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ORIGINAL ARTICLE

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A two-temperature generalized magneto-thermoelastic formulation for a rotating medium with thermal shock under hydrostatic initial stress

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Abstract The interaction between magnetic field and thermal field in an elastic half-space, homogeneous and isotropic under two temperature and initial stress are investigated using a normal mode method in the framework of the Lord–Şhulman theory, with thermal shock and rotation. The medium rotates with a uniform angular velocity, and it is considered to be permeated by a uniform magnetic field and hydrostatic initial stress. The general solution we obtain is finally applied to a specific problem. The variations in temperature, the dynamical temperature, the stress and the strain distributions through the horizontal distance are calculated by an appropriate numerical example and graphically illustrated.

Keywords Generalized thermoelasticity · Thermal shock · Two-temperature problem · Lord–Şhulman theory

List of symbols

δ_{ij}	Kronecker delta function
α_t	Coefficient of linear thermal expansion
T	Absolute temperature
T_0	Reference temperature chosen so that $\left\lvert \frac{T-T_0}{T_0} \right\rvert < 1$
$\phi = \phi_0 - T$	Conductive temperature
η	Hydrostatic initial stress
λ, μ	Lame's constants
μ_0	Magnetic permeability
$\theta = T - T_0$	Thermodynamical temperature
ρ	Density of the medium
σ_{ij}	Components of the stress tensor
τ_0	Thermal relaxation time
a	Two-temperature parameter
C_E	Specific heat at constant strain
e	Cubical dilatation

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e_{ij}	Components of the strain tensor
F_i	Lorentz force
K	Thermal conductivity
P	Initial pressure
u_i	Components of the displacement vector
F_i	Lorentz body force

1 Introduction

Recently, the linear theory of micropolar elasticity has become of primary importance in many fields. Concern the ultrasonic waves that its elastic vibrations have small wavelengths and high frequencies, the presence of the microstructure is fundamental. Its influence on the propagation of waves results in the development of new type of waves, which are not described in the framework of the classical theory of elasticity. Metals, polymers, composites, solids, rocks, concrete are typical media with microstructures. The thermoelasticity theory which the strain rate term introduction and formed in the Fourier heat conduction equation which leads to a parabolic heat conduction equation type, is called the diffusion equation and the theory called classical dynamical coupled theory derived by Biot [1]. This theory predicts a finite speed for elastic waves propagation and an infinite speed for a thermal wave propagation, this make a conflict meaning with the practical theory measures in labs. Lord and Șhulman [2], and Green and Lindsay [3] proposed new models of thermoelasticity in generalized forms solved the paradox of the difference between the theoretical and experimental results and advocating the existence of finite thermal wave speed for propagation in solids.

The two generalized theories proposed one or two relaxation times in the thermoelastic interaction that modified Fourier′s heat conduction equation to measure a finite speed for a thermal wave propagation, on the other hand modified by correcting Neumann–Duhamel relation and the energy equation. For the modifications made in the generalized two theories, the heat propagation displayed as a wave phenomenon rather than a diffusion one, which usually referred to the effect of second sound. The features of these two theories are the different of their structures from one another, so, cannot obtained as a special case of one to another. Many researchers applied these two generalized thermoelastic theories and pointed out new works revealed and interesting thermoelastic phenomena with new external parameters. In Refs. [4,5], the researches made a new brief reviews considering this topic and detail the generalization of these theories. The interaction between Maxwell electromagnetic field and the deformable solids of motion considered by many researchers trying for possibility application related to geology and geophysical problems and certain topics in acoustics, sound, and optics. Moreover, the nature of the earth material may be conducting or semiconducting electrically and subject to its own magnetic field, so, the earth's material may affect the propagation of waves. The propagation of waves in solids considering the electromagnetic field and thermal field has been investigated and interested to many authors. Plane waves in thermoelasticity and magneto-thermoelasticity are discussed in [6]. In the field of generalized magneto-thermoelastic equations, in [7] a study of the propagation of plane waves in a solid under the influence of an electromagnetic field is presented, obtaining the governing equations in the general case and the solution for some particular cases. These results have been extended to rotating media in [8]. Refs. [9,10] investigated the effect of thermal shock parameter on the generalized magneto-thermoelastic waves phenomenon under assumption a perfectly conducting half-space. In most problems, we deal the classical or generalized thermoelastic phenomena under the displacement potential function approach defined Lame's potential must considered. However, in [11,12] several disadvantages of the potential function approach are discussed.

Secondly, more assumptions that are stringent must be made on the behavior of potential functions than on the actual physical quantities. Finally, it was concluded that many integral representations of physical quantities are convergent in the classical sense while their potential function representations only converge in the mean. In [13–15], a generalized thermoelasticity problem for an infinitely long cylinder in two dimensions is investigated.

The development of initial stresses in the medium can be due to many reasons, for example, resulting from variation in temperature, a process of quenching, overburden layer, and slow process of creep, gravitation, weight, largeness, and so forth. It is of great interest to study the influence of these stresses on the propagation of stress waves. It was the achievement of Biot [16] to show that acoustic propagation under initial stresses would be fundamentally different from that under stress-free state and could not be represented merely by introducing stress-dependent elastic coefficients in the classical theory. He has obtained the velocities of longitudinal and transverse waves propagating along the coordinate axes only. Reflection and refraction phenomena of plane waves in an unbounded isotropic medium in linear thermoelasticity under initial stresses in the context of different theories of generalized thermoelasticity are studied in [17–24].

Some further problems in thermoelastic rotating media considering different thermoelasticity theories are investigated in [25–34]. In the field of generalized materials, as the ones studied in [35–39], described by higher gradient models [40–44], propagation of waves has been investigated both numerically [45–47] and experimentally [48,49]. Interesting effects of the methods introduced in this article can be emerge when considering their applications in the field of the so-called metamaterials [50–52].

The present work investigates the effect of hydrostatic initial stress, rotation, initial stress, thermal shock and magnetic field on a thermoelastic medium with two temperatures in the context of the normal mode analysis. Numerical results have been calculated considering a copper as a thermoelastic example, and displaying graphically the variations of the displacements, temperature, strain, conductive temperature, and stresses with respect to horizontal axis under thermal shock and two temperatures.

2 Formulation of the problem

Following, we consider the medium is a perfect electric conductor, the Maxwell equations, taking into account absence of the displacement current (SI) [31] take the form:

$$\begin{aligned}
&\text{curl}\vec{h} = \vec{J} \\
&\text{curl}\vec{E} = -\mu_e \frac{\partial \vec{h}}{\partial t} \\
&\text{div}\vec{h} = 0, \text{div}\vec{E} = 0 \\
&\text{where } \vec{h} = \text{curl}(\vec{u} \times \vec{H}_0), \vec{H} = \vec{H}_0 + \vec{h}(x, y, t)
\end{aligned} \tag{1}$$

The heat conduction equation takes the form [22]

$$K\varphi_{,ii} = \left(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2}\right)(\rho C_E T + \gamma T_0 u_{i,j}) \tag{2}$$

The stress strain relation as follows:

$$\sigma_{ij} = \lambda e \delta_{ij} + 2\mu e_{ij} - \gamma T \delta_{ij} - P(\delta_{ij} + \omega_{ij}) \tag{3}$$

where $e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \omega_{ij} = \frac{1}{2}(u_{j,i} - u_{i,j})$

Since the medium is rotating uniformly with an angular velocity $\underline{\Omega} = \Omega \underline{n}$, where $\underline{n}$ is a unit vector representing the direction of the axis of rotation, the displacement equation of motion in the rotating frame of reference has two additional terms, centripetal acceleration $\underline{\Omega} \times (\underline{\Omega} \times \underline{u})$, due to time-varying motion only and the Coriolis acceleration $2\underline{\Omega} \times \underline{\dot{u}}$, where $\underline{u}$ is the dynamic displacement vector.

The equation of motion takes the form

$$\rho[\ddot{u}_i + \{\underline{\Omega} \times (\underline{\Omega} \times \underline{u})\}_i + (2\underline{\Omega} \times \underline{\dot{u}})_i] = \sigma_{ij,j} + F_i \tag{4}$$

where $\vec{F} = \vec{J} \times \vec{B}, \vec{B} = \mu_e \vec{H}_0$

The equation relates between the dynamical heat and heat conduction written as:

$$\varphi - T = a\varphi_{,ii}, \tag{5}$$

where $a > 0$ two-temperature parameter, as in [22].

Assumption an elastic homogenous half-space $x \geq 0$ rotates uniformly with angular velocity Ω, in the presence of constant magnetic field $\vec{H}_0$ directed along the z-axis and of an initial compression P obeying to Eqs. (1)–(4). The displacement components for the 2D medium have the form

$$u_x = u(x, y, t), \quad u_y = v(x, y, t), \quad u_z = 0. \tag{6}$$

The heat conduction equation takes the form

$$K\left(\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2}\right) = \left(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2}\right)\rho C_E T + \gamma T_0 \left(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2}\right)\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right). \tag{7}$$

The constitutive equations can be rewritten as

$$\sigma_{xx} = (2\mu + \lambda)\frac{\partial u}{\partial x} + \lambda\,\frac{\partial v}{\partial y} - \gamma T - P, \tag{8}$$

$$\sigma_{yy} = (2\mu + \lambda)\frac{\partial v}{\partial y} + \lambda\,\frac{\partial u}{\partial x} - \gamma T - P, \tag{9}$$

$$\sigma_{xy} = \left(\mu + \frac{1}{2}P\right)\frac{\partial u}{\partial y} + \left(\mu - \frac{1}{2}P\right)\frac{\partial v}{\partial x}, \quad \sigma_{yx} = \left(\mu + \frac{1}{2}P\right)\frac{\partial v}{\partial x} + \left(\mu - \frac{1}{2}P\right)\frac{\partial u}{\partial y}. \tag{10}$$

The Maxwell's equations take the form

$$\tau_{ij} = \mu_e[H_i h_j + H_j h_i - H_k.h_k\delta_{ij}] \tag{11}$$

The equation of motion can eventually be written as

$$\rho\left(\frac{\partial^2 u}{\partial t^2} - \Omega^2 u - 2\Omega\frac{\partial v}{\partial t}\right) = (\lambda + 2\mu + \mu_e H_0^2)\frac{\partial^2 u}{\partial x^2} + \left(\lambda + \mu + \frac{1}{2}P + \mu_e H_0^2\right)\frac{\partial^2 v}{\partial x \partial y} + \left(\mu - \frac{1}{2}P\right)\frac{\partial^2 u}{\partial y^2} - \gamma T_0\frac{\partial T}{\partial x} \tag{12}$$

$$\rho\left(\frac{\partial^2 v}{\partial t^2} - \Omega^2 v + 2\Omega\frac{\partial u}{\partial t}\right) = (\lambda + 2\mu + \mu_e H_0^2)\frac{\partial^2 v}{\partial y^2} + \left(\lambda + \mu + \frac{1}{2}P + \mu_e H_0^2\right)\frac{\partial^2 u}{\partial x \partial y} + \left(\mu - \frac{1}{2}P\right)\frac{\partial^2 v}{\partial x^2} - \gamma T_0\frac{\partial T}{\partial y} \tag{13}$$

The equation relates between the dynamical heat and heat conduction rewritten as:

$$\varphi - T = a\left(\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2}\right) \tag{14}$$

For simplicity, we will use the non-dimensional variables (A.1)–(A.8) reported in Appendix. Hence, we have (dropping the dashed for convenience)

$$\nabla^2\varphi - \left(1 + \tau_0\frac{\partial}{\partial t}\right)\frac{\partial \theta}{\partial t} - \varepsilon\left(1 + \tau_0\frac{\partial}{\partial t}\right)\frac{\partial e}{\partial t} = 0 \tag{15}$$

$$\varphi - \theta = \beta\left(\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2}\right), \tag{16}$$

where $\varepsilon = \frac{\gamma}{\rho C_E}$ and $\beta = a\eta^2 c_0^2$. The equation of motion (12) and (13) takes the form

$$\frac{\partial^2 u}{\partial t^2} - \Omega^2 u - 2\Omega\frac{\partial v}{\partial t} = \frac{(\lambda + 2\mu + \mu_e H_0^2)}{\rho C_0^2}\frac{\partial^2 u}{\partial x^2} + \frac{(\lambda + \mu + \frac{1}{2}P + \mu_e H_0^2)}{\rho C_0^2}\frac{\partial^2 v}{\partial x\,\partial y} + \frac{\left(\mu - \frac{1}{2}P\right)}{\rho C_0^2}\frac{\partial^2 u}{\partial y^2} - \frac{\gamma}{\rho C_0^2}\frac{\partial \theta}{\partial x} \tag{17}$$

$$\frac{\partial^2 v}{\partial t^2} - \Omega^2 v + 2\Omega\frac{\partial u}{\partial t} = \frac{(\lambda + 2\mu + \mu_e H_0^2)}{\rho C_0^2}\frac{\partial^2 v}{\partial y^2} + \frac{(\lambda + \mu + \frac{1}{2}P + \mu_e H_0^2)}{\rho C_0^2}\frac{\partial^2 u}{\partial x\,\partial y} + \frac{\left(\mu - \frac{1}{2}P\right)}{\rho C_0^2}\frac{\partial^2 v}{\partial x^2} - \frac{\gamma}{\rho C_0^2}\frac{\partial \theta}{\partial y} \tag{18}$$

Let consider now the scalar potential functions $\Pi(x, y, t)$ and $\psi(x, y, t)$ defined by the relations in the non-dimensional form

$$u = \frac{\partial \Pi}{\partial x} + \frac{\partial \psi}{\partial y}, \quad v = \frac{\partial \Pi}{\partial y} - \frac{\partial \psi}{\partial x} \tag{19}$$

Substitute from equations (19) and (15) in Eqs. (12) and (13), it follows that

$$\left[\nabla^2 - \frac{1}{a_2}\frac{\partial^2}{\partial t^2} + \frac{\Omega^2}{a_2}\right]\Pi - \frac{2\Omega}{a_2}\frac{\partial \psi}{\partial t} - a^*\theta = 0 \tag{20}$$

$$\left(\nabla^2 - \frac{1}{a_1}\frac{\partial^2}{\partial t^2} + \frac{\Omega^2}{a_1}\right)\psi + \frac{2\Omega}{a_1}\frac{\partial \Pi}{\partial t} = 0 \tag{21}$$

where the a_i parameters are given in Appendix (A.9)–(A.13). Moreover, Eq. (15) takes the form

$$\nabla^2\varphi - \left(1 + \tau_0\frac{\partial}{\partial t}\right)\frac{\partial \theta}{\partial t} - \varepsilon\left(1 + \tau_0\frac{\partial}{\partial t}\right)\frac{\partial}{\partial t}(\nabla^2\Pi) = 0 \tag{22}$$

To get the solution of these equations, it is useful to adopt the following decomposition in terms of normal modes

$$[\Pi, \psi, \varphi, \theta, \sigma_{ij}](x, y, t) = [u^*(x), \varphi^*(x), \theta^*(x), \sigma^*_{ij}(x)]\exp(\omega t + iby). \tag{23}$$

where ω is the (complex) time constant, $i = \sqrt{-1}$, b a wave number and $u^*(x)$, $\varphi^*(x)$, $\theta^*(x)$ and $\sigma^*_{ij}(x)$ the amplitudes of the field quantities. By manipulation of Eqs. (20)–(23) and (18), one finally obtains

$$[D^2 - A_1]\Pi^* + A_0\psi^* - A_2\theta^* = 0, \tag{24}$$

$$[D^2 - b^2]\varphi^* - A\theta^* + B[D^2 - b^2]\Pi^* = 0, \tag{25}$$

$$[D^2 - A_3]\varphi^* = -\beta^*\theta^*, \tag{26}$$

$$[D^2 - m^2]\psi^* + A_4\Pi^* = 0, \tag{27}$$

where all the parameters' definitions are listed in Appendix (A.14)–(A.22).

Eliminating $\Pi^*(x)$, $\psi^*(x)$ and $\varphi^*(x)$ in Eqs. (24)–(27), we obtain the partial differential equation satisfied by $\theta^*(x)$

$$[D^6 - ED^4 + FD^2 - G]\theta^*(x) = 0 \tag{28}$$

where the constants E, F and G are reported in Appendix (A.23)–(A.25). In a similar manner, we get

$$[D^6 - ED^4 + FD^2 - G](\Pi^*, \varphi^*)(x) = 0. \tag{29}$$

The above equation can be factorized as

$$\left(D^2 - k_1^2\right)\left(D^2 - k_2^2\right)(D^2 - k_3^2)\theta^*(x) = 0 \tag{30}$$

where $k_n^2 (n = 1, 2, 3)$ are the roots of the following characteristic equation

$$k^6 - Ek^4 + Fk^2 - G = 0. \tag{31}$$

The solution of Eq. (31) is

$$\begin{aligned}
k_1^2 &= \frac{2^{\frac{4}{3}}R_1 + R_3(2^{\frac{2}{3}}R_3 + 2E)}{6R_3}\\
k_2^2 &= \frac{2^{\frac{4}{3}}R_1(-1 + i\sqrt{3}) + R_3(4E - 2^{\frac{2}{3}}R_3(1 + i\sqrt{3}))}{12R_3}\\
k_3^2 &= \frac{2^{\frac{4}{3}}R_1(-1 - i\sqrt{3}) + R_3(4E - 2^{\frac{2}{3}}R_3(1 - i\sqrt{3}))}{12R_3}
\end{aligned} \tag{32}$$

with

$$R_1 = E^2 - 3F, \quad R_2 = 2E^3 - 9EF + 27, \quad R_3 = \left(R_2 + \sqrt{-4R_1^3 + R_2^2}\right)^{\frac{1}{3}}$$

Solution of Eq. (30) which is bounded as $x \to \infty$ is given by

$$\theta^*(x) = \sum_{n=1}^{3} M_n(b, \beta, \omega) \exp(-k_n x) \tag{33}$$

Similarly,

$$\varphi^*(x) = \sum_{n=1}^{3} M'_n(b, \beta, \omega) \exp(-k_n x) \tag{34}$$

$$\Pi^*(x) = \sum_{n=1}^{3} M''_n(b, \beta, \omega) \exp(-k_n x) \tag{35}$$

$$\psi^*(x) = \sum_{n=1}^{3} M'''_n(b, \beta, \omega) \exp(-k_n x), \tag{36}$$

since

$$u^*(x) = D\Pi^* + i\mathrm{b}\psi^*, \tag{37}$$

$$v^*(x) = i\mathrm{b}\Pi^* - D\psi^*, \tag{38}$$

$$e^*(x) = Du^* + i\mathrm{b}\,v^*. \tag{39}$$

To determine the amplitude of the horizontal and vertical displacements u and v, which are bounded as $x \to \infty$, then substitute from Eqs. (34) and (35), into Eqs. (37) and (38), one get

$$u^*(x) = \sum_{n=1}^{3} [-k_n M''_n(b, \beta, \omega) + ibM'''_n(b, \beta, \omega)]e^{-k_n x}, \tag{40}$$

$$v^*(x) = \sum_{n=1}^{3} [ibM''_n(b, \beta, \omega) + k_n M'''_n(b, \beta, \omega)]e^{-k_n x}. \tag{41}$$

where M_n, M'_n, M''_n and M'''_n are some parameters depending on β, b and ω. Substituting from Eqs. (33)–(36) into Eqs. (24)–(27), we have

$$M'_n(b, \beta, \omega) = H_{1n} M_n(b, \beta, \omega), \quad n = 1, 2, 3 \tag{42}$$

$$M''_n(b, \beta, \omega) = H_{2n} M_n(b, \beta, \omega), \quad n = 1, 2, 3 \tag{43}$$

$$M'''_n(b, \beta, \omega) = H_{3n} M_n(b, \beta, \omega), \quad n = 1, 2, 3 \tag{44}$$

where the H_{in} factors are reported in Appendix (A.26)–(A.28).

Thus, we have

$$\varphi^*(x) = \sum_{n=1}^{3} H_{1n} M_n(b, \beta, \omega) \exp(-k_n x) \tag{45}$$

$$\Pi^*(x) = \sum_{n=1}^{3} H_{2n} M_n(b, \beta, \omega) \exp(-k_n x) \tag{46}$$

$$\psi^*(x) = \sum_{n=1}^{3} H_{3n} M_n(b, \beta, \omega) \exp(-k_n x) \tag{47}$$

Substitution of Eqs. (40) and (41) into Eqs. (8)–(10), we get

$$\sigma_{xx}^* = \sum_{n=1}^{3} h_n M_n(b, \beta, \omega) \exp(-k_n x) - P, \tag{48}$$

$$\sigma_{yy}^* = \sum_{n=1}^{3} h'_n M_n(b, \beta, \omega) \exp(-k_n x) - P \tag{49}$$

$$\sigma_{xy}^* = \sum_{n=1}^{3} h''_n M_n(b, \beta, \omega) \exp(-k_n x) \tag{50}$$

$$\tau_{xx}^* = \sum_{n=1}^{3} g_n M_n(b, \beta, \omega) \exp(-k_n x), \quad \tau_{yy}^* = \sum_{n=1}^{3} g_n M_n(b, \beta, \omega) \exp(-k_n x) \tag{51}$$

where h_n, h'_n, h''_n and g_n are defined in Appendix (A.29)–(A.32).

The normal mode analysis consists, in fact, looking for the solution in the Fourier transformed domain, considering all the field quantities are sufficiently smooth on the real line such that normal mode analysis of these functions exists.

3 Application: the thermal shock problem

In this application, we will calculate $M_n (n = 1, 2, 3)$. Physically, the positive exponentials at infinity are vanish. So, the parameters M_1, M_2 and M_3 determined such the boundary conditions on the surface at given as follows:

(i) Thermal field on the surface of the half-space subjected under thermal shock as the form

$$\theta(0, y, t) = f(0, y, t) \tag{52}$$

(ii) Sum of normal stresses including mechanical and Maxwell stresses on the surface of the half-space equal the compression stress applied

$$\sigma_{xx}(0, y, t) + \tau_{xx}(0, y, t) = -P \tag{53}$$

(iii) Sum of shear stresses including mechanical and Maxwell stresses on the surface of the half-space is traction free

$$\sigma_{xy}(0, y, t) + \tau_{xy}(0, y, t) = 0. \tag{54}$$

By using the expressions calculated in the previous sections on the above boundary conditions, we can obtain three algebraic Eqs. as follows

$$\sum_{n=1}^{3} M_n(b, \beta, \omega) = f^*(y, t) \tag{55}$$

$$\sum_{n=1}^{3} (h_n + g_n) M_n(b, \beta, \omega) = 0 \tag{56}$$

$$\sum_{n=1}^{3} h''_n M_n(b, \beta, \omega) = 0 \tag{57}$$

Invoking the boundary conditions (55)–(57) at the surface $x = 0$ of the plate, we obtain a system of three equations. After applying the inverse matrix method, we get the values of the three constants M_j, $j = 1, 2, 3$. Hence, we obtain the expressions of displacements, temperature distribution and another physical quantity of the plate.

4 Numerical results

With a view to illustrate the analytical procedure presented earlier, we now consider physical quantities for thermoelastic material. The results depict the variation of temperature, displacement components, and stress considering Lord-Shulman theory. The physical constants for the copper material, used in the numerical example, are reported in Table 1.

The computations were carried out for the constant values $t = 0.1$, $\Omega = 10^7$, $\beta = 0.1$, $H = 10^5$, $P = 10^{10}$, $b = 0.25$. The numerical technique is run for the distribution of the real part of ϕ and the thermal temperature θ, the displacement u, v, strain e the stresses $(\sigma_{xx}, \sigma_{xy}, \sigma_{yy})$ and Maxwell's stress τ_{xx}. All components depend on space x and time t, thermal relaxation time τ, magnetic field, rotation, initial stress, also, b and β parameters.

Figures 1 and 2 display the variation in ϕ and θ with respect to the axis x for different values of t, b, β, τ, H, P and Ω. It can be observed that all the parameters have the same effect on ϕ and θ except P; it has to be remarked that ϕ and θ start from a maximum value at the wall $(x = 0)$ and decrease to zero as x tends to infinity. It is clear that ϕ and θ increase with an increasing of t, β and τ, but decrease with the increasing values of H and Ω; also, it is obvious that ϕ decreases with increasing of P, but increases with θ.

Figure 3 plots the variation in the displacement u with respect to axis x for different values of t, b, β, τ, H and P. It is clear from the plots that the displacement u starts from a minimum value increasing to its maximum, and after it goes to zero as x tends to infinity. Moreover, it can be observed that u decreases and then increases nearly to $x = 0.8$ with the variation in t and β. Instead, u does not affect by variation in P when one observes its behavior for different values of b, τ and H.

In Fig. 4, the variation in vertical velocity v with respect to x axis, for different values of t, b, β, τ, H and P, is shown. One can observe that v starts from zero at $x = 0$ (center of the medium)—this means that the vertical velocity vanishes at the center of the medium—decreasing to its minimum value tending to zero as x tends to infinity. It has to be remarked that v decreases with the increasing values of t, b and β, but increases with an increasing of τ and H; also, it is clear that v is not affected by variation in P.

Figure 5 clears the variation in the strain e with respect to x-axis taking into account different values of t, b, β, τ, H and P. It can be observed that e starts from its maximum value at $x = 0$ decreasing to its minimum tending to zero as x tends to infinity. Moreover, e increases with increasing values of t and β, but decreases with an increasing of b, τ and H. Finally, it illustrates that v is not affected by variation in P.

Figure 6 represents the variation in the axial stress σ_{xx} axis with respect to the x for different values of t, b, β, τ, H and P. One can observe that σ_{xx} has a maximum value at $x = 0$, decreases to its minimum and increases again tending to zero as x tends to infinity. It appears that σ_{xx} decreases with the increasing values of t, b, β, H and P but increases with an increasing of τ.

In Fig. 7, it is displayed the variation in shear stress σ_{xy} with respect to x-axis taking into account the variation in t, b, β, τ, H and P. One can observe that σ_{xy} starts from zero at $x = 0$ (center of the medium) decreasing to its maximum and finally tending to zero as x tends to infinity. It is seen that σ_{xy} decreases with the increasing values of t, b and β, but increases with an increasing of H and P. It has to be remarked that σ_{xy} increases with an increasing of τ near the medium center but decreases for large values of x.

Figure 8 displays the variation in the stress σ_{yy} with respect to x-axis with variation in t, b, β, τ, H and P. It is clear from the figures that σ_{yy} starts from its minimum value at $x = 0$ (center of the medium), increasing to its maximum and finally tending to zero as x tends to infinity. It has to be remarked that σ_{yy} decreases with

Table 1 Constants used for the numerical implementation

λ	7.59×10^9 N/m^2	ρ	7800 kg/m^2
μ	3.86×10^{10} kg/ms^2	K	386 N/Ks
C_E	383.1 J/(kg k)	τ_0	0.02
α	-1.28×10^9 N/m^2	f^*	1
a	1	T_0	293 K
a_t	1.78×10^{-5} N/m^2	ω	$\omega_0 + i\xi$
ω_0	2	ξ	1
η	8886.73 m/s^2	ε	0.0168

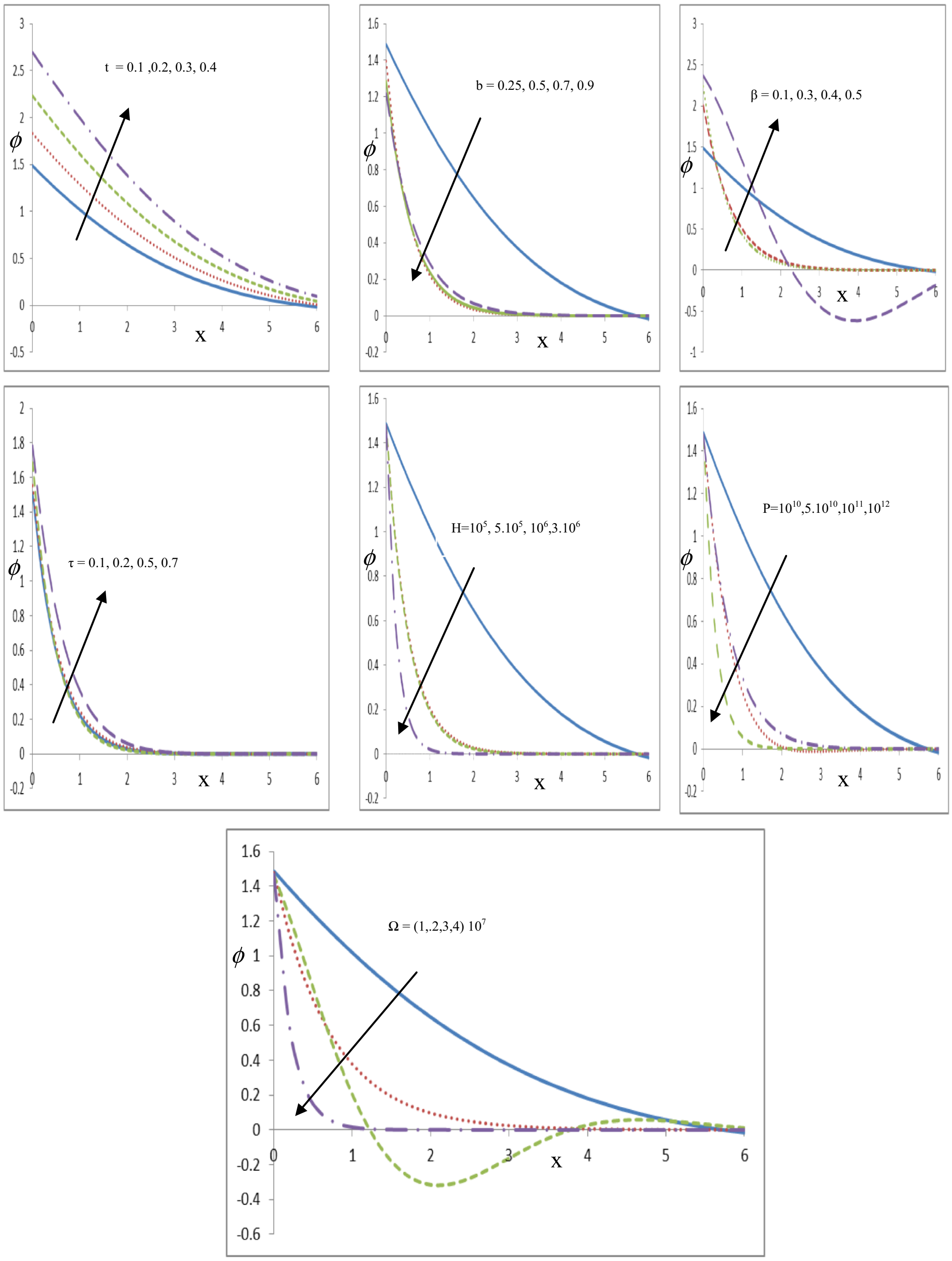

Fig. 1 Distribution of the conductive temperature ϕ with different values at $y = -1$

the increasing values of t, b, β and P, but increases with an increase in τ; also, it shows that σ_{yy} affects slightly with an increasing of H.

Finally, Fig. 9 displays the variation in the Maxwell's stress τ_{xx} with respect to x, t, b, β, τ, H and P. It can be observed that τ_{xx} starts from its maximum value at $x = 0$, decreasing to its minimum and finally tending to zero as x tends to infinity. It is shown that τ_{xx} increases with increasing values of t, H and β, but decreases with an increasing of b and τ; also, it seems that τ_{xx} is not affected by variation in P.

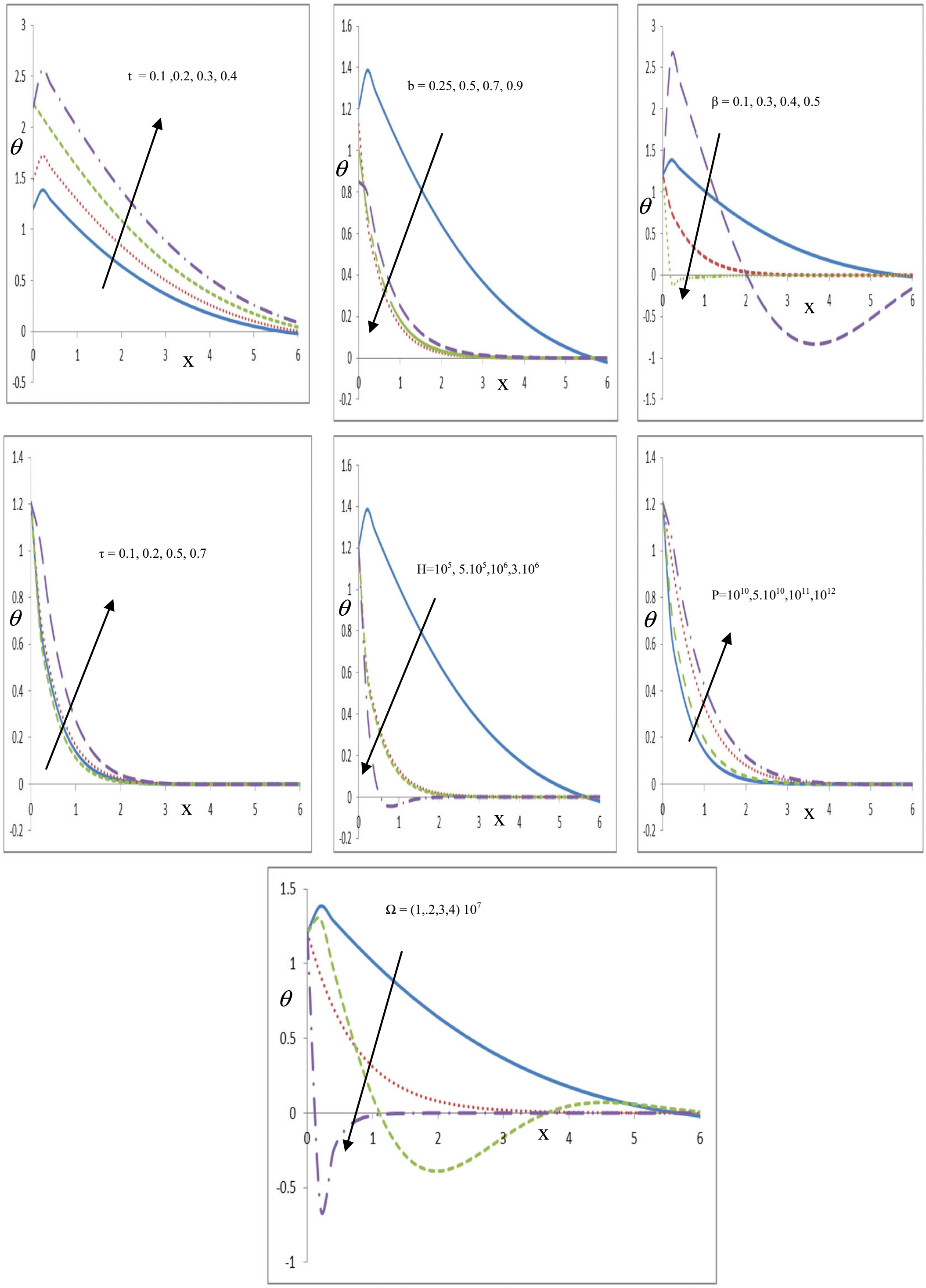

Fig. 2 Distribution of the thermodynamical temperature θ with different values at $y = -1$

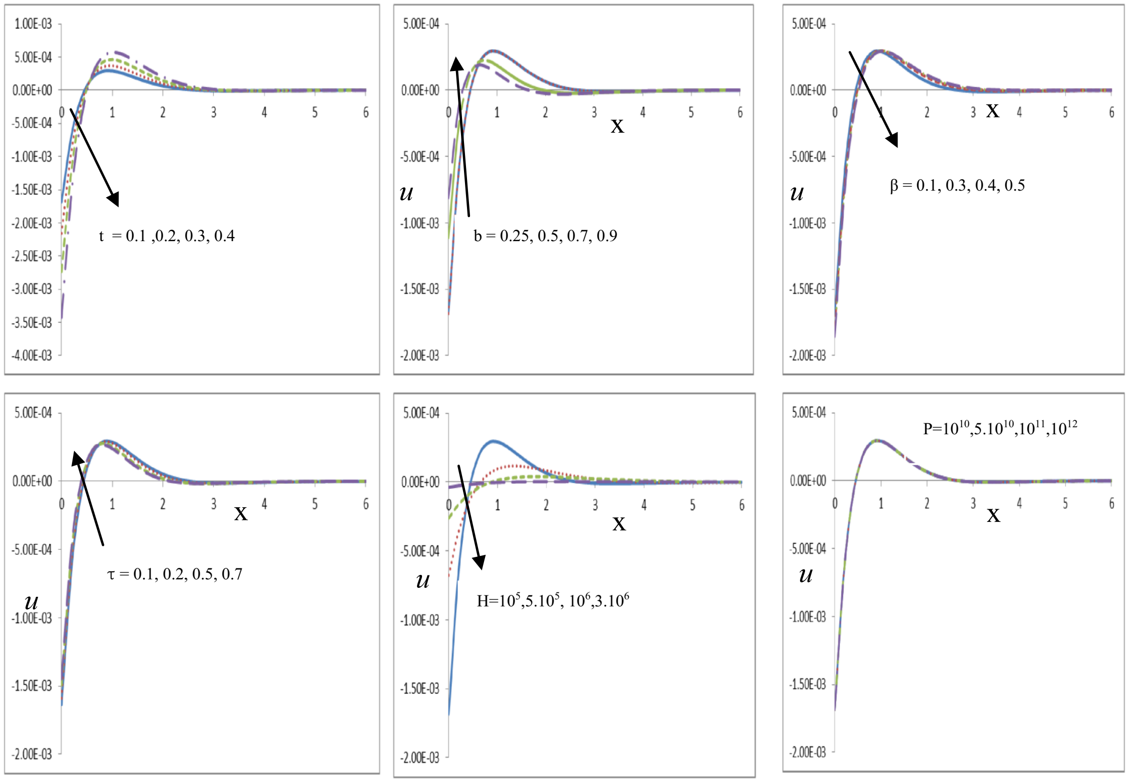

Fig. 3 Distribution of the displacement u with different values at $y = -1$

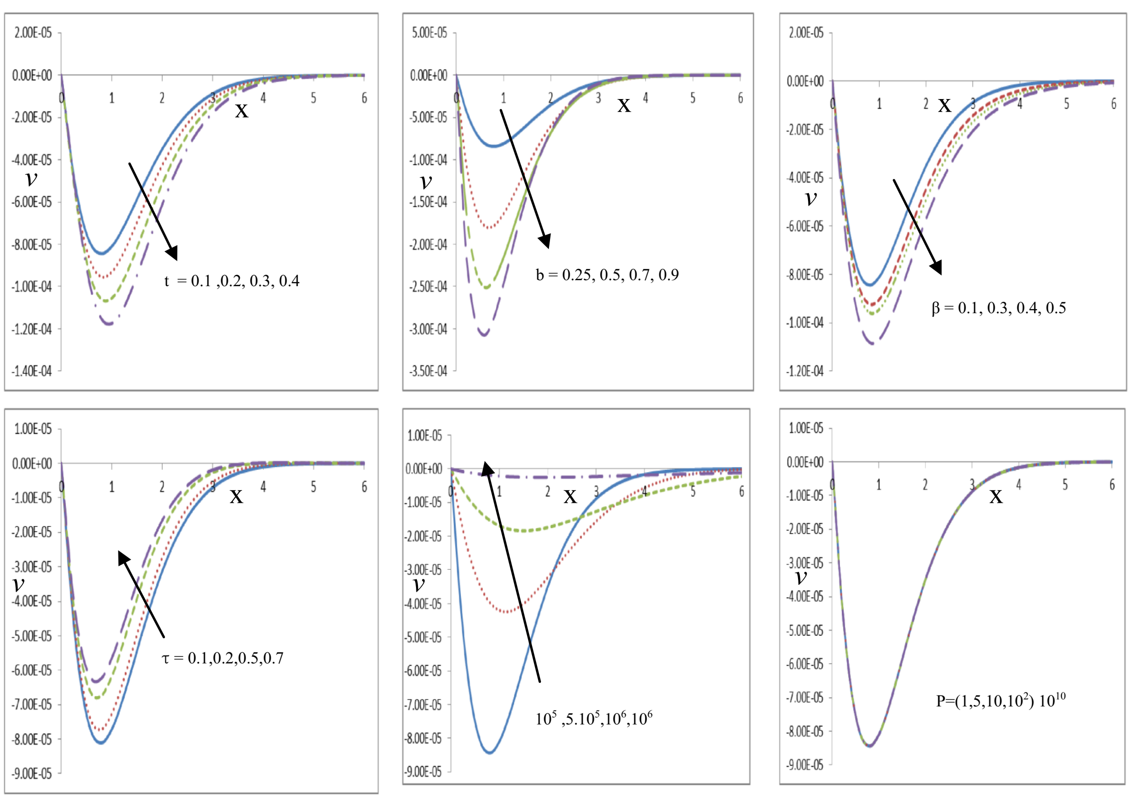

Fig. 4 Distribution of displacement v with different values at $y = -1$

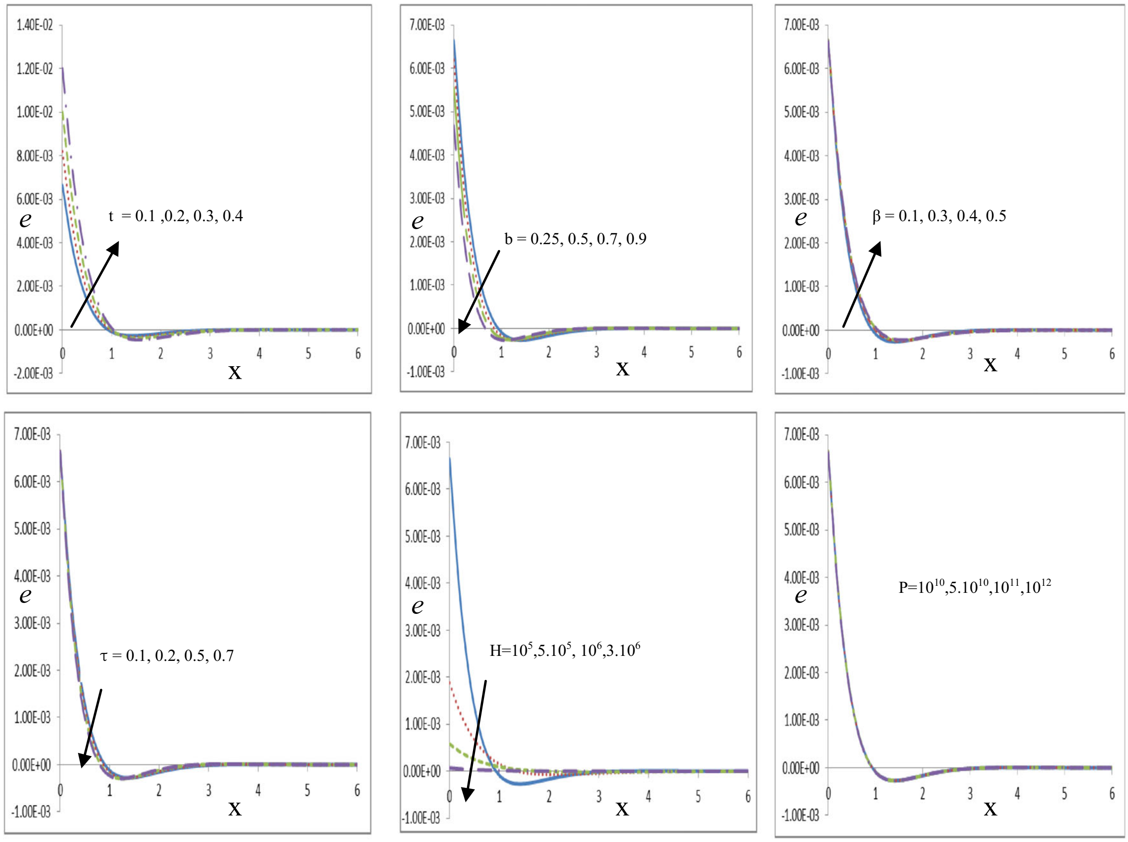

Fig. 5 Distribution of the strain e with different values at $y = -1$

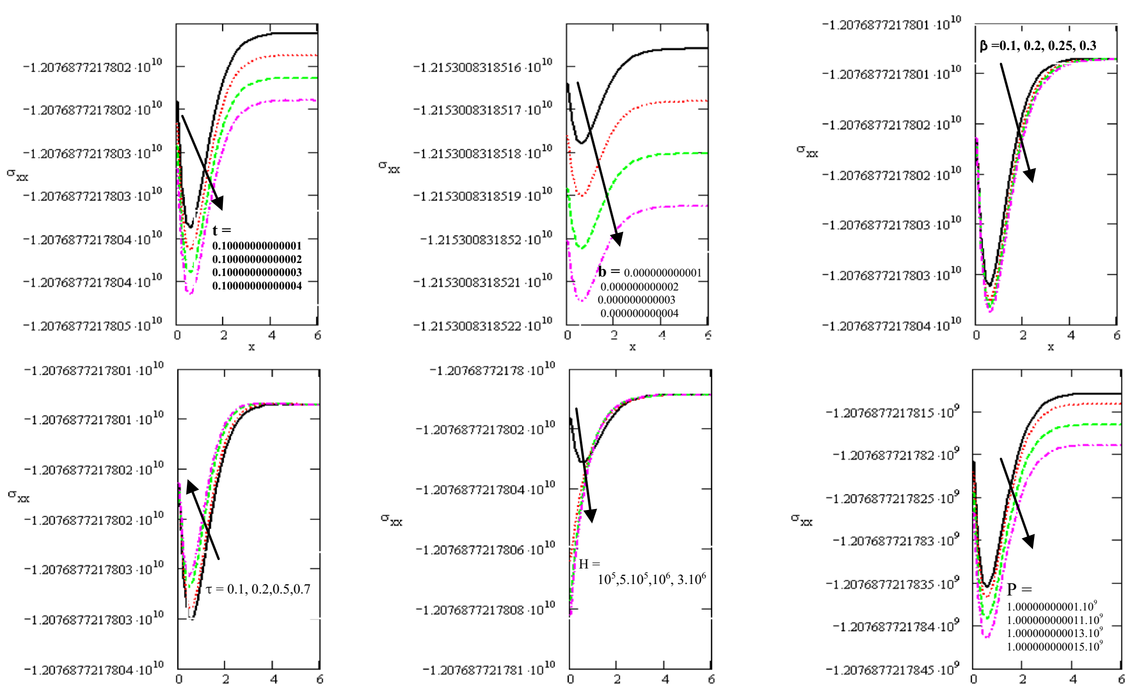

Fig. 6 Distribution of the stress σ_{xx} with different values at $y = -1$

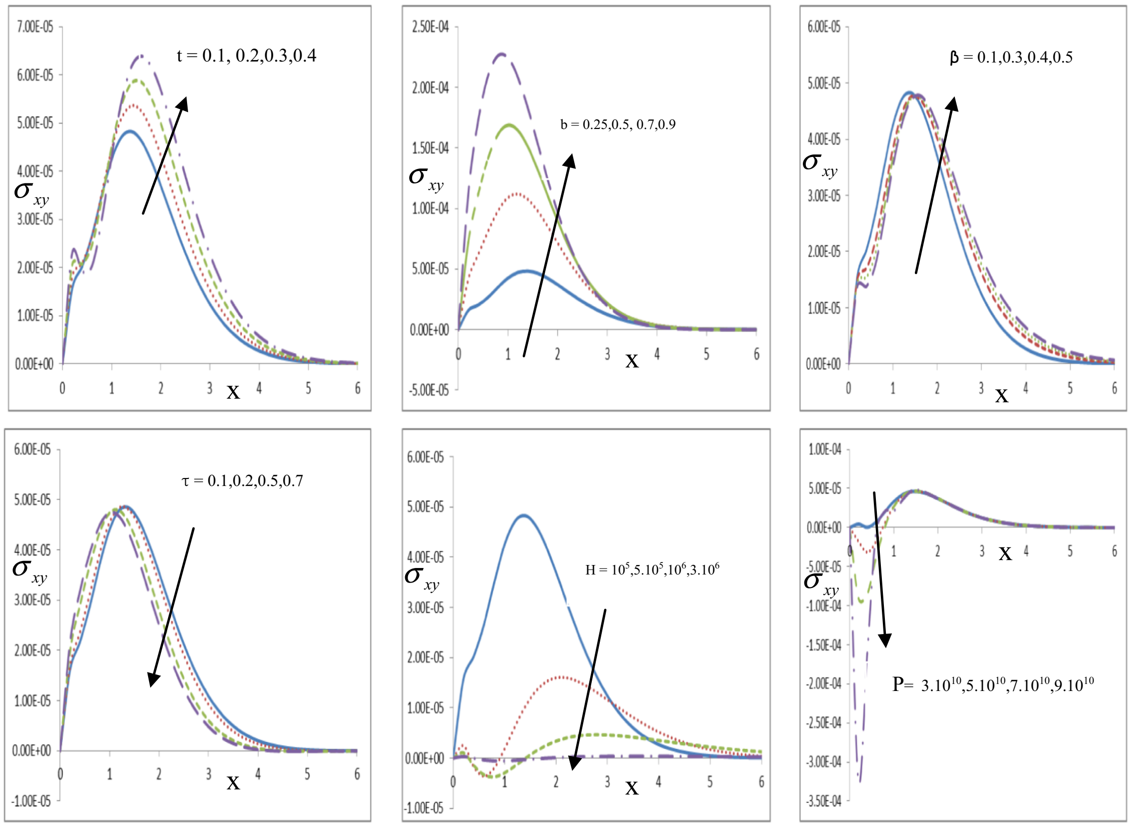

Fig. 7 Distribution of the stress σ_{xy} with different values at $y = -1$

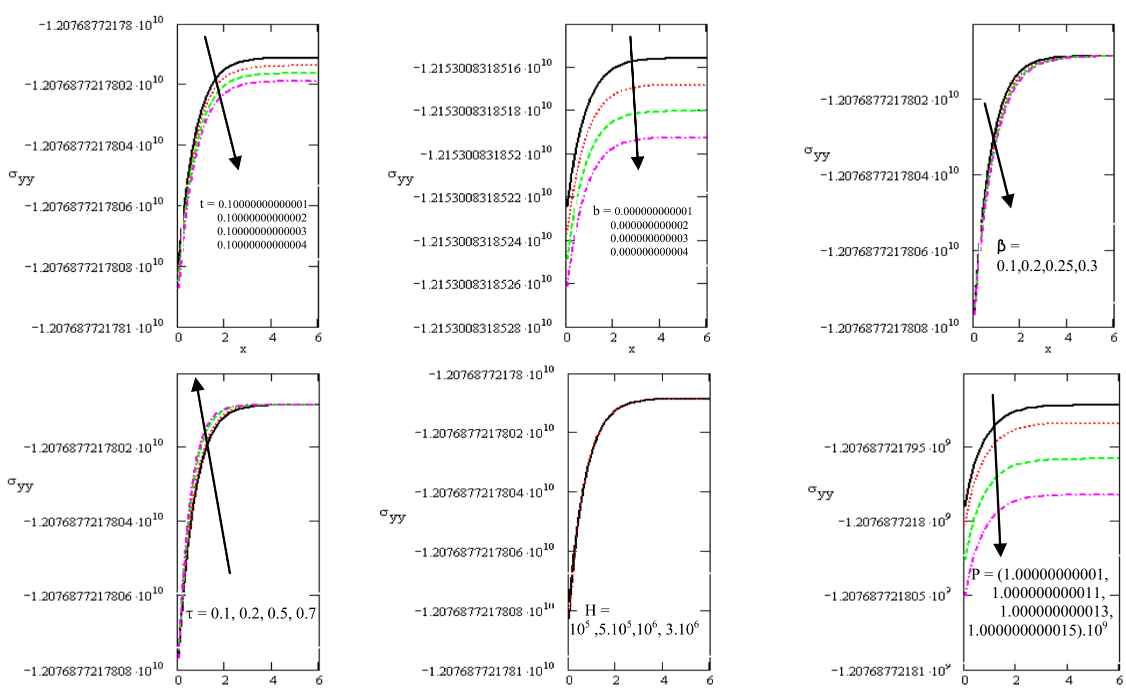

Fig. 8 Distribution of the stress σ_{yy} with different values at $y = -1$

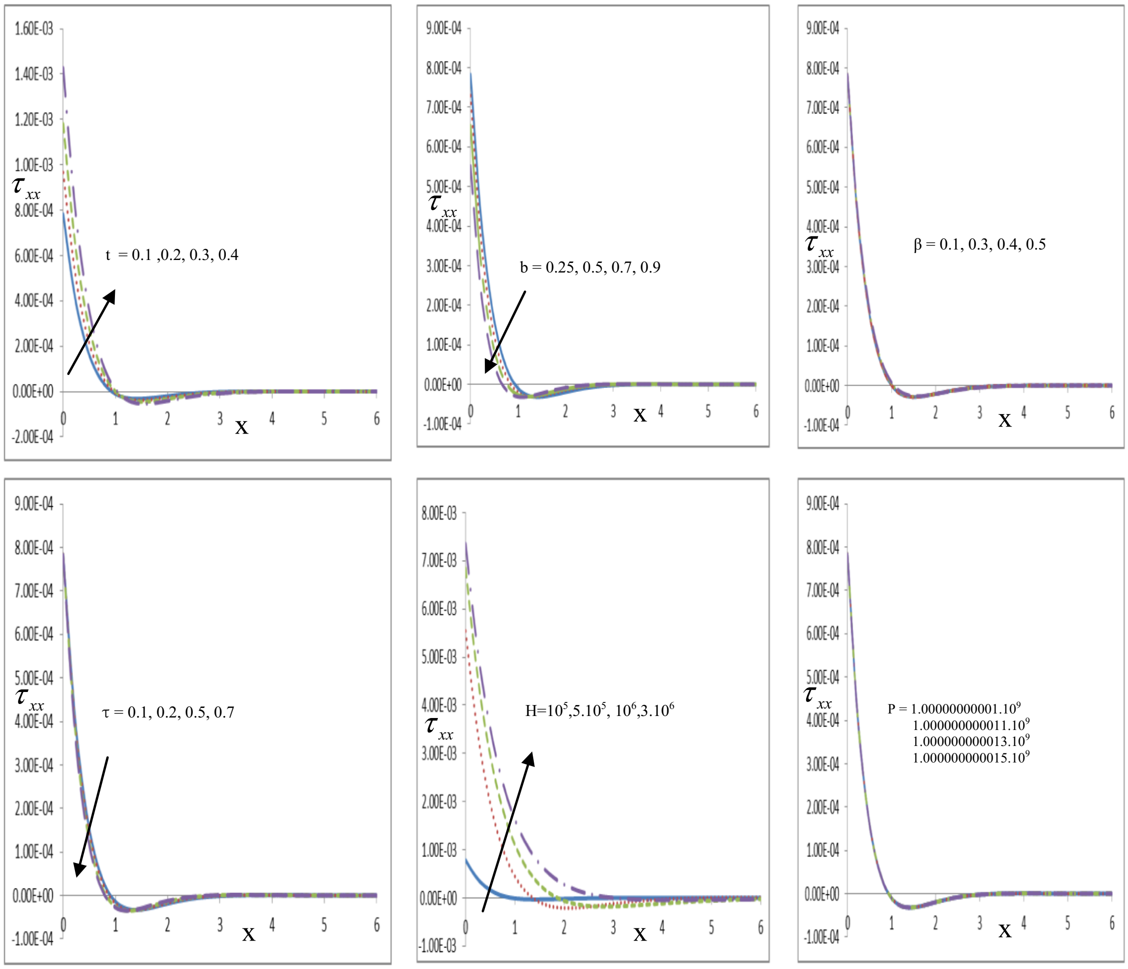

Fig. 9 Distribution of the Maxwell stress τ_{xx} with different values at $y = -1$

5 Conclusion

In this article, we have studied the magneto-thermoelastic interactions in an initially stressed isotropic homogeneous elastic half-space with two temperatures, in the framework of the Lord–Şhulman (LS) theory, with thermal shock and rotation. The general solution we have obtained has been finally applied to a specific problem: the variations in temperature, the dynamical temperature, the stress and the strain distributions through the horizontal distance have been calculated by an appropriate numerical example and graphically illustrated. We can therefore conclude that: (1) the value of many physical quantities converges to zero with an increasing in the distance x; (2) the relaxation time, initial stress, rotation, magnetic field, thermal shock and two temperatures have a significant role on the physical meaning of the phenomena; (3) the vertical velocity v starts from zero at $x = 0$ and this fact indicates that it vanishes at the center of the medium; (4) the components of u, v, e and τ_{xx} are not affected by the initial stress P.

Generalized thermoelastic theories can be useful when investigating the properties of generalized materials, as the ones studied in [53–62]. A further development is represented by the study of the arising and evolution of damage in these materials. Useful preliminary results in this field can be found in [63–67]. The problem treated in this paper can in perspective be applied to the study of plates and shells [68–73]. A further case which could be investigated by using the same methods is represented by composite materials, as the ones studied in [74–76].

Appendix

The non-dimensional variables are

$$(x', y', u', v') = c_0\eta(x, y, u, v), \tag{A.1}$$
$$(t', \tau_0') = c_0^2\eta(t, \tau_0), \tag{A.2}$$
$$(\theta', \varphi') = \frac{(T, \varphi) - T_0}{T_0}, \tag{A.3}$$
$$\sigma_{ij}' = \frac{\sigma_{ij}}{2\mu + \lambda}, \tag{A.4}$$
$$h' = \frac{h}{2\mu + \lambda}, \tag{A.5}$$
$$P' = \frac{P}{2\mu + \lambda}, \tag{A.6}$$
$$\tau' = \frac{\tau}{2\mu + \lambda}, \tag{A.7}$$
$$\Omega' = \frac{\Omega}{c_0^2\eta} \tag{A.8}$$

where $\eta = \frac{\rho C_E}{K}$, $C_2^2 = \frac{\mu}{\rho}$ and $c_0^2 = \frac{2\mu+\lambda}{\rho}$.
Definition of the parameters a_i

$$a_0 = \frac{\gamma T_0}{\rho C_0^2}, \tag{A.9}$$
$$a_2 = 1 + R_h^2, \tag{A.10}$$
$$R_h^2 = \frac{\mu_e H_0^2}{\rho C_0^2}, \tag{A.11}$$
$$a^* = \frac{a_0}{a_2}, \tag{A.12}$$
$$a_1 = \frac{\mu - \frac{1}{2}P}{\rho C_0^2} \tag{A.13}$$

and R_h^2 is called Alfven speed.
The parameters A_i are defined as

$$A = \omega\,(1 + \omega\tau_0)\,, \tag{A.14}$$
$$A_1 = b^2 + \frac{\omega^2}{a_2} - \frac{\Omega^2}{a_2}, \tag{A.15}$$
$$A_2 = a^*, \tag{A.16}$$
$$A_3 = \left(\beta b^2 + 1\right)/\beta,\ B = -\varepsilon A, \tag{A.17}$$
$$\beta^* = \frac{1}{\beta}, \tag{A.18}$$
$$m^2 = b^2 + \frac{\omega^2}{a_1} - \frac{\Omega^2}{a_1}, \tag{A.19}$$
$$A_0 = -\frac{2\Omega\omega}{a_2}, \tag{A.20}$$
$$A_4 = \frac{2\Omega\omega}{a_1} \tag{A.21}$$
$$D = \frac{d}{dx} \tag{A.22}$$

The constants E, F and G

$$E=\frac{b^2+\beta AA_3+(m^2+A_1)(1+\beta A)-\beta BA_2(A_3+m^2+b^2)}{1+\beta A-\beta BA_2}, \tag{A.23}$$

$$F=\frac{(m^2+A_1)(b^2+\beta AA_3)+(A_1m^2-A_0A_4)(1+\beta A)-\beta BA_2(m^2A_3+b^2A_3+b^2m^2)}{1+\beta A-\beta BA_2}, \tag{A.24}$$

$$G=\frac{(A_1m^2-A_0A_4)(b^2+\beta AA_3)-\beta BA_2b^2m^2A_3}{1+\beta A-\beta BA_2}, \tag{A.25}$$

The H_{in} factors are

$$H_{1n}=\left(\frac{\beta^*}{A_3-k_n^2}\right),\quad n=1,2,3 \tag{A.26}$$

$$H_{2n}=\frac{A+(b^2-k_n^2)H_{1n}}{B(k_n^2-b^2)},\quad n=1,2,3 \tag{A.27}$$

$$H_{3n}=\frac{A_4H_{2n}}{m^2-k_n^2},\quad n=1,2,3 \tag{A.28}$$

$$h_n=-\left(ibk_nH_{3n}-H_{2n}k_n^2-\frac{\lambda}{2\mu+\lambda}(ibk_nH_{3n}-b^2H_{2n})+\frac{\gamma T_0}{2\mu+\lambda}\right) \tag{A.29}$$

$$h'_n=\left(ibk_nH_{3n}-b^2H_{2n}-\frac{\lambda}{2\mu+\lambda}(-k_n^2H_{2n}+ibk_nH_{3n})-\frac{\gamma T_0}{2\mu+\lambda}\right) \tag{A.30}$$

$$h''_n=\frac{\mu+(1/2)(\lambda+2\mu)P}{2\mu+\lambda}(ib)(-k_nH_{2n}+ibH_{3n}) -\frac{\mu-(1/2)(\lambda+2\mu)P}{2\mu+\lambda}(k_n)(ibH_{2n}+k_nH_{3n}) \tag{A.31}$$

$$g_n=\frac{\mu_eH_0^2}{2\mu+\lambda}(k_n^2-b^2)H_{2n} \tag{A.32}$$

References

1. Biot, M.A.: Thermoclasticity and irreversible thermodynamics. J. Appl. Phys. **27**, 240–253 (1956)
2. Lord, H.W., Shulman, Y.: A generalized dynamical theory of thermolasticity. J. Mech. Phys. Solids. **15**, 299–306 (1967)
3. Green, A.E., Lindsay, K.A.: Thermoelasticity. J. Elast. **2**, 1–7 (1972)
4. Chandrasekharaiah, D.S., Srinath, K.S.: Thermoelastic interactions without energy dissipation due to a point heat source. J. Elast. **50**, 97–108 (1998)
5. Chandrasekharaiah, D.S., Murthy, H.N.: Temperature-rate-dependent thermo-elastic interactions due to a line heat source. Acta. Mech. **89**, 1–12 (1991)
6. Puri, P.: Plane waves in thermoelasticity and magneto-thermoelasticity. Int. J. Eng. Sci. **10**, 467–476 (1972)
7. Nayfeh, A., Nemat-Nasser, S.: Transient thermoelastic waves in half-space with thermal relaxation. ZAMP **23**, 52–68 (1972)
8. Roy Choudhuri, S.K., Mukhopdhyay, S.: Effect of rotation and relaxation on plane waves in generalized thermo-viscoelasticity. Int. J. Math. Math. Sci. **23**, 479–505 (2000)
9. Ezzat, M.A., Othman, M.I.A.: Electromagneto-thermoelastic plane waves with two relaxation times in a medium of perfect conductivity. Int. J. Eng. Sci. **38**, 107–120 (2000)
10. Ezzat, M., Othman, M.I.A., El-Karamany, A.S.: Electromagneto-thermoelastic plane waves with thermal relaxation in a medium of perfect conductivity. J. Therm. Stresses **24**, 411–432 (2001)
11. Bahar, L.Y., Hetnarski, R.: State space approach to thermoelasticity. J. Therm. Stresses **1**, 135–145 (1978)
12. Sherief, H.: State space formulation for generalized thermoelasticity with one relaxation time including heat sources. J. Therm. Stresses **16**, 163–176 (1993)
13. Sherief, H., Anwar, M.: A two dimensional generalized thermoelasticity problem for an infinitely long cylinder. J. Therm. Stresses **17**, 213–227 (1994)
14. Youssef, H.M., El-Bary, A.A.: Mathematical model for thermal shock problem of a generalized thermoelastic layered composite material with variable thermal conductivity. Commun. Methods Sci. Technol. **12**(2), 165–171 (2006)
15. Elsibai, K., Youssef, H.: State space formulation to the vibration of gold nano-beam induced by ramp type heating without energy dissipation in femtoseconds scale. J. Therm. Stresses **34**, 244–263 (2011)
16. Biot, M.A.: Mechanics of Incremental Deformations. Wiley, New York, NY (1965)
17. Chattopadhyay, A., Bose, S., Chakraborty, M.: Reflection of elastic waves under initial stress at a free surface: P and SV motion. J. Acoust. Soc. Am. **72**(1), 255–263 (1982)

18. Montanaro, A.: On singular surfaces in isotropic linear thermoelasticity with initial stress. J. Acoust. Soc. Am. **106**(3I), 1586–1588 (1999)
19. Othman, M.I.A., Song, Y.: Reflection of plane waves from an elastic solid half-space under hydrostatic initial stress without energy dissipation. Int. J. Solids Struct. **44**(17), 5651–5664 (2007)
20. Singh, B.: Effect of hydrostatic initial stresses on waves in a thermoelastic solid half-space. Appl. Math. Comput. **198**(2), 494–505 (2008)
21. Chen, P.J., Gurtin, M.E.: On a theory of heat conduction involving two temperatures. Zamp. **19**, 614–627 (1968)
22. Youssef, H.M.: Theory of two-temperature-generalized thermoelasticity. IMA J. Appl. Math. **71**, 383–390 (2006)
23. Youssef, H.M., Al-Lehaibi, E.A.: State-space approach of two-temperature generalized thermoelasticity of one-dimensional problem. Int. J. Solids Struct. **44**, 1550–1562 (2007)
24. Youssef, H.: Problem of generalized thermoelastic infinite medium with cylindrical cavity subjected to a ramp-type heating and loading. Arch. Appl. Mech. **75**, 553–565 (2006)
25. Schoenberg, M., Censor, D.: Elastic waves in rotating media. Quart. Appl. Math. **31**, 15–125 (1973)
26. Puri, P.: Plane thermoelastic waves in rotating media. Bull. DeL'Acad. Polon. Des. Sci. Ser. Tech. **XXIV**, 103–110 (1976)
27. Roy Choudhuri, S.K., Debnath, L.: Magneto-thermoelastic plane waves in rotating media. Int. J. Eng. Sci. **21**, 155–163 (1983a)
28. Roy Choudhuri, S.K., Debnath, L.: Magneto-elastic plane waves in infinite rotating media. J. Appl. Mech. **50**, 283–288 (1983b)
29. Abouelregal, A.E., Abo-Dahab, S.M.: Dual-phase-lag diffusion model for Thomson's phenomenon on elctromagneto-thermoelastic an infinitely long solid cylinder. J. Comput. Theor. Nanosci. **11**(4), 1031–1039 (2014)
30. Lotfy, K., Abo-Dahab, S.M.: Two-dimensional problem of two temperature generalized thermoelasticity with normal mode analysis under thermal shock problem. J. Comput. Theor. Nanosci. **27**(1), 67–91 (2017)
31. Roy Choudhuri, S.K., Banerjee, S.: Magneto-thermoelastic interactions in an infinite viscoelastic cylinder of temperaturerate dependent material subjected to a periodic loading. Int. J. Eng. Sci. **36**(5/6), 635–643 (1998)
32. Abo-Dahab, S.M., Abd-Alla, A.M., Alqarni, A.J.: A two-dimensional problem in generalized thermoelasticity with rotation and magnetic field. Result Phys. **7**, 2742–2751 (2017)
33. Abd-alla, A.E.N.N., Alshaikh, F., Del Vescovo, D., Spagnuolo, M.: Plane waves and eigenfrequency study in a transversely isotropic magneto-thermoelastic medium under the effect of a constant angular velocity. J. Therm. Stresses **40**(9), 1079–1092 (2017)
34. Abbas, I.A., Abdalla, A.E.N.N., Alzahrani, F.S., Spagnuolo, M.: Wave propagation in a generalized thermoelastic plate using eigenvalue approach. J. Therm. Stresses **39**(11), 1367–1377 (2016)
35. dell'Isola, F., Andreaus, U., Placidi, L.: At the origins and in the vanguard of peridynamics, non-local and higher-gradient continuum mechanics: an underestimated and still topical contribution of Gabrio Piola. Math. Mech. Solids **20**(8), 887–928 (2015)
36. dell'Isola, F., Seppecher, P., Madeo, A.: How contact interactions may depend on the shape of Cauchy cuts in Nth gradient continua: approach "à la D'Alembert". Zeitschrift für angewandte Mathematik und Physik **63**(6), 1119–1141 (2012)
37. Sciarra, G., dell'Isola, F., Coussy, O.: Second gradient poromechanics. Int. J. Solids Struct. **44**(20), 6607–6629 (2007)
38. dell'Isola, F., Steigmann, D.: A two-dimensional gradient-elasticity theory for woven fabrics. J. Elast. **118**(1), 113–125 (2015)
39. dell'Isola, F., Romano, A.: On the derivation of thermomechanical balance equations for continuous systems with a nonmaterial interface. Int. J. Eng. Sci. **25**(11–12), 1459–1468 (1987)
40. Alibert, J.J., Seppecher, P., dell'Isola, F.: Truss modular beams with deformation energy depending on higher displacement gradients. Math. Mech. Solids **8**(1), 51–73 (2003)
41. Pideri, C., Seppecher, P.: A second gradient material resulting from the homogenization of an heterogeneous linear elastic medium. Contin. Mech. Thermodyn. **9**(5), 241–257 (1997)
42. dell'Isola, F., Seppecher, P.: Edge contact forces and quasi-balanced power. Meccanica **32**(1), 33–52 (1997)
43. Seppecher, P., Alibert, J.J., dell'Isola, F.: Linear elastic trusses leading to continua with exotic mechanical interactions. J. Phys. Conf. Ser. **319**(1), 012018 (2011)
44. Seppecher, P.: Second-gradient theory: application to Cahn–Hilliard fluids. In: Continuum Thermomechanics. Springer, Dordrecht, pp. 379–388 (2000)
45. Berezovski, A., Giorgio, I., Della Corte, A.: Interfaces in micromorphic materials: wave transmission and reflection with numerical simulations. Math. Mech. Solids **21**(1), 37–51 (2016)
46. dell'Isola, F., Giorgio, I., Andreaus, U.: Elastic pantographic 2D lattices: a numerical analysis on the static response and wave propagation. Proc. Estonian Acad. Sci. **64**(3), 219 (2015)
47. Giorgio, I., Galantucci, L., Hamdan, A.M., Del Vescovo, D.: Wave reflection at a free interface in an anisotropic pyroelectric medium with nonclassical thermoelasticity. Contin. Mech. Thermodyn. **28**(1–2), 67–84 (2016)
48. Laudato, M., Manzari, L., Barchiesi, E., Di Cosmo, F., Göransson, P.: First experimental observation of the dynamical behavior of a pantographic metamaterial. Mech. Res. Commun. **94**, 125–127 (2018)
49. Barchiesi, E., Laudato, M., Di Cosmo, F.: Wave dispersion in non-linear pantographic beams. Mech. Res. Commun. **94**, 128–132 (2018)
50. Barchiesi, E., Spagnuolo, M., Placidi, L.: Mechanical metamaterials: a state of the art. Math. Mech. Solids 1081286517735695 (2018)
51. Di Cosmo, F., Laudato, M., Spagnuolo, M.: Acoustic metamaterials based on local resonances: homogenization, optimization and applications. In: Generalized Models and Non-classical Approaches in Complex Materials, vol. 1, pp. 247–274. Springer, Cham (2018)
52. dell'Isola, F., Seppecher, P., Alibert, J.J., Lekszycki, T., Grygoruk, R., Pawlikowski, M., Gołaszewski, M., et al.: Pantographic metamaterials: an example of mathematically driven design and of its technological challenges. Contin. Mech. Thermodyn. 1–34 (2018)

53. Andreaus, U., Spagnuolo, M., Lekszycki, T., Eugster, S.R.: A Ritz approach for the static analysis of planar pantographic structures modeled with nonlinear Euler-Bernoulli beams. Contin. Mech. Thermodyn. 1–21 (2018)
54. Spagnuolo, M., Andreaus, U.: A targeted review on large deformations of planar elastic beams: extensibility, distributed loads, buckling and post-buckling. Math. Mech. Solids 1081286517737000 (2018)
55. Placidi, L., Barchiesi, E., Turco, E., Rizzi, N.L.: A review on 2D models for the description of pantographic fabrics. Zeitschrift für angewandte Mathematik und Physik **67**(121), 1–20 (2016)
56. Barchiesi, E., Placidi, L.: A review on models for the 3D statics and 2D dynamics of pantographic fabrics. Wave Dynamics and Composite Mechanics for Microstructured Materials and Metamaterials. Advanced Structured Materials, vol. 59, pp. 239–258. Springer, Singapore (2017)
57. Placidi, L., Barchiesi, E., Battista, A.: An inverse method to get further analytical solutions for a class of metamaterials aimed to validate numerical integrations. Mathematical Modelling in Solid Mechanics. Advanced Structured Materials, vol. 69, pp. 193–2110. Springer, Singapore (2017)
58. Placidi, L., Barchiesi, E., Della Corte, A.: Identification of two-dimensional pantographic structures with a linear d4 orthotropic second gradient elastic model accounting for external bulk double forces. Mathematical Modelling in Solid Mechanics. Advanced Structured Materials, vol. 69, pp. 211–232. Springer, Singapore (2017)
59. Andreaus, U., Giorgio, I., Lekszycki, T.: A 2-D continuum model of a mixture of bone tissue and bio-resorbable material for simulating mass density redistribution under load slowly variable in time. ZAMM Journal of Applied Mathematics and Mechanics/Zeitschrift für Angewandte Mathematik und Mechanik **94**(12), 978–1000 (2014)
60. Andreaus, U., Giorgio, I., Madeo, A.: Modeling of the interaction between bone tissue and resorbable biomaterial as linear elastic materials with voids. Zeitschrift für angewandte Mathematik und Physik **66**(1), 209–237 (2015)
61. Giorgio, I., Andreaus, U., Scerrato, D., dell'Isola, F.: A visco-poroelastic model of functional adaptation in bones reconstructed with bio-resorbable materials. Biomech. Model. Mechanobiol. **15**(5), 1325–1343 (2016)
62. Giorgio, I., Andreaus, U., Lekszycki, T., Della Corte, A.: The influence of different geometries of matrix/scaffold on the remodeling process of a bone and bioresorbable material mixture with voids. Math. Mech. Solids **22**(5), 969–987 (2017)
63. Spagnuolo, M., Barcz, K., Pfaff, A., Dell'Isola, F., Franciosi, P.: Qualitative pivot damage analysis in aluminum printed pantographic sheets: numerics and experiments. Mech. Res. Commun. **83**, 47–52 (2017)
64. Turco, E., Misra, A., Sarikaya, R., Lekszycki, T.: Quantitative analysis of deformation mechanisms in pantographic substructures: experiments and modeling. Contin. Mech. Thermodyn. **31**(1), 209–223 (2018)
65. Placidi, L., Misra, A., Barchiesi, E.: Two-dimensional strain gradient damage modeling: a variational approach. Zeitschrift für angewandte Mathematik und Physik **69**(3), 56 (2018)
66. Placidi, L., Barchiesi, E.: Energy approach to brittle fracture in strain-gradient modelling. Proc. R. Soc. A **474**(2210), 20170878 (2018)
67. Placidi, L., Barchiesi, E., Misra, A.: A strain gradient variational approach to damage: a comparison with damage gradient models and numerical results. Math. Mech. Complex. Syst. **6**(2), 77–100 (2018)
68. Altenbach, J., Altenbach, H., Eremeyev, V.A.: On generalized Cosserat-type theories of plates and shells: a short review and bibliography. Arch. Appl. Mech. **80**(1), 73–92 (2010)
69. Eremeyev, V.A., Pietraszkiewicz, W.: The nonlinear theory of elastic shells with phase transitions. J. Elast. **74**(1), 67–86 (2004)
70. Altenbach, H., Eremeyev, V.A.: On the shell theory on the nanoscale with surface stresses. Int. J. Eng. Sci. **49**(12), 1294–1301 (2011)
71. Pietraszkiewicz, W., Eremeyev, V., Konopińska, V.: Extended non-linear relations of elastic shells undergoing phase transitions. ZAMM-Journal of Applied Mathematics and Mechanics/Zeitschrift für Angewandte Mathematik und Mechanik: Applied Mathematics and Mechanics **87**(2), 150–159 (2007)
72. Eremeyev, V., Zubov, L.: On constitutive inequalities in nonlinear theory of elastic shells. ZAMM-Journal of Applied Mathematics and Mechanics/Zeitschrift für Angewandte Mathematik und Mechanik: Applied Mathematics and Mechanics **87**(2), 94–101 (2007)
73. Altenbach, H., Eremeyev, V.A. (Eds.).: Shell-Like Structures: Non-classical Theories and Applications, vol. 15. Springer, New York (2011)
74. Franciosi, P., Spagnuolo, M., Salman, O.U.: Mean Green operators of deformable fiber networks embedded in a compliant matrix and property estimates. Contin. Mech. Thermodyn. **31**(1), 101–132 (2018)
75. Franciosi, P.: A Decomposition method for obtaining global mean Green operators of inclusions patterns. Application to parallel infinite beams in at least transversally isotropic media. Int. J. Solids Struct. **147**(15), 1–19 (2018)
76. Franciosi, P., Lormand, G.: Using the radon transform to solve inclusion problems in elasticity. Int. J. Solids Struct. **41**(3–4), 585–606 (2004)