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Original article

The onset of Taylor-like vortices in the flow induced by an impulsively started rotating cylinder*

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Abstract. The onset of instability in the flow by an impulsively started rotating cylinder is analyzed under linear theory. It is well-known that at the critical Taylor number $T_c = 1695$ the secondary flow in form of Taylor vortices sets in under the narrow-gap approximation. Here the dimensionless critical time τ_c to mark the onset of instability for $T \gg T_c$ is presented as a function of the Taylor number T. Available experimental data of water indicate that deviation of the velocity profiles from the primary flow occurs starting from a certain time $\tau \cong 4\tau_c$. It seems evident that during $\tau_c \leq \tau \leq 4\tau_c$ the secondary flow is very weak and the primary state of time-dependent annular Couette flow is maintained.

Key words: Taylor-like vortices, stability analysis, time-dependent Couette flow, propagation theory

1 Introduction

In a rotating Couette flow apparatus a fluid is initially at rest. At a time $t = 0$, the angular velocity of the inner cylinder is suddenly increased to a high velocity Ω_i. The ensuing unsteady Couette flow evolves into the secondary flow of Taylor-like vortices at a certain time. Chen and Christensen [2] reported their experimental results that the average spacing between the vortices and the detection time for the first appearance of vortices decreases as Ω_i increases. In the present study we present the results of an instability analysis of this flow phenomenon.

Chen and Kirchner [3] examined the above instability problem by employing the initially distributed perturbations of a given wavelength and monitoring their growth under linear theory. The detection time t_D of the secondary flow is usually defined as the time at which the ratio of the transient kinetic energy of disturbances to the assumed initial one reaches 10^3. In addition to this so-called amplification theory they employed the quasi-steady approach, in which the stability of the instantaneous velocity profile of the primary flow is analyzed. This is called the frozen-time model. In this model, the growth rate of disturbances is usually assumed to be zero and the stability criteria are obtained under the principle of the exchange of stabilities. For a given Ω_i the smallest time t_c to satisfy the perturbation equations is obtained. Recently Tan

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and Thorpe [17] defined a transient Taylor number and the critical conditions were determined by assuming the maximum value of this transient-Taylor number as the well-known steady-state critical Rayleigh number by considering the similarity between the time-dependent Rayleigh-Bénard instability problem (for brevity, the R-B problem) and the time-dependent Taylor one. Since it is well-known that the governing equations and the boundary conditions for these two problems are very similar (Chandrasekhar [1], Zierep [20]), the above-mentioned methods have originally been developed for the R-B problem.

Another model which has analyzed the time-dependent convective instability problem is the propagation theory (Choi [4], Yang and Choi [18], Kim et al. [12]). This theory has dealt with R-B problems in initially motionless fluid layers heated rapidly from below. The resulting stability criteria have represented well experimental data of various systems. This model is extended to the flow system of fluid in an impulsively started rotating cylinder. According to this model it is here assumed that at $t = t_c$ infinitesimal disturbances of the angular velocity are propagated mainly within the hydrodynamical boundary-layer thickness Δ of the primary Couette flow. The length scales in disturbance variables and the stability parameters are rescaled with the characteristic length Δ. In usual deep-pool systems of $\Delta \propto \sqrt{\nu t}$, the most important parameter becomes the time-dependent Taylor number, which is yielded by replacing the length scale in the usual Taylor number with Δ. Here ν is the kinetic viscosity. The resulting theoretical results will be compared with available experimental data.

2 Stability analysis

2.1 Governing equations

The system considered here is a Newtonian fluid confined between two concentric cylinders of radii R_i and $R_o(> R_i)$. Let the axis of the inner cylinder be along the z' axis of the cylindrical coordinates (r', θ, z'). At time $t = 0$, the inner cylinder is impulsively started and maintained at a constant surface speed $V_0'(= R_i\Omega_i)$ and the outer cylinder is kept stationary ($\Omega_o = 0$). Here Ω_i and Ω_o are the angular velocities of the inner and the outer cylinder, respectively. The schematic diagram of the primary system is shown in Fig. 1. The governing equations of the present flow field is expressed by

$$\nabla \cdot \mathbf{U} = 0, \tag{1}$$

$$\left\{\frac{\partial}{\partial t} + \mathbf{U}\cdot\nabla\right\}\mathbf{U} = -\frac{1}{\rho}\nabla P + \nu\nabla^2\mathbf{U} \tag{2}$$

where $\mathbf{U}$, P and ρ represent the velocity vector, the dynamic pressure and the density, respectively.

The important parameters for describing the present system are the Taylor number T, the Reynolds number Re, the ratio μ of angular speeds and the radius ratio η defined as

$$T = \frac{V_0'^2 d^3}{\nu^2 R_i}, \quad Re = \frac{R_i^2\Omega_i}{\nu}, \quad \mu = \frac{\Omega_o}{\Omega_i}, \quad \eta = \frac{R_i}{R_o} \tag{3}$$

In case of a very slow rotating speed the primary velocity profile finally becomes time-independent and Taylor vortices appear at $T = T_c$ (Chandrasekhar [1]):

$$T_c = 1695 \quad \text{for} \quad \eta \to 1 \ \text{ and } \ \mu \to 0 \tag{4}$$

which means a narrow-gap system.

But for systems of large T, the secondary flow sets in before the primary-flow field becomes fully-developed and time-independent. In this transient system it is important to predict the critical time to mark the onset of Taylor-like vortices. The primary velocity field of developing Couette flow is represented for the case of constant physical properties in dimensionless form:

$$\frac{\partial v_0}{\partial \tau} = \frac{\partial}{\partial r}\left(\frac{\partial}{\partial r} + \frac{1}{r}\right) v_0 \tag{5}$$

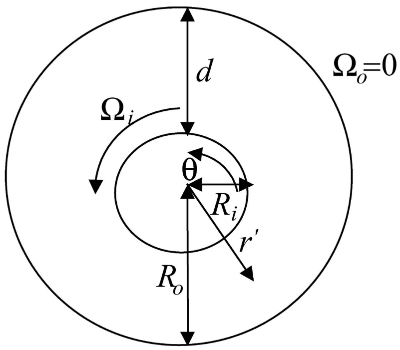

Fig. 1. Schematic diagram of system considered here

with the following initial and boundary conditions,

$$v_0=0 \quad \text{at} \quad \tau=0 \tag{6}$$
$$v_0=0 \quad \text{at} \quad r=1/(1-\eta)$$
$$v_0=1 \quad \text{at} \quad r=\eta/(1-\eta)$$

where $\tau=\nu t/d^2$, $v_0=V_0/V_0'$, $r=r'/d$ and $d=(R_o'-R_i')$.

Tranter (1956) obtained the exact solutions of Eqs. (5) and (6) by using the Hankel transform as

$$v_0(x,\tau_1)=\frac{\eta}{\xi}\left(\frac{\xi^2-1}{\eta^2-1}\right)+\sum_{i=0}^{\infty}Q\left(\lambda_i,\eta\right)\left[J_1\left(\lambda_i\frac{\xi}{\eta}\right)Y_1\left(\lambda_i\right)-J_1\left(\lambda_i\right)Y_1\left(\lambda_i\frac{\xi}{\eta}\right)\right]\exp\left(-\lambda_i^2\tau_1\right) \tag{7}$$

where $x=\left\{(r'-R_i)+(r'-R_0)\right\}/2d$ and $\tau_1=\nu t/R_i^2$. Here J_1 and Y_1 are the 1st kind Bessel functions and the functions ξ and Q are

$$\xi=(1+\eta)/2+(1-\eta)x \tag{8}$$

$$Q\left(\lambda_i,\eta\right)=\pi\left\{\left[J_1(\lambda_i)/J_1(\lambda_i/\eta)\right]^2-1\right\}^{-1} \tag{9}$$

The λ_i's are the roots of the equation

$$J_1\left(\frac{\lambda}{\eta}\right)Y_1\left(\lambda\right)-J_1\left(\lambda\right)Y_1\left(\frac{\lambda}{\eta}\right)=0 \tag{10}$$

The present study concerns the case of $\eta\to 1$, i.e. narrow-gap systems and $\tau\to 0$, i.e. large T. For the former limiting case, the gap size $d(=(R_o-R_i))$ is small compared to the mean radius $(R_o+R_i)/2$ and there is no difference between $\partial/\partial r+1/r$ and $\partial/\partial r$ (Chandrasekhar [1]), i.e. no effect of curvature. Since the above solution does not work so well for the case of $\eta\to 1$, the following primary-flow solution is useful (Schlichting and Gersten [15]):

$$v_0=\sum_{i=0}^{\infty}\left\{\operatorname{erfc}\left(\frac{i}{\sqrt{\tau}}+\frac{\zeta}{2}\right)-\operatorname{erfc}\left(\frac{i+1}{\sqrt{\tau}}-\frac{\zeta}{2}\right)\right\} \qquad \text{as } \eta\to 1 \tag{11}$$

where $\tau=\nu t/d^2$, $\zeta=y/\sqrt{\tau}$ and $y=(r'-R_i)/d$ with $d=R_o-R_i$. Here erfc denotes the complementary error function. For deep-pool systems of small τ, where the boundary-layer thickness is much smaller than the gap size, the above solution reduces to

$$v_0=\operatorname{erfc}\left(\frac{\zeta}{2}\right) \quad \text{with} \quad \delta=3.65\sqrt{\tau} \qquad \text{as } \tau\to 0 \tag{12}$$

where $\delta(=\Delta/d)$ denotes the usual dimensionless boundary-layer thickness with $v_0(\delta)=0.01$. In the present study Eq. (11) is used since it involves Eq. (12).

2.2 Stability equation

The typical disturbances of the secondary flow which are observed experimentally are the axisymmetric ones having the following forms (Chandrasekhar [1]):

$$(U_1, V_1, P_1) = (u', v', p')\cos kz' \qquad (13)$$
$$W_1 = w' \sin kz'$$

where k is the wavenumber and the primed quantities representing amplitude functions are a function of r' and t. Under linear theory the stability equations are obtained when w' and p' are eliminated. Under the narrow-gap approximation, where $\partial/\partial r + 1/r \cong \partial/\partial r$ (Chandrasekhar, 1961), the resulting dimensionless disturbance equations are represented by

$$\left(\frac{\partial^2}{\partial r^2} - a^2 - \frac{\partial}{\partial \tau}\right)\left(\frac{\partial^2}{\partial r^2} - a^2\right) u = v_0 a^2 v \qquad (14)$$

$$\left(\frac{\partial^2}{\partial r^2} - a^2 - \frac{\partial}{\partial \tau}\right) v = 2T\frac{\partial v_0}{\partial r} u \qquad (15)$$

with the proper boundary conditions,

$$u = \partial u/\partial r = v = 0 \qquad \text{at } y = 0 \text{ and } 1 \qquad (16)$$

where $u = d^2u'/(\nu R_\mathrm{i})$, $v = 2v'/V_0'$ and $a = kd$. The subscript '0' denotes the primary state and a represents the dimensionless vertical wavenumber.

2.3 Propagation theory

The propagation theory employed to find the onset time of the secondary flow, i.e., the critical time t_c, is based on the assumption that in deep-pool systems of small time the perturbed angular velocity component V_1 is propagated mainly within the hydrodynamic boundary-layer thickness $\Delta(\propto \sqrt{\nu t})$ of the primary flow near the onset time t_c of the secondary flow and the following scale relations are valid for perturbed quantities from the linearized equations of equation (2):

$$\nu\frac{u'}{\Delta^2} \sim \frac{V_0'}{R_\mathrm{i}} v' \qquad (17)$$
$$u'\frac{\partial V_0'}{\partial r} \sim \nu\frac{v'}{\Delta^2} \qquad (18)$$

from the balance between viscous and inertia terms in equation (2). Now, based on the relation (17), the following amplitude relation is obtained in dimensionless form:

$$\left|\frac{u}{v}\right| \sim \delta^2 \qquad (19)$$

where $\delta(\propto \tau^{1/2})$ is the usual dimensionless boundary-layer thickness. The relation (18) yields

$$\frac{\partial V_0}{\partial r} \sim \frac{\nu R_\mathrm{i}}{\Delta^4 V_0'} = \frac{V_0'}{\Delta}\left(\frac{\Delta^3 V_0'^2}{\nu R_\mathrm{i}}\right)^{-1} = \frac{V_0'}{\Delta} T_\Delta^{-1} \qquad (20)$$

where T_Δ is the Taylor number based on the boundary-layer thickness Δ. With increasing T, both the onset time t_c and its corresponding thickness Δ become smaller and the characteristic value of $T(\Delta/d)^3$, i.e. $T\tau^{3/2}$, will become a constant. For small time, the modified Taylor number $T^*(= T\tau^{3/2})$ has been used in stability analyses (Otto [14]). There are many possible forms of dimensionless amplitude functions of disturbances like

$$[u(\tau, y), v(\tau, y)] = \left[\tau^{n+1} u^*(\tau, y), \tau^n v^*(\tau, y)\right] \tag{21}$$

which satisfy the relation (19).

At this stage the criterion to determine n is necessary. Shen [16] suggested the momentary instability condition: the temporal growth rate of the kinetic energy of the perturbation velocity should exceed that of the primary velocity at the onset time of secondary flow. In the present system the dimensionless kinetic energy is defined as

$$E(t) = \frac{1}{2} \parallel \mathbf{u} \parallel^2 \tag{22}$$

where $\parallel \cdot \parallel$ denotes the norm. Here his concept is extended, based on the work of Choi et al. (2003) who treated the R-B problem. Since there is no primary flow in the θ- and the z'-direction and the condition of $|u/v| \to 0$ is valid for $\tau \to 0$ (see the relation (19)), the dimensionless kinetic energy can be divided into the basic one and its perturbed one:

$$E_0(t) = \frac{1}{2} \parallel v_0 \parallel^2, \quad E_1(t) = \frac{1}{2} \parallel v \parallel^2 \tag{23}$$

Then the temporal growth rate of the basic kinetic energy (r_0) and the perturbed one (r_1) are obtained as the root-mean-square quantities in the present two dimensional fields:

$$r_0(\tau) = \frac{1}{\langle v_0 \rangle} \frac{\mathrm{d}\langle v_0 \rangle}{\mathrm{d}\tau}, \quad r_1(\tau) = \frac{1}{\langle v \rangle} \frac{\mathrm{d}\langle v \rangle}{\mathrm{d}\tau} \tag{24}$$

where $\langle \cdot \rangle = \sqrt{\left(\int_A (\cdot)^2 dA\right)/A}$ and $A = Sdr'$ with $S = \pi d/a$. For the case of $n = 0$ the condition of $r_0 = r_1$ is satisfied at $\tau = \tau_c$. This condition has been suggested as the critical condition in R-B problems (Choi et al. [5], Chung [6]). In the present flow system it is assumed that at the marginal state $T^* =$ constant and $v = v^*(\zeta)$. This means that the amplitude function of the perturbed velocity in the radial direction follows the behavior of the primary flow for small τ (see Eq. (12)). Furthermore the relation of $T^* =$ constant is shown even in theoretical results from the amplification theory (Chen and Kirchner [3]) and the maximum-transient-Taylor-number criterion (Tan and Thorpe [17]). By the above reasoning we set $u = \tau u^*(\zeta)$ and $v = v^*(\zeta)$.

For deep-pool systems of $\delta \propto \sqrt{\tau}$, the dimensionless time τ is related with the time for development of the boundary-layer thickness, which plays dual roles of time and length. By using relations (19) and (20) the following self-similar stability equations are obtained with $\partial/\partial\tau = (-\zeta/2\tau)D$ and $\partial^2/\partial r^2 = (1/\tau)D^2$ from Eqs. (14) and (15),

$$\left\{\left(D^2 - a^{*2}\right)^2 + \frac{1}{2}\left(\zeta D^3 - a^{*2}\zeta D + 2a^{*2}\right)\right\} u^* = v_0 a^{*2} v^* \tag{25}$$

$$\left(D^2 + \frac{1}{2}\zeta D - a^{*2}\right) v^* = 2T^* u^* D v_0 \tag{26}$$

where $D = \mathrm{d}/\mathrm{d}\zeta$ and $a^* = a\sqrt{\tau}$. The proper boundary conditions of no slip are

$$u^* = Du^* = v^* = 0 \qquad \text{at } \zeta = 0 \text{ and } \infty \tag{27}$$

For a given τ, T^* and a^* are treated as eigenvalues and the minimum value of T^* is found in the plot of T^* vs. a^* under the principle of exchange of stabilities. This produces the earliest time τ_c and its corresponding wavenumber a_c.

The conventional frozen-time model neglects the terms involving $\partial/\partial\tau$ in Eqs. (14) and (15) in amplitude coordinates τ and y. This results in $\left(D^2 - a^{*2}\right)^2 u^* = v_0 a^{*2} v^*$ and $\left(D^2 - a^{*2}\right) v^* = 2T^* u^* D v_0$ instead of Eqs. (25) and (26). The minimum T-value is obtained for a given τ_c.

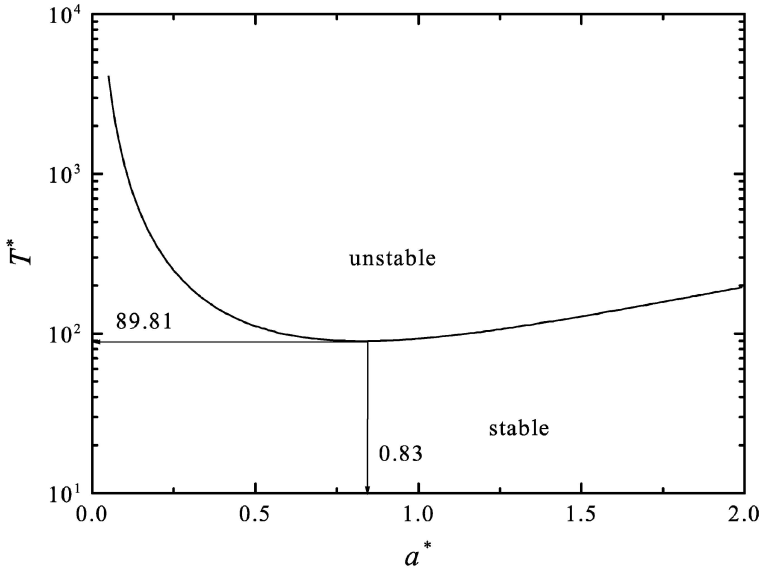

Fig. 2. Marginal stability curve under the principle of exchange of stabilities for small time of $\tau_c \to 0$ from the propagation theory

3 Solution method

In the present study the stability Eqs. (25)~(27) are solved by employing the outward shooting scheme. In order to integrate the stability equations the proper values of D^2v^*, D^3v^* and Du^* at $\zeta = 0$ are assumed for a given a^*. Since the stability equations and the boundary conditions are all homogeneous, the value of $D^2v^*(0)$ can be assigned arbitrarily and the T^*-value is assumed. This procedure can be understood easily by taking into account of characteristics of the eigenvalue problem. After all the values at $\zeta = 0$ are provided, this eigenvalue problem can be proceeded numerically.

Integration is performed from the inner cylinder $\zeta = 0$ to a fictitious outer boundary with the fourth order Runge–Kutta-Gill method. If guessed values of T^*, $D^3v^*(0)$ and $u^*(0)$ are correct, v^*, Dv^* and u^* will vanish at the outer boundary. To improve the initial guesses the Newton-Raphson iteration is used. When convergence is achieved, the outer boundary is increased by a predetermined value and the above procedure is repeated. Since the disturbances decay exponentially outside the boundary-layer thickness, the incremental change of T^* also decays fast with an increase in outer boundary depth. This behavior enables us to extrapolate the eigenvalue T^* to the infinite depth by using the Shank transform. The results of this procedure are presented in Fig. 2, as a plot of T^* vs. a^*. The minimum value of T^*, i.e., $T_c^* = 89.81$ at $a_c^* = 0.83$, will mark the onset of vortices. In the case of the frozen-time model a similar technique is employed and the characteristic values are obtained.

4 Results and discussion

For a single-mode instability the stability criteria to mark the onset of secondary flow in form of Taylor-like vortices, based on Eq. (12), are obtained from Fig. 2:

$$\tau_c = 20.05T^{-2/3}, \quad a_c = 0.19T^{1/3} \quad \text{as } \tau_c \to 0 \tag{28}$$

The resulting normalized amplitude functions of u^* and v^* are shown in Fig. 3. It is shown that v' is propagated mainly within the boundary-layer thickness of the primary Couette flow (see Eq. (12)). For a given T, a fastest growing mode of infinitesimal disturbances in form of Taylor-like vortices would set in with $a = a_c$ at $\tau = \tau_c$. The above equations show that τ_c decreases with an increase in T. From distributions of

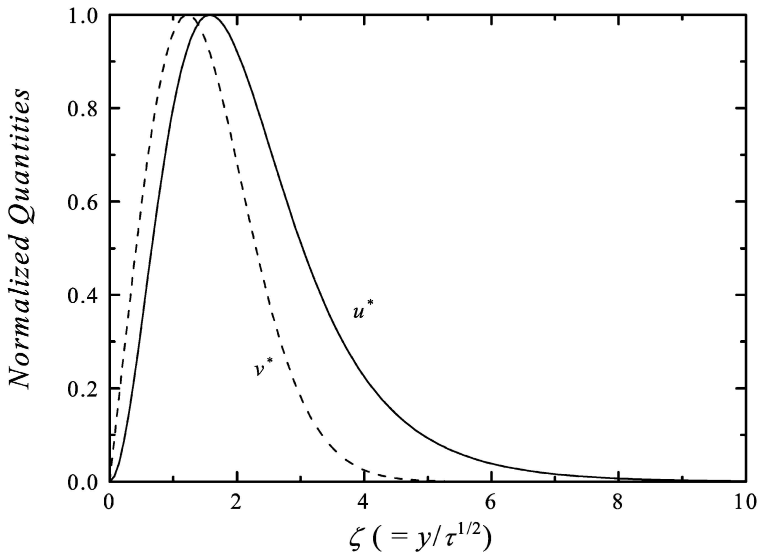

Fig. 3. Amplitude profiles at $\tau = \tau_c$ for small time of $\tau_c \to 0$ from the propagation theory

the primary flow (Eq. (12)) and the perturbation quantities, we can obtain the following relation:

$$r_0 = r_1 = \frac{1}{4\tau_c} \quad \text{as} \quad \tau \to 0 \tag{29}$$

which satisfies the condition of marginal state of instabilities we have suggested.

Now, the domain of time is extended to finite τ with $\eta \to 1$ by keeping Eqs. (25) and (26) and using Eq. (11). In Eq. (27) the infinite upper boundary is replaced with the finite one $y = 1$, i.e. $\zeta = 1/\sqrt{\tau_c}$ and in Eqs. (25) and (26) T^* and a^* are replaced with $\tau_c^{3/2}T$ and $a\tau_c^{1/2}$. Also, in Eq. (11) τ is fixed as τ_c but ζ is maintained. Since τ is the fixed parameter, the resulting stability equations are a function of ζ only and the spirit of relations (19) and (20) is still alive. For a given τ_c, the minimum T-value and its corresponding wavenumber a_c are obtained. The results are summarized in Fig. 4, wherein those obtained from the conventional frozen-time model are also shown. For $\tau_c < 0.01$ with $\eta \to 1$ the present predictions are the same as those in deep-pool systems (Eq. (28)). For large τ_c they approach the well-known critical value of $T_c = 1695$ in the steady state (see Eq. (4)). It is known that for small τ the frozen-time model yields the lower bound of τ_c and the terms involving $\partial/\partial\tau$ in Eqs. (25) and (26) stabilize the system. It is interesting that the propagation theory yields smoothly the stability criteria over the whole domain of time like those in the R-B problem (Yang and Choi [18]).

Foster [8] commented that with correct dimensional relations the relation of $\tau_m \approx 4\tau_c$ would be kept for a small time in the R-B problem. This means that a fastest growing mode of instabilities, which sets in at $\tau = \tau_c$, will grow with time until manifest convection is detected near the whole bottom boundary at $\tau = \tau_m$. For the present system, based on the amplification theory pioneered by Foster [7], Chen and Kirchner [3] reported a similar trend for the present system. Their results show that the characteristic time τ_i at which disturbances first tend to grow, is about one-fourth of the time at which the secondary flow is clearly observable experimentally. Figure 5a illustrates that the present predictions of $4\tau_c$ with $\eta \to 1$ or $\tau \to 0$ compare well with Liu's [13] experimental data for $T > 5 \times 10^5$ and Kasagi and Hirata's [11] ones for the whole-T domain. The former experiment was conducted with $\eta = 0.2$ and the latter one with $\eta = 0.75$. Recently Tan and Thorpe [17] suggested a simple instability analysis. They assumed that at the detection time of manifest convection, τ_m, the following relation is maintained, based on Eq. (12):

$$\text{Maximum of} \left\{ \frac{d^3 V_0'^2}{\nu^2 R_i} y^5 \left(\frac{\partial v_0}{\partial y} \right)^2 \right\} = 1100 \tag{30}$$

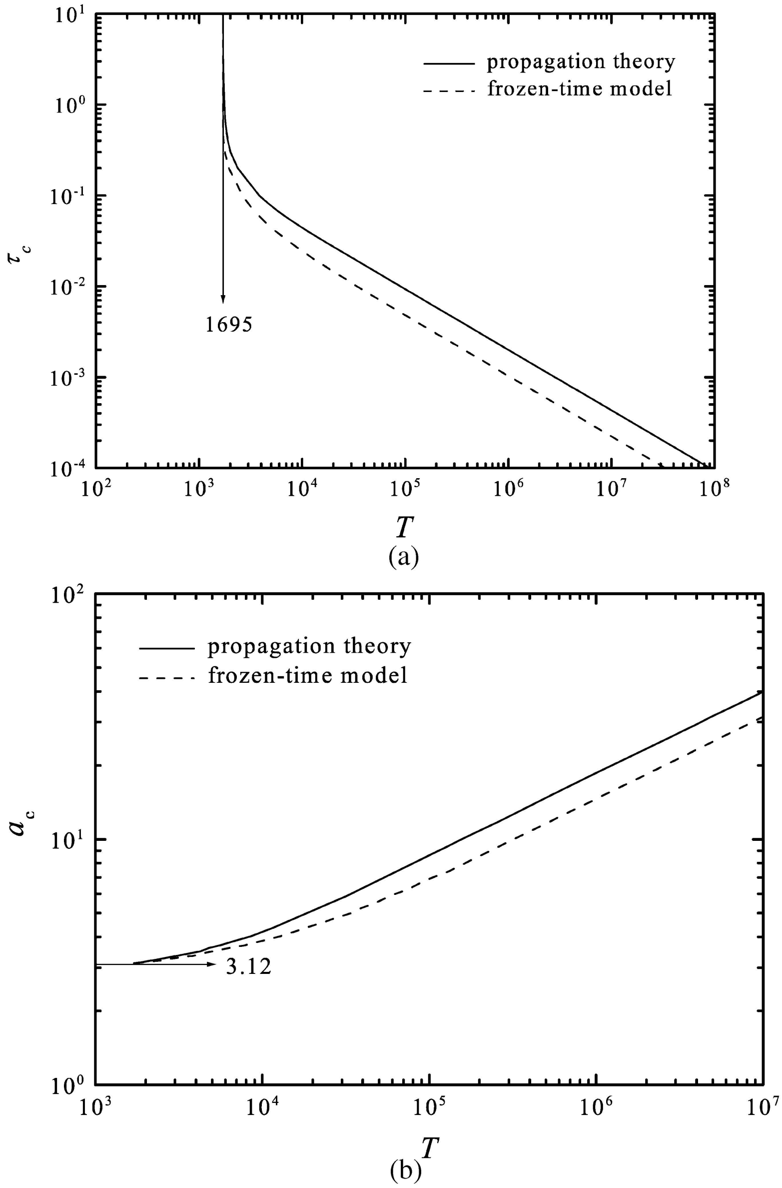

Fig. 4a,b. Stability conditions for $\eta \to 1$: (**a**) critical time and (**b**) critical wavenumber

which is satisfied by $y_{max} = \sqrt{5\tau}$. The value 1100 is the critical Rayleigh number in the isothermally heated R-B problem with a free upper boundary. This results in $\tau_m = 82.9T^{-2/3}$, which corresponds to the system of $\eta \to 1$ and large T. Their τ_m-predictions agree very well with the present $4\tau_c$-values for large T but they deviates with decreasing T. It is interesting that a common relation is involved in the above results: $T^* =$ constant. In Fig. 5b the critical wavenumbers predicted from the above models are compared with the available experimental data. The present predictions agree well with experimental data in the whole T-range. It seems that a_c is a weak function of η.

It is evident that during $\tau_c \le \tau \le \tau_m$ flow patterns would not change significantly with time. For small η, e.g., $\eta = 0.2$ in Fig. 5a the curvature effect becomes important in predicting the stability criteria. Therefore, the present prediction is invalid for $T < 5 \times 10^5$ with $\eta = 0.2$ (see (1), (2) and (6a) in Fig. 5a). But incorporating the results from the frozen-time model with those from the propagation theory brings reasonable stability criteria for the whole T-domain even in the case of $\eta = 0.75$ like those for $\eta = 0.2$. As shown in Figs. 4 and 5, the frozen-time model yields the minimum bound of the stability criteria. Once in-

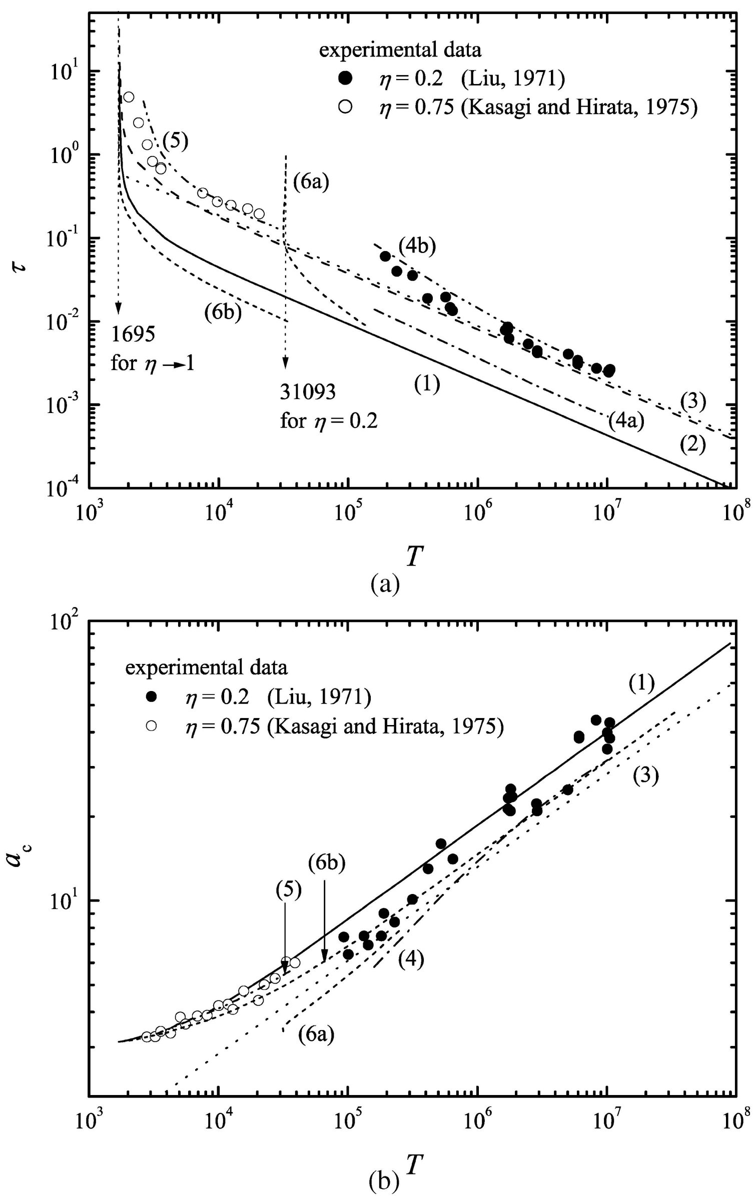

Fig. 5a,b. Comparison of predictions with experimental data of $\eta = 0.2$ and 0.75; (**a**) characteristic times and (**b**) critical wavenumbers. With $\eta \to 1$: (1), present τ_c; (2), present $4\tau_c$; (3), Tan and Thorpe's τ_m. Chen and Kirchner's (1971) predictions: (4a), τ_i; (4b), τ_m from the amplification theory with $\eta = 0.2$; (5), prediction from the amplification theory with $\eta = 0.75$. From the frozen-time model, τ_c: (6a), $\eta = 0.2$; (6b), $\eta = 0.75$

finitesimal secondary flow sets in at $\tau = \tau_c$, it should grow until detected at $\tau \cong 4\tau_c$. This behavior has been shown in the time-dependent Bénard-type convection problems (Yoon and Choi [19], Kang and Choi [9], Kang et al. [10])

Considering the above theoretical results, it is known that the present predictions provide the reasonable stability criteria for deep-pool systems of $\tau_c \to 0$ and also for systems of $\eta \to 1$. It seems that the propagation theory is a powerful method to predict the stability criteria reasonably well in the simple systems, hydrodynamic or thermal, of which the primary states involve a similarity variable like ζ in time-dependent diffusion processes. Their τ_m-values compare well with the present $4\tau_c$-values. This means that incipient instabilities should grow until the secondary flow is detected at $\tau = \tau_m$.

5 Conclusions

The onset of the secondary flow in the flow by an impulsively started rotating cylinder has been analyzed by using linear stability theory. The propagation theory has been employed to predict the critical time τ_c to mark the onset of convective instability for $\tau \to 0$ or $\eta \to 1$. Even though the propagation theory is a rather simple model, the relation of $\tau_m \simeq 4\tau_c$ is consistent with experimental measurements for large T. The present results show that the infinitesimal disturbance sets in at $\tau = \tau_c$ and grows until detected around $\tau \cong 4\tau_c$. This means that secondary flow is very weak during $\tau_c \leq \tau \leq \tau_m$. More refined studies on the η-effect and the nonlinear growth of disturbances are now under progress.

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