for fine-scale turbulence noise Tam *et al.* focused on the noise radiated at χ = 90°. In this direction, there is practically no large turbulence structures noise. Figure 13 shows a plot of $10\log S$ versus $\log(V_j/a_\infty)$ with T_r/T_∞ as a parameter at r/D_j = 100. Again the data corresponding to different jet temperature ratios can be adequately approximated by straight lines. A good fit to the data is (in dB per 1 Hz band)

$$10\log\left(\frac{B}{p_{\mathrm{ref}}^2}\right) = 83.2 + \frac{19.3}{(T_r/T_\infty)^{0.62}} + 10\log\left(\frac{V_j}{a_\infty}\right)^n, \tag{5}$$

where

$$n = 6.4 + \frac{1.2}{(T_r/T_\infty)^{1.4}}. \tag{6}$$

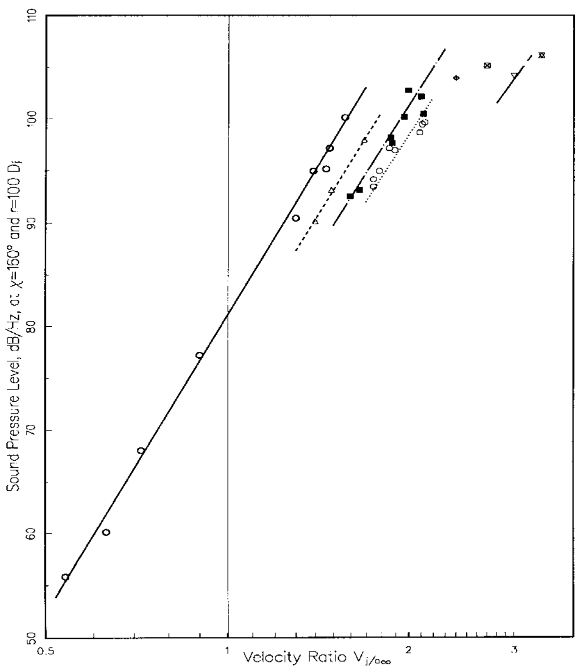

Figure 13. Variation of the peak amplitude of the noise spectrum of fine-scale turbulence at χ = 90° with jet velocity and temperature ratios.

T_r/T_∞	1.0	1.4	1.8	2.2	2.7	3.3	4.1	4.9
Measurements	○	△	■	◯	⊕	⊗	▽	⧖
Equation (5)	——	– – –	—·—	· · · · · ·			—-—	

According to this empirical fit, the velocity exponent is equal to 7.6 for cold jets. This is close to the well-known subsonic jet value of 8. However, the jet temperature exerts a fairly strong effect. At a jet temperature ratio of 2, the value of n drops to 6.85.

It is worthwhile to point out that in Figures 12 and 13, the subsonic jet data of Ahuja (1973) forms a natural extension of the supersonic jet noise data from the NASA Langley Research Center. This reinforce the belief that the noise generation mechanisms are the same regardless of whether the jet is subsonic or supersonic.

5. The Future

It is clear that considerable progress has been made in our understanding of jet mixing noise since the work of Lighthill. As yet we are still unable to predict, even with a good deal of empirical input, the noise spectra of the two basic components. Given our limited understanding of jet turbulence at the present time, the prospect of successfully formulating a first principle noise prediction theory is not very encouraging. However, it seem possible, perhaps, in a few years time, that a semi-empirical theory based on the turbulence modeling approach could be developed. At the present time, the noise generation processes of the large turbulence structures of the jet flow appears to be reasonably well understood. It is hoped that future work will clarify how fine-scale turbulence produces noise.

Recently, computational aeroacoustics has made impressive advances. This new methodology should be able to assist in the simulation of large turbulence structures in jet flows. It would not be surprising that, before too long, jet noise from the large turbulence structures can be accurately predicted by direct numerical simulation. However, the same may not be true for fine-scale turbulence noise. To be able to resolve this type of turbulent motion accurately, an exceedingly large computer memory is required. It seems that the requirement far exceeds what could become available in the near future. Thus, it is likely that jet noise, just as turbulence, well remain an unfinished business for quite some time to come.

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