

Original Article

A combined Chapman-Enskog and Grad method. III. Polyatomic gases in magnetic fields

A.S. Fernandes, S. Reinecke and G.M. Kremer

Departamento de Física, Universidade Federal do Paraná, Caixa Postal, 19081, 81.531-990, Curitiba, Brasil

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A kinetic theory is developed for rarefied polyatomic gases of spherical and rough molecules with rotational energy in the presence of an external constant magnetic field. A method of solution of Boltzmann equation that combines features of the methods of Chapman-Enskog and Grad is used to determine transport coefficients that depend on the external magnetic field (Senftleben-Beenakker effect).

1 Introduction

In this work we develop a kinetic theory of rarefied polyatomic gases consisting of spherical molecules with internal rotational energy in an external constant magnetic field. We assume that the energy of the internal rotational variable can be treated classically [1] but a similar procedure could also be applied if the energy of the internal variable were treated quantum-mechanically [2].

The standard methods used in kinetic theory of transport processes are the Chapman-Enskog [3] and the Grad method [4]. In the Chapman-Enskog method the deviation from equilibrium is found as a solution of an integral equation that follows from the Boltzmann equation. In the Grad's method the deviation from equilibrium of the distribution function is written in terms of moments of the distribution function. The transport coefficients of the latter are then obtained from the field equations of the moments through an iteration scheme.

An alternative method was proposed by Bezerra, Reinecke and Kremer [5] that combines the features of the Chapman-Enskog and Grad methods. It requires neither a solution of the integral equation nor the use of the field equations for the moments. As in Grad's method the deviation of equilibrium of the distribution function is written in terms of the moments of the distribution function whereas the constitutive equations follow directly from the Boltzmann equation. The combined Chapman-Enskog and Grad method was applied to a monatomic gas and mixtures [5] and to ionized gases [6].

The aim of this work is to apply the combined Chapman-Enskog and Grad method to polyatomic gases and to determine the changes of thermal conductivity and shear viscosity coefficients in a presence of a constant magnetic field (Senftleben-Beenakker effect) [7]. The model of the rough spheres of Bryan [3] is used for molecular collisions; the principal feature of this model is the reversing of the relative velocity of the points which come into contact in a binary collision. The theory is based on thirty-seven scalar fields that are the moments of the distribution function. The influence of dominant polarizations $\omega_{\langle i \omega_j \rangle} C_k$ and $\omega_{\langle i \omega_j \rangle}$ [7] are considered and the thermal conductivity tensor and the shear viscosity tensor are obtained as functions of the external magnetic field. The inversion of a fourth-order tensor is used to determine the shear viscosity tensor. The results obtained agree with those that follow from the Chapman-Enskog method [8, 9].

2 Dynamics of a binary collision and Boltzmann equation

We consider a rarefied polyatomic gas of perfectly rough, perfectly elastic and spherical molecules with mass m , diameter a and moment of inertia I . We denote by $(\mathbf{c}, \mathbf{c}_1)$ and $(\boldsymbol{\omega}, \boldsymbol{\omega}_1)$ the linear and the angular velocities of two molecules before collision while $(\mathbf{c}', \mathbf{c}'_1)$ and $(\boldsymbol{\omega}', \boldsymbol{\omega}'_1)$ are these velocities after collision. \mathbf{k} is the unit vector in the direction of the line that joins the two molecular centers at collision, pointing from the center of the molecule labelled 1 to the other. If we apply the conservations laws of linear and angular momentum to the molecules we obtain the impact equations

$$\begin{aligned} m\mathbf{c}' &= m\mathbf{c} - \mathbf{J}, & m\mathbf{c}'_1 &= m\mathbf{c}_1 + \mathbf{J}, \\ I\boldsymbol{\omega}' &= I\boldsymbol{\omega} + \frac{a}{2}\mathbf{k} \times \mathbf{J}, & I\boldsymbol{\omega}'_1 &= I\boldsymbol{\omega}_1 + \frac{a}{2}\mathbf{k} \times \mathbf{J}, \end{aligned} \quad (2.1)$$

where \mathbf{J} is the impulse exerted on the molecule 1 by the molecule without label. In a binary collision of rough spheres the relative velocity of the points of contact is reversed after collision and the impulse is given by

$$\mathbf{J} = -\frac{m\kappa}{\kappa+1} \left[\mathbf{g} - \frac{a}{2}\mathbf{k} \times (\boldsymbol{\omega} + \boldsymbol{\omega}_1) + \frac{1}{\kappa}(\mathbf{k} \cdot \mathbf{g})\mathbf{k} \right], \quad (2.2)$$

where $\mathbf{g} = \mathbf{c}_1 - \mathbf{c}$ is the relative linear velocity between the centers of the molecules and $\kappa = \frac{4I}{ma^2}$ is the dimensionless moment of inertia which may range from 0 to $2/3$ corresponding respectively to a concentration of mass at the center of the molecule and to a uniform distribution of the mass at the surface of the spherical molecule.

Unlike the case of smooth spheres, no inverse collision exists for rough spheres, i.e., collision in which molecules with initial velocities $(\mathbf{c}', \boldsymbol{\omega}')$ and $(\mathbf{c}'_1, \boldsymbol{\omega}'_1)$ result molecules with final velocities $(\mathbf{c}, \boldsymbol{\omega})$ and $(\mathbf{c}_1, \boldsymbol{\omega}_1)$. Hence we denote by $(\mathbf{c}^*, \boldsymbol{\omega}^*)$ and $(\mathbf{c}_1^*, \boldsymbol{\omega}_1^*)$ the initial velocities of molecules that correspond to the final velocities $(\mathbf{c}, \boldsymbol{\omega})$ and $(\mathbf{c}_1, \boldsymbol{\omega}_1)$.

The state of a polyatomic rarefied gas of spherically symmetrical and rough spheres is characterized by the one-particle distribution function $f(\mathbf{x}, \mathbf{c}, \boldsymbol{\omega}, t)$ such that $f(\mathbf{x}, \mathbf{c}, \boldsymbol{\omega}, t) d\mathbf{x} d\mathbf{c} d\boldsymbol{\omega}$ represents the number of molecules in the volume element between \mathbf{x} and $\mathbf{x} + d\mathbf{x}$, with linear velocities between \mathbf{c} and $\mathbf{c} + d\mathbf{c}$ and angular velocities between $\boldsymbol{\omega}$ and $\boldsymbol{\omega} + d\boldsymbol{\omega}$, at time t . The distribution function obeys the Boltzmann equation

$$\begin{aligned} \frac{\partial f}{\partial t} + c_i \frac{\partial f}{\partial x_i} + \frac{\tau_i}{I} \frac{\partial f}{\partial \omega_i} &= \frac{\partial f}{\partial t} + c_i \frac{\partial f}{\partial x_i} + \gamma (\boldsymbol{\omega} \times \mathbf{H})_i \frac{\partial f}{\partial \omega_i} = \\ &= \int (f_1^* f^* - f_1 f) a^2 (\mathbf{g} \cdot \mathbf{k}) d\mathbf{k} d\mathbf{c}_1 d\boldsymbol{\omega}_1. \end{aligned} \quad (2.3)$$

In Eq.(2.3) we have neglected external body forces but considered the external torque $\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{H}$ exerted by a magnetic field \mathbf{H} upon the magnetic moment of a molecule $\boldsymbol{\mu} = \gamma I \boldsymbol{\omega}$, where $\gamma = 2\pi g \mu_N / h$ is the molecular gyromagnetic ratio related to the rotational Landé g -factor, μ_N the nuclear magneton and h the Planck constant. Moreover, $d\mathbf{k}$ is an element of solid angle and the following abbreviations were introduced

$$\begin{aligned} f^* &= f(\mathbf{x}, \mathbf{c}^*, \boldsymbol{\omega}^*, t), & f &= f(\mathbf{x}, \mathbf{c}, \boldsymbol{\omega}, t), \\ f_1^* &= f(\mathbf{x}, \mathbf{c}_1^*, \boldsymbol{\omega}_1^*, t), & f_1 &= f_1(\mathbf{x}, \mathbf{c}_1, \boldsymbol{\omega}_1, t). \end{aligned} \quad (2.4)$$

3 Basic fields and Grad distribution function

The macroscopic state of a polyatomic gas will be described by thirty-seven basic scalar fields defined by

$$\varrho(\mathbf{x}, t) = \int m f d\mathbf{c} d\boldsymbol{\omega}, \quad \varrho v_i(\mathbf{x}, t) = \int m c_i f d\mathbf{c} d\boldsymbol{\omega}, \quad (3.1)$$

$$T(\mathbf{x}, t) = \frac{m}{3k\varrho} \int \left(\frac{1}{2} m C^2 + \frac{1}{2} I \omega^2 \right) f d\mathbf{c} d\boldsymbol{\omega}, \quad (3.2)$$

$$q_i^T(\mathbf{x}, t) = \int \frac{1}{2} m C^2 C_i f d\mathbf{c} d\boldsymbol{\omega}, \quad q_i^R(\mathbf{x}, t) = \int \frac{1}{2} I \omega^2 C_i f d\mathbf{c} d\boldsymbol{\omega}, \quad (3.3)$$

$$p_{\langle ij \rangle}(\mathbf{x}, t) = \int m C_{\langle i} C_{j \rangle} f d\mathbf{c} d\boldsymbol{\omega}, \quad \Pi(\mathbf{x}, t) = \frac{1}{3} \int \left(\frac{1}{2} m C^2 - \frac{1}{2} I \omega^2 \right) f d\mathbf{c} d\boldsymbol{\omega}, \quad (3.4)$$

$$q_{\langle ij \rangle}(\mathbf{x}, t) = \int I \omega_{\langle i} \omega_{j \rangle} f d\mathbf{c} d\boldsymbol{\omega}, \quad h_{\langle ij \rangle k}(\mathbf{x}, t) = \int I \omega_{\langle i} \omega_{j \rangle} C_k f d\mathbf{c} d\boldsymbol{\omega}, \quad (3.5)$$

where k is the Boltzmann constant and $C_i = c_i - v_i$ is the peculiar linear velocity. In Eqs. (3.1)–(3.5) we have denoted by ρ the mass density, ρv_i the linear momentum density, T the temperature, q_i^T the translational heat flux, q_i^R the rotational heat flux, $p_{\langle ij \rangle} = p_{ij} - \frac{1}{3} p_{rr} \delta_{ij}$ the pressure deviator and $\Pi = \frac{1}{3} p_{rr} - \rho \frac{k}{m} T$ the dynamic pressure. The basic fields $q_{\langle ij \rangle}$ and $h_{\langle ij \rangle k}$, which are related to the polarizations $\omega_{\langle i} \omega_{j \rangle}$ and $\omega_{\langle i} \omega_{j \rangle} C_k$, are the dominant fields in the calculations of the coefficients of shear viscosity and thermal conductivity, respectively.

We get the distribution function close to equilibrium by seeking the extremum of the entropy density [10], given by

$$\rho s = -k \int f \ln f d\mathbf{c} d\boldsymbol{\omega}, \quad (3.6)$$

under the constraints (3.1)–(3.5). This problem is equivalent to finding the extremum of the following functional without constraints

$$\begin{aligned} F = - \int \left[k \ln f + \Lambda m + \Lambda_i^v m C_i + \lambda \left(\frac{1}{2} m C^2 + \frac{1}{2} I \omega^2 \right) + \lambda_i^T \frac{m C^2}{2} C_i \right. \\ \left. + \lambda_i^R \frac{I \omega^2}{2} C_i + \Lambda_{ij} m C_{\langle i} C_{j \rangle} + \lambda' \left(\frac{1}{2} m C^2 - \frac{1}{2} I \omega^2 \right) \right. \\ \left. + \lambda_{ij} I \omega_{\langle i} \omega_{j \rangle} + \Lambda_{ijk} I \omega_{\langle i} \omega_{j \rangle} C_k \right] f d\mathbf{c} d\boldsymbol{\omega}, \end{aligned} \quad (3.7)$$

where Λ , Λ_i^v , λ , λ_i^T , λ_i^R , Λ_{ij} , λ' , λ_{ij} and Λ_{ijk} are Lagrange multipliers which do not depend on the distribution function f . By setting $\frac{\partial F}{\partial f} = 0$ in Eq. (3.7) we obtain the distribution function that maximizes the entropy density under the constraints (3.1)–(3.5). The determination of the Lagrange multipliers follows from the use of the Eqs. (3.1)–(3.5) and by considering processes close to equilibrium:

$$\begin{aligned} f = f^{(0)} \left[1 + \frac{1}{2\rho} \left(\frac{m}{kT} \right)^2 p_{\langle ij \rangle} C_i C_j + \frac{1}{2\rho} \left(\frac{m}{kT} \right)^2 \Pi \left(C^2 - \frac{I \omega^2}{m} \right) \right. \\ \left. + \frac{I}{2m\rho} \left(\frac{m}{kT} \right)^2 q_{\langle ij \rangle} \omega_i \omega_j + \frac{1}{\rho} \left(\frac{m}{kT} \right)^2 \left(\frac{m C^2}{5kT} - 1 \right) q_i^T C_i \right. \\ \left. + \frac{1}{\rho} \left(\frac{m}{kT} \right)^2 \left(\frac{I \omega^2}{3kT} - 1 \right) q_i^R C_i + \frac{I}{2m\rho} \left(\frac{m}{kT} \right)^3 h_{\langle ij \rangle k} \omega_i \omega_j C_k \right] = f^{(0)} (1 + \phi), \end{aligned} \quad (3.8)$$

where

$$f^{(0)} = \frac{\rho}{m} \frac{(mI)^{3/2}}{(2\pi kT)^3} \exp \left\{ - \left(\frac{m C^2}{2kT} + \frac{I \omega^2}{2kT} \right) \right\} \quad (3.9)$$

is the Maxwellian distribution function. Eq. (3.8) is the Grad distribution function for the basic fields (3.1)–(3.5).

4 Combined Chapman-Enskog and Grad method

By using the Chapman-Enskog method [3] we insert the distribution function (3.8) into the Boltzmann equation (2.3) and get

$$\frac{\partial f^{(0)}}{\partial t} + c_i \frac{\partial f^{(0)}}{\partial x_i} + \gamma (\boldsymbol{\omega} \times \mathbf{H})_i \frac{\partial \phi}{\partial \omega_i} = \mathcal{T}^* [\phi], \quad (4.1)$$

where we have introduced the notation

$$\mathcal{T}^* [\phi] = \int f_1^{(0)} f^{(0)} (\phi_1^* + \phi^* - \phi_1 - \phi) a^2 (\mathbf{g} \cdot \mathbf{k}) d\mathbf{k} d\mathbf{c}_1 d\boldsymbol{\omega}_1. \quad (4.2)$$

In Eq.(4.1) we have retained the term proportional to $\frac{\partial \phi}{\partial \omega_i}$ since this term gives the coupling between the magnetic field \mathbf{H} and the fields defined by Eqs.(3.5)_{1,2}.

The multiplication of Eq. (4.1) by an arbitrary function $\varphi(\mathbf{x}, \mathbf{c}, \boldsymbol{\omega}, t)$ and the integration over all values of velocities \mathbf{c} and $\boldsymbol{\omega}$ leads to

$$\begin{aligned} \int \varphi \left[\frac{\partial f^{(0)}}{\partial t} + c_i \frac{\partial f^{(0)}}{\partial x_i} + \gamma (\boldsymbol{\omega} \times \mathbf{H})_i \frac{\partial \phi}{\partial \omega_i} \right] d\mathbf{c} d\boldsymbol{\omega} \\ = \int \varphi \mathcal{T}^* [\phi] d\mathbf{c} d\boldsymbol{\omega} = \int \phi \mathcal{T} [\varphi] d\mathbf{c} d\boldsymbol{\omega}, \end{aligned} \quad (4.3)$$

where

$$\mathcal{T} [\phi] = \int f_1^{(0)} f^{(0)} (\phi_1' + \phi' - \phi_1 - \phi) a^2 (\mathbf{g} \cdot \mathbf{k}) d\mathbf{k} d\mathbf{c}_1 d\boldsymbol{\omega}_1, \quad (4.4)$$

with $\phi_1' = \phi(\mathbf{x}, \mathbf{c}_1', \boldsymbol{\omega}_1', t)$ and so on. The term on the right-hand side of Eq.(4.3) follows by the use of the symmetry properties of the collision integral.

If we identify in Eq. (4.3) the arbitrary function φ with the summational invariants m , mc_i and $(\frac{mC^2}{2} + \frac{I\omega^2}{2})$ and by performing the integrations over all values of velocities \mathbf{c} and $\boldsymbol{\omega}$, we obtain the field equations for the mass density ρ , linear momentum density ρv_i and temperature for a Euler fluid, in which $p_{<ij>} = 0$, $q_i^T = q_i^R = 0$. These field equations are used to eliminate the time derivatives of ρ , v_i and T in Eq.(4.3), and we have

$$\begin{aligned} \int \varphi f^{(0)} \left\{ \frac{m}{kT} C_i C_j \frac{\partial v_{<i>}}{\partial x_{j>}} + \frac{m}{6kT} \left(C^2 - \frac{I\omega^2}{m} \right) \frac{\partial v_r}{\partial x_r} \right. \\ \left. + \frac{1}{T} \left[\left(\frac{mC^2}{2kT} - \frac{5}{2} \right) + \left(\frac{I\omega^2}{2kT} - \frac{3}{2} \right) \right] C_i \frac{\partial T}{\partial x_i} \right. \\ \left. + \gamma \left[\frac{I}{m} \frac{q_{<ps>}}{\rho} \left(\frac{m}{kT} \right)^2 \omega_s + \frac{I}{m} \frac{h_{<ps>k}}{\rho} \left(\frac{m}{kT} \right)^3 \omega_s C_k \right] (\boldsymbol{\omega} \times \mathbf{H})_p \right\} \\ = \int \phi \mathcal{T} [\varphi] d\mathbf{c} d\boldsymbol{\omega}. \end{aligned} \quad (4.5)$$

In the next sections we shall use the Eq. (4.5) to determine the constitutive equations for the heat flux vector $q_i = q_i^T + q_i^R$ and for the pressure deviator tensor $p_{<ij>}$ that depend on the magnetic field \mathbf{H} .

5 Heat flux and coefficients of thermal conductivity

If we identify in Eq. (4.5) the arbitrary function $\varphi(\mathbf{x}, \mathbf{c}, \boldsymbol{\omega}, t)$ with $\frac{I\omega^2}{2} C_i$, $\frac{mC^2}{2} C_i$ and $I\omega_{<i>}\omega_{j>} C_k$ and integrate over all values of \mathbf{c} and $\boldsymbol{\omega}$, we obtain the following system of equations:

$$A_1 H_{<il>} + A_2 Q_i^T + A_3 Q_i^R = A_4 \frac{\partial T}{\partial x_i}, \quad (5.1)$$

$$B_1 H_{<il>l} + B_2 Q_i^T + B_3 Q_i^R = B_4 \frac{\partial T}{\partial x_i}, \quad (5.2)$$

$$\begin{aligned} & [D_1 \delta_{ij} \delta_{pk} - D_2 (\delta_{ik} \delta_{pj} + \delta_{jk} \delta_{ip})] H_{<pl>l} - D_3 H_{<ij>k} - D_4 (H_{<ik>j} + H_{<jk>i}) \\ & - D_5 \left(\delta_{ik} \delta_{pj} + \delta_{jk} \delta_{ip} - \frac{2}{3} \delta_{ij} \delta_{kp} \right) Q_p^T + D_6 \left(\delta_{ik} \delta_{js} + \delta_{jk} \delta_{is} - \frac{2}{3} \delta_{ij} \delta_{ks} \right) Q_s^R \\ & = D_7 (\delta_{il} \varepsilon_{pir} + \delta_{jl} \varepsilon_{pir}) \mathcal{H}_r H_{<pl>k}, \end{aligned} \quad (5.3)$$

where we have introduced the reduced quantities

$$\begin{aligned} Q_i^T &= \frac{\sqrt{\pi} a^2 T}{m} \left(\frac{m}{kT} \right)^{3/2} q_i^T, & Q_i^R &= \frac{\sqrt{\pi} a^2 T}{m} \left(\frac{m}{kT} \right)^{3/2} q_i^R, \\ H_{<ij>k} &= \frac{\sqrt{\pi} a^2 T}{m} \left(\frac{kT}{m} \right)^{3/2} h_{<ij>k}, & \mathcal{H}_i &= \frac{\gamma m}{\rho a^2} \left(\frac{m}{\pi kT} \right)^{1/2} H_i \end{aligned} \quad (5.4)$$

and the coefficients

$$\begin{aligned} A_1 &= 3(2\kappa + 1), & A_2 &= -10\kappa, & A_3 &= 10(2\kappa^2 + 2\kappa + 1), \\ A_4 &= -\frac{45}{8}(\kappa + 1)^2, & B_1 &= 15\kappa, & B_2 &= 6(17\kappa + 4), \\ B_3 &= -50\kappa, & B_4 &= -\frac{225}{8}(\kappa + 1)^2, & D_1 &= 4(2\kappa + 1), \\ D_2 &= 11\kappa + 3, & D_3 &= 2(20\kappa^2 + 16\kappa + 11), & D_4 &= \kappa + 3, \\ D_5 &= 4\kappa, & D_6 &= -4(2\kappa + 1), & D_7 &= \frac{15}{2}(\kappa + 1)^2. \end{aligned} \quad (5.5)$$

The solution of this system of equations for the reduced heat flux vector $Q_i(\mathcal{H}) = Q_i^T(\mathcal{H}) + Q_i^R(\mathcal{H})$ leads to the generalized Fourier law

$$Q_i(\mathcal{H}) = -\lambda_{ij}(\mathcal{H}) \frac{\partial T}{\partial x_j} \quad (5.6)$$

where $\lambda_{ij}(\mathcal{H})$ is the thermal conductivity tensor.

The procedure used to solve the system of equations (5.1)–(5.3) is the following: first we solve the system for $\mathcal{H} = 0$ and then we subtract from the original system the field-free equations and solve the set of equations that are field-dependent [11]. For this purpose we introduce the decompositions:

$$\begin{aligned} H_{<ij>k}(\mathcal{H}) &= H_{<ij>k}^{(0)} + H_{<ij>k}^{(1)}(\mathcal{H}), \\ Q_i^T(\mathcal{H}) &= Q_i^{T(0)} + Q_i^{T(1)}(\mathcal{H}), \\ Q_i^R(\mathcal{H}) &= Q_i^{R(0)} + Q_i^{R(1)}(\mathcal{H}), \end{aligned} \quad (5.7)$$

where

$$H_{<ij>k}^{(1)}(0) = 0, \quad Q_i^{T(1)}(0) = 0 \quad \text{and} \quad Q_i^{R(1)}(0) = 0. \quad (5.8)$$

5.1 Field-free system of equations

When $\mathcal{H} = 0$ the system of equations (5.1)–(5.3) reduces to

$$A_1 H_{<il>l}^{(0)} + A_2 Q_i^{T(0)} + A_3 Q_i^{R(0)} = A_4 \frac{\partial T}{\partial x_i}, \quad (5.9)$$

$$B_1 H_{<il>l}^{(0)} + B_2 Q_i^{T(0)} + B_3 Q_i^{R(0)} = B_4 \frac{\partial T}{\partial x_i}, \quad (5.10)$$

$$\begin{aligned} & [D_1 \delta_{ij} \delta_{pk} - D_2 (\delta_{ik} \delta_{pj} + \delta_{jk} \delta_{ip})] H_{<pl>l}^{(0)} - D_3 H_{<ij>k}^{(0)} \\ & - D_4 (H_{<ik>j}^{(0)} + H_{<jk>i}^{(0)}) - D_5 \left(\delta_{ik} \delta_{pj} + \delta_{jk} \delta_{ip} - \frac{2}{3} \delta_{ij} \delta_{kp} \right) Q_p^{T(0)} + D_6 \left(\delta_{ik} \delta_{js} + \delta_{jk} \delta_{is} - \frac{2}{3} \delta_{ij} \delta_{ks} \right) Q_s^{R(0)} = 0. \end{aligned} \quad (5.11)$$

From Eqs. (5.9)–(5.10) it follows that

$$Q_i^{T(0)} = E_1 H_{<il>l}^{(0)} + E_3 \frac{\partial T}{\partial x_i}, \quad Q_i^{R(0)} = E_2 H_{<il>l}^{(0)} + E_4 \frac{\partial T}{\partial x_i} \quad (5.12)$$

where we have introduced the coefficients

$$\begin{aligned} E_1 &= \frac{-15\kappa(\kappa+1)^2}{(102\kappa^3 + 101\kappa^2 + 75\kappa + 12)}, \\ E_2 &= -\frac{3}{10} \frac{(127\kappa^2 + 75\kappa + 12)}{(102\kappa^3 + 101\kappa^2 + 75\kappa + 12)}, \\ E_3 &= -\frac{225}{16} \frac{(\kappa+1)^3(2\kappa+1)}{(102\kappa^3 + 101\kappa^2 + 75\kappa + 12)}, \\ E_4 &= -\frac{9}{4} \frac{(\kappa+1)^2(19\kappa+3)}{(102\kappa^3 + 101\kappa^2 + 75\kappa + 12)}. \end{aligned} \quad (5.13)$$

We consider that in a linearized theory the vector $H_{<il>l}$ is proportional to the gradient of temperature [see Eqs. (5.24)–(5.25)], hence we write

$$H_{<il>l}^{(0)} = \frac{10}{3} \beta_1 \frac{\partial T}{\partial x_i}, \quad (5.14)$$

where β_1 is a parameter to be determined. From Eqs. (5.11)–(5.14), we get

$$\beta_1 = \frac{3}{4} \frac{(\kappa+1)(50\kappa^3 + 227\kappa^2 + 125\kappa + 12)}{(1360\kappa^4 + 2266\kappa^3 + 1707\kappa^2 + 853\kappa + 116)}. \quad (5.15)$$

The Fourier law follows from Eqs. (5.12), (5.13) and (5.15)

$$Q_i = Q_i^{T(0)} + Q_i^{R(0)} = \left[E_3 + E_4 + \frac{10}{3} (E_1 + E_2) \beta_1 \right] \frac{\partial T}{\partial x_i} = -\lambda_0 \frac{\partial T}{\partial x_i} \quad (5.16)$$

where λ_0 is the dimensionless coefficient of thermal conductivity

$$\lambda_0 = \frac{3}{16} \frac{(\kappa+1)^2 (2000\kappa^4 + 9490\kappa^3 + 13449\kappa^2 + 7336\kappa + 1121)}{(1360\kappa^5 + 3626\kappa^4 + 3973\kappa^3 + 2560\kappa^2 + 969\kappa + 116)}. \quad (5.17)$$

Eq. (5.17) agrees with that obtained in ref. [12]. The Pidduck approximation [13] follows from Eqs. (5.1)–(5.2) when $H_{<il>l} = 0$

$$\lambda_0 = \frac{9}{16} \frac{(\kappa+1)^2 (50\kappa^2 + 151\kappa + 37)}{(102\kappa^3 + 101\kappa^2 + 75\kappa + 12)}. \quad (5.18)$$

5.2 Field-dependent equations

By subtracting the field-free equations (5.9)–(5.11) from Eqs. (5.1)–(5.3) lead to

$$A_1 H_{<il>l}^{(1)} + A_2 Q_i^{T(1)} + A_3 Q_i^{R(1)} = 0, \quad (5.19)$$

$$B_1 H_{<il>l}^{(1)} + B_2 Q_i^{T(1)} + B_3 Q_i^{R(1)} = 0, \quad (5.20)$$

$$\begin{aligned} & [D_1 \delta_{ij} \delta_{pk} - D_2 (\delta_{ik} \delta_{pj} + \delta_{jk} \delta_{ip})] H_{<pl>l}^{(1)} - D_3 H_{<ij>k}^{(1)} - D_4 (H_{<ik>j}^{(1)} + H_{<jk>i}^{(1)}) \\ & - D_5 \left(\delta_{ik} \delta_{pj} + \delta_{jk} \delta_{ip} - \frac{2}{3} \delta_{ij} \delta_{kp} \right) Q_p^{T(1)} + D_6 \left(\delta_{ik} \delta_{js} + \delta_{jk} \delta_{is} - \frac{2}{3} \delta_{ij} \delta_{ks} \right) Q_s^{R(1)} \\ & = D_7 (\delta_{il} \varepsilon_{pjr} + \delta_{jl} \varepsilon_{pir}) \cdot \mathcal{H}_r (H_{<pl>k}^{(0)} + H_{<pl>k}^{(1)}) . \end{aligned} \quad (5.21)$$

From Eqs. (5.19)–(5.20) it follows that the reduced heat flux vector is given by

$$Q_i^{(1)} = Q_i^{T(1)} + Q_i^{R(1)} = (E_1 + E_2) H_{<il>l}^{(1)}. \quad (5.22)$$

To determine the constitutive equation for the heat flux vector $Q_i = -\lambda_{ij} \frac{\partial T}{\partial x_j}$, we must know the tensor $H_{<ij>k}$ which is solution, through Eqs. (5.19)–(5.21), of the following expression

$$\begin{aligned} & \left[D_1 \delta_{ij} \delta_{pk} - D_2 (\delta_{ik} \delta_{pj} + \delta_{jk} \delta_{ip}) + (E_2 D_6 - E_1 D_5) (\delta_{ik} \delta_{jp} + \delta_{jk} \delta_{ip} \right. \\ & \quad \left. - \frac{2}{3} \delta_{ij} \delta_{kp}) \right] H_{<pl>l}^{(1)} - D_3 H_{<ij>k}^{(1)} - D_4 (H_{<ik>j}^{(1)} + H_{<jk>i}^{(1)}) \\ & = D_7 (\delta_{il} \varepsilon_{pjr} + \delta_{jl} \varepsilon_{pir}) \cdot \mathcal{H}_r (H_{<pl>k}^{(0)} + H_{<pl>k}^{(1)}) . \end{aligned} \quad (5.23)$$

This is a tensorial equation for the components of the tensor (5.7)₁.

We consider the tensor $H_{<ij>k} = H_{<ij>k}^{(0)} + H_{<ij>k}^{(1)}$ that is linear in the temperature gradient $\frac{\partial T}{\partial x_i}$ and that depends on the antisymmetric tensor $B_{ij} = \frac{1}{\mathcal{H}} \varepsilon_{ijk} \cdot \mathcal{H}_k$. We write it as a function of a fourth-order tensor:

$$H_{<ij>k} = t_{<ij>km} \frac{\partial T}{\partial x_m} = (t_{<ij>km}^{(0)} + t_{<ij>km}^{(1)}) \frac{\partial T}{\partial x_m}, \quad (5.24)$$

where

$$t_{<ij>km}^{(0)} = \beta_1 \left(\delta_{ik} \delta_{jm} + \delta_{im} \delta_{jk} - \frac{2}{3} \delta_{ij} \delta_{km} \right) \quad (5.25)$$

and $t_{<ij>km}^{(1)}(0) = 0$.

By multiplying (5.23) by δ_{ij} and inserting Eq. (5.24) in the resulting equation, we get

$$\begin{aligned} & E \left(\delta_{ik} t_{<jr>rm}^{(1)} + \delta_{jk} t_{<ir>rm}^{(1)} - \frac{2}{3} \delta_{ij} t_{<kr>rm}^{(1)} \right) - D_3 t_{<ij>km}^{(1)} - D_4 \left(t_{<ik>jm}^{(1)} + t_{<jk>im}^{(1)} - \frac{2}{3} \delta_{ij} t_{<kr>rm}^{(1)} \right) \\ & = D_7 \left[\left(t_{<ir>km}^{(0)} + t_{<ir>km}^{(1)} \right) \varepsilon_{rjt} \mathcal{H}_t + \left(t_{<jr>km}^{(0)} + t_{<jr>km}^{(1)} \right) \varepsilon_{rit} \mathcal{H}_t \right]. \end{aligned} \quad (5.26)$$

In the above expression we have introduced

$$E = - \frac{(5310\kappa^4 + 4961\kappa^3 + 3678\kappa^2 + 1191\kappa + 108)}{5(102\kappa^3 + 101\kappa^2 + 75\kappa + 12)}. \quad (5.27)$$

We can reduce the order of the above tensorial equation by multiplying it successively by δ_{jk} , $B_{jk} = \frac{1}{\mathcal{H}} \varepsilon_{jkr} \mathcal{H}_r$, $\frac{1}{\mathcal{H}^2} \mathcal{H}_i \mathcal{H}_j$ and $\frac{1}{\mathcal{H}^2} \mathcal{H}_j \mathcal{H}_k$. Thus follows the system of equations

$$D_{im} = W_1 C_{im} - B_{ri} C_{rm} + 5\beta_1 B_{im}, \quad (5.28)$$

$$E_{im} = C_{im} + W_2 B_{ri} C_{rm} + W_3 D_{im} - B_{ri} D_{rm} + 2\beta_1 (2\delta_{im} - \frac{1}{\mathcal{H}^2} \mathcal{H}_i \mathcal{H}_m), \quad (5.29)$$

$$F_{im} = -W_4 C_{im} + \frac{2E}{D_3 \mathcal{H}^2} \mathcal{H}_i \mathcal{H}_r C_{rm} - \frac{2D_4}{D_3} E_{im}, \quad (5.30)$$

$$D_4 F_{im} + (D_3 + D_4) E_{im} + D_7 \mathcal{H} B_{ri} E_{rm} - E C_{im} - \frac{1}{\mathcal{H}^2} W_5 \mathcal{H}_i \mathcal{H}_r C_{rm} - D_7 \beta_1 \mathcal{H} B_{im} = 0. \quad (5.31)$$

In the above equations we have introduced the notation

$$C_{im} = t_{<il>lm}^{(1)}, \quad E_{im} = \frac{\mathcal{H}_j \mathcal{H}_k}{\mathcal{H}^2} t_{<ij>km}^{(1)}, \quad (5.32)$$

$$D_{im} = B_{jk} t_{<ij>km}^{(1)}, \quad F_{im} = \frac{\mathcal{H}_j \mathcal{H}_k}{\mathcal{H}^2} t_{<jk>im}^{(1)}$$

and the coefficients

$$W_1 = -\frac{45(\kappa + 1)^3 (1360\kappa^4 + 2266\kappa^3 + 1707\kappa^2 + 853\kappa + 116)}{2\mathcal{H} (102\kappa^3 + 101\kappa^2 + 75\kappa + 12)}, \quad (5.33)$$

$$W_2 = -\frac{2(1838\kappa^4 + 1925\kappa^3 + 1478\kappa^2 + 555\kappa + 60)}{15\mathcal{H} (\kappa + 1)^2 (102\kappa^3 + 101\kappa^2 + 75\kappa + 12)}, \quad (5.34)$$

$$W_3 = -\frac{2(40\kappa^2 + 31\kappa + 19)}{15\mathcal{H} (\kappa + 1)^2}, \quad (5.35)$$

$$W_4 = -\frac{4(485\kappa^4 + 583\kappa^3 + 464\kappa^2 + 198\kappa + 24)}{5(20\kappa^2 + 16\kappa + 11)(102\kappa^3 + 101\kappa^2 + 75\kappa + 12)}, \quad (5.36)$$

$$W_5 = -\frac{(1430\kappa^4 + 297\kappa^3 - 34\kappa^2 - 393\kappa - 84)}{5(102\kappa^3 + 101\kappa^2 + 75\kappa + 12)}. \quad (5.37)$$

The representation of a second-order tensor that is a function of an axial vector \mathcal{H} is given by

$$C_{im} = \sigma_1 \delta_{im} + \frac{\sigma_2}{\mathcal{H}} \varepsilon_{iml} \mathcal{H}_l + \sigma_3 \frac{\mathcal{H}_i \mathcal{H}_m}{\mathcal{H}^2} \quad (5.38)$$

where σ_1 , σ_2 and σ_3 are parameters to be determined.

If we insert Eq. (5.38) in the system (5.28)–(5.31) and solve it we get σ_1 , σ_2 and σ_3 and therefore the vector $H_{<il>l} = C_{im} \frac{\partial T}{\partial x_m}$. Hence follows from Eq. (5.22) the generalized Fourier law

$$Q_i^{(1)} = -\lambda_{ij}^{(1)} (\mathcal{H}) \frac{\partial T}{\partial x_j} \quad (5.39)$$

where the dimensionless thermal conductivity tensor is given by

$$\lambda_{ij}^{(1)} (\mathcal{H}) = \lambda_{ij} (\mathcal{H}) - \lambda_0 \delta_{ij} = -(E_1 + E_2) (\sigma_1 \delta_{ij} + \frac{\sigma_2}{\mathcal{H}} \varepsilon_{ijl} \mathcal{H}_l + \sigma_3 \frac{\mathcal{H}_i \mathcal{H}_j}{\mathcal{H}^2}). \quad (5.40)$$

To illustrate this procedure we choose, without loss of generality, the applied magnetic field to lie along the x -axis of a coordinate system fixed in some point of the gas. We introduce as usual the coefficients of thermal conductivity parallel (λ_{\parallel}), perpendicular (λ_{\perp}) and transverse (λ_{tr}) defined as the components of the matrix of the thermal conductivity tensor:

$$\left(\lambda^{(1)} \right)_{ij} = \begin{pmatrix} \lambda_{\parallel}^{(1)} & 0 & 0 \\ 0 & \lambda_{\perp}^{(1)} & \lambda_{tr}^{(1)} \\ 0 & -\lambda_{tr}^{(1)} & \lambda_{\perp}^{(1)} \end{pmatrix} \quad (5.41)$$

where

$$\lambda_{\parallel}^{(1)} = \lambda_{xx}^{(1)} = -(E_1 + E_2)(\sigma_1 + \sigma_3), \quad (5.42)$$

$$\lambda_{\perp}^{(1)} = \lambda_{yy}^{(1)} = \lambda_{zz}^{(1)} = -(E_1 + E_2)\sigma_1, \quad (5.43)$$

$$\lambda_{tr}^{(1)} = \lambda_{yz}^{(1)} = -\lambda_{zy}^{(1)} = -(E_1 + E_2)\sigma_2. \quad (5.44)$$

The relative changes in the parallel and perpendicular thermal conductivity coefficients are given by

$$\begin{aligned} \frac{\Delta\lambda_{\parallel}}{\lambda_0} &= \frac{\lambda_{\parallel}^{(1)}}{\lambda_0} = [6(E_1 + E_2)(D_3 + 2D_4)D_7^2\beta_1\mathcal{H}^2] \\ &\times \frac{1}{\lambda_0 [(D_3 + 2D_4)(D_3 - D_4)(3D_3 - 10E + D_4) + D_7^2(3D_3 + 4D_4 - 4E)\mathcal{H}^2]} \end{aligned} \quad (5.45)$$

and

$$\begin{aligned} \frac{\Delta\lambda_{\perp}}{\lambda_0} &= \frac{\lambda_{\perp}^{(1)}}{\lambda_0} = -\frac{3D_7^2\beta_1}{\delta\lambda_0}(E_1 + E_2) \\ &\times [3(D_4 - D_3)(D_3 + 2D_4)^2(9D_3 - 7D_4 - 5E) - 4D_7^2(3D_3 + D_4)(3D_3 + D_4 - E)\mathcal{H}^2] \mathcal{H}^2, \end{aligned} \quad (5.46)$$

where

$$\begin{aligned} \delta &= (D_3 + 2D_4)^2(D_4 - D_3)^2(10E - 3D_3 - D_4)^2 \\ &+ D_7^2 [45D_3^4 + 102D_3^3D_4 - 19D_3^2D_4^2 - 100D_3D_4^3 + 72D_4^4 \\ &- E(138D_3^3 + 274D_3^2D_4 + 160D_3D_4^2 - 72D_4^3) \\ &+ E^2(185D_3^2 + 260D_3D_4 + 180D_4^2)] \mathcal{H}^2 + 4D_7^4(E - 3D_3 - D_4)^2 \mathcal{H}^4. \end{aligned} \quad (5.47)$$

The relative change in the transversal thermal conductivity coefficient is:

$$\begin{aligned} \frac{\lambda_{tr}}{\lambda_0} &= \frac{\lambda_{tr}^{(1)}}{\lambda_0} = -\frac{3D_7\beta_1}{\delta\lambda_0}(E_1 + E_2)(3D_3 + D_4 - 10E) [5(D_4 - D_3)^2 \\ &\times (D_3 + 2D_4)^2 + 4D_7^2(2D_3^2 + 2D_3D_4 + D_4^2)\mathcal{H}^2] \mathcal{H}. \end{aligned} \quad (5.48)$$

In Figs. 1, 2 and 3 the three independent components of the thermal conductivity tensor are plotted for some values of the dimensionless moment of inertia $\kappa = \frac{4I}{ma^2}$. The field parameter is the dimensionless quantity $\mathcal{H} = \frac{\gamma m}{\rho a^2} \left(\frac{m}{\pi kT}\right)^{1/2} H$. These curves reproduce the universal behaviour observed for polyatomic gases in the presence of external magnetic fields [7] and they agree with the results obtained through the Chapman-Enskog method [8]. We note, however, that the saturation values shown in Figs. 1 and 2 and the maximal values shown in Figs. 3 are much larger than those obtained experimentally [7]. This fact is due mainly to the molecular model used, since for collisions between rough spherical molecules even the slightest of grazing collisions leads to a large deflection.

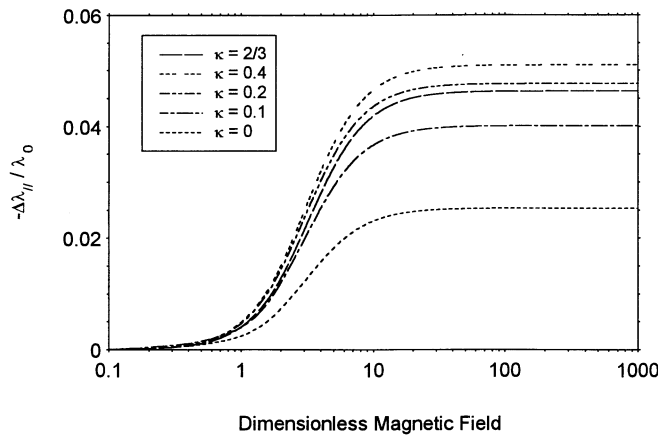


Fig. 1. Decrease of the parallel thermal conductivity

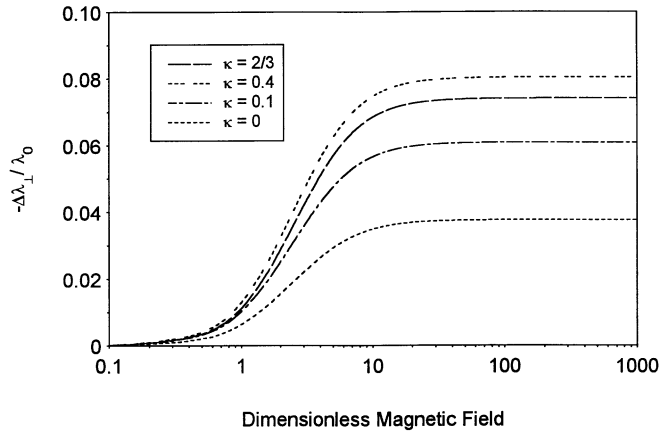


Fig. 2. Decrease of the perpendicular thermal conductivity

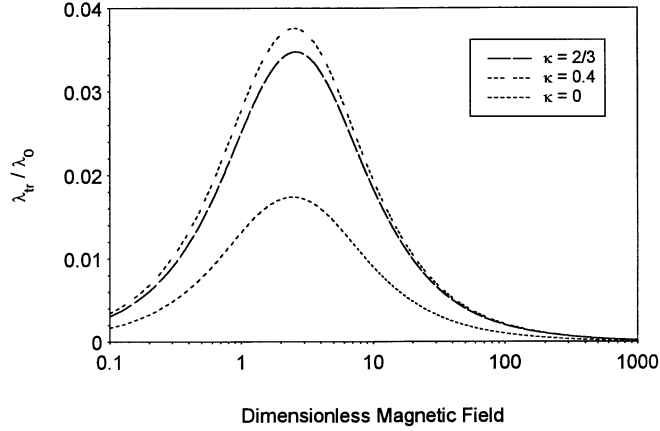


Fig. 3. Relative changes of the transverse thermal conductivity

6 Pressure tensor deviator and the coefficients of the shear viscosity

If we identify in Eq. (4.5) the arbitrary function $\varphi(\mathbf{x}, \mathbf{c}, \omega, t)$ with $C_{\langle k} C_{l \rangle}$ and $\omega_{\langle k} \omega_{l \rangle}$ and integrate over all values of \mathbf{c} and ω , we obtain the following system of equations

$$\frac{\partial v_{\langle i}}{\partial x_{j \rangle}} = -\frac{4a^2}{15} \left(\frac{\pi}{mkT} \right)^{1/2} \frac{1}{(\kappa + 1)^2} [(13\kappa + 6)p_{\langle ij \rangle} + 5\kappa q_{\langle ij \rangle}], \quad (6.1)$$

$$q_{\langle jk \rangle} \varepsilon_{kij} \mathcal{H}_q + q_{\langle ik \rangle} \varepsilon_{kjq} \mathcal{H}_q + \frac{8\kappa}{15(\kappa + 1)^2} \left[5p_{\langle ij \rangle} + \frac{(10\kappa + 3)}{\kappa} q_{\langle ij \rangle} \right] = 0 \quad (6.2)$$

for the components of the tensors $p_{\langle ij \rangle}$ and $q_{\langle ij \rangle}$. Our aim is to obtain from this system the deviatoric pressure tensor in a form of a Navier-Stokes law

$$p_{\langle ij \rangle}(\mathcal{H}) = -2\eta_{\langle ij \rangle \langle kl \rangle}(\mathcal{H}) \frac{\partial v_{\langle k}}{\partial x_{l \rangle}} \quad (6.3)$$

where $\eta_{\langle ij \rangle \langle kl \rangle}(\mathcal{H})$ is the shear viscosity tensor.

6.1 Field-free equations

When $\mathcal{H} = 0$ Eqs. (6.1)–(6.2), yields

$$\frac{\partial v_{\langle i}}{\partial x_{j \rangle}} = -\frac{4a^2}{15} \left(\frac{\pi}{mkT} \right)^{1/2} \frac{1}{(\kappa + 1)^2} [(13\kappa + 6)p_{\langle ij \rangle} + 5\kappa q_{\langle ij \rangle}], \quad (6.4)$$

$$\frac{8\kappa}{15(\kappa+1)^2} \left[5p_{\langle ij \rangle} + \frac{(10\kappa+3)}{\kappa} q_{\langle ij \rangle} \right] = 0 \quad (6.5)$$

and it follows that $p_{\langle ij \rangle}(0) = -2\eta_0 \frac{\partial v_{\langle i}}{\partial x_{j \rangle}}$, with the shear viscosity coefficient given by

$$\eta_0 = \frac{5}{8a^2} \left(\frac{mkT}{\pi} \right)^{1/2} \frac{(\kappa+1)^2 (10\kappa+3)}{(35\kappa^2+33\kappa+6)}. \quad (6.6)$$

Eq. (6.6) agrees with the result obtained in ref. [12]. The Pidduck approximation [13] follows from Eq. (6.4) when $q_{\langle ij \rangle} = 0$

$$\eta_0 = \frac{15}{8a^2} \left(\frac{mkT}{\pi} \right)^{1/2} \frac{(\kappa+1)^2}{(13\kappa+6)}. \quad (6.7)$$

6.2 Field-dependent equations

We consider the fourth-order tensor

$$\begin{aligned} A_{\langle ij \rangle \langle kl \rangle} &= a_1 \left(\delta_{jk} \varepsilon_{liq} \mathcal{H}_q + \delta_{ik} \varepsilon_{ljq} \mathcal{H}_q + \delta_{jl} \varepsilon_{kiq} \mathcal{H}_q + \delta_{il} \varepsilon_{kj q} \mathcal{H}_q \right) \\ &\quad + b_1 \left(\delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk} - \frac{2}{3} \delta_{ij} \delta_{kl} \right), \end{aligned} \quad (6.8)$$

traceless in the indices ij and kl where a_1 and b_1 are coefficients that do not depend on the magnetic field. Hence, Eqs. (6.1)–(6.2) can be rewritten as

$$\frac{\partial v_{\langle i}}{\partial x_{j \rangle}} = \alpha_1 p_{\langle ij \rangle} + \alpha_2 q_{\langle ij \rangle}, \quad (6.9)$$

$$\alpha_3 p_{\langle ij \rangle} + A_{\langle ij \rangle \langle kl \rangle} q_{\langle kl \rangle} = 0 \quad (6.10)$$

where

$$\begin{aligned} \alpha_1 &= -\frac{4a^2}{15} \left(\frac{\pi}{mkT} \right)^{1/2} \frac{(13\kappa+6)}{(\kappa+1)^2}, & \alpha_3 &= \frac{16}{3} \frac{\kappa}{(\kappa+1)^2}, \\ \alpha_2 &= -\frac{4a^2}{3} \left(\frac{\pi}{mkT} \right)^{1/2} \frac{\kappa}{(\kappa+1)^2}, & \alpha_4 &= b_1 = \frac{8}{15} \frac{(10\kappa+3)}{(\kappa+1)^2} \end{aligned} \quad (6.11)$$

and $a_1 = 1$. From Eqs. (6.8)–(6.10) it follows that

$$\frac{\partial v_{\langle i}}{\partial x_{j \rangle}} = A'_{\langle ij \rangle \langle kl \rangle} q_{\langle kl \rangle} \quad (6.12)$$

where the fourth-order tensor $A'_{\langle ij \rangle \langle kl \rangle}$ is given by

$$\begin{aligned} A'_{\langle ij \rangle \langle kl \rangle} &= \frac{1}{2\alpha_3} \left[(\alpha_2 \alpha_3 - 2\alpha_1 \alpha_4) \left(\delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk} - \frac{2}{3} \delta_{ij} \delta_{kl} \right) - \right. \\ &\quad \left. 2\alpha_1 \left(\delta_{jk} \varepsilon_{liq} \mathcal{H}_q + \delta_{ik} \varepsilon_{lj q} \mathcal{H}_q + \delta_{jl} \varepsilon_{ki q} \mathcal{H}_q + \delta_{il} \varepsilon_{kj q} \mathcal{H}_q \right) \right]. \end{aligned} \quad (6.13)$$

The inverse tensor from (6.13) is found from identity

$$\left(A'^{-1} \right)_{\langle mn \rangle \langle ij \rangle} A'_{\langle ij \rangle \langle kl \rangle} = \frac{1}{2} \left(\delta_{mk} \delta_{nl} + \delta_{ml} \delta_{nk} - \frac{2}{3} \delta_{mn} \delta_{kl} \right) \quad (6.14)$$

and from the representation of a fourth-order tensor that is a function of an axial vector \mathcal{H} . It is given by [6]

$$\begin{aligned}
(A'^{-1})_{\langle kl \rangle \langle ij \rangle} &= \frac{1}{4(b_2^2 + a_2^2 \mathcal{H}^2)} \left\{ b_2 \left(\delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk} - \frac{2}{3} \delta_{ij} \delta_{kl} \right) \right. \\
&\quad + a_2 (\delta_{ik} \varepsilon_{ljr} \mathcal{H}_r + \delta_{jk} \varepsilon_{lir} \mathcal{H}_r + \delta_{il} \varepsilon_{kjr} \mathcal{H}_r + \delta_{jl} \varepsilon_{kir} \mathcal{H}_r) \\
&\quad + \frac{3a_2^2 b_2}{b_2^2 + a_2^2 \mathcal{H}^2} \left(\delta_{ik} \mathcal{H}_j \mathcal{H}_l + \delta_{jk} \mathcal{H}_i \mathcal{H}_l + \delta_{il} \mathcal{H}_j \mathcal{H}_k + \delta_{jl} \mathcal{H}_i \mathcal{H}_k - \frac{4}{3} \delta_{ij} \mathcal{H}_k \mathcal{H}_l \right. \\
&\quad \left. - \frac{4}{3} \delta_{kl} \mathcal{H}_i \mathcal{H}_j + \frac{4}{9} \mathcal{H}^2 \delta_{kl} \delta_{ij} \right) + \frac{3a_2^3}{b_2^2 + a_2^2 \mathcal{H}^2} (\varepsilon_{kir} \mathcal{H}_j \mathcal{H}_l \mathcal{H}_r + \varepsilon_{kjr} \mathcal{H}_i \mathcal{H}_l \mathcal{H}_r \\
&\quad + \varepsilon_{lir} \mathcal{H}_j \mathcal{H}_k \mathcal{H}_r + \varepsilon_{ljr} \mathcal{H}_i \mathcal{H}_k \mathcal{H}_r) + \frac{12a_2^4}{b_2(b_2^2 + a_2^2 \mathcal{H}^2)} \\
&\quad \left. \times \left(\mathcal{H}_i \mathcal{H}_j \mathcal{H}_k \mathcal{H}_l - \frac{1}{3} \mathcal{H}^2 \delta_{ij} \mathcal{H}_k \mathcal{H}_l - \frac{1}{3} \mathcal{H}^2 \delta_{kl} \mathcal{H}_i \mathcal{H}_j + \frac{1}{9} \mathcal{H}^4 \delta_{ij} \delta_{kl} \right) \right\}. \quad (6.15)
\end{aligned}$$

In Eq. (6.15) we have

$$a_2 = \frac{a^2}{20} \left(\frac{\pi}{mkT} \right)^{1/2} \frac{(13\kappa + 6)}{\kappa} \quad \text{and} \quad b_2 = \frac{2a^2}{25} \left(\frac{\pi}{mkT} \right)^{1/2} \frac{(35\kappa^2 + 33\kappa + 6)}{\kappa(\kappa + 1)^2}. \quad (6.16)$$

From Eqs. (6.12) and (6.15) it follows that

$$q_{\langle kl \rangle} = (A'^{-1})_{\langle kl \rangle \langle ij \rangle} \frac{\partial v_{\langle i}}{\partial x_{j \rangle}}. \quad (6.17)$$

Now with Eqs. (6.8), (6.15) and (6.17) we can write Eq. (6.10) in the form of Navier-Stokes law (6.3) with the shear viscosity tensor given by

$$\begin{aligned}
\eta_{\langle ij \rangle \langle kl \rangle} &= \frac{1}{4\alpha_3 (b_2^2 + 4a_2^2 \mathcal{H}^2)} \left\{ (b_2 \alpha_4 + 4a_2 \mathcal{H}^2) \left(\delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk} - \frac{2}{3} \delta_{ij} \delta_{kl} \right) \right. \\
&\quad + \frac{1}{2} (b_2 - a_2 \alpha_4) (\delta_{il} \varepsilon_{kjq} \mathcal{H}_q + \delta_{ik} \varepsilon_{ljq} \mathcal{H}_q + \delta_{jl} \varepsilon_{kiq} \mathcal{H}_q + \delta_{jk} \varepsilon_{liq} \mathcal{H}_q) \\
&\quad - 3a_2 \left(\delta_{il} \mathcal{H}_j \mathcal{H}_k + \delta_{ik} \mathcal{H}_j \mathcal{H}_l + \delta_{jl} \mathcal{H}_i \mathcal{H}_k + \delta_{jk} \mathcal{H}_i \mathcal{H}_l - \frac{4}{3} \delta_{ij} \mathcal{H}_k \mathcal{H}_l \right. \\
&\quad \left. - \frac{4}{3} \delta_{kl} \mathcal{H}_i \mathcal{H}_j + \frac{4}{9} \mathcal{H}^2 \delta_{ij} \delta_{kl} \right) \\
&\quad + \frac{3a_2^2 b_2}{b_2^2 + a_2^2 \mathcal{H}^2} \left[\alpha_4 \left(\delta_{il} \mathcal{H}_j \mathcal{H}_k + \delta_{ik} \mathcal{H}_j \mathcal{H}_l + \delta_{jl} \mathcal{H}_i \mathcal{H}_k + \delta_{jk} \mathcal{H}_i \mathcal{H}_l - \frac{4}{3} \delta_{ij} \mathcal{H}_k \mathcal{H}_l \right. \right. \\
&\quad \left. \left. - \frac{4}{3} \delta_{kl} \mathcal{H}_i \mathcal{H}_j + \frac{4}{9} \mathcal{H}^2 \delta_{ij} \delta_{kl} \right) \right. \\
&\quad \left. + (\varepsilon_{kiq} \mathcal{H}_j \mathcal{H}_l \mathcal{H}_q + \varepsilon_{liq} \mathcal{H}_j \mathcal{H}_k \mathcal{H}_q + \varepsilon_{kjq} \mathcal{H}_i \mathcal{H}_l \mathcal{H}_q + \varepsilon_{liq} \mathcal{H}_k \mathcal{H}_i \mathcal{H}_q) \right] \\
&\quad + \frac{3a_2^3}{b_2^2 + a_2^2 \mathcal{H}^2} \left[\alpha_4 (\varepsilon_{ilr} \mathcal{H}_j \mathcal{H}_k \mathcal{H}_r + \varepsilon_{ikr} \mathcal{H}_j \mathcal{H}_l \mathcal{H}_r + \varepsilon_{jlr} \mathcal{H}_k \mathcal{H}_i \mathcal{H}_r + \varepsilon_{jkr} \mathcal{H}_l \mathcal{H}_i \mathcal{H}_r) \right. \\
&\quad \left. + \mathcal{H}^2 \delta_{il} \mathcal{H}_j \mathcal{H}_k + \mathcal{H}^2 \delta_{ik} \mathcal{H}_l \mathcal{H}_j + \mathcal{H}^2 \delta_{jl} \mathcal{H}_i \mathcal{H}_k + \mathcal{H}^2 \delta_{jk} \mathcal{H}_i \mathcal{H}_l - 4 \mathcal{H}_i \mathcal{H}_j \mathcal{H}_k \mathcal{H}_l \right] \\
&\quad \left. + \frac{12a_2^4 \alpha_4}{b_2 (b_2^2 + a_2^2 \mathcal{H}^2)} \mathcal{H}_{\langle i} \mathcal{H}_{j \rangle} \mathcal{H}_{\langle k} \mathcal{H}_{l \rangle} \right\}. \quad (6.18)
\end{aligned}$$

Without loss of generality we put the magnetic field in the x -axis of a coordinate system in the gas. By neglecting the terms related to the divergence of the velocity we obtain the scheme of de Groot and Mazur [14] for the viscosity coefficients, which is shown in Table 1. The coefficients η_1 through η_5 are given by

Table 1. Shear viscosity coefficients

	$\frac{\partial v_{<x}}{\partial x_{>}}$	$\frac{\partial v_{<y}}{\partial x_{>}}$	$\frac{\partial v_{<z}}{\partial x_{>}}$	$\frac{\partial v_{<y}}{\partial x_{>}}$	$\frac{\partial v_{<z}}{\partial x_{>}}$	$\frac{\partial v_{<x}}{\partial x_{>}}$
$p_{<xx>}$	$-2\eta_1$	0	0	0	0	0
$p_{<yy>}$	0	$-2\eta_2$	$-2(\eta_1 - \eta_2)$	$-2\eta_4$	0	0
$p_{<zz>}$	0	$-2(\eta_1 - \eta_2)$	$-2\eta_2$	$2\eta_4$	0	0
$p_{<yz>}$	0	η_4	$-\eta_4$	$2\eta_1 - 4\eta_2$	0	0
$p_{<zx>}$	0	0	0	0	$-2\eta_3$	$-2\eta_5$
$p_{<xy>}$	0	0	0	0	$2\eta_5$	$-2\eta_3$

$$\eta_1 = \eta_0 = \frac{5}{8a^2} \left(\frac{mkT}{\pi} \right)^{1/2} \frac{(\kappa + 1)^2 (10\kappa + 3)}{(35\kappa^2 + 33\kappa + 6)}, \quad (6.19)$$

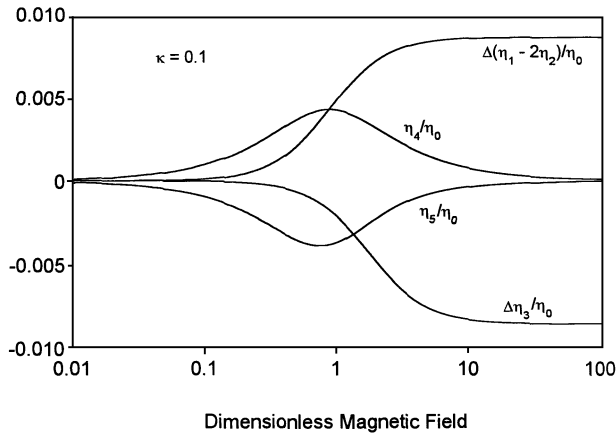
$$\eta_2 = \frac{1}{2\alpha_3 (b_2^2 + 4a_2^2 \mathcal{H}^2)} \left[b_2 \alpha_4 + 2a_2 \left(1 + \frac{a_2 b_2 \alpha_4}{b_2^2 + a_2^2 \mathcal{H}^2} \right) \mathcal{H}^2 + \frac{2a_2^4 \alpha_4}{b_2 (b_2^2 + a_2^2 \mathcal{H}^2)} \mathcal{H}^4 \right], \quad (6.20)$$

$$\eta_3 = \frac{1}{2\alpha_3 (b_2^2 + 4a_2^2 \mathcal{H}^2)} \left[b_2 \alpha_4 + a_2 \left(1 + \frac{3a_2^2 b_2 \alpha_4}{b_2^2 + a_2^2 \mathcal{H}^2} \right) \mathcal{H}^2 + \frac{3a_2^3}{b_2^2 + a_2^2 \mathcal{H}^2} \mathcal{H}^4 \right], \quad (6.21)$$

$$\eta_4 = \frac{(a_2 \alpha_4 - b_2)}{\alpha_3 (b_2^2 + 4a_2^2 \mathcal{H}^2)} \mathcal{H}, \quad (6.22)$$

$$\eta_5 = \frac{(b_2 - a_2 \alpha_4)}{2\alpha_3 (b_2^2 + 4a_2^2 \mathcal{H}^2)} \left(1 + \frac{3a_2^2}{b_2^2 + a_2^2 \mathcal{H}^2} \right) \mathcal{H}. \quad (6.23)$$

Therefore, from the twenty-five components of the tensor (6.18) only five are independent. η_1 , η_2 and η_3 are longitudinal components related to the direction of the gas flux while η_4 and η_5 are transversal components related to the directions mutually perpendicular both to the gas flux and magnetic field [7]. We observe that in this model the coefficient η_1 does not depend on the magnetic field. Besides, for the field-free condition we have for the longitudinal coefficients $\eta_1 = \eta_2 = \eta_3 = \eta_0$ and for the transversal coefficients $\eta_4 = \eta_5 = 0$. In Fig. 4 we have plotted the relative changes of shear viscosity coefficients for the value $\kappa = 0.1$ of the dimensionless moment of inertia. The parameter of the field is the dimensionless quantity $\mathcal{H} = \frac{\gamma m}{\rho a^2} \left(\frac{m}{\pi kT} \right)^{1/2} H$. These results agree with those of ref. [9].

**Fig. 4.** Relative changes of the shear viscosity coefficients

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