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ORIGINAL ARTICLE

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Moore–Gibson–Thompson thermoelasticity in the context of double porous materials

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Abstract The present study approaches the theory of Moore–Gibson–Thompson thermoelasticity in the context of the materials with double porosity structure. The main results of the present study are based on a reciprocity theorem for the thermoelastic materials with double porosity that leads us in determining of the uniqueness theorems for the solution of mixed problems for the materials with double porosity. The reciprocity theorem is a Betti-type result that has the main goal to establish the connection between the external action systems and their thermoelastic states. In order to obtain the uniqueness results, it was introduced a new form of energy equation.

Keywords MGT thermoelasticity · Double porosity bodies · Uniqueness

1 Introduction

In the last years, the Moore–Gibson–Thompson (MGT) equations have drawn the attention of many researchers. Although initially, the MGT equation modeled the physical phenomena from fluid mechanics, [1], in the last years it was approached under the thermoelasticity theory from the MGT perspective, where the heat transfer is governed by an integro-differential equation, [2]. The MGT thermoelasticity with two temperatures is studied in [3]. Many scholars approached this theory from a theoretical point of view [4–7] and also from a practical point of view [7–11]. The domain of influence under the MGT thermoelasticity theory was studied recently, [16,17]. Some papers deal with the linear elasticity theory for bodies with a dipolar structure [14,15]. In the present study, we considered the MGT theory in the context of materials with double porosity. The bodies with double porosity are encountered in many technical domains having applicability in building materials or geology, [18,19] as well as in medicine, having applications in the study of the bones [20].

The materials with double porosity structure present a high interest, fact that is proved by the numerous papers that deal this type of media, [21–28]. Also, this type of material was approached by us in some previous studies, [29–31].

The present paper is structured as follows. In Sect. 2 is defined the mixed problem for MGT thermoelasticity with double porosity structure. Section 3 highlights the main results of the present study that are based on a reciprocity theorem for the anisotropic thermoelastic materials with double porosity that leads us in determining

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of the uniqueness theorems for the solution of mixed problems for the materials with double porosity. The reciprocity theorem is a Betti-type result that has the main goal to establish the connection between the external action systems and their thermoelastic states. In order to obtain the uniqueness results, it was introduced a new form of energy equation.

2 Basic equations

The motion equations of a double porous material are:

$$\rho \ddot{u}_i = t_{ji,j} + \rho F_i. \tag{1}$$

The balance equations are:

$$\begin{aligned} \kappa_1 \ddot{\phi} &= \sigma_{j,j} + p + \rho M, \\ \kappa_2 \ddot{\psi} &= \tau_{j,j} + r + \rho N. \end{aligned} \tag{2}$$

In the equations that govern the linear thermoelasticity of the double porous material (1), (2), we have the following notations: ρ is the density of the body, κ_1, κ_2 are the equilibrated inertia coefficients, τ_j and σ_j are the stress vectors and t_{ij} is the stress tensor on the surface of the body $\partial\Omega$. The forces that act on the double porous body are:

- the direct forces F_i;
- the forces that act on the pores: p is the intrinsic force and M is the extrinsic force;
- the forces that act on the cracks: r is the intrinsic force and N is the extrinsic force.

In these equations, we have noted by dot the variation in report with time of the displacement field u_i and the volume fraction fields ϕ, ψ.

The differential Moore–Gibson–Thompson (MGT) equation for thermoelasticity was obtained by Quintanilla (MMS/2019) for the relaxation parameter $\gamma > 0$ in the context of Maxwell and Cattaneo heat conduction:

$$\gamma \cdot c(x) \frac{d^3\theta}{dt^3} + c(x) \frac{d^2\theta}{dt^2} = \left(K_{ij}(x)\theta_{,i} + K^*_{ij}(x)\theta_{,i} \right)_{,j}, \tag{3}$$

where K_{ij} is the conductivity rate tensor, θ is the temperature and c is the thermal capacity.

In order to have a mixed-initial boundary value problem, we have the initial conditions:

$$\theta(x,0) = \theta_0(x); \ \frac{d\theta}{dt}(x,0) = \theta_1(x); \ \frac{d^2\theta}{dt^2}(x,0) = \theta_2(x), \tag{4}$$

for all $x \in \Omega$, where Ω is the three dimensional domain that is considered smooth enough in order to apply the divergence theorem.

The Dirichlet boundary conditions are:

$$\theta(x,t) = 0, (\forall) x \in \partial\Omega, t > 0. \tag{5}$$

Further, we consider the following assumptions:

(a) The thermal capacity $c(x)$ is a positive function such as there is a positive constant $c_0 > 0 : c(x) \geq c_0 > 0$;

(b) For every vector ζ_i, there is a positive constant k^* such as:

$$K^*_{ij}\zeta_i\zeta_j \geq k^*\zeta_i\zeta_i; \tag{a.1}$$

(c) For every vector ζ_i, there is a positive constant $\tilde{k}$ such as

$$h_{ij}\zeta_i\zeta_j \geq \tilde{k}\zeta_i\zeta_i, \tag{a.2}$$

where $h_{ij}(x) = K_{ij}(x) - \gamma K^*_{ij}(x)$;

(d) For every vector ζ_i, there is a positive constant k^0 such as:

$$K_{ij}\zeta_i\zeta_j \geq k^0\zeta_i\zeta_i. \tag{a.3}$$

The deformations of the bodies with double porosity are given by the following variables: $u_i(x,t)$, $\phi(x,t)$, $\psi(x,t)$, $\theta(x,t)$, $(\forall)(x,t) \in \Omega \times [0,\infty)$.

The energy function has the following quadratic form in the context of the linear thermoelasticity for materials with double porosity structure:

$$\begin{aligned}\mathcal{E} =&\frac{1}{2}C_{ijkl}u_{k,l}u_{i,j} + B_{ij}\phi u_{i,j} + D_{ij}\psi u_{i,j}\\ &+ \frac{1}{2}\alpha_{ij}\phi_{,i}\phi_{,j} + b_{ij}\phi_{,i}\psi_{,j} + \frac{1}{2}\gamma_{ij}\psi_{,i}\psi_{,j} + \frac{1}{2}\alpha_1\phi^2\\ &+ \alpha_3\phi\psi + \frac{1}{2}\alpha_2\psi^2 - \frac{1}{2}c(\gamma\ddot{\theta} + \dot{\theta})^2 + \frac{1}{2}K_{ij}\theta_{,i}\theta_{,j}\\ &- (\beta_{ij}u_{i,j} + a_1\phi + a_2\psi)(\gamma\ddot{\theta} + \dot{\theta}).\end{aligned} \tag{6}$$

The constitutive equations for the bodies with double porosity in the context of linear thermoelasticity are:

$$\begin{aligned}t_{ij} =&C_{ijkl}u_{k,l} + B_{ij}\phi + D_{ij}\psi - \beta_{ij}(\gamma\ddot{\theta} + \dot{\theta});\\ \sigma_{ij} =&\alpha_{ij}\phi_{,j} + b_{ij}\psi_{,j};\\ \tau_i =&b_{ij}\phi_{,j} + \gamma_{ij}\psi_{,j};\\ p =&- B_{ij}u_{i,j} - \alpha_1\phi - \alpha_3\psi + a_1((\gamma\ddot{\theta} + \dot{\theta});\\ r =&- D_{ij}u_{i,j} - \alpha_3\phi - \alpha_2\psi + a_2(\gamma\ddot{\theta} + \dot{\theta});\\ \eta =&\beta_{ij}u_{i,j} + a_1\phi + a_2\psi + c(\gamma\ddot{\theta} + \dot{\theta});\\ q_i =&K_{ij}\theta_{,j}.\end{aligned} \tag{7}$$

The relations (7) are obtained based on the quadratic form of internal energy (6), i.e.,

$$\begin{aligned}&t_{ij} = \frac{\partial\mathcal{E}}{\partial u_{i,j}};\ \sigma_{ij} = \frac{\partial\mathcal{E}}{\partial\phi_{,i}};\ \tau_i = \frac{\partial\mathcal{E}}{\partial\psi_{,i}};\ \xi = -\frac{\partial\mathcal{E}}{\partial\phi};\ \zeta = -\frac{\partial\mathcal{E}}{\partial\psi};\\ &\eta = -\frac{\partial\mathcal{E}}{\partial(\gamma\ddot{\theta} + \dot{\theta})};\ q_i = \frac{\partial\mathcal{E}}{\partial\theta_{,i}}.\end{aligned}$$

The energy equation is:

$$\rho T_0\dot{\eta} = q_{i,i} + \rho\delta, \tag{8}$$

where δ is the heat supply, q_i is the heat flux, η is the entropy and T_0 is the absolute temperature that is considered constant in the reference configuration. Taking into account the constitutive equation regarding the entropy $(7)_6$ and the flux equation $(7)_7$, the energy equation will have the following form:

$$\rho T_0\left[\beta_{ij}\dot{u}_{i,j} + a_1\dot{\phi} + a_2\dot{\psi} + c(\gamma\dddot{\theta} + \ddot{\theta})\right] = (K_{ij}\theta_{,j})_{,i} + \rho\delta. \tag{9}$$

In the context of MGT theory of thermoelasticity for materials with double porosity structure, it is necessary to insert the initial conditions for $t = 0$:

$$\begin{aligned}&u_i(x,0) = 0;\ \phi(x,0) = 0;\ \psi(x,0) = 0;\ \theta(x,0) = 0;\\ &\dot{u}_i(x,0) = 0;\ \dot{\phi}(x,0) = 0;\ \dot{\psi}(x,0) = 0;\ \dot{\theta}(x,0) = 0;\ \ddot{\theta}(x,0) = 0.\end{aligned} \tag{10}$$

The mixed problem for the MGT theory has the boundary conditions:

$$\begin{aligned}&u_i = u_i^* \text{ on } \partial\Omega_1 \times [0,t^*);\ t_i = t_i^* \text{ on } \partial\Omega_1^c \times [0,t^*)\\ &\phi = \phi^* \text{ on } \partial\Omega_2 \times [0,t^*);\ \lambda = \lambda^* \text{ on } \partial\Omega_2^c \times [0,t^*)\\ &\psi = \psi^* \text{ on } \partial\Omega_3 \times [0,t^*);\ m = m^* \text{ on } \partial\Omega_3^c \times [0,t^*)\\ &\theta = \theta^* \text{ on } \partial\Omega_4 \times [0,t^*);\ \nu = \nu^* \text{ on } \partial\Omega_4^c \times [0,t^*)\end{aligned} \tag{11}$$

where u_i^*, t_i^*, ϕ^*, ψ^*, m^*, θ^*, ν^* are prescribed functions for a specific moment of time t^* that can be finite or infinite.

The boundary of the domain Ω is divided in four surfaces $\partial\Omega_1$, $\partial\Omega_2$, $\partial\Omega_3$, $\partial\Omega_4$. The complement of the considered surface is noted by superscript index "c" such that: $\partial\Omega_i \cap \partial\Omega_i^c = \emptyset$ and $\partial\Omega_i \cup \partial\Omega_i^c = \partial\Omega$, $\forall i = \overline{1,4}$.

The components of the surface force traction, surface couple and flux are expressed by:

$$t_i = t_{ij}n_i;\;\; \lambda = \sigma_i n_i;\;\; m = \tau_i n_i;\;\; \nu = q_i n_i. \tag{12}$$

In the context of MGT theory of thermoelasticity for materials with double porosity structure, the mixed problem is formed from Eqs. (1), (2), (7) and (8) with initial conditions (10) and the boundary conditions (11).

The solution of the considered problem is the ordered array $(u_i, \phi, \psi, \theta)$ and it is represented by the effect of the external actions (E_A) on the system that are defined by the ordered array:

$$E_A = (F_i, M, N, \delta, u_i^*, \phi^*, \psi^*, \theta^*, t_i^*, \lambda^*, m^*, \nu^*).$$

The thermoelastic state (T_S) generated by the external forces is defined by:

$$T_S = (u_i, \phi, \psi, \theta, t_{ij}, \sigma_i, \tau_i, p, r, q_i, \eta).$$

Further, we will consider that on the double porous material act two different loading systems:

$$E_A^{(i)} = (F_i^{(i)}, M^{(i)}, N^{(i)}, \delta^{(i)}, u_i^{*(i)}, \phi^{*(i)}, \psi^{*(i)}, \theta^{*(i)}, t_i^{*(i)}, \lambda^{*(i)}, m^{*(i)}, \nu^{*(i)}), i = \overline{1,2},$$

and for each loading system, we have two thermoelastic states:

$$T_S^{(i)} = (u_i^{(i)}, \phi^{(i)}, \psi^{(i)}, \theta^{(i)}, t_{ij}^{(i)}, \sigma_i^{(i)}, \tau_i^{(i)}, p^{(i)}, r^{(i)}, q_i^{(i)}, \eta^{(i)}), i = \overline{1,2}.$$

3 Reciprocity and uniqueness theorems

In this section, we will enunciate some reciprocity theorems that are of Betti's type. These theorems are useful in order to determine a reciprocity relation between the external action systems and their corresponding thermoelastic states. Further, we consider the convolution product $*$.

Theorem 1 *Let us consider the external action systems $E_A^{(i)}$ for two different types of loadings and their thermoelastic states $T_S^{(i)}$, respectively. The following reciprocity relation takes place:*

$$\begin{aligned}
&\int_\Omega \rho(F_i^{(1)}*u_i^{(2)} - F_i^{(2)}*u_i^{(1)} + M^{(1)}*\phi^{(2)} - M^{(2)}*\phi^{(1)} + N^{(1)}*\psi^{(2)} - N^{(2)}*\phi^{(1)})dV \\
&\quad - \int_{\partial\Omega_1^c} (t_i^{*(2)}*u_i^{(1)} - t_i^{*(1)}*u_i^{(2)})dA - \int_{\partial\Omega_1} (t_{ji}^{(2)}*u_i^{*(1)} - t_{ji}^{(1)}*u_i^{*(2)})n_j dA \\
&\quad - \int_{\partial\Omega_2^c} (\lambda^{*(2)}*\phi^{(1)} - \lambda^{*(1)}*\phi^{(2)})dA - \int_{\partial\Omega_2} (\sigma_j^{(2)}*\phi^{*(1)} - \sigma_j^{(1)}*\phi^{*(2)})n_j dA \\
&\quad - \int_{\partial\Omega_3^c} (m^{*(2)}*\psi^{(1)} - m^{*(1)}*\psi^{(2)})dA - \int_{\partial\Omega_3} (\tau_j^{(2)}*\psi^{*(1)} - \tau_j^{(1)}*\psi^{*(2)})n_j dA \\
&= \frac{1}{\rho T_0}\left\{\int_{\partial\Omega_4^c} \left[\nu^{*(1)} * \left(\theta^{(2)} + \alpha\frac{\partial\theta^{(2)}}{dt}\right) - \nu^{*(2)} * \left(\theta^{(1)} + \alpha\frac{\partial\theta^{(1)}}{dt}\right)\right] dA\right. \\
&\quad + \int_{\partial\Omega_4} \left[q_i^{(1)} * \left(\theta^{*(2)} + \alpha\frac{\partial\theta^{*(2)}}{dt}\right) - q_i^{(2)} * \left(\theta^{*(1)} + \alpha\frac{\partial\theta^{*(1)}}{dt}\right)\right] n_i dA
\end{aligned}$$

$$+\int_{\partial\Omega}\left[q_i^{(2)}*\left(\theta_{,i}^{(1)}+\alpha\frac{\partial\theta_{,i}^{(1)}}{dt}\right)-q_i^{(1)}*\left(\theta_{,i}^{(2)}+\alpha\frac{\partial\theta_{,i}^{(2)}}{dt}\right)\right]dV$$
$$\left.+\rho\int_{\partial\Omega}\left[\delta^{(1)}*\left(\theta^{(2)}+\alpha\frac{\partial\theta^{(2)}}{dt}\right)-\delta^{(2)}*\left(\theta^{(1)}+\alpha\frac{\partial\theta^{(1)}}{dt}\right)\right]dV\right\}. \tag{13}$$

Proof In order to obtain the reciprocity relation of Betti type, we will apply the Laplace transform to Eqs. (1) and (2). Let us consider that the image through the Laplace transform of the original function $f(x,t)$ will be $\overline{f}(x,t)$. Therefore, the governing equations for the double porous materials will have the following form:

$$\begin{aligned}
&\rho s^2\overline{u}_i^{(i})=\overline{t}_{ji,j}^{(i)}+\rho\overline{F}_i^{(i)}\\
&\kappa_1 s^2\overline{\phi}^{(i)}=\overline{\sigma}_{j,j}^{(i)}+\overline{p}^{(i)}+\rho\overline{M}^{(i)}, \qquad i=\overline{1,2}\\
&\kappa_2 s^2\overline{\psi}^{(i)}=\overline{\tau}_{j,j}^{(i)}+\overline{r}^{(i)}+\rho\overline{N}^{(i)}.
\end{aligned} \tag{14}$$

The energy equation (9) through the Laplace transform will be:

$$\rho T_0\left[\beta_{ij}s\overline{u}_i^{(i)}+a_1 s\overline{\phi}^{(i)}+a_2 s\overline{\psi}^{(i)}+c(\gamma s^3\overline{\theta}^{(i)}+s^2\overline{\theta}^{(i)})\right]=K_{ij}\overline{\theta}_{,ij}^{(i)}+\rho\overline{\delta}^{(i)}, i=\overline{1,2}. \tag{15}$$

The constitutive equations through the Laplace transform will have the following form:

$$\begin{aligned}
\overline{t}_{ij}^{(i)}&=C_{ijkl}\overline{u}_{k,l}^{(i)}+B_{ij}\overline{\phi}^{(i)}+D_{ij}\overline{\psi}^{(i)}-\beta\left(\gamma s^2\overline{\theta}^{(i)}+s\overline{\theta}^{(i)}\right)\\
\overline{\sigma}_{ij}^{(i)})&=\alpha_{ij}\overline{\phi}_{,j}^{(i)})+b_{ij}\overline{\psi}_{,j}^{(i)}\\
\overline{\tau}_i^{(i)}&=b_{ij}\overline{\phi}_{,j}^{(i)}+\gamma_{ij}\overline{\psi}_{,j}^{(i)}\\
\overline{p}^{(i)}&=-B_{ij}\overline{u}_{i,j}^{(i)}-\alpha_1\overline{\phi}^{(i)}-\alpha_3\overline{\psi}^{(i)}+a_1\left(\gamma s^2\overline{\theta}^{(i)}+s\overline{\theta}^{(i)}\right)\\
\overline{r}^{(i)}&=-D_{ij}\overline{u}_{i,j}^{(i)}-\alpha_3\overline{\phi}^{(i)}-\alpha_2\overline{\psi}^{(i)}+a_2\left(\gamma s^2\overline{\theta}^{(i)}+s\overline{\theta}^{(i)}\right)\\
\overline{\eta}^{(i)}&=\beta_{ij}\overline{u}_{i,j}^{(i)}+a_1\overline{\phi}^{(i)}+a_2\overline{\psi}^{(i)}+c\left(\gamma s^2\overline{\theta}^{(i)}+s\overline{\theta}^{(i)}\right)\\
\overline{q}_i^{(i)}&=K_{ij}\overline{\theta}_{,j}^{(i)}.
\end{aligned} \tag{16}$$

The boundary conditions (11) through the Laplace transform become:

$$\begin{aligned}
&\overline{u}_i=\overline{u}_i^{*(i)} \text{ on } \partial\Omega_1\times[0,t^*);\ \overline{t}_i=\overline{t}_i^{*(i)} \text{ on } \partial\Omega_1^c\times[0,t^*)\\
&\overline{\phi}=\overline{\phi}^{*(i)} \text{ on } \partial\Omega_2\times[0,t^*);\ \overline{\lambda}=\overline{\lambda}^{*(i)} \text{ on } \partial\Omega_2^c\times[0,t^*)\\
&\overline{\psi}=\overline{\psi}^{*(i)} \text{ on } \partial\Omega_3\times[0,t^*);\ \overline{m}=\overline{m}^{*(i)} \text{ on } \partial\Omega_3^c\times[0,t^*)\\
&\overline{\theta}=\overline{\theta}^{*(i)} \text{ on } \partial\Omega_4\times[0,t^*);\ \overline{v}=\overline{v}^{*(i)} \text{ on } \partial\Omega_4^c\times[0,t^*).
\end{aligned} \tag{17}$$

Writing Eq. $(14)_1$ for each loading and multiplying with $\overline{u}_i^{(2)}$ and $\overline{u}_i^{(1)}$, respectively, and after that subtract and integrate on the considered domain, we obtain:

$$\begin{aligned}
&\int_{\Omega}\rho\left(\overline{F}_i^{(1)}\overline{u}_i^{(2)}-\overline{F}_i^{(2)}\overline{u}_i^{(1)}\right)dV\\
&=\int_{\Omega}\left(\overline{t}_{ji}^{(2)}\overline{u}_i^{(1)}-\overline{t}_{ji}^{(1)}\overline{u}_i^{(2)}\right)_{,j}dV+\int_{\Omega}\left(\overline{t}_{ji}^{1)}\overline{u}_{i,j}^{(2)}-\overline{t}_{ji}^{(2)}\overline{u}_{i,j}^{(1)}\right)dV.
\end{aligned} \tag{18}$$

In the first integral from the right side of (18), we apply the divergence theorem, and in the second integral from the right side of (18), we will take into account the constitutive Eq. (16). Therefore, Eq. (18) will have the following form:

$$\begin{aligned}&\int_{\Omega} \rho\left(\overline{F}_i^{(1)}\overline{u}_i^{(2)}-\overline{F}_i^{(2)}\overline{u}_i^{(1)}\right)dV\\&=\int_{\partial\Omega^c}\left(\overline{t}_i^{*(2)}\overline{u}_i^{(1)}-\overline{t}_i^{*(1)}\overline{u}_i^{(2)}\right)dA+\int_{\partial\Omega_1}\left(\overline{t}_{ji}^{(2)}u_i^{*(1)}-\overline{t}_{ji}^{(1)}u_i^{*(2)}\right)n_j dA\\&+\int_{\Omega}\Big[B_{ij}\left(\overline{\phi}^{(1)}\overline{u}_{i,j}^{(2)}-\overline{\phi}^{(1)}\overline{u}_{i,j}^{(2)}\right)+D_{ij}\left(\overline{\psi}^{(1)}\overline{u}_{i,j}^{(2)}-\overline{\psi}^{(1)}\overline{u}_{i,j}^{(2)}\right)\\&-\beta_{ij}s(1+\gamma s)\left(\overline{\theta}^{(1)}\overline{u}_{i,j}^{(2)}-\overline{\theta}^{(2)}\overline{u}_{i,j}^{(1)}\right)\Big]dV.\end{aligned}\tag{19}$$

Next, we write Eq. $(14)_2$ for the both loading systems and compute the subtraction after each equation is multiplied by $\overline{\phi}^{(2)}, \overline{\phi}^{(1)}$, respectively. Integrating on Ω, we obtain:

$$\begin{aligned}&\int_{\Omega}\rho\left(\overline{M}^{(1)}\overline{\phi}^{(2)}-\overline{M}^{(2)}\overline{\phi}^{(1)}\right)dV\\&=\int_{\Omega}\left(\overline{\sigma}_{j,j}^{(2)}\overline{\phi}^{(1)}-\overline{\sigma}_{j,j}^{(1)}\overline{\phi}^{(2)}\right)dV+\int_{\Omega}\left(\overline{p}^{(2)}\overline{\phi}^{(1)}-\overline{p}^{(1)}\overline{\phi}^{(2)}\right)dV.\end{aligned}\tag{20}$$

In the first integral from the right side, we apply the divergence theorem, and for both integrals, we take into account the constitutive equations. Therefore, the relation (20) will have the following form:

$$\begin{aligned}&\int_{\Omega}\rho\left(\overline{M}^{(1)}\overline{\phi}^{(2)}-\overline{M}^{(2)}\overline{\phi}^{(1)}\right)dV=\int_{\partial\Omega_2^c}\left(\overline{\lambda}^{*(2)}\overline{\phi}^{(1)}-\overline{\lambda}^{*(1)}\overline{\phi}^{(2)}\right)dA\\&+\int_{\partial\Omega_2}\left(\overline{\sigma}_j^{(2)}\overline{\phi}^{*(1)}-\overline{\sigma}_j^{(1)}\overline{\phi}^{*(2)}\right)n_j dA+\int_{\Omega}b_{ij}\left(\overline{\psi}_{,j}^{(1)}\overline{\phi}_{,j}^{(2)}-\overline{\psi}_{,j}^{(2)}\overline{\phi}_{,j}^{(1)}\right)dV\\&-\int_{\Omega}\Big[B_{ij}\left(\overline{u}_{i,j}^{(2)}\overline{\phi}^{(1)}-\overline{u}_{i,j}^{(1)}\overline{\phi}^{(2)}\right)+\alpha_3\left(\overline{\psi}^{(2)}\overline{\phi}^{(1)}-\overline{\psi}^{(1)}\overline{\phi}^{(2)}\right)\\&-a_1 s(\gamma s+1)\left(\overline{\theta}^{(2)}\overline{\phi}^{(1)}-\overline{\theta}^{(1)}\overline{\phi}^{(2)}\right)\Big]dV.\end{aligned}\tag{21}$$

At last, we multiply the equations from $(14)_3$ with $\overline{\psi}^{(2)}$ and $\overline{\psi}^{(1)}$ written for each load of the system, making the subtraction and integrate on the considered domain. Now we have:

$$\begin{aligned}&\int_{\Omega}\rho\left(\overline{N}^{(1)}\overline{\psi}^{(2)}-\overline{N}^{(2)}\overline{\psi}^{(1)}\right)dV\\&=\int_{\Omega}\left(\overline{\tau}_{j,j}^{(2)}\overline{\psi}^{(1)}-\overline{\tau}_{j,j}^{(1)}\overline{\psi}^{(2)}\right)dV+\int_{\Omega}\left(\overline{r}^{(2)}\overline{\psi}^{(1)}-\overline{r}^{(1)}\overline{\psi}^{(2)}\right)dV.\end{aligned}\tag{22}$$

Using the divergence theorem and the constitutive equations and taking into account the boundary conditions (11), the relation (22) will have the form:

$$\begin{aligned}&\int_{\Omega}\rho\left(\overline{N}^{(1)}\overline{\psi}^{(2)}-\overline{N}^{(2)}\overline{\psi}^{(1)}\right)dV=\int_{\partial\Omega_3^c}\left(\overline{m}^{*(2)}\overline{\psi}^{(1)}-\overline{m}^{*(1)}\overline{\psi}^{(2)}\right)dA\\&+\int_{\partial\Omega_3}\left(\overline{\tau}_j^{(2)}\overline{\psi}^{*(1)}-\overline{\tau}_j^{(1)}\overline{\psi}^{*(2)}\right)n_j dA+\int_{\Omega}b_{ij}\left(\overline{\phi}_{,j}^{(2)}\overline{\psi}_{,j}^{(1)}-\overline{\phi}_{,j}^{(1)}\overline{\psi}_{,j}^{(2)}\right)dV\end{aligned}$$

$$-\int_{\Omega}\left[D_{ij}\left(\overline{u}_{i,j}^{(2)}\overline{\psi}^{(1)}-\overline{u}_{i,j}^{(1)}\overline{\psi}^{(2)}\right)+\alpha_3\left(\overline{\phi}^{(2)}\overline{\psi}^{(1)}-\overline{\phi}^{(1)}\overline{\psi}^{(2)}\right)\right.$$
$$\left.-a_2 s(\gamma s+1)\left(\overline{\theta}^{(2)}\overline{\psi}^{(1)}-\overline{\theta}^{(1)}\overline{\psi}^{(2)}\right)\right]dV. \tag{23}$$

Next, we write the energy equation (15) for both loadings and we multiply them by $\overline{\theta}^{(2)}$ and $\overline{\theta}^{(1)}$, respectively, and after that subtracting the relations and integrating on the domain Ω, we have:

$$\int_{\Omega}\left[\beta_{ij}s\left(\overline{u}_{i,j}^{(1)}\overline{\theta}^{(2)}-\overline{u}_{i,j}^{(2)}\overline{\theta}^{(1)}\right)+a_1 s\left(\overline{\phi}^{(1)}\overline{\theta}^{(2)}-\overline{\phi}^{(2)}\overline{\theta}^{(1)}\right)\right.$$
$$\left.+a_2 s\left(\overline{\psi}^{(1)}\overline{\theta}^{(2)}-\overline{\psi}^{(2)}\overline{\theta}^{(1)}\right)\right]dV$$
$$=\frac{1}{\rho T_0}\int_{\Omega}K_{ij}\left(\overline{\theta}_{,ij}^{(1)}\overline{\theta}^{(2)}-\overline{\theta}_{,ij}^{(2)}\theta^{(1)}\right)dV+\frac{1}{T_0}\int_{\Omega}\left(\overline{\delta}^{(1)}\overline{\theta}^{(2)}-\overline{\delta}^{(2)}\overline{\theta}^{(1)}\right)dV. \tag{24}$$

Using the divergence theorem and taking into account the boundary conditions, the relation (24) becomes:

$$\int_{\Omega}\left[\beta_{ij}s\left(\overline{u}_{i,j}^{(1)}\overline{\theta}^{(2)}-\overline{u}_{i,j}^{(2)}\overline{\theta}^{(1)}\right)+a_1 s\left(\overline{\phi}^{(1)}\overline{\theta}^{(2)}-\overline{\phi}^{(2)}\overline{\theta}^{(1)}\right)\right.$$
$$\left.+a_2 s\left(\overline{\psi}^{(1)}\overline{\theta}^{(2)}-\overline{\psi}^{(2)}\overline{\theta}^{(1)}\right)\right]dV$$
$$=\frac{1}{\rho T_0}\int_{\partial\Omega_4^c}\left(\overline{\theta}^{(2)}\overline{v}^{*(1)}-\overline{\theta}^{(1)}\overline{v}^{*(2)}\right)dA+\frac{1}{\rho T_0}\int_{\partial\Omega_4}\left(\overline{\theta}^{*(2)}\overline{q}_i^{(1)}-\overline{\theta}^{*(1)}\overline{q}_i^{(2)}\right)n_i dA$$
$$-\frac{1}{\rho T_0}\int_{\Omega}\left(\overline{\theta}_{,i}^{(2)}\overline{q}_i^{(1)}-\overline{\theta}_{,i}^{(1)}\overline{q}_i^{(2)}\right)dV+\frac{1}{T_0}\int_{\Omega}\left(\overline{\delta}^{(1)}\overline{\theta}^{(2)}-\overline{\delta}^{(2)}\overline{\theta}^{(1)}\right)dV. \tag{25}$$

We may observe that in Eq. (25) appear the terms from (19), (21) and (23); therefore, the relation (25) will take the following form:

$$\int_{\Omega}\left[\rho\left(\overline{F}_i^{(1)}\overline{u}_i^{(2)}-\overline{F}_i^{(2)}\overline{u}_i^{(1)}\right)+\rho\left(\overline{M}^{(1)}\overline{\phi}^{(2)}-\overline{M}^{(2)}\overline{\phi}^{(1)}\right)+\rho\left(\overline{N}^{(1)}\overline{\psi}^{(2)}-\overline{N}^{(2)}\overline{\psi}^{(1)}\right)\right]dV$$
$$-\int_{\partial\Omega_1^c}\left(\overline{t}_i^{*(2)}\overline{u}_i^{(1)}-\overline{t}_i^{*(1)}\overline{u}_i^{(2)}\right)dA-\int_{\partial\Omega_1}\left(\overline{t}_{ji}^{(2)}\overline{u}_i^{*(1)}-\overline{t}_{ji}^{(1)}\overline{u}_i^{*(2)}\right)n_j dA$$
$$-\int_{\partial\Omega_2^c}\left(\overline{\lambda}^{*(2)}\overline{\phi}^{(1)}-\overline{\lambda}^{*(1)}\overline{\phi}^{(2)}\right)dA-\int_{\partial\Omega_2}\left(\overline{\sigma}_j^{(2)}\overline{\phi}^{*(1)}-\overline{\sigma}_j^{(1)}\overline{\phi}^{*(2)}\right)n_j dA$$
$$-\int_{\partial\Omega_3^c}\left(\overline{m}^{*(2)}\overline{\psi}^{(1)}-\overline{m}^{*(1)}\overline{\psi}^{(2)}\right)dA-\int_{\partial\Omega_3}\left(\overline{\tau}_j^{(2)}\overline{\psi}^{*(1)}-\overline{\tau}_j^{(1)}\overline{\psi}^{*(2)}\right)n_j dA$$
$$=\frac{1+\gamma s}{\rho T_0}\left[\int_{\partial\Omega_4^c}\left(\overline{\theta}^{(2)}\overline{v}^{*(1)}-\overline{\theta}^{(1)}\overline{v}^{*(2)}\right)dA+\int_{\partial\Omega_4}\left(\overline{\theta}^{*(2)}\overline{q}_i^{*(1)}-\overline{\theta}^{*(1)}\overline{q}_i^{*(2)}\right)n_i dA\right.$$
$$\left.+\int_{\Omega}\left(\overline{\theta}_{,i}^{(1)}\overline{q}_i^{(2)}-\overline{\theta}_{,i}^{(2)}\overline{q}_i^{(1)}\right)dV+\int_{\Omega}\rho\left(\overline{\delta}^{(1)}\overline{\theta}^{(2)}-\overline{\delta}^{(2)}\overline{\theta}^{(1)}\right)dV\right]. \tag{26}$$

In the relation (26), we apply the inverse Laplace transform $\mathcal{L}$ and the convolution product noted by $*$ is obtained. Thus, we use:

$$\overline{F}_i^{(1)}\overline{u}_i^{(2)}=\mathcal{L}[F_i^{(1)}](s)\cdot\mathcal{L}[u_i^{(2)}](s)=F_i^{(1)}*u_i^{(2)}$$

$$s\overline{\theta}^{(2)}\nu^{*(1)} = \mathcal{L}[\nu^{*(1)}](s)\cdot\mathcal{L}\left[\frac{\partial\theta^{(2)}}{\partial t}\right](s) = \nu^{*(1)} * \frac{\partial\theta^{(2)}}{\partial t}.$$

The result from Theorem 1 is obtained. □

Based on the theory of Green and Lindsay, let us introduce the scalar function:

$$\Phi = T_0 + \dot{\theta} + \gamma\ddot{\theta} + \dot{\theta}\ddot{\theta} + \frac{1}{2}d\ddot{\theta}^2. \tag{27}$$

The energy function of Moore–Gibson–Thompson thermoelasticity will have the expression:

$$\mathbb{E} = E_i - \eta\Phi, \tag{28}$$

where according to Biot, the generalized free energy function $\mathcal{E}$ is given by:

$$\mathcal{E} = E_i - \eta T_0, \tag{29}$$

where E_i is the internal energy. Taking into account the initial conditions of the MGT thermoelasticity from (10), the energy function $\mathbb{E}$ will have the following quadratic expression:

$$\begin{aligned}\mathbb{E}=&\frac{1}{2}C_{ijkl}u_{i,j}u_{k,l}+B_{ij}\phi u_{i,j}+D_{ij}\psi u_{i,j}+\frac{1}{2}\phi_{,i}\phi_{,j}+b_{ij}\phi_{,i}\psi_{,j}+\frac{1}{2}\gamma_{ij}\psi_{,i}\psi_{,j}\\&+\frac{1}{2}\alpha_1\phi^2+\alpha_3\phi\psi+\frac{1}{2}\alpha_2\psi^2+\frac{1}{2}K_{ij}\theta_{,i}\theta_{,j}-A\dot{\theta}^2-B\dot{\theta}\ddot{\theta}+C\ddot{\theta}^2,\end{aligned} \tag{30}$$

where $A = \frac{3}{2}c,\ B = \frac{3}{2}c\gamma + b\mathcal{M}, C = \frac{3}{2}c\gamma^2 + \frac{d}{2}\mathcal{M}$ and $\mathcal{M} = \beta_{ij}u_{i,j} + a_1\phi + a_2\psi$. The kinetic energy per mass unit is given by:

$$E_{\mathbb{K}}(t) = \frac{1}{2}\left[\rho\dot{u}_i(t)\dot{u}_i(t) + \kappa_1\dot{\phi}(t)\dot{\phi}(t) + \kappa_2\dot{\psi}(t)\dot{\psi}(t)\right]. \tag{31}$$

Based on the relation of function of energy (30) and kinetic energy (31), we can enunciate the following theorem:

Theorem 2 *The variation of energy in the MGT thermoelasticity for double porous materials is expressed by:*

$$\begin{aligned}\frac{d}{dt}\int_{\Omega}(E_{\mathbb{K}}+\mathbb{E})dV=&\rho\int_{\Omega}\left(F_i\dot{u}_i + M\dot{\phi} + N\dot{\psi}\right)dV + \int_{\partial\Omega}\left(t_{ji}\dot{u}_i+\sigma_j\dot{\phi}+\tau_j\dot{\psi}\right)n_j dA\\&+\int_{\Omega}\left(q_i\dot{\theta}_{,j} + \frac{1}{\rho T_0}(q_{i,i}+\rho\delta)(\gamma\ddot{\theta}+\dot{\theta})\right)dV\\&-\int_{\Omega}\left(\frac{2}{3}A(\gamma\ddot{\theta}+\dot{\theta})(\gamma\dddot{\theta}+\ddot{\theta})+(2A+B)\dot{\theta}\ddot{\theta}+B\ddot{\theta}^2+2C\ddot{\theta}\cdot\dddot{\theta}\right)dV.\end{aligned} \tag{32}$$

Proof The variation of the kinetic energy in report with time is:

$$\dot{E}_{\mathbb{K}} = \rho\dot{u}_i(t)\ddot{u}_i(t) + \kappa_1\dot{\phi}(t)\ddot{\phi}(t) + \kappa_2\dot{\psi}(t)\ddot{\psi}(t). \tag{33}$$

The terms from (34) are obtained from the governing equations for the double porous bodies (1), (2), multiplied by $\dot{u}_i$, $\dot{\phi}$ and $\dot{\psi}$, respectively. Therefore, by integration on the Ω domain, using the divergence theorem and taking into account the constitutive Eq. (7), Eq. (34) takes the following form:

$$\begin{aligned}\int_{\Omega}\dot{E}_{\mathbb{K}}(t)dt =& \int_{\Omega}\rho(F_i\dot{u}_i + M\dot{\phi} + N\dot{\psi})dV + \int_{\partial\Omega}(t_{ji}\dot{u}_i + \sigma_j\dot{\phi} + \tau_j\dot{\psi})n_j dA\\&-\int_{\Omega}\left[B_{ij}u_{i,j}\dot{\phi} + \alpha_1\phi\dot{\phi} + \alpha_3\psi\dot{\phi} - a_1(\gamma\ddot{\theta}+\dot{\theta})\dot{\phi}\right.\end{aligned}$$

$$+ D_{ij}u_{i,j}\dot{\psi} + \alpha_3\phi\dot{\psi} + \alpha_2\psi\dot{\psi} - a_2(\gamma\ddot{\theta} + \dot{\theta})\dot{\psi}$$
$$+ C_{ijkl}u_{k,l}\dot{u}_{i,j} + B_{ij}\phi\dot{u}_{i,j} + D_{ij}\psi\dot{u}_{i,j} - \beta\dot{u}_{i,j}(\gamma\ddot{\theta} + \dot{\theta})$$
$$+\alpha_{ij}\phi_{,j}\dot{\phi}_{,j} + b_{ij}\psi_{,j}\dot{\phi}_{,j} + b_{ij}\phi_{,j}\dot{\psi}_{,j} + \gamma_{ij}\psi_{,j}\dot{\psi}_{,j}\big]dV. \tag{34}$$

Integrating on the domain Ω the variation of the function of energy (30) in report with time, we have:

$$\int_\Omega \dot{\mathbb{E}}(t)dV = \int_\Omega \big[C_{ijkl}u_{i,j}\dot{u}_{k,l} + B_{ij}\dot{\phi}u_{i,j} + B_{ij}\phi\dot{u}_{i,j} + D_{ij}\dot{\psi}u_{i,j} + D_{ij}\psi\dot{u}_{i,j} + \alpha_{ij}\phi_{,i}\dot{\phi}_{,j}$$
$$+ \alpha_3\phi\dot{\psi} + \alpha_3\dot{\phi}\psi + b_{ij}\dot{\phi}_{,i}\psi_{,j} + b_{ij}\phi_{,i}\dot{\psi}_{,j} + \gamma_{ij}\phi_{,i}\dot{\psi}_{,j} + \alpha_1\phi\dot{\phi}$$
$$+\alpha_2\psi\dot{\psi} + K_{ij}\theta_{,i}\dot{\theta}_{,j} - 2A\dot{\theta}\ddot{\theta} - B\ddot{\theta}^2 - b\dot{\theta}\dddot{\theta} + 2C\ddot{\theta}\cdot\dddot{\theta}\big]dV. \tag{35}$$

Taking into account the energy equation given in (9), we have:

$$\beta_{ij}\dot{u}_{i,j} + a_1\dot{\phi} + a_2\dot{\psi} = \frac{1}{\rho T_0}(q_{i,i} + \rho\delta) - c(\gamma\dddot{\theta} + \ddot{\theta}).$$

Summing the relations (34) and (35) on the domain Ω, we obtain the variation of the energy in the context of the MGT thermoelasticity for double porous materials. Therefore, the result of Theorem 2 is obtained. □

Theorem 3 *If the energy function $\mathbb{E}$ from (30) is positive defined, then the considered mixed problem for double porous materials in the MGT thermoelasticity contexts admits only one solution.*

Proof In order to prove the uniqueness of the solution of the mixed problem for double porous materials in the context of MGT thermoelasticity, we assume that the considered problem admits two solutions: $(u_{i1}, \phi_1, \psi_1, \theta_1)$ and $(u_{i2}, \phi_2, \psi_2, \theta_2)$.

For null loadings F_i, M and N and for zero boundary conditions: $u_i^* = 0, \phi^* = 0, \psi^* = 0, t_i^* = 0, \lambda^* = 0, m^* = 0, \nu^* = 0$, the difference between these solutions $(u_{i1} - u_{i2}, \phi_1 - \phi_2, \psi_1 - \psi_2, \theta_1 - \theta_2)$ is also a solution of the mixed problem for double porous materials in the MGT thermoelasticity context.

Therefore, the energy equation (35) will have the following expression:

$$\int_\Omega (\dot{E}_{\mathbb{K}} + \dot{\mathbb{E}})dV = -\int_\Omega \left(\frac{2}{3}A(\gamma\ddot{\theta}+\dot{\theta})(\gamma\dddot{\theta} + \ddot{\theta}) + (2A + B)\dot{\theta}\ddot{\theta} + B\ddot{\theta}^2 + 2C\ddot{\theta}\cdot\dddot{\theta}\right)dV \leq 0.$$

Taking into account the null initial conditions for $t = 0$ from (10), we observe that the kinetic energy from (31) and the energy function from (30) are zero: $E_{\mathbb{K}} = 0$; $\mathbb{E} = 0$. Hence, their sum is also zero. Based on the above inequality, on the fact that the energy and the kinetic energy are positive defined, we may draw the conclusion that the difference between the considered solution is null. Therefore, the mixed problem with initial and boundary conditions for the double porous materials in the MGT thermoelasticity context is unique. □

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